Earth's Control Panel (aka Modeling Planetary Climates with Python)

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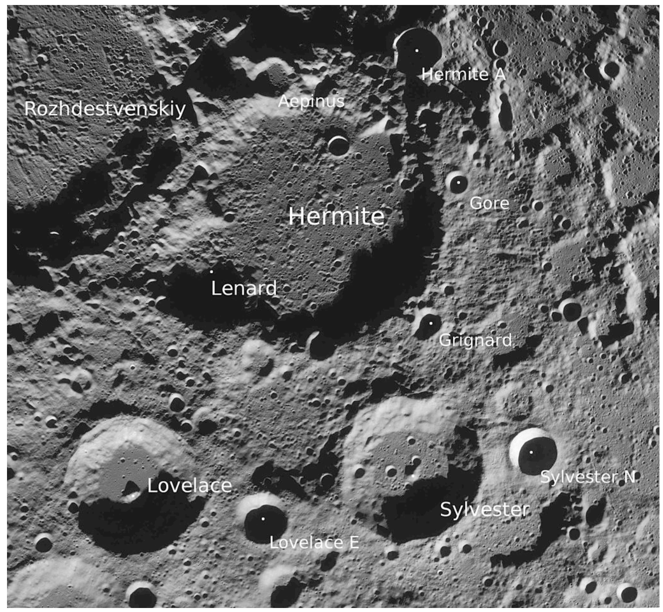
Part I: Using observations from airless bodies in the solar system to develop planetary climate models

Naked planet base case: Earth's little airless sibling, the Moon



Visual imagery of the Moon's Hermite crater, at 85 degrees north latitude (Image source: ...)

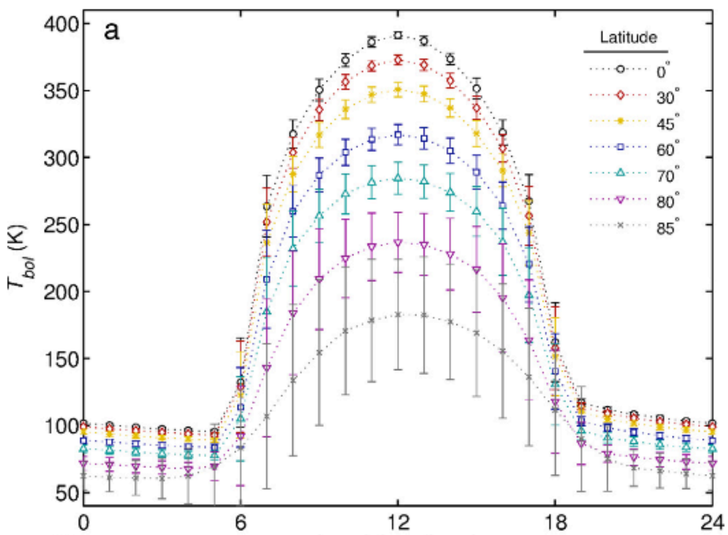
About forty-five miles (75 km) from the Moon's north pole, beneath the south rim of the Hermite crater, there is a spot that probably hasn't seen the light of day for millions of years. Unlike Earth, where the 23.5 degree tilt of the planet's rotational axis causes the polar regions to see half a year of sunshine and half a year of nighttime darkness, the Moon has an inclination of only five degrees relative to the ecliptic, meaning that the topography of certain craters is sufficient to cast permanent shadows over parts of the Moon's airless surface.

In addition to the complete lack of solar energy at certain points beneath the rims of polar craters, the Moon is a rock in space with no atmosphere to trap what little heat there is or bring it in from warmer latitudes. Therefore we have long expected the Hermite crater to be very, very cold. But we didn't know precisely how cold until just the last few years.

In 2009 NASA launched a robot with a variety of instruments to map and observe the surface of the Moon. The Lunar Reconnaissance Orbiter (LRO) and its Diviner lunar radiometer have been in an eccentric polar orbit around the Moon ever since. Scientists were excited to find that Hermite crater harbors the coldest known place in the entire solar system, with surface temperatures measured as low as 26 degrees above absolute zero (26 Kelvins, -247 C, -413 F), which is even colder than distant Pluto (Freeberg and Jones, 2009).

The Moon is an essentially airless body, with less gas pressure on its surface than the $< 10^{-12}$ atm ultra-high vacuum (UHV) systems used for scientific research on Earth. Since there is no atmosphere to absorb or scatter incoming sunlight, at its equator the Moon's surface has been observed to reach temperatures as high as 399 K (124 C, 255 F) (Vasavada *et al.*, 2012). Likewise, without an atmosphere to trap nighttime thermal radiation, temperatures can dip below 100 K (-173 C, -180 F) at the same equatorial locations.

The nights are very long on the Moon, with a synodic day of 29.53 days. In other words, if you were on the surface of the Moon, it would take 29 1/2 Earth days for the Sun to return to the same



Zonal average temperature observations for each lunar hour on the Moon at various latitudes as measured by the Diviner radiometer, aboard NASA's Lunar Reconnaissance Orbiter (From ...)

position in the sky. Nighttime is nearly two Earth weeks long. Diviner radiometry observations have shown that the Moon's surface cools rapidly in the days before sunset and then cools more slowly throughout the nightuntil sunrise starts the diurnal temperature cycle all over again. Except in those shady places where the sun never shines, which simply stay cold all the time.

Orbiting the sun at the same average distance as Earth, the Moon is something like what our planet would be without an atmosphere, with extreme temperature swings that would be intolerable to life, at least not without extensive protection. Compare the Moon's extremes to Earth's coldest ***in situ*** temperature -89.2 C (-128.6 F) at Vostok in Antarctica, and its hottest ***in situ*** temperature of 56.7 C (134.1 F) at Death Valley in California:

	Tmax (K)	Tmin (K)
Earth	330	184
Moon	399	26

On average, the Moon receives exactly the same amount of solar radiation as Earth, with a tiny amount of variation as it moves into its orbital extremes slightly closer to the Sun and slightly farther away. This makes the Moon a powerful test case for developing our physical understanding of Earth's climate.

Naked rock energy balance model

Consider a square meter of rock at the subsolar point on the Moon. In this idealized case, the Sun is directly overhead at lunar noon, halfway through the Moon's synodic period of 29.53 Earth days.

At thermal equilibrium, the radiant flux in watts per square meter received by our rock surface f_{in} and the power radiated back out to space f_{out} are equal:

$$P_{in} = P_{out}$$

The total power radiated by our sun, the solar luminosity $L_{\odot}$, is $L_{\odot} = 3.83 \times 10^{26}\ W$. At a distance d from the sun, the irradiance I(d), expressed in W/m^2, is

$$P_{in} = I(d) = \frac{L_{\odot}}{4\pi d^2}$$

For this simple model, can use the mean orbital distance of the Earth (the semimajor axis of Earth's elliptical orbit) $d_E = 149.6 \times 10^9\ m$ since the Moon is on average exactly as far away from the Sun as Earth is, with variation of up to $\pm 0.271\ \%$ as the Moon occasionally reaches its own perihelion and aphelion.

As we know from 19th century physics, a body with emissivity ϵ will radiate energy proportional to the fourth power of its temperature T (Stefan-Boltzmann law).

We can write the following expression for the power radiating outward (in W/m^2) from the Moon's surface at the subsolar point and at an equilibrium temperature T_s, where σ is the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$:

$$P_{out} = \epsilon \sigma T_s^4$$

$$\frac{L_\odot}{4\pi d_E^2} = \epsilon \sigma T_s^4$$

Solving for T_s,

$$T_s = \left(\frac{L_\odot}{4\pi \epsilon \sigma d_E^2} \right)^{1/4}$$

We should verify that the units on both side of our equation correspond with each other:

$$[K] = \left(\frac{[W]}{[Wm^{-2}K^4][m]^2} \right)^{1/4}$$

$$[K] = [\left(K^4\right)^{1/4}]$$

$$[K] = [K]$$

Inserting our data and assuming emissivity of 1.0,

$$T_s = \left(\frac{3.83 \times 10^{26}\ W}{4\pi(1.0)(5.67 \times 10^{-8}\ Wm^{-2}K^{-4})(149 \times 10^9\ m)^2} \right)^{1/4}$$

$$T_s = 394\ K$$

This is remarkably close to the maximum temperature of 399 K that the Diviner radiometer has measured on the Moon's surface!

Guemard (2018) and others have provided an updated value of the solar "constant" for Earth $S_E = 1361.1 \pm 0.5\ Wm^{-2}$, the radiant flux at the top of Earth's atmosphere. We can determine whether this affects our estimate by reformulating the energy balance equation to include S_E on the P_{in} side of the equation:

$$P_{in} = P_{out}$$

$$S_E = \epsilon \sigma T_s^4$$

giving us

$$T_s = \left(\frac{S_E}{\epsilon\sigma}\right)^{1/4}$$

$$T_s = \left(\frac{1361.1\ Wm^{-2}}{(1.0)(5.67 \times 10^{-8}\ Wm^{-2}K^{-4})}\right)^{1/4}$$

$$T_s = 394\ K$$

A couple of important parameters are necessary for simulating the Moon's maximum and minimum temperatures: the measured reflectivity and emissivity of the Moon's surface. Defining the normal albedo α as the fraction of insolation that is not absorbed (Bond albedo), Vasavada *et al.* (2012) used Diviner data to calculate mean albedo values of 0.07 and 0.16 for mare and highland areas of the Moon, respectively, and T_7 (IR/thermal channel 7) spectral emissivity $\epsilon_7 = 0.98$. The energy balance with albedo becomes:

$$(1-\alpha)s_E = \epsilon \sigma T_s^4$$

$$T_s = \left(\frac{(1-0.115)(1361.1\ Wm^{-2})}{(0.98)(5.67 \times 10^{-8}\ Wm^{-2}K^{-4})}\right)^{1/4}$$

$$T_s = 384\ K$$

What about the minimum of 26 K in the Hermite crater? How can we model the temperature of the eternal shade under the rims of craters on the polar Moon?

Shaded from the sun, P_{in} for such an environment should only have two terms that we can ordinarily neglect under sunny conditions: q_g, the surface heat flux from the Moon's interior (primarily from radioactive decay), and q_c, the radiant flux from the cosmic microwave background radiation. Siegler and Smrekar (2014) tuned radiogenic thermal conduction models using observations taken by the Apollo, GRAIL, and Selene missions and estimated global lunar surface heat flux values between $9 - 13\ mW\ m^{-2}$. Fixsen (2009) calculated a cosmic background flux of $q_c = 3.13 \times 10^{-6}\ Wm^{-2}$.

With a mean value of $q_g = 11 \times 10^{-3}\ Wm^{-2}$, $q_c \approx 0$ since $q_c \ll q_g$. Therefore $P_{in} \approx 11 \times 10^{-3}\ Wm^{-2}$ and the energy balance for permanent shadow on the moon becomes:

$$f_{in} \approx q_g = \epsilon \sigma T_s^4$$

$$T_s = \left(\frac{q_g}{\epsilon\sigma}\right)^{1/4}$$

$$T_s = 21K$$

which is also remarkably close to the observed extreme low of 26 K in Hermite crater! The fact that our simple model underestimates the temperature slightly suggests that either the local surface heat flux is higher than typical estimated lunar values and/or there is a residual nighttime radiant flux from nearby line-of-sight regions within the crater that do experience some sunlight.

Time-dependent modeling

A first step to simulating the diurnal temperature cycle on the surface of the Moon is to simply assume instantaneous thermal equilibrium and recalculate new modified black body temperatures at each new time step.

For this we want to derive an expression for input flux to the surface as a function of time $f_{in}(t)$. On a horizontal surface at the subsolar point,

$$f_{in}(t) = S_E(t) + q_g + q_c$$

Since $S_E \gg q_g, q_c$, between sunrise and sunset,

$$f_{in}(t) \approx S_E(t)$$

The geothermal and cosmic heating terms become important at night, however, so for our time-dependent model we should leave them.

For the solar equator (the path of the subsolar point across the surface of the moon)

$$cos\ \Theta_0(t) = cos\ H(t)$$

where H as the local hour angle and Θ_0 as the local zenith angle of the Sun.

$$H(t) = \frac{360^\circ}{24}(t_0 - 12) = (15^\circ)(t_0 - 12)$$

and in radian form:

$$H(t) = \frac{2\pi}{24}(t_0 - 12) = \frac{\pi}{12}(t_0 - 12)$$

Since

$$cos\ \Theta_0(t) = cos\ H(t) = \frac{S_{E,z}(t)}{S_E}$$

$$S_{E,z}(t) = S_E\ cos\ H(t)$$

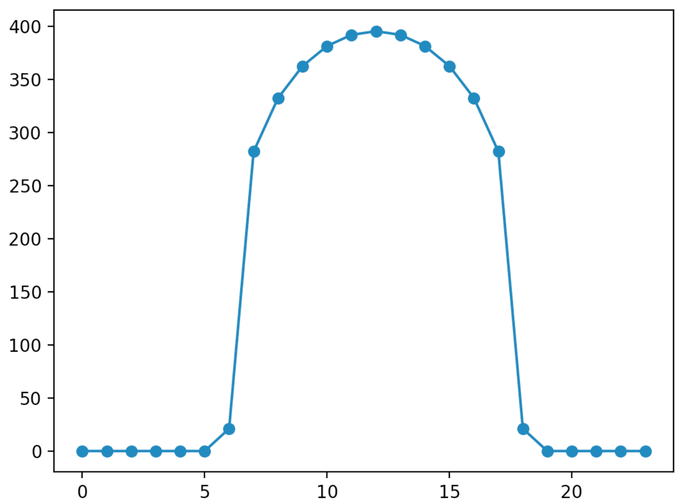
$$S_{E,z}(t) = S_E\ cos\ \big((15^\circ)(t_0 - 12)\big)$$

From this we can express the surface temperature as a function of time at the solar equator on the Moon as:

$$T_s(t) = \left(\frac{S_{E,z}(t)}{\epsilon\sigma}\right)^{1/4}$$

$$T_s(t) = \left(\frac{S_E\, cos\ \left((15^\circ)(t-12)\right)}{\epsilon\sigma}\right)^{1/4}$$

$$T_s(t) = \left(\frac{S_E\, cos\ \left(\frac{\pi}{12}(t-12)\right)}{\epsilon\sigma}\right)^{1/4}$$



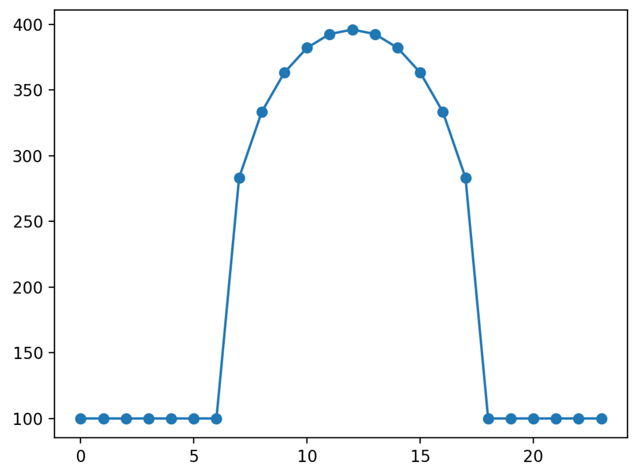
Simulated equatorial temperature on the Moon for each lunar hour with no nighttime heat sources. Note the nocturnal minimum: absolute zero. (todo: add units and labels to axes, format

To more closely approach the diurnal temperature curve that Diviner found on the moon, we need to extend our model to include the effects of thermal diffusion within the regolith. The slow nocturnal cooldown to nonzero temperatures suggests that the subsurface absorbs solar energy during the lunar day, providing a slow, time-dependent soil heat flux $q_s(t)$, largely determined by the bulk thermal inertia $(k\rho c)^{1/2}$ of the near-surface. The slab of Moon soil acting as this nocturnal heat source is rather thin: Vasavada *et al.* (1999) determined that the variation of temperature beyond a depth of 0.80 m is negligible over the course of a lunar day.

Observing that the nighttime temperature at the lunar equator is around 100 K, and assuming that the only significant flux to the surface is from daytime heating of the subsurface, as a first approximation we can use the energy balance equation to estimate the average soil heat flux q_s.

$$q_s = \epsilon\sigma T_s^4$$

$$q_s \approx 5.56\ Wm^{-2}$$



Simulated equatorial temperature on the Moon for each lunar hour with constant nighttime heating of 5.56 W/m2. Note the slightly warmer nocturnal minimum of 100 K. (todo: add units

Simply inserting this value into the input side of our time-dependent temperature model is a bit of a punt, since we are forcing the soil heat flux to be unrealistically static. But this does get us a step closer. The simulated diurnal time series has a realistic daytime temperature peak of 393 K, and nighttime temperatures only drop to 100 K, which is about what the Diviner observations have shown.

To simulate a transient transfer of energy into soil with enough thermal inertia to cause nighttime cooling, we need to describe the thermodynamics of the lunar regolith.

Heat conduction into (and back out of) the lunar regolith is a nonlinear process that requires sophisticated numerical techniques to model precisely.

Energy conservation for a differential volume of lunar ground can be described as

$$\frac{dE_{soil}}{dt} = \int_V \rho \frac{\partial e}{\partial t} dV$$

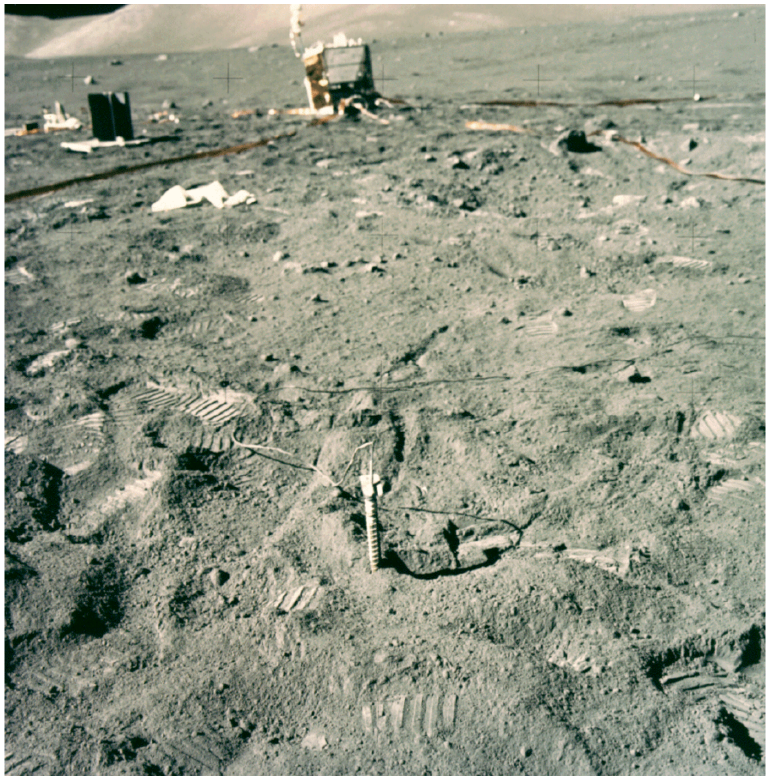
where E_{soil} is the total (extensive) energy of the volume of soil, ρ is the volumetric mass density, e is the specific energy, and the integral of is over the entire volume. The l.h.s. expresses the Lagrangian derivative for the system as a whole, and the r.h.s. includes the Eulerian local derivative for each point within the system.

Since $\frac{\partial e}{\partial t} = c\frac{\partial T}{\partial t}$, where c is the specific heat,

$$\frac{dE_{soil}}{dt} = \int_V \rho c \frac{\partial T}{\partial t} dV$$

Letting $\dot{Q}$ be the rate at which energy enters the volume (rate of heat transfer), another expression of energy conservation would be

$$\frac{dE_{soil}}{dt} = \dot{Q}$$



Apollo in-situ soil heat flux measurement (Image source:..."

Assuming no internal heat source, the rate of heat transfer in turn can be written in vector form as

$$\dot{Q} = -\int_A \mathbf{q}'' \cdot \mathbf{n} \, dA$$

where $\mathbf{q}''$ is the heat flux vector, $\mathbf{n}$ is the normal outward surface vector of the surface element dA, and the area integral is over the surface area of the system.

The divergence theorem allows us to transform the area integral into a volume integral:

$$\int_A \mathbf{q}'' \cdot \mathbf{n} \, dA = \int_V \nabla \cdot \mathbf{q}''$$

Now we can write another expression for energy conservation in terms of the heat flux and temperature:

$$\int_V \rho c \frac{\partial T}{\partial t} dV = -\int_V \nabla \cdot \mathbf{q}'' dV$$

$$\int_V \rho c \frac{\partial T}{\partial t} dV + \int_V \nabla \cdot \mathbf{q}'' dV = 0$$

$$\int_V \left(\rho c \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q}''\right) dV = 0$$

$$\rho c \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q}'' = 0$$

Here we make use of Fourier's law of heat conduction (cf. Ingersoll and Zobel, 1913: "*When different parts of a solid body are at different temperatures, heat flows from the hotter to the colder portions by a process of transference – probably from molecule to molecule – known as conduction*.")

$$\mathbf{q}'' = -k \nabla T$$

where $\mathbf{q}''$ is the heat flux vector, k is mean thermal conductivity of the slab in $Wm^{-2}K^{-1}$, and ∇T is the gradient of the temperature field: $\nabla T = \frac{\partial T}{\partial x}\mathbf{i}, \frac{\partial T}{\partial y}\mathbf{j}, \frac{\partial T}{\partial z}\mathbf{k}$. The negative sign in Fourier's law indicates that the heating flows down the temperature gradient, from hotter to colder regions. Neglecting horizontal heat flows, Fourier's law reduces to a vertical gradient:

$$\mathbf{q}'' = -k \frac{\partial T}{\partial z}\mathbf{i}$$

Now using this reduced form of Fourier's law, we can rewrite the energy conservation equation as

$$\rho c \frac{\partial T}{\partial t} - (\nabla \cdot k \frac{\partial T}{\partial z}\mathbf{i}) = 0$$

$$\rho c \frac{\partial T}{\partial t} - (\nabla \cdot k \frac{\partial T}{\partial z}\mathbf{i}) = 0$$

$$\rho c \frac{\partial T}{\partial t} - \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) = 0$$

which gives us the one-dimensional soil heat conduction equation (HCE)

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right)$$

Although this is a canonical parabolic partial differential equation, the quantities ρ, c, and k vary throughout the subsurface as a function of depth and temperature. The resulting nonlinearity excludes an analytical solution. A numerical solution is required to calculate the transient heat flux and resulting temperatures.

Hayne *et al*. (2017) describe an observation-validated numerical model that I make extensive use of here. Soil heat fluxes from solar forcing are calculated for a multilayer regolith slab by rewriting the heat conduction equation in terms of conserved conducted heat flux q_s, which although varying over the course of a year, averages out to the geothermal heat flux q_g ($\bar{q}_s = q_g$):

---(to be continued - contains some inconsistently named variables)

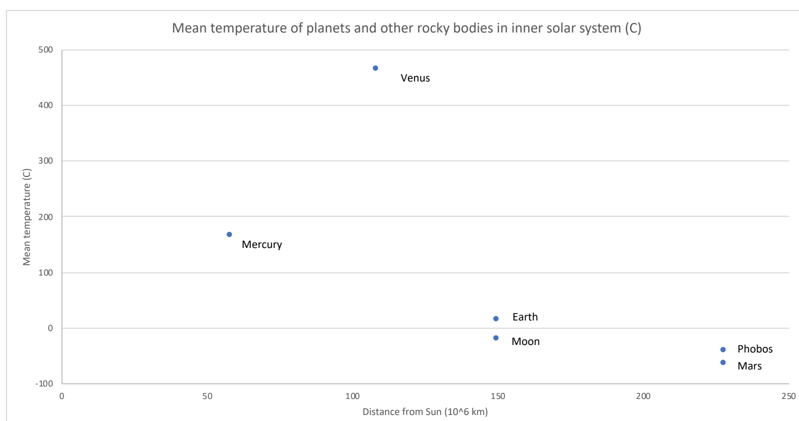
Global equilibrium temperatures of the terrestrial bodies

In the 1970s and 80s, for example, several probes successfully landed on the surface of Venus and transmitted data back to Earth. These probes found a carbon dioxide-rich atmosphere with 92 times Earth's atmospheric pressure at sea level and a surface temperature of about 462 oC. Subsequent observations have shown that the nighttime side of Venus as well as the poles have the same surface temperature, which is hot enough to melt lead.

An important paradox is that even though Venus is nearly twice as far from the sun as Mercury, the closest planet to the sun is actually colder than the second closest. Mercury's average surface temperature is 167 oC, ranging from 427 oC on the day side to as low as -180 oC on the night side of the planet.

In 1924, Petit and Nicholson at the Mount Wilson Observatory discovered that there is practically as much thermal radiation on the night side of Venus as on the day side.

If the planets behave as black bodies, their temperatures should vary inversely with the square of their distance from the sun, as solar irradiance decreases. It is useful to take a look at the temperature observations that we have. Data for the Moon is useful because it is at about the same distance from the sun as Earth, but essentially lacks an atmosphere. Mars' larger moon Phobos is also instructive for the same reason.



There are a few features of this data that are instructive. First, Venus is obviously some special case with regard to its surface temperature, lying far above the inverse square curve that characterizes the temperatures of the other inner planetary bodies. Second, the Moon is colder on average than Earth, even though it receives the same amount of sunlight. In fact Meanwhile Phobos is actually warmer than Mars.

A bit of 19th century physics can help us understand these differences.

As a body in a solar system receives energy from its star, its temperature will increase until it radiates energy back out at an equivalent rate. At this thermal equilibrium,

$$P_{in} = P_{out}$$

where P_{in} and P_{out} are the incoming and outgoing radiative power.

The total power radiated by our sun, the solar luminosity $L_{\odot}$, is $L_{\odot} = 3.83 \times 10^{26}\ W$. At a distance d from the sun, the irradiance I(d), expressed in W/m2, is

$$I(d) = \frac{L_\odot}{4\pi d^2}$$

A spherical body with mean radius a_p^2, whether it is rotating or not, will receive a total power P_{in} equal to the irradiance at its orbital distance multiplied by its cross-sectional area πa_p^2.

$$P_{in} = I(d)(\pi a_p^2) = \frac{L_\odot(\pi a_p^2)}{4\pi d^2}$$

$$P_{in} = \frac{L_\odot a_p^2}{4d^2}$$

The Stefan-Boltzmann quantification of Kirchoff's law of thermal radiation describes this condition: the total energy emitted per unit area (radiant emittance) by a black body at thermal equilibrium is directly proportional to the fourth power of that body's temperature (T):

$$j^* = \sigma T_{eq}^4 \qquad [\mathrm{W \cdot m^{-2}}]$$

where T_{eq} is the equilibrium temperature of the planet and σ, the Stefan-Boltzmann constant, is a constant of proportionality which has been measured empirically to be $\sigma = 5.67\times10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$.

On a rapidly rotating planet, the incoming solar radiation is received by the day side and emitted by the entire sphere, while on a non-rotating planet, only the day side of the planet absorbs and emits energy.

For a rapidly rotating planet, total power emitted by the entire planet P_{out} is the radiant emittance multiplied by the surface area of the planet:

$$P_{out} = \sigma T_{eq}^4 (4\pi a_p^2)$$

With $P_{in} = P_{out}$, we can solve for the equilibrium temperature T_{eq}:

$$\frac{L_\odot a_p^2}{4d^2} = \sigma T_{eq}^4 (4\pi a_p^2)$$

$$\frac{L_\odot}{4\pi d^2} = 4\sigma T_{eq}^4$$

$$\frac{L_\odot}{16\pi \sigma d^2} = T_{eq}^4$$

$$T_{eq} = \left[\frac{L_\odot}{16\pi\sigma d^2}\right]^{1/4}$$

For a non-rotating planet, the total power emitted P_{out} is the radiant emittance multiplied by the cross-sectional area of the sphere:

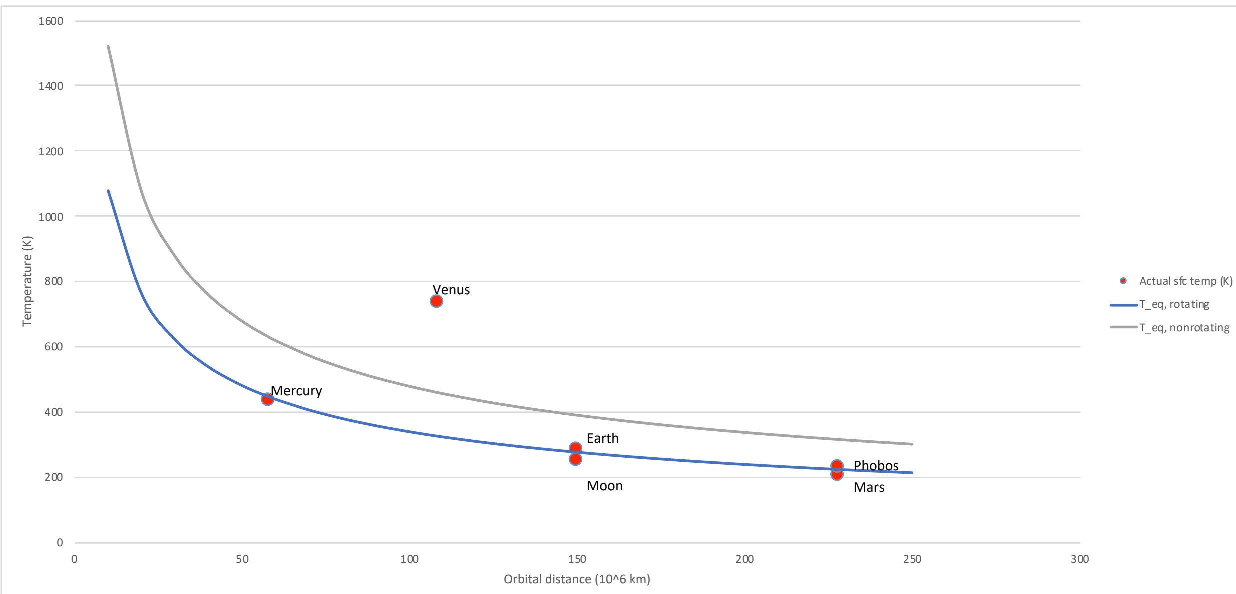
$$P_{out} = \sigma T_{eq}^4 (\pi a_p^2)$$

with the result

$$T_{eq} = \left[\frac{L_\odot}{4\pi\sigma d^2} \right]^{1/4}$$

Note that in both cases the size of the planet is irrelevant, and that the equilibrium temperature is merely a function of orbital distance.

Now using these expressions, we can replot our observed planetary temperature data against theoretical equilibrium temperature curves to see how much the planets deviate from black body behavior:



Observed mean surface temperatures of inner solar system bodies and equilibrium temperature curves.

Here we see that the equilibrium temperature curve for a rotating spherical body at orbital distance *d* does a fairly good job of approximating the actual observed surface temperature of the inner solar system bodies, with the exception of Venus. We'll come back to Venus later. But we know that there is a huge difference between the environment on the Earth and the Moon. With a mean surface temperature of -20 C, Earth would be a snowball and life as we know it would be impossible.

Here is the comparison of observed surface temperature to blackbody temperature in tabular form:

Planet	Mean orbital distance (10^6 km)	Mean orbital distance (AU)	Teq, rotating (K)	Teq, nonrotating (K)	Tsfc, observed (K)

Mercury	57.9	0.387	447	633	440
Venus	108.2	0.723	327	463	737
Earth	149.6	1	278	394	288
Moon	149.6	1	278	394	253
Mars	227.9	1.52	226	319	210
Phobos	227.9	1.52	226	319	233

At first glance, we might conclude that Mercury's observed temperature best matches the thermodynamics of a rotating body, where sunlight is distributed over the planet over the course of an Earth-like day. But this is not at all the case: Mercury rotates upon its axis with respect to distant stars quite slowly, only once every 58.6 days. This causes extreme variations in temperature across its surface: during perihelion, when the planet is closest to the sun, the temperature can reach up to 700 K (426 C), or nearly the melting point of aluminum on Earth, at the subsolar point (where the sun is directly overhead). In contrast, on the dark side, temperatures can plummet to as low as 103 K or -170 C, which is very nearly the boiling point of oxygen on Earth.

It turns out that there is of course much more to the story of why the planets have the thermal characteristics that they do.

One of the most important is the reflectivity of each body, called the albedo. For the purposes of calculating equilibrium temperature, we require the Bond albedo, which is the fraction of the radiant flux (radiant power) to a body that is scattered back to space. It follows that the Bond albedo can be used to directly characterize what fraction of incoming solar radiation (insolation) a body will absorb. Since the planets have a nonzero Bond albedo, and therefore do not absorb all insolation, we need to revise our equilibrium temperature equations to account for albedo.

At thermal equilibrium where

$$P_{in} = P_{out}$$

a body with a Bond albedo A_b will have an incoming radiant flux

$$P_{in} = (1 - A_b)\frac{L_\odot a_p^2}{4d^2}$$

giving

$$T_{eq} = \left[\frac{(1 - A_b)L_\odot}{16\pi\sigma d^2}\right]^{1/4}$$

for a rapidly rotating body and

$$T_{eq} = \left[\frac{(1 - A_b)L_\odot}{4\pi\sigma d^2}\right]^{1/4}$$

for a nonrotating body.

With this information included, we can recalculate the equilibrium temperatures:

Planet	Mean orbital distance (AU)	Bond albedo	Teq, rotating (K)	Teq, nonrotating (K)	Tsfc, observed (K)
Mercury	0.387	0.088	437	618	440
Venus	0.723	0.76	**229**	324	**737**
Earth	1	0.306	**254**	359	**288**
Moon	1	0.11	270	382	**253**
Mars	1.52	0.25	209	297	210
Phobos*	1.52	0.107	219	310	233

*Only geometric albedo has been determined for Phobos. However, under the assumption of ideal Lambertian reflectance, where the brightness of a surface is the same regardless of the observer's angle of view, the value of the Bond albedo A_b is approximately 3/2 that of the geometric albedo A_g (Dyudina et al. 2016).

$A_b \approx \frac{3}{2} A_g$

It is with these calculations that we can see where simple black body physics comes up short when it comes to describing the temperatures of the planets. Especially after accounting for its very high albedo, Venus is far hotter than a body ought to be at its distance from the sun. Earth is also substantially warmer than its albedo and orbital distance would predict. If Earth were a perfect absorber of insolation, it would have a temperature of just 5 oC. Factor in the reflectivity and scattering caused by its real surfaces, including clear atmosphere, clouds, ice, snow, dirt, rock, vegetation, urban landscapes, farmlands, and of course water, and its energy balance predicts a temperature which is far too cold to sustain life, 254 K, (-19 oC or -2.5 oF). Note how closely this matches the actual observed mean temperature on the Moon, 253 K. This is likely confirmation of the validity of our radiative balance estimates of temperature so far.

The Moon has practically no atmosphere and is cold. Earth, at the same distance from the sun, is much warmer. Venus, meanwhile, with its very high albedo, has an observed temperature that greatly exceeds the temperature it would have if it emitted radiation as a black body. Something must be modifying the emissivity ϵ of these bodies. For a perfect radiator, $\epsilon = 1$.

By assuming that the observed surface temperature represents the actual equilibrium state of the planet, we can rearrange our equations to solve for the emissivity ϵ.

Let

$$P_{out} = \epsilon \sigma T_{obs}^4 (4\pi a_p^2)$$

Once again for a rapidly rotating planet with Bond albedo A_b,

$$P_{in} = (1 - A_b) \frac{L_\odot (\pi a_p^2)}{4\pi d^2}$$

$$(1 - A_b) \frac{L_\odot (\pi a_p^2)}{4\pi d^2} = \epsilon \sigma T_{obs}^4 (4\pi a_p^2)$$

Solving for ϵ,

$$\epsilon = \frac{(1 - A_b)L_{\odot}}{16\pi d^2 \sigma T_{obs}^4}$$

Planet	epsilon
Mercury	0.975
Venus	0.00934
Earth	0.606
Moon	1.30
Mars	0.998
Phobos	0.784

ref = ?

(to be continued)---

References

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Appendix A:

Python scripts

(todo: incorporate code into text with explanation)

nakedplanet_1qm_timedependent_solareqonly.py

```
"""

Simple zero-dimensional model for calculating the equilibrium
temperature over the course of a local day at the solar equator
on an airless terrestrial body (in this case, the Moon).

Selected Moon/Earth parameters included

Python 3.6

"""

from scipy import constants
import math
import numpy as np
import pandas as pd
import matplotlib.pyplot


# Physical constants

sigma = constants.sigma
pi = constants.pi

# Solar system parameters

S_E = 1361.1 # Solar constant at Earth TOA, +/- 0.5 [W m^-2]
(Gueymard, 2018)
L_sun = 3.83e26 # Luminosity of the sun [W]
d_E = 149.6e9 # Semimajor axis of Earth's orbit [m]
q_g = 11e-3 # Mean lunar surface flux [W m^-2] (Siegler and
Smrekar (2014)
```

```
q_c = 3.13e-6 # Cosmic background radiation flux [W m^-2]
(Fixsen, 2009)
d_E_perihelion = 147.5e9 # Earth at perihelion [m]
d_moon_apogee = 40.55e9 # Moon at apogee [m]

# Local parameters
epsilon = .98 # Lunar emissivity (Vasavada et al., 2012)
alpha = 0.115 # Lunar Bond albedo, 0.07 for mare and 0.16 for
highland areas (Vasavada et al., 2012)
alpha_highland = 0.16

# Surface temperature at each lunar hour at the solar equator,
no soil conductivity
# todo: use numpy arrays to do away with for loop via vector
operations
time_list = []
temperature_list = []

for t in range (0,24):
    T_s = ( (S_E*math.cos((math.radians(15))*(t-12)) + q_g + q_c)
/epsilon/sigma)**(1/4)
    if type(T_s) == complex:
        T_s = 0

    time_list.append(t)
    temperature_list.append(T_s)
    print(t, "{0:1.3g}".format(T_s))

matplotlib.pyplot.scatter( time_list, temperature_list)
matplotlib.pyplot.plot( time_list, temperature_list)
matplotlib.pyplot.show()
```

Appendix B.

Greek letters for those who don't know them yet

α, Α	alpha
β, Β	beta
γ, Γ	gamma
δ, Δ	delta
ε, Ε	epsilon
ζ, Ζ	zeta
η, Η	eta
θ, Θ	theta
ι, Ι	iota
ϰ, Κ	kappa
λ, Λ	lambda
μ, Μ	mu
ν, Ν	nu
ξ, Ξ,	xi
ο, Ο	omicron
π, Π	pi
ϱ, Ρ	rho
σ, Σ	sigma
τ, Τ	tau
υ, Υ	upsilon
ϕ, Φ	phi
χ, Χ	chi
ψ, Ψ	psi
ω, Ω	omega