MTMG19: Tropical Weather Systems
Assessed Assignment
Student #: 23865130

Q1 a) Calculate Coriolis param @ 5°S, 5°N, 10°N

$\Omega = 7.292 \times 10^{-5}$ rad s^{-1}
$a = 6.37 \times 10^{6}$ m

$f = 2\Omega \sin\phi$
$f(\phi = 5°S) = -1.27 \times 10^{-5}\ s^{-1}$
$f(\phi = 5°N) = 1.27 \times 10^{-5}\ s^{-1}$
$f(\phi = 10°N) = 2.53 \times 10^{-5}\ s^{-1}$

b) Calculate % error at above lats using β-plane approx.

$f_\beta = f_0 + \beta y$, $f_0 = 0$ (at equator)
$\Rightarrow f_\beta = \beta y$
$\beta = 2\Omega\cos\phi / a$
$= 2.29 \times 10^{-11}\ m^{-1} s^{-1}$ at equator

$y = \frac{\pi}{180}\phi a$ (in °)

$PE(f, f_\beta) = \left| \frac{f - f_\beta}{f} \right| \times 100$ (% error)

$PE(\phi = 5°S) = 0.12\%$
$PE(\phi = 5°N) = 0.12\%$
$PE(\phi = 10°N) = 0.51\%$

c) Calculate when the % error exceeds 5%

See Figure 1

$\phi_{crit} = 30.85° \simeq 31°N$ or $31°S$

Q1 d) i) suitability of β-plane approx. for equatorially trapped (Kelvin) waves

Equatorially trapped Kelvin waves have the following latitudinal velocity dependence: ~~y_e~~ $u = u_0 \exp\left(-\frac{\beta y^2}{2c}\right)$, giving an e-folding distance of $y_e = \sqrt{\frac{2c}{\beta}}$. ~~Using~~ ($u$: velocity, c: phase speed of wave). Using $c = 80\ ms^{-1}$ (from Q3) gives an e-folding dist $y_e = 2644$ km or $\underline{\phi = 23.8°}$. Therefore the β-plane approx. should not incur significant inaccuracies.

ii) suitability of β-plane approx for Held-Hou Hadley circulation model

When sun is directly ~~of~~ over equator at equinoxes, extent of Hadley circ. according to Held-Hou model is ~ ~~2~~ 20°S - 20°N (from notes). So β-plane approx is good <u>in this case</u>. However, when the sun is above one of the Tropics at the solstices, the extent of the Hadley circ. in the winter hemisphere ~~can~~ is much larger, & the β-plane approx ~~not~~ will cause large (unacceptable) inaccuracies.

Q2 a) Estimate how much the heaviest rainfall events might increase if the Indian Ocean SST increases by 2.5K.

Take SST = 300K, increasing SST will increase e_s by Clausius-Clapeyron: $\frac{de_s}{dT} = \frac{Le_s}{R_v T^2}$ (where $p \gg e$)

$$\Rightarrow \% \text{ change: } \frac{1}{e_s}\frac{de_s}{dT}\Delta T \times 100 = \frac{L}{R_v T^2}\Delta T \times 100 = 15.0\%$$

Alternatively, calculate using Tetens' formula:

$$e_{s,T}(T[°C]) = 6.112 \exp\left(\frac{17.67T}{T+243.5}\right)$$
[hPa]

similar values, good!

$$\Rightarrow \% \text{ change 2: } \frac{e_{s,T}(29.35) - e_{s,T}(26.85)}{e_{s,T}(26.85)} \times 100 = 15.7\%$$

Now, assume heaviest rainfall events remove all water vapour from atm.:

$$q = \frac{e_s}{\frac{m_d}{m_v}(p-e_s) - e_s} \overset{p \gg e}{\simeq} \frac{e_s}{\frac{m_d}{m_v}p}$$ $\Leftarrow$ denom const. (take p const)

$\Rightarrow$ Under above assumptions heaviest rainfall events will increase by same ~~amount~~ % as e_s from above the Indian Ocean that is advected (by the Monsoon) over India, i.e. around 15%.

b) θ_e: This is the temperature that a parcel of air would have if it were ~~the~~ lifted dry-adiabatically to its LCL, then pseudo-moist adiabatically ~~to~~ until it is completely dry (all water vapour has condensed out, raising its temperature), then lowered dry-adiabatically to the reference pressure ($p = 1000$ hPa typically).

Q2 b) θ_{es}: The same as θ_e but the initial parcel is taken to be saturated. Therefore it follows the pseudo-moist adiabat at the start as it is lifted. $\theta_{es} \geq \theta_e \geq \theta$ where the first inequality holds if the parcel is saturated & the second holds if the parcel is dry.

i) Calculate θ_e for $T = 25°C$, $r_v = 17\,g\,kg^{-1}$

~~Take~~ Consider a 1.017 kg parcel of air (1 kg air, 17g water vapour) Raising & lowering this parcel will condense out all the water, heating the parcel ~~...~~ due to latent heat released.

$LH = 17\times10^{-3} \times 2.5\times10^{6} = 4.25\times10^{4}\,J$

Heating of air $= LH/(c_p \times \underset{(1kg}{M}) = 42.3K$

$\Rightarrow \theta_e = 273.15 + 25 + 42.3 = \underline{340.5K}$

Tephigram (see attached) gives $\theta_e = 74°C = \underline{347.15K}$

ii) Calc. θ_e for $T = 40$, $r_v = 7\,g\,kg^{-1}$

$LH = 7\times10^{-3} \times 2.5\times10^{6} = 17500J$

Heating $= 17.49K$

$\underline{\theta_e = 330.6K}$

Tephigram gives $\theta_e = 61°C = \underline{334.15K}$

In both cases there is a reasonably large discrepency. The calculation could be improved by calculating the LCL numerically then using a formula such as eq 6.16 in Maarten Ambaum's book to calculate θ_e more accurately.

Q3 a) Upper level zonal winds associated with dry, equatorially trapped Kelvin waves vary between +10 ms^{-1} & −10 ms^{-1}, KW has a phase speed of 80 ms^{-1}, estimate the variation in geopotential height and hence of surface pressure. State any additional assumptions.

wind speed — $u = \frac{\phi}{c}$ — ϕ – geopotential, c – phase speed , $c = 80\,ms^{-1}$, $u = \pm 10\,ms^{-1}$

$\Rightarrow \phi = \pm 800\,m^2s^{-2} = g_0 z$

$\underline{z = \pm 81.55\,m}$

$\frac{dp}{dz} = -\rho g_0$, take $\rho = 1.22\,kgm^{-3}$ (i.e. const over z)

$\Rightarrow \Delta p = -\rho_0 g_0 \Delta z$

$= -1.22 \times 9.81 \times 163.10$

$\underline{\Delta p = -19.52\,hPa}$

b) Latitudinal variation of windspeed for equatorially trapped KW:

$u(y) = u_0 e^{-\frac{\beta y^2}{2c}}$

$\phi = 12° = 0.209$ rad

$y = 1.33 \times 10^6$

$\beta = 2.29 \times 10^{-11}\,s^{-1}$

$\Rightarrow u(y = 1.33 \times 10^6) = 7.75\,ms^{-1}$

$\phi = \pm 620.1\,m^2s^{-2}$

$z = \pm 63.2\,m$

$\underline{\Delta p = -15.13\,hPa}$

Q3 c) Equatorial Kelvin Waves (EKW) travel eastwards with a speed 3 times greater than Rossby waves (RW), which travel westwards. This can be seen in the diagram, where the eastward extent of the response is approximately 3x further than the westward extent. In (Gill, 1980) a dissipative frictional force is included which explains why the waves die out. The eastwards extent is caused by the EKW, & the westward by the RW, hence ~~this is t~~ the speed difference ~~3x (?)~~ (taken with the frictional force) is the reason for the eastward extent being 3x further.

Both EKW & RW cause wind towards $x=0$. The heating also provides a vortex stretching term like $\frac{D\zeta}{Dt} = f\frac{\partial w}{\partial z}$, which induces regions of +ve vorticity (i.e. cyclones, lows) off-equatorially (because $f=0$ on equator, rising air wont induce vortex). The return flow is associated with the RW, hence is farther west. This also explains why cyclones are ~~further west~~ on the western side of the heating.

Q4 a) What impact will more rapid warming of the poles than the equator have on the Hadley circ. (using the Held-Hou model)?

The poles warming more than the equator will reduce the pole-to-equator temperature gradient, represented by $\Delta\theta$ in the Held-Hou model. This will have impacts on the extent of the Hadley circ, given by

$Y = \sqrt{\frac{5 \Delta\theta g H}{3 \Omega^2 \theta_0}}$, and the difference between the equilibrium & actual temperatures, given by $\theta_{E0} - \theta_{M0} = \frac{5 \Delta\theta^2 g H}{18 a^2 \Omega^2 \theta_0}$. The Held-Hou ~~tu~~ model therefore predicts that the extent will decrease proportional to $\sqrt{\Delta\theta}$, and the temp. diff. will decrease ~~according~~ proportional to $\Delta\theta^2$ (all other params being held const). The temp. diff reduction implies less heat will be transferred polewards.

With global warming, θ_0 (reference potential temp.) will increase ~~and~~. H (height of the upper-level winds) will also increase due to thermal expansion of the atm.
θ_0 increasing will act in ~~opposition to~~ the same way as $\Delta\theta$ decreasing for Y & $\theta_{E0} - \theta_{M0}$, & H increasing will act in opposition in both cases.

b) Estimate the upper-level zonal wind at 5°N, 10°N & 25°N

$u(5°N) = 3.54 \text{ ms}^{-1}$

$u(10°N) = 14.15 \text{ ms}^{-1}$ $\qquad u = \frac{\Omega y^2}{a}$

$u(25°N) = 88.43 \text{ ms}^{-1}$

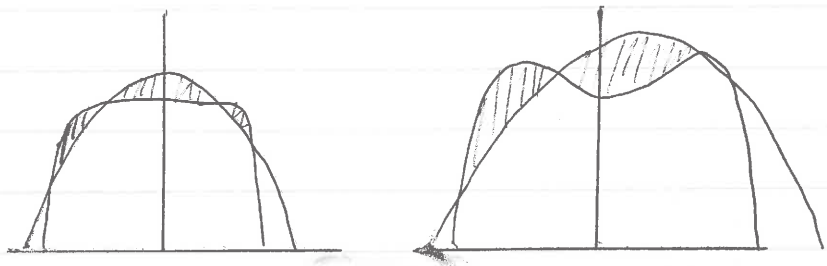
Q4 c) $u = \frac{\Omega}{a}\left(y^2 - y_c^2\right)$

$y = 0,\ y_c = 12°N \Rightarrow \underline{u = -20.38\ ms^{-1}}$

For $-y_c < y < y_c$ u will be -ve (easterly)

d)

(bad copy of slide in notes!)



$\phi_0 = 0$ $\phi_0 \sim 6°$

The mass transport is proportional to the ~~????~~ energy transport, which is given by the shaded areas in the diagrams above. As can be seen, in the off-equatorial heating case this mass transport to the winter hemisphere is far larger than the transport to either hemisphere in the symmetrical case. This meanse that when off-equatorial heating is modelled, the mass transport to both hemispheres over the course of a full year will be larger, as the extra mass transport during winter more than compensates for the reduction in summer.

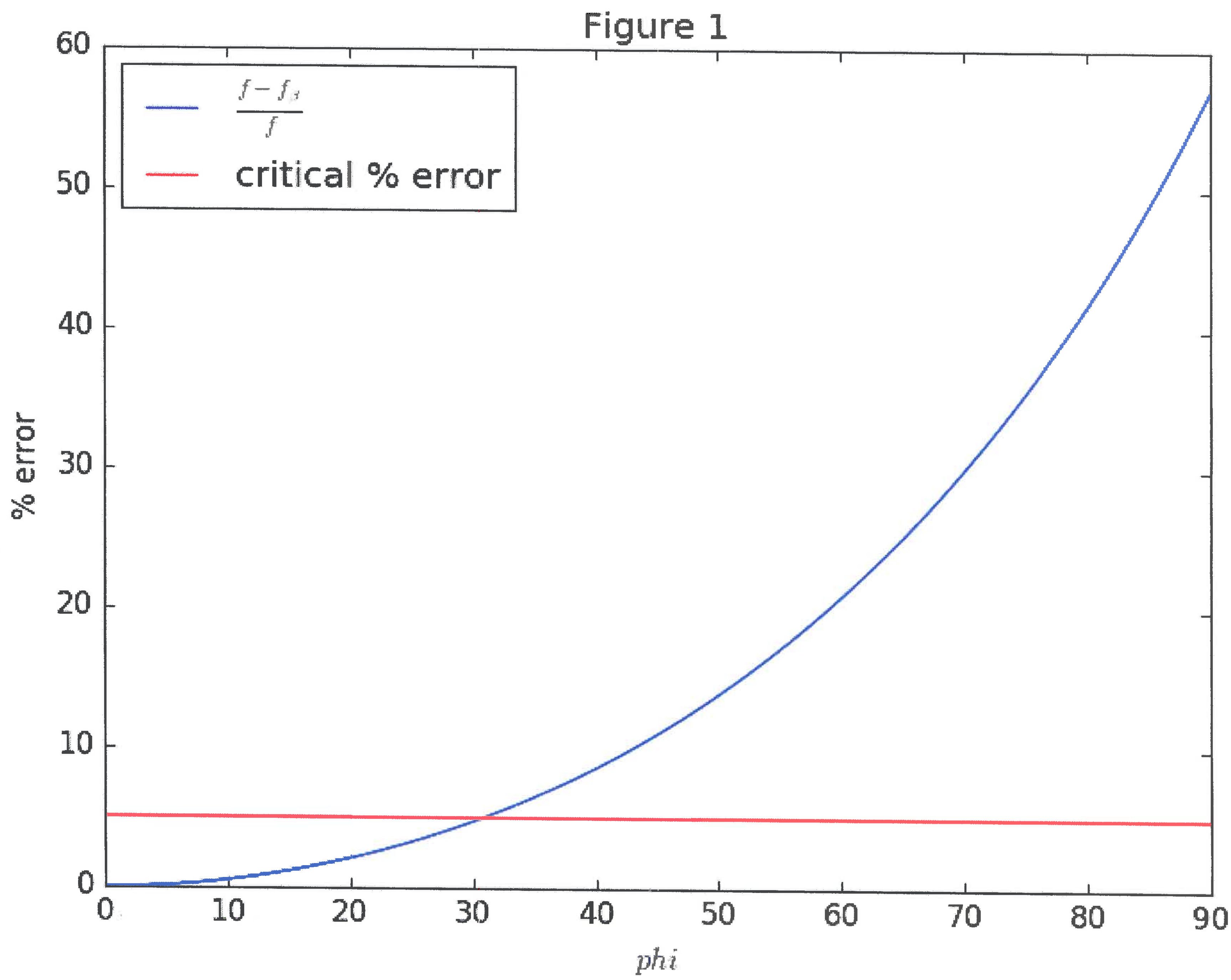


Figure 1
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