Using GPGO to find electricity tariffs that induce optimal load profiles

1 Scenario

The premise is that I am an electricity supplier / distributor on the national grid in the near future. I have a set of customer that draw power from the grid and pay for this power according to a tariff that is linear in power but the gradient may vary over time. I have the ability to set this tariff. I may also have a set of power sources that I can use to supply power but I cannot control these sufficiently to match demand so any imbalance must be purchased from or sold to the grid at a poor rate. I therefore wish to set the tariff so that demand is matched as closely as possible my supply. This is possible if a significant proportion of consumer devices have some autonomous capability and are able to alter their actions in response to variation in the daily tariff so that their cost of operation is minimized.

2 Agent specification

Each customer has a number of devices which consume power. Some (and at present almost all) of these devices do not have any autonomy. They draw power according to their use pattern by the consumer. This power use profile is random but very well studied, particularly the sum over all customers. Let each consumer be $c \in C$ and their power use be $pc(t) \sim randomprocess$. The power used by all customers is therefore

$$P_c(t) = \sum_{c \in C} p_c(t) \tag{1}$$

The power that can be supplied at a low cost $P_G(t)$is mostly uncontrollable if we are concerned with green power supplies but can be reasonably well predicted in advance. Some consumer devices will be able to alter their power use profile in order to exploit price variations. Let θ be a parameter vector with sufficient information to derive the instantaneous tariff at any time according to

$$\tau = f(t, \theta) \tag{2}$$

where f is some know function. Let each device with the ability to independently alter their profile in response to θ be defined as an agent $a \in A$. Each agent has private information $\phi_a \in \Phi$ set by the user which constrains the agent operation to behaviors that allow it to fulfill its function. It must chose a schedule $s_a \in S_a$ from the set of all schedules that satisfy its constraints. Not all choices of s_a are equally good to the user so the agent also has a utility function $u_s : S, \Phi \to \mathbb{R}$ which allows it to determine the value to the user of choosing a particular action. If the power use induced under s is given by $p_a(s,t)$then the total cost of electricity under that schedule is

$$u_a^e = \int\limits_t p(s_a, t)\tau(\theta, t)\mathrm{d}t \tag{3}$$

The schedule chose by an intelligent agent will therefore be

$$s_a = \underset{s \in S_a}{\arg\min}\, u_a^e + u_a^s \tag{4}$$

The power used by all agents over time is therefore

$$P_a(t) = \sum_{a \in A} p(s_a, t) \tag{5}$$

The power imbalance that must be met by sale/purchase of excess is given by

$$\Delta P(t) = P_a(t) + P_c(t) - P_G(t) \tag{6}$$

We wish to minimize this under some cost mapping i.e. integral of the 2-norm

$$y = G(\theta) = \int\limits_t P(t)^2 \mathrm{d}t \tag{7}$$

which is a multiple input single output function $G : \mathbb{R}^N \to \mathbb{R}$ Since ϕ, u may be any arbitrary values and arbitrarily complex functions for each agent we must treat G as a black box process which may be multi-modal. We obviously cannot experiment on a real electricity network with a realistic number of customers so we must use simulations. Since the total power use is the sum of a large number of individual profiles each of which is determined by an agent running an optimization routine we must treat this black box function as expensive and therefore use appropriate optimization techniques for functions that are expensive to evaluate.

3 The simplest agent

The most simple agent is a shiftable static load. Washers and dryers are an example of this category. They will consume power in a fixed profile from the start time until the end of their cycle of length T . The only scheduling variable to be changed is the start time and the only constraint is the latest permissible finish time tf . The agent is considered to be initialized by the user at time t_0 for which the utility of starting is zero and decreases linearly according to a gradient parameter α. The utility of scheduling is therefore

$$ua(s = t_s) = \alpha(t_s t_0) \qquad S = [t_0, t_f T) \tag{8}$$

This makes the optimization a 1D search with in a finite region so relatively simple to implement. The values of α, t_0, t_f and the load profile and duration should be drawn from some distribution based on ownership and use of various devices.

4 More complex agents

It may be useful to implement various load profiles and nonlinear utility functions. It is easy to imagine a device being set to finish in time for x but do not run between y and z unless w Thermal loads These are much more complex. The agents require to maintaining a temperature T as close as possible to some target for all the time they are operating with a utility cost for deviation. This must be maintained despite thermal loss to the environment using a heater/heatpump. The simplest reasonable model is to consider a fixed thermal mass cm with a binary power input $k_1\delta(on)$and loss to the environment $k_2(T - T_{ext})$. A suggested utility for deviation from the target is asymmetric quadratic cost. For a reasonably simple case such as this the optimal control input under a given tariff can be found using quadratic solvers. There is considerable scope for refinement in modeling thermal loads. The heat pump power, thermal mass and insulation rating can all be investigated, the external temperature is variable in geography and strongly linked to daily weather conditions. considerable work could be done in putting together a good predictive model of demand for a given population using surveys and observation of the response.

5 Tariff specification

One of the common electricity pricing schemes proposed for the smart grid is time of use (TOC) pricing in which prices vary over time but are fixed and communicated to consumers in advance. The method we propose is to communicate the information for an entire 24 hour period in advance of the start of that period . This allows agent sufficient time to reschedule loads to the optimum within operation constraints and simplifies the problem by making it episodic. A common proposal for TOU tariffs is to split the day into 48 half hour periods with constant rates during those blocks. We will consider this method, but we will also consider continuously varying tariffs defined by finite support vectors such as Gaussian mixture models with varying size of support vector. Various methods for real time control have been proposed. However these are limited in effectiveness as they can only influence devices which have real time flexibility in operation. Providing price information ahead of time opens an entire set of devices up to influence by price variations. Methods to incentivize agents to report back predicted so that purchase of power to balance excess in advance can be more accurate. I see no obvious reason this cannot also be done in conjunction with time of use pricing. *cite* propose a method for shifting load by time of use pricing, but their method relies on agents being slow to converge to the optimum in the prevention of new peaks forming. There seems to be no reason for an individual agent not to learn faster to save more money and if all agents were to do this the system would not function.

6 Plan

6.1 1st step

The tariff will be a GMM with 8 support points (looping around). All $\sim 10^3$ agents in the system will have a 1 unit load for 1 hour and must finish by the end of the day. The agent start time is drawn from a distribution with a distinct peak in the middle of the day. The power supplies that I own provide a flat profile equal to the average consumption at a constant cost. Excess supply incurs a small quadratic cost. Excess demand incurs a larger quadratic cost. This should be very easy to implement, the agents themselves provide an obvious parallelization. I have python GPGO code already. With no apparent reason to do otherwise I suppose I should use a Matern kernel. The prior is more debatable. If the pdf from which the agent start times are drawn is known I could say my prior belief on the induced load is demand/supply and integrate this under the cost function to find a prior on the output but this would only really be tractable at this level, it probably wouldn't work for more complex agents. Once this seems to be working: variation of number of support points. is there a good value to use? Switch to constant block tariff, how does it compare. How does the variance in the result correspond to the number of agents? What should I be using as a performance metric, ratio of cost to flat tariff? Could also look at max/min ratio reduction rather than overall cost if the optimization is changed to have this as its objective.

6.2 Moving Further

Realistic power profiles and start times. Make a thermal load agent and test that under similar simple circumstances. Realistic thermal load parameters. Match load to a given supply profile rather than just flattening (tidal/wind/wave)

6.3 Even Further

Combine consumer loads, static loads and thermal loads in realistic proportions to make a model of a real customer set. Moving focus away from the tariff setting somewhat: build models of populations with parameters that are affected by location/weather/timeofyear. Try to do inference on the distribution of agent parameters based on the response to a tariff. Example: combining response with weather maps over several days I might be able to work out the geographical distribution of heating/cooling system powers and setpoints and so have a better model for demand prediction under weather variations in future.