Using GPGO to find electricity tariffs that induce optimal load profiles

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The premise is that I am an electricity suplier / distributor on the national grid in the near future. I have a set of customer that draw power from the grid and pay for this power according to a tariff that is linear in power but the gradient may vary over time. I have hte ability to set this tariff. I may also have a set of power sources that I can use to supply power but I cannot control these sufficiently to match demand so any imbalance must be purchased from or sold to the grid at a poor rate. I therefor ich to set the tariff so that demand is matched as closely as possible my supply.

Each customer has a number of devices which consume power. Some (and at present almost all) of these devices do not have any autonomy. They draw power acording to their use pattern by the consumer. This power use profile is random but very well studied, particularly the sum over all customers. Let each consumer be $c \in C$ and their power use be $p_c\left(t\right) \sim randomprocess$. The power used by all customers is therefore

$$P_c\left(t\right) = \sum_{c \in C} p_c\left(t\right) \tag{1}$$

The power that can be supplied at a low cost $P_G\left(t\right)$is mostly uncontollable if we are concerned with green power suplies but can be reasonably well predicted in advance.

Some consumer devices will be able to alter their powe use profile in order to exploit price variations. Let θ be a parameter vector with sufficient information to derive the instantaneous tariff at any time according to

$$\tau = \tau\left(t, \theta\right) \tag{2}$$

where f is some know function. Let each device with the ability to independantly alter their profile in response to θ be defined as an agent $a \in A$. Each agent has private information $\phi_a \in \Phi$ set by the user which constrains the agent operation to behaviors that allow it to fulfill its function. It must chose a schedule $s_a \in S_a$from the set of all schedules that satisfy its constraints. Not all choices of s_a are equally good to the user so the agent also has a utility function $u^s_a : S, \Phi \rightarrow \mathbb{R}$which allows it to determine the value to the user of choosing a particular action. If the power use given by is $p_a(s,t)$ then the total cost of electricity so that schedule is

$$u_a^e = \int_t p(s_a,t)\,\tau(\theta,t)\,\mathrm{d}t \tag{3}$$

The schedule chose by an intelligent agent will therefore be

$$s_a = \arg\min_{s\in S_a} u_a^e + u_a^s \tag{4}$$

The power used by all agents over time is therefore

$$P_a(t) = \sum_{a\in A} p(s_a,t) \tag{5}$$

The power imbalance that must be met bu sale/purchase of excess is given by

$$\Delta P(t) = P_a(t) + P_c(t) - P_G(t) \tag{6}$$

We wish to minimise this under some cost mapping i.e. integral of the 2-norm

$$y = G(\theta) = \int_t \|\triangle P(t)\|_2\,\mathrm{d}t \tag{7}$$

which is a multiple input single output function $G : \mathbb{R}^N \to \mathbb{R}$

Since ϕ, u may be any arbitrary values and arbitrarily complex functions for each agent we must treat G as a black box process which may be multimodal. We obviously cannot experiment on a real electricity network with a realistic number of customers so we must use simulations. Since hte total power use is the sum of a large number of individual profiles each of which is determinied by an agent running an optimisation routine we must treat this black box function as expensive and therefore use appropriate optimisation techniques for functions that are expensive to evaluate.

One of the common electricity pricing schemes proposed for the smart grid is time of use (TOC) pricing in which prices vary over time but are fixed and communicated to consumers in advance. The method we propose is to communicate the information for an entire 24 hour period in advance of the start of that period . This allows agent sufficient time to reschedule loads to the optimum within operation constraints and simplifies the problem by making it episodic.

Simplest model

The most simple agent is a shiftable static load. Washers and dryers are an example of this category. They will consume power in a fixed profile from the start time until the ned of their cycle of lenght T. The only scheduling variable to be changed is the start time and the only constraint is the latest permissible finish time t_f. The agent is considered to be initialised by the user at time

$t_0$for which the utility of starting is zero and decreases linearly according to a gradient parameter α. The utility of scheduling is therefore

$$u_a(s = t_s) = -\alpha(t_s - t_0)\ S = [t_0, t_f - T) \tag{8}$$

This makes the optimisation a 1D search with in a finite region so relatively simple to implement. The values of α, t_0, t_f and the load profile and duration should be drawn from some distrbution based on ownership and use of various devices.

A common proposal for TOU tariffs is to split the day into 48 half hour periods with constant rates during those blocks. We wil consider this method, but we will also consider continuously varying tariffs defined by finite support vectors such as Gasussian mixture models with varying size of support vector.

Existing methods. Various methods for real time control have been proposed. However these are limited in effectiveness as they can only influence devices which have real time flexibility in operation. Providing price information ahead of time opens an entire set of devices up to influence by price variations. Methods to incentivise agents to report back predicted so that purchase of power to balance excess in advance can be more accurate. I see no obvious reason this cannot also be done in conjunction with time of use pricing. *cite* rpopose a method for shifting load by time of use pricing, but hteir method relys on agents being slow to converge to the optimum in the prevention of new peaks forming. There seems to be no reason fo an individual agent not to learn faster to save more money and if all agents were to do this hte system would not function.

More complex model

Upgrades to SSL

It may be useful to implement various load profiles and nonlinear utility functions. It is easy to imagine a device being set to "finish in time for x but do not run between y and z unless w"

Thermal loads

These are much more complex. The agents require to maintaing a temperature T as close as possible to some target for all the time they are operating with a utility cost for deviation. This must be maintained despite thermal loss to the environment using a heater/heatpump. The simplest reasonable model is to consider a fixed thermal mass cm with a binary power input $k_1\delta$ (on)and loss to the environment $k_2(T - T_{ext})$. A suggested utility for deviation from the target is asymettric quadratic cost. For a reasonably simple case such as this the optimal control input under a given tariff can be found using quadratic solvers.

There is considerable scope for refinement in modelling thermal loads. The heat pump power, thermal mass and insulation rating can all be investigated, the external temperature is variable in geography and strongly linked to daily weather conditions. considerable work could be done in putting together a good predictive model of demand for a given population using surveys and observation of the response.