MTMW14 Project 2: Using Shallow Water Equations to Model Ocean Gyres

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Introduction

In this project a model of a large scale ocean gyre is developed. The model is based on the Shallow Water Equations (SWEs), linearised about a resting state:

$$\frac{\partial \eta}{\partial t} = -H(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}), \tag{1}$$

$$\frac{\partial u}{\partial t} = +(f_0 + \beta y)v - g\frac{\partial \eta}{\partial x} - \gamma u + \frac{\tau_x}{\rho H}, \tag{2}$$

$$\frac{\partial v}{\partial t} = -(f_0 + \beta y)u - g\frac{\partial \eta}{\partial y} - \gamma v + \frac{\tau_y}{\rho H}. \tag{3}$$

Here η represents the height perturbation, u and v represent the column averaged zonal and meridional velocity perturbations. H is the average height, f_0 and β are the Coriolis and β parameters, g is the acceleration due to gravity, *gamma* represents drag processes, and τ_x and τ_y represent the wind stress forcings. The equations were solved using a square domain, with the sides being given by L. The values used for the parameters were:

parameter	value	unit
L	1×10^6	m
H	1000	m
f_0	1×10^{-4}	s^{-1}
β	1×10^{-11}	m^{-1} s^{-1}
g	10	m s^{-2}
γ	1×10^{-6}	s^{-1}
ρ	1000	kg m^{-3}
τ_x	$-cos(\frac{\pi y}{L})$	XX
τ_y	0	XX

Following (XX Matsuno Beckers Deleersnijder) the SWEs are solved on an Arakawa-C grid using the forward-backward time scheme. The Arakawa-C grid was chosen so that e.g. spatial derivatives of u in the x direction would be available at the points where η is calculated. The domain is taken to be the size of the η grid points, as can be seen in Figure 1. First η, u then v are calculated:

$$\eta^{n+1} = \eta^n - H\Delta t(\frac{\partial u^n}{\partial x} + \frac{\partial v^n}{\partial y}), \tag{4}$$

$$u^{n+1} = u^n + (f_0 + \beta y)\Delta t v^n - g\Delta t\frac{\partial \eta^{n+1}}{\partial x} - \gamma \Delta t u^n + \Delta t\frac{\tau_x}{\rho H}, \tag{5}$$

$$v^{n+1} = v^n - (f_0 + \beta y)\Delta t u^{n+1} - g\Delta t\frac{\partial \eta^{n+1}}{\partial y} - \gamma \Delta t v^n + \Delta t\frac{\tau_y}{\rho H}. \tag{6}$$

Second η, v then u are calculated:

$$\eta^{n+2} = \eta^{n+1} - H\Delta t(\frac{\partial u^{n+1}}{\partial x} + \frac{\partial v^{n+1}}{\partial y}), \tag{7}$$

$$v^{n+2} = v^{n+1} - (f_0 + \beta y)\Delta t u^{n+1} - g\Delta t\frac{\partial \eta^{n+2}}{\partial y} - \gamma\Delta t v^{n+1} + \Delta t\frac{\tau_y}{\rho H}, \tag{8}$$

$$u^{n+2} = u^{n+1} + (f_0 + \beta y)\Delta t v^{n+2} - g\Delta t\frac{\partial \eta^{n+1}}{\partial x} - \gamma\Delta t u^{n+1} + \Delta t\frac{\tau_x}{\rho H}. \tag{9}$$

Task A

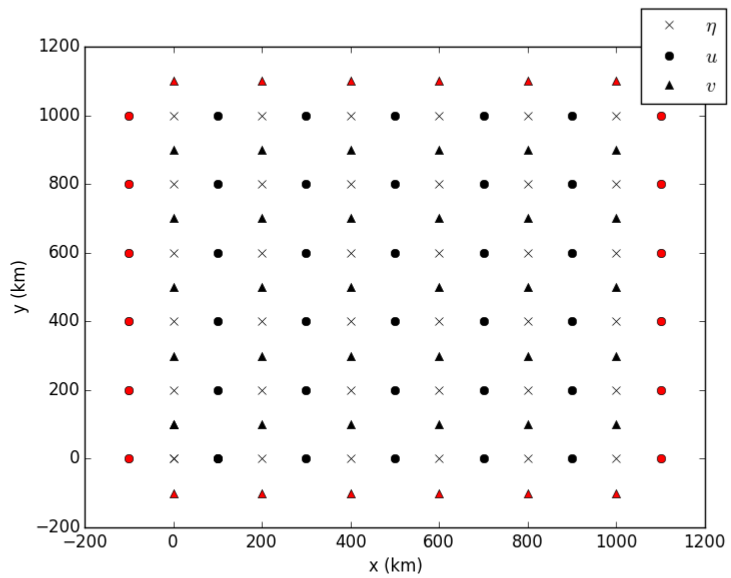

Figure 1: Shows where η, u and v are calculated on the Arakawa-C grid for $\Delta x = \Delta y = 250000m$ (lower than the lowest resolution used in this project, for illustration only). Note, u is one bigger in the x direction than η (and similar for v in y direction), and therefore the minimum and maximum u coordinates lie $\frac{\Delta x}{2}$ outside the domain (and similar for v). The red grid-point values show values which are held at 0 due to the boundary conditions.

Dispersive nature of CTCS

The shortwave oscillations in the CTCS solutions, as seen by oscillations to the left of the largest changes in gradient, are caused by the dispersive nature of CTCS. This can be explained by the dispersion relationship for CTCS. This dispersion relationship is derived by considering what happens to the amplification factor for CTCS, and can be used to show that the numerical phase velocity of the waves is given by:

$$\sin\alpha = c\sin k\Delta x \tag{10}$$

$$\frac{u_n}{u} = \pm\frac{\alpha}{ck\Delta x} \tag{11}$$

Appendix A

All code can be downloaded from the following link:
https://github.com/markmuetz/mtmw14