Regular Lattice. Hexagonal Symmetry

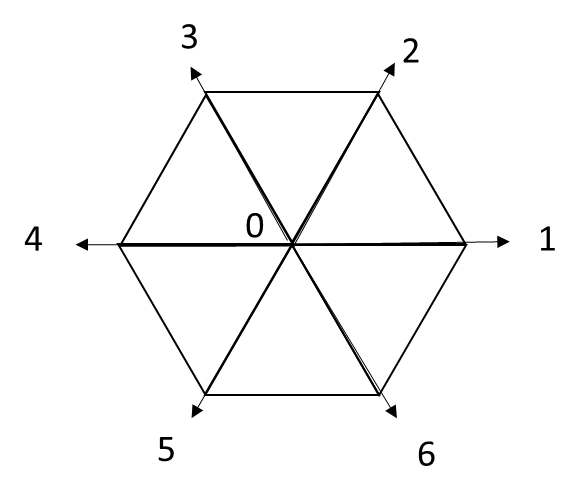
Each one with 6 neighbors.

Connected by vectors

Each lattice site can have up to 6 particles.

Each particle has mass .

Each particle can move only along one of the 6 directions



Each time cycle the particles hop to the nearest neighbor ONLY via a discreet vector ***cia***. Longer jumps can’t happen, which means the particles have the same energy. No 2 particles can move along the same vector (exclusion principle).

How do I determine which particle goes where? (Guess: particle 1 goes down1, etc. If particle 1 is 0, it doesn’t go. Maybe?)

Use a single digit integer per site (short) for number of particles. No. Now they say 6 bits. Makes more sense.

No need to store direction. That is indicated by the particles *i* index.

Occupation number is . I guess there are 6 of these per site.? Yes. Makes sense. ***x*** is the site. *i* is both the index of the particle and the direction it’s about to go. So

This will produce 6N occupation numbers (where N is the number of lattice sites).

This creates a “Boolean Field” which is just the collection of the 6N Booleans indicating the presence of a particle in a slot at a site, the slot dictating where it will move at the next time step.

The *a* parameter could be the *x, y* coordinate of the location of the cell. Maybe. Might not be right.

Looks like both. X is the x,y position, and i=1,2 is the direction of the speed of the particle.

Evolution

Free-streaming—particle transfers from site to site according to discrete speeds *cia*.

I.E. The *ith* occupation number at location ***x*** + ***cia*** at a subsequent time step is the same as the occupation number at location x at the time before it. Therefore

Transcript of Boltzmann free-streaming operator.

The change in the *ith* occupation number is the *ith* occupation number at ***x*** + ***cia*** at a subsequent time step minus the *ith* occupation number at ***x*** at the current time.

Collisions

When at a site, the particles interact and re-shuffle their momenta along with conservation of energy and momenta. Since the speeds all have the same magnitude (c=1) and mass (m=1) they all have the same energy.

Conserve particle number

Conserve total momentum

Collision operator: Change from

Phase space can be depicted by

Doing it that way, we can define a transition matrix, where each element identifies an allowed collision (1) and disallowed collision (2).

Unfortunately, a single input into the transformation can have more than one output (2 in this case). So we define a probability. We will say equal probability.

E.G

For instance

Here’s the matrix