# **Preliminaries**

```
Clear["Global`*"]
AbundSeries = "GoldenRatio";

Which[
   AbundSeries == "GoldenRatio",
        SetDirectory[FileNameJoin[{NotebookDirectory[], "../results/GoldenRatio/"}]],
   AbundSeries == "Arithmetic",
        SetDirectory[FileNameJoin[{NotebookDirectory[], "../results/Arithmetic/"}]]
];
```

## Define functions to compute fisher matrix and expected fisher matrix

Definition using Hessian (applies under mild regularity conditions):

```
In[*]:= fisher[fun_, pars_] := -D[Apply[fun, pars], {pars, 2}]
    Expected Fisher Matrix given experimental design (unused)

In[*]:= EFisher[FisherMatrix_, func_, PDistE_, NDistE_, subs_] :=
    Expectation[
    Expectation[
    FisherMatrix /. subs, x ≈ PoissonDistribution[func /. subs]],
    {P ≈ PDistE, N ≈ NDistE}]
```

#### Min and max limits on expected count of prey eaten

```
<code>ln[*]:= SqrtDetEFlim[func_, Det_, Nvals_, Pvals_, subs_, FminMult_, FmaxMult_] :=</code>
                      Piecewise[
                           {{Sqrt[Det],
                                   Max[func /. subs /. P → Pvals /. N → Nvals] ≤ Max[Nvals] * FmaxMult &&
                                       Min[func /. subs /. P \rightarrow Pvals /. N \rightarrow \{Max[Nvals]\}] \ge 1 * FminMult
                               }}];
              SqrtDetEFlimSN1[func_, Det_, Nvals_, Pvals_, subs_, FminMult_, FmaxMult_] :=
                       Piecewise[
                           {{Sqrt[Det],
                                   \label{eq:max_func} {\sf Max[func /. subs /. P \rightarrow Pvals /. N \rightarrow Nvals]} \ \le \ {\sf Max[Nvals] * FmaxMult \&\& Max[Nvals] * FmaxMult &\& Max[Nvals] * FmaxMult && Max[Nvals] * Fmax[Nvals] * Fmax[Nvals]
                                       Min[func /. subs /. P → Pvals /. N → {Max[Nvals]}] ≥ 1 * FminMult &&
                                       dbMax[func /. subs /. P \rightarrow Pvals /. N \rightarrow Nvals] \leq 1
                               }}];
               SqrtDetEFlimSN2[func_, Det_, Nvals_, Pvals_, subs_, FminMult_, FmaxMult_] :=
                      Piecewise[
                           {{Sqrt[Det],
                                   Max[func /. subs /. P → Pvals /. N → Nvals] ≤ Max[Nvals] * FmaxMult &&
                                       Min[func /. subs /. P \rightarrow Pvals /. N \rightarrow \{Max[Nvals]\}] \ge 1 * FminMult &&
                                         b Max[func /. subs /. P \rightarrow Pvals /. N \rightarrow Nvals] \le 1
                              }}];
```

## Define Functional Responses - Count of prey eaten

```
In[*]:= (* k=1 *)
     H1 = aNPT; (* Holling Type I *)
    LR = a = \frac{N}{P} PT; (* Linear ratio-dependent *)
    BWL1 = a \frac{\sqrt{N}}{\sqrt{P}} PT; (* Barbier, Wojcik & Loreau 2021 *)
    H2 = \frac{aN}{1+ahN} PT; (* Holling Type II *)
    MM = \frac{aN}{b \cdot N} PT; (* Michaelis-Menten *)
    HV = a = \frac{N}{D^{V}} PT; (* Hassell-Varley *)
     R = a N^u PT; (* Rosenzweig '71 *)
    AG = \frac{a \frac{N}{p}}{1 + a b \frac{N}{p}} PT; (* Arditi-Ginzburg, Sutherland *)
```

$$\begin{split} & \text{CDAO} = \frac{a^{\frac{N}{\sqrt{p}}}}{1+ab^{\frac{N}{\sqrt{p}}}} \text{PT; (* Cosner et al. 1999 *)} \\ & \text{GI} = \frac{1}{b} \left( 1 - \text{Exp}[-a \, \text{N}] \right) \, \text{PT;} \\ & \text{GIA} = \frac{1}{b} \left( 1 - \text{Exp}[-a \, \text{N}] \right) \, \text{PT; (* Gause-Ivlev modified by Aldebert *)} \\ & \text{GB} = \frac{1}{b} \left( 1 - \text{Exp}[-a \, \text{N} \, / \, \text{P}] \right) \, \text{PT; (* Gutierrez \& Baumgaertner 1984 *)} \\ & \text{HT} = \frac{1}{b} \, \text{Tanh}[a \, b \, \text{N}] \, \text{PT; (* Jassby \& Platt 1976 *)} \\ & \text{HT} = \frac{1}{b} \, \text{Exp}[2 \, a \, b \, \text{N}] + 1} \, \text{PT;} \\ & \text{(* Alternative parameterization of Jassby \& Platt 1976 *)} \\ & \text{H3} = \frac{a \, \text{N}^2}{1+ab \, \text{N}^2} \, \text{PT; (* Holling Type III *)} \\ & \text{AGK} = \frac{a \, \left( \frac{N}{p} \right)^2}{1+ab \, \left( \frac{N}{p} \right)^2} \, \text{PT; (* Kratina et al. 2008 *)} \\ & \text{AO} = \frac{a \, \text{N}}{1+ab \, \sqrt{N}} \, \text{PT; (* } \, \frac{a \, \text{N}}{\sqrt{a \, \text{L} + \sqrt{a} \, b \, \text{N}}} \, \text{PT; *) (* Abrams 1982 *)} \\ & \text{A1} = \sqrt{\frac{a \, \text{N}}{1+ab \, \text{N}}} \, \text{PT; (* } \, \frac{a \, \text{N}}{\sqrt{a \, (1+ch \, \text{N})}} \, \text{PT *) (* Abrams 1990 *)} \\ & \text{A3} = \frac{a \, \sqrt{N}}{1+ab \, \sqrt{N}} \, \text{PT; (* Sokol \& Howell 1987 *)} \\ & \text{SH} = \frac{a \, \text{N}}{b + N^2} \, \text{PT; (* Sokol \& Howell 1987 *)} \\ & \text{SH} = \frac{a \, \text{N}}{1+ab \, N^2} \, \text{PT; (* Holling-form Sokol \& Howell 1987 *)} \\ & \text{CM} = \frac{a \, \text{N}}{1+ab \, \text{N} + c \, (P-1)} \, \text{PT; (* Beddington-DeAngelis *)} \\ & \text{CM} = \frac{a \, \frac{N}{p}}{1+ab \, \frac{N}{p^{\nu}}} \, \text{PT; (* Arditi-Akcakaya *)} \\ \end{aligned}$$

BWL2 = a N<sup>u</sup> P<sup>v</sup> T; (\* Barbier, Wojcik & Loreau 2021 \*)

$$\text{H3R} = \frac{\text{a N}^{\text{H}}}{1 + \text{a b N}^{\text{H}}} \text{P T; (* Holling Type III *) }$$

$$\text{A2} = \frac{\text{a N}}{1 + \text{a b N}} \text{P T; (* Holling Type III *)}$$

$$\text{S3} = \left(\frac{\text{a N}}{1 + \text{a b N}}\right)^{\text{U}} \text{P T; (* new, generalized A1 *)}$$

$$\text{S3b} = \left(\frac{\text{a N}}{\text{b + N}}\right)^{\text{U}} \text{P T; (* new, generalized A1 in MM form*)}$$

$$\text{HLB} = \frac{\text{a N}^2}{1 + \text{c N} + \text{a b N}^2} \text{P T; (* Hassell, Lawton \& Beddington 1977 *)}$$

$$\text{HLBb} = \frac{\text{a N}}{1 + \text{a b N} + \text{c N}^2} \text{P T; (* Michaelis-Mentend-form of Hassell, Lawton \& Beddington 1977 *)}$$

$$\text{MH} = \frac{\text{a N}}{1 + \text{a b N} + \text{c N}^2} \text{P T; (* Holling-form Monod-Haldane; Andrews 1968 *)}$$

$$\text{MHb} = \frac{\text{a N}}{1 + \text{a b N} + \text{c N}^2} \text{P T; (* Monod-Haldane; Andrews 1968 *)}$$

$$\text{MHC} = \frac{\text{a N}}{\text{b + N + c N}^2} \text{P T; (* original Monod-Haldane; Andrews 1968 *)}$$

$$\text{T0} = \frac{\text{a N}}{1 + \text{a b N + c N}^2} \text{P T; (* Tostowaryk '72 *)}$$

$$\text{a n Exp[d N]}$$

$$\text{FHM} = \frac{\text{a N}}{1 + \text{a b N + c N}^2} \text{P T; (* Fujii, Holling \& Mace '86 *)}$$

$$\text{W} = \frac{1}{\text{b}} \left(1 - \text{Exp}[-\text{a N} / \text{P}^{\text{V}}]\right) \text{P T; (* Watt 1959 *)}$$

$$\text{TTA} = \frac{\text{a N}}{1 + \text{a b N + c P - (1 - \text{Exp}[-\text{cP}])}} \text{P T; (* Tyutyunov, Titova \& Arditi 2008 *)}$$

$$\text{SSB} = \frac{\text{a N}}{1 + \text{a b N + c P - (1 - \text{Exp}[-\text{cP}])}} \text{P T; (* Schenk, Bersier \& Bacher 2005 *)}$$

$$\text{SSS} = \frac{2 \text{a N}}{1 + \text{a (b + c) N + \sqrt{(1 + \text{a (b + c) N) (1 + \text{a (b + c + 4 b c) N)}}}} \text{P T; (* Jeschke et al. 2002 using citardauq Formula *)}$$

$$\text{RGD} = \frac{2 \text{a N}}{1 + \text{a b N + \sqrt{(1 + \text{a b N})^2 + 8 \text{a c (P - 1)}}} \text{P T; }$$

(\*k=4\*)

```
BDOR = \frac{a N^u}{1 + a b N^u + c (P-1)} PT; (* Okuyama & Ruyle 2011 *)
CMOR = \frac{a \, N^{u}}{1 + a \, b \, N^{u} + c \, (P - 1) + a \, b \, c \, N^{u} \, (P - 1)} \, P \, T; \quad (* \, Okuyama \, \& \, Ruyle \, 2011 \, *)
AAOR = \frac{a \frac{N^{u}}{p^{v}}}{1 + a b \frac{N^{u}}{p^{v}}} PT; (* Okuyama & Ruyle 2011 *)
SN1 = \frac{a N}{1 + a b N + c (P-1) + a b c (1-d) N (P-1)} PT; (* Stouffer & Novak 2021 *)
SN2 = \frac{a N (1 + c (1 - d) (P - 1))}{1 + a b N + c (P - 1) + a b c (1 - d) N (P - 1)} PT;
(* new, but see Stouffer & Novak 2021 *)
models = {
    H1, LR, BWL1,
    H2, MM, HV, R, AG, CDAO, GI, GIA, GB, HT, HTb, H3, AGK, A0, A1, A3, SH, SHb,
    BD, CM, AA, BWL2, H3R, A2, S3, S3b,
    HLB, HLBb, MH, MHb, MHc, To, FHM, W, TTA, SBB, SSS, RGD,
    BDOR, CMOR, AAOR, SN1, SN2};
modelNames = (Trace@{
         H1, LR, BWL1,
         H2, MM, HV, R, AG, CDAO, GI,
         GIA, GB, HT, HTb, H3, AGK, A0, A1, A3, SH, SHb,
         BD, CM, AA, BWL2, H3R, A2, S3, S3b, HLB, HLBb, MH,
         MHb, MHc, To, FHM, W, TTA, SBB, SSS, RGD,
          BDOR, CMOR, AAOR, SN1, SN2}) [[All, 1]] [[1;; Length[models]]];
DumpSave["../Models.mx",
   {models, modelNames,
    H1, LR, BWL1,
    H2, MM, HV, R, AG, CDAO, GI, GIA, GB, HT, HTb, H3, AGK, A0, A1, A3, SH, SHb,
    BD, CM, AA, BWL2, H3R, A2, S3, S3b,
    HLB, HLBb, MH, MHb, MHc, To, FHM, W, TTA, SBB, SSS, RGD,
    BDOR, CMOR, AAOR, SN1, SN2}];
```

Method to assess whether model parameters are identifiable (a "necessary but not sufficient" test)

```
In[•]:= ( *
    model = \frac{a N}{c+a b N} P T;
    parms={a, b,c};
    sensitivity=D[model,{parms}]
    GatherBy[Range@Length[sensitivity],sensitivity[[#]]&]
    *)
```

#### **Define Likelihood functions**

```
ln[\cdot]:= PoislL[func_] := -n func + Log[func] x /. {n \to 1}
    lLH1[a_] := PoislL[H1]
    lLLR[a_] := PoislL[LR]
    lLBWL1[a_] := PoislL[BWL1]
    lLH2[a_, b_] := PoislL[H2]
    lLMM[a_, b_] := PoislL[MM]
    lLHV[a_, v_] := PoislL[HV]
    lLR[a_, u_] := PoislL[R]
    lLAG[a_, b_] := PoislL[AG]
    lLCDAO[a_, b_] := PoislL[CDAO]
    lLGI[a_, b_] := PoislL[GI]
    lLGIA[a_, b_] := PoislL[GIA]
    lLGB[a_, b_] := PoislL[GB]
    lLHT[a_, b_] := PoislL[HT]
    lLHTb[a_, b_] := PoislL[HTb]
    lLH3[a_, b_] := PoislL[H3]
    lLAGK[a_, b_] := PoislL[AGK]
    lLA0[a_, b_] := PoislL[A0]
    lLA3[a_, b_] := PoislL[A3]
    lLA1[a_, b_] := PoislL[A1]
    lLSH[a_, b_] := PoislL[SH]
    lLSHb[a_, b_] := PoislL[SHb]
    lLBD[a_, b_, c_] := PoislL[BD]
    lLCM[a_, b_, c_] := PoislL[CM]
    lLAA[a_, b_, v_] := PoislL[AA]
    lLBWL2[a_, v_, u_] := PoislL[BWL2]
    lLH3R[a_, b_, u_] := PoislL[H3R]
    lLA2[a_, b_, c_] := PoislL[A2]
    lLHLB[a_, b_, c_] := PoislL[HLB]
    lLHLBb[a_, b_, c_] := PoislL[HLBb]
```

```
lLMH[a_, b_, c_] := PoislL[MH]
lLMHb[a_, b_, c_] := PoislL[MHb]
lLMHc[a_, b_, c_] := PoislL[MHc]
lLTo[a_, b_, c_] := PoislL[To]
lLFHM[a_, b_, d_] := PoislL[FHM]
lLW[a_, b_, v_] := PoislL[W]
lLTTA[a_, b_, c_] := PoislL[TTA]
lLSBB[a_, b_, v_] := PoislL[SBB]
lLSSS[a_, b_, c_] := PoislL[SSS]
lLRGD[a_, b_, c_] := PoislL[RGD]
lLS3[a_, b_, u_] := PoislL[S3]
lLS3b[a_, b_, u_] := PoislL[S3b]
lLBDOR[a_, b_, c_, u_] := PoislL[BDOR]
lLCMOR[a_, b_, c_, u_] := PoislL[CMOR]
lLAAOR[a_, b_, v_, u_] := PoislL[AAOR]
lLSN1[a_, b_, c_, d_] := PoislL[SN1]
lLSN2[a_, b_, c_, d_] := PoislL[SN2]
```

## Define master Geometric Complexity function

```
ClearAll[GeomComplex]
GeomComplex[
  Nvalues,
  Pvalues_,
  Model_] :=
 Module[
  {
   Nvals = Nvalues,
   Pvals = Pvalues,
   Tval = 1,
   NIntMethod = {"LocalAdaptive", "SingularityHandler" → Automatic},
   minRec = 150,
   maxRec = 500,
   accgoal = 3,
   precgoal = 3,
   FminMult = 1, (* 0.1 or 1 *)
   FmaxMult = 1 (* 10 or 1 *)
  },
  Nprobs = ConstantArray[1 / Length[Nvals], Length[Nvals]];
  Pprobs = ConstantArray[1 / Length[Pvals]];
  NDistE = EmpiricalDistribution[Nprobs → Nvals];
  PDistE = EmpiricalDistribution[Pprobs → Pvals];
```

```
subs = {T → Tval};
ParmRange = {
  {a, 0, Infinity},
  {b, 0, Infinity},
  {c, 0, Infinity},
  {v, 0, 1, Infinity},
  {u, 0, 1, Infinity},
  {d, -Infinity, -1, 0, 1, Infinity}
 };
Which[
 Model == "H1",
 DetH1 = Det[EFisher[fisher[lLH1, {a}], H1, PDistE, NDistE, subs]];
 NIntH1 =
  Log[
   NIntegrate[
    SqrtDetEFlim[H1, DetH1, Nvals, Pvals, subs, FminMult, FmaxMult],
    ParmRange[[1]],
    AccuracyGoal → accgoal,
    PrecisionGoal → precgoal,
    Method → NIntMethod,
    MaxRecursion → maxRec]]
 Model == "LR",
 DetLR = Det[EFisher[fisher[lLLR, {a}], LR, PDistE, NDistE, subs]];
 NIntLR =
  Log[
   NIntegrate[
    SqrtDetEFlim[LR, DetLR, Nvals, Pvals, subs, FminMult, FmaxMult],
    ParmRange[[1]],
    AccuracyGoal → accgoal,
    PrecisionGoal → precgoal,
    Method → NIntMethod,
    MaxRecursion → maxRec]]
 Model == "BWL1",
 DetBWL1 = Det[EFisher[fisher[lLBWL1, {a}], BWL1, PDistE, NDistE, subs]];
 NIntBWL1 =
  Log[
   NIntegrate[
    SqrtDetEFlim[BWL1, DetBWL1, Nvals, Pvals, subs, FminMult, FmaxMult],
    ParmRange[[1]],
    AccuracyGoal → accgoal,
```

```
PrecisionGoal → precgoal,
   Method → NIntMethod,
   MaxRecursion → maxRec]]
Model == "H2",
DetH2 = Det[EFisher[fisher[lLH2, {a, b}], H2, PDistE, NDistE, subs]];
NIntH2 =
 Log[
  NIntegrate[
   SqrtDetEFlim[H2, DetH2, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "MM",
DetMM = Det[EFisher[fisher[lLMM, {a, b}], MM, PDistE, NDistE, subs]];
NIntMM =
 Log[
  NIntegrate[
   SqrtDetEFlim[MM, DetMM, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "HV",
DetHV = Det[EFisher[fisher[lLHV, {a, v}], HV, PDistE, NDistE, subs]];
NIntHV =
 Log[
  NIntegrate[
   SqrtDetEFlim[HV, DetHV, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[4]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
```

```
MaxRecursion → maxRec]]
Model == "R",
DetR = Det[EFisher[fisher[lLR, {a, u}], R, PDistE, NDistE, subs]];
 Log[
  NIntegrate[
   SqrtDetEFlim[R, DetR, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[5]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "H3",
DetH3 = Det[EFisher[fisher[lLH3, {a, b}], H3, PDistE, NDistE, subs]];
NIntH3 =
 Log[
  NIntegrate[
   SqrtDetEFlim[H3, DetH3, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "AG",
DetAG = Det[EFisher[fisher[lLAG, {a, b}], AG, PDistE, NDistE, subs]];
NIntAG =
 Log[
  NIntegrate[
   SqrtDetEFlim[AG, DetAG, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
,
```

```
Model == "CDAO",
DetCDAO = Det[EFisher[fisher[lLCDAO, {a, b}], CDAO, PDistE, NDistE, subs]];
NIntCDAO =
 Log[
  NIntegrate[
   SqrtDetEFlim[CDAO, DetCDAO, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "GI",
DetGI = Det[EFisher[fisher[lLGI, {a, b}], GI, PDistE, NDistE, subs]];
NIntGI =
 Log[
  NIntegrate[
   SqrtDetEFlim[GI, DetGI, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion \rightarrow 0,
   MaxRecursion → maxRec]]
Model == "GIA",
DetGIA = Det[EFisher[fisher[lLGIA, {a, b}], GIA, PDistE, NDistE, subs]];
NIntGIA =
 Log[
  NIntegrate[
   SqrtDetEFlim[GIA, DetGIA, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "GB",
DetGB = Det[EFisher[fisher[lLGB, {a, b}], GB, PDistE, NDistE, subs]];
```

```
NIntGB =
 Log[
  NIntegrate[
   SqrtDetEFlim[GB, DetGB, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → 0,
   MaxRecursion → maxRec]]
Model == "HT",
DetHT = Det[EFisher[fisher[lLHT, {a, b}], HT, PDistE, NDistE, subs]];
NIntHT =
 Log[
  NIntegrate[
   SqrtDetEFlim[HT, DetHT, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "HTb",
DetHTb = Det[EFisher[fisher[lLHTb, {a, b}], HTb, PDistE, NDistE, subs]];
NIntHTb =
 Log[
  NIntegrate[
   SqrtDetEFlim[HTb, DetHTb, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "A0",
DetA0 = Det[EFisher[fisher[lLA0, {a, b}], A0, PDistE, NDistE, subs]];
NIntA0 =
 Log[
```

```
NIntegrate[
   SqrtDetEFlim[A0, DetA0, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "A3",
DetA3 = Det[EFisher[fisher[lLA3, {a, b}], A3, PDistE, NDistE, subs]];
NIntA3 =
 Log[
  NIntegrate[
   SqrtDetEFlim[A3, DetA3, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "AGK",
DetAGK = Det[EFisher[fisher[lLAGK, {a, b}], AGK, PDistE, NDistE, subs]];
NIntAGK =
 Log[
  NIntegrate[
   SqrtDetEFlim[AGK, DetAGK, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "A1",
DetA1 = Det[EFisher[fisher[lLA1, {a, b}], A1, PDistE, NDistE, subs]];
NIntA1 =
 Log[
  NIntegrate[
   SqrtDetEFlim[A1, DetA1, Nvals, Pvals, subs, FminMult, FmaxMult],
```

```
ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "SH",
DetSH = Det[EFisher[fisher[lLSH, {a, b}], SH, PDistE, NDistE, subs]];
NIntSH =
 Log[
  NIntegrate[
   SqrtDetEFlim[SH, DetSH, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec] ]
Model == "SHb",
DetSHb = Det[EFisher[fisher[lLSHb, {a, b}], SHb, PDistE, NDistE, subs]];
NIntSHb =
 Log[
  NIntegrate[
   SqrtDetEFlim[SHb, DetSHb, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec] ]
Model == "BD",
DetBD = Det[EFisher[fisher[lLBD, {a, b, c}], BD, PDistE, NDistE, subs]];
NIntBD =
 Log[
  NIntegrate[
   SqrtDetEFlim[BD, DetBD, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
```

```
ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "CM",
DetCM = Det[EFisher[fisher[lLCM, {a, b, c}], CM, PDistE, NDistE, subs]];
NIntCM =
 Log[
  NIntegrate[
   SqrtDetEFlim[CM, DetCM, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "AA",
DetAA = Det[EFisher[fisher[lLAA, {a, b, v}], AA, PDistE, NDistE, subs]];
NIntAA =
 Log[
  NIntegrate[
   SqrtDetEFlim[AA, DetAA, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[4]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "BWL2",
DetBWL2 = Det[EFisher[fisher[lLBWL2, {a, v, u}], BWL2, PDistE, NDistE, subs]];
NIntBWL2 =
 Log[
  NIntegrate[
   SqrtDetEFlim[BWL2, DetBWL2, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
```

```
ParmRange[[4]],
   ParmRange[[5]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "H3R",
DetH3R = Det[EFisher[fisher[lLH3R, {a, b, u}], H3R, PDistE, NDistE, subs]];
NIntH3R =
 Log[
  NIntegrate[
   SqrtDetEFlim[H3R, DetH3R, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[5]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "A2",
DetA2 = Det[EFisher[fisher[lLA2, {a, b, c}], A2, PDistE, NDistE, subs]];
NIntA2 =
 Log[
  NIntegrate[
   SqrtDetEFlim[A2, DetA2, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "HLB",
DetHLB = Det[EFisher[fisher[lLHLB, {a, b, c}], HLB, PDistE, NDistE, subs]];
NIntHLB =
 Log[
  NIntegrate[
   SqrtDetEFlim[HLB, DetHLB, Nvals, Pvals, subs, FminMult, FmaxMult],
```

```
ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "HLBb",
DetHLBb = Det[EFisher[fisher[lLHLBb, {a, b, c}], HLBb, PDistE, NDistE, subs]];
NIntHLBb =
 Log[
  NIntegrate[
   SqrtDetEFlim[HLBb, DetHLBb, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "MH",
DetMH = Det[EFisher[fisher[lLMH, {a, b, c}], MH, PDistE, NDistE, subs]];
NIntMH =
 Log[
  NIntegrate[
   SqrtDetEFlim[MH, DetMH, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "MHb",
DetMHb = Det[EFisher[fisher[lLMHb, {a, b, c}], MHb, PDistE, NDistE, subs]];
NIntMHb =
 Log[
  NIntegrate[
```

```
SqrtDetEFlim[MHb, DetMHb, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "MHc",
DetMHc = Det[EFisher[fisher[lLMHc, {a, b, c}], MHc, PDistE, NDistE, subs]];
NIntMHc =
 Log[
  NIntegrate[
   SqrtDetEFlim[MHc, DetMHc, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "To",
DetTo = Det[EFisher[fisher[lLTo, {a, b, c}], To, PDistE, NDistE, subs]];
NIntTo =
 Log[
  NIntegrate[
   SqrtDetEFlim[To, DetTo, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "FHM",
DetFHM = Det[EFisher[fisher[lLFHM, {a, b, d}], FHM, PDistE, NDistE, subs]];
NIntFHM =
 Log[
```

```
NIntegrate[
   SqrtDetEFlim[FHM, DetFHM, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[6]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "W",
DetW = Det[EFisher[fisher[lLW, {a, b, v}], W, PDistE, NDistE, subs]];
NIntW =
 Log[
  NIntegrate[
   SqrtDetEFlim[W, DetW, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[4]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "TTA",
DetTTA = Det[EFisher[fisher[lLTTA, {a, b, c}], TTA, PDistE, NDistE, subs]];
NIntTTA =
 Log[
  NIntegrate[
   SqrtDetEFlim[TTA, DetTTA, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "SBB",
DetSBB = Det[EFisher[fisher[lLSBB, {a, b, v}], SBB, PDistE, NDistE, subs]];
NIntSBB =
```

```
Log[
  NIntegrate[
   SqrtDetEFlim[SBB, DetSBB, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[4]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "SSS",
DetSSS = Det[EFisher[fisher[lLSSS, {a, b, c}], SSS, PDistE, NDistE, subs]];
NIntSSS =
 Log[
  NIntegrate[
   SqrtDetEFlim[SSS, DetSSS, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "RGD",
DetRGD = Det[EFisher[fisher[lLRGD, {a, b, c}], RGD, PDistE, NDistE, subs]];
NIntRGD =
 Log[
  NIntegrate[
   SqrtDetEFlim[RGD, DetRGD, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "S3",
DetS3 = Det[EFisher[fisher[lLS3, {a, b, u}], S3, PDistE, NDistE, subs]];
```

```
NIntS3 =
 Log[
  NIntegrate[
   SqrtDetEFlim[S3, DetS3, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[5]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "S3b",
DetS3b = Det[EFisher[fisher[lLS3b, {a, b, u}], S3b, PDistE, NDistE, subs]];
NIntS3b =
 Log[
  NIntegrate[
   SqrtDetEFlim[S3b, DetS3b, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[5]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → minRec,
   MaxRecursion → maxRec]]
Model == "BDOR",
DetBDOR = Det[EFisher[fisher[lLBDOR, {a, b, c, u}], BDOR, PDistE, NDistE, subs]];
NIntBDOR =
 Log[
  NIntegrate[
   SqrtDetEFlim[BDOR, DetBDOR, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   ParmRange[[5]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → 300,
   MaxRecursion → 1000]]
,
```

```
Model == "CMOR",
DetCMOR = Det[EFisher[fisher[lLCMOR, {a, b, c, u}], CMOR, PDistE, NDistE, subs]];
NIntCMOR =
 Log[
  NIntegrate[
   SqrtDetEFlim[CMOR, DetCMOR, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   ParmRange[[5]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → 300,
   MaxRecursion → 1000]]
Model == "AAOR",
DetAAOR = Det[EFisher[fisher[lLAAOR, {a, b, v, u}], AAOR, PDistE, NDistE, subs]];
NIntAAOR =
 Log[
  NIntegrate[
   SqrtDetEFlim[AAOR, DetAAOR, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[4]],
   ParmRange[[5]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
   Method → NIntMethod,
   MinRecursion → 300,
   MaxRecursion → 1000]]
Model == "SN1",
DetSN1 = Det[EFisher[fisher[lLSN1, {a, b, c, d}], SN1, PDistE, NDistE, subs]];
NIntSN1 =
 Log[
  NIntegrate[
   SqrtDetEFlimSN1[SN1, DetSN1, Nvals, Pvals, subs, FminMult, FmaxMult],
   ParmRange[[1]],
   ParmRange[[2]],
   ParmRange[[3]],
   ParmRange[[6]],
   AccuracyGoal → accgoal,
   PrecisionGoal → precgoal,
```

```
Method → NIntMethod,
     MinRecursion → 300,
     MaxRecursion → 1000]]
  Model == "SN2",
  DetSN2 = Det[EFisher[
     fisher[lLSN2, {a, b, c, d}], SN2, PDistE, NDistE, subs]];
  NIntSN2 =
   Log[
    NIntegrate[
     SqrtDetEFlimSN2[SN2, DetSN2, Nvals, Pvals, subs, FminMult, FmaxMult],
     ParmRange[[1]],
     ParmRange[[2]],
     ParmRange[[3]],
     ParmRange[[6]],
     AccuracyGoal → accgoal,
     PrecisionGoal → precgoal,
     Method → NIntMethod,
     MinRecursion → 300,
     MaxRecursion → 1000]]
 ]
]
```

# Define functions to create experimental designs

The "GoldenRatio" series of abundance levels (starting at 3 for prey and 1 for predators) enabling the generation of arbitrary numbers of equidistant points between min and (specified) max abundances.

The series will be approximately Fibonacci when PreyMax is set to Fibonacci[n] for a desired series length *n*.

The "Arithmetic series" spaces out levels equidistantly in arithmetic space to the same maximum as used for the "GoldenRatio" series.

```
Infer:= logGRSpace[a_, b_, n_] := Round[GoldenRatio^Range[a, b, (b - a) / (n - 1)]]
    PreyVals[n_, PreyMax_, AbundSeries_] :=
     Which[
      AbundSeries == "GoldenRatio",
      logGRSpace[2, Log[GoldenRatio, PreyMax] + 3, n],
      AbundSeries == "Arithmetic",
      Round[Range[3, Max[logGRSpace[2, Log[GoldenRatio, PreyMax] + 3, n]],
         (Max[logGRSpace[2, Log[GoldenRatio, PreyMax] + 3, n]] - 3) / (n - 1)]]
     1
    PredVals[n_, PredMax_, AbundSeries_] :=
     If [n = 1,
      {1}, (* If only a single level is requested,
      specify a single predator individual *)
      Which[ (* Otherwise, determine predator levels according to GoldenRatio
        or Arithmetic series beginning with 1 predator individual *)
       AbundSeries == "GoldenRatio",
       logGRSpace[0, Log[GoldenRatio, PredMax] + 1, n],
       AbundSeries == "Arithmetic",
       Round[Range[1, Max[logGRSpace[0, Log[GoldenRatio, PredMax] + 1, n]],
          (Max[logGRSpace[0, Log[GoldenRatio, PredMax] + 1, n]] - 1) / (n - 1)]]
      11
```

## Specify AbundanceSeries and determine and export designs

Be sure to copy numbers to GeomComp[] function below.

```
In[*]:= PreyMinLevels = 5;
    PreyMaxLevelsVar = 10;
    PreyMaxLevelsFix = 10;
    PredMinLevels = 1;
    PredMaxLevelsVar = 5;
    PredMaxLevelsFix = 4;
```

#### Export "Var" designs - increasing length and increasing maximum value

```
In[*]:= VarDesigns = Table[
        {PreyVals[i, Fibonacci[i], AbundSeries],
         PredVals[j, Fibonacci[j], AbundSeries]},
        {i, PreyMinLevels, PreyMaxLevelsVar},
        {j, PredMinLevels, PredMaxLevelsVar}
      ];
    TableForm[VarDesigns];
    Dimensions[VarDesigns]
     (*
    Export["DesignsVar1.txt",TeXForm[VarDesigns[[All,1;;3,All]]]];
    Export["DesignsVar2.txt",TeXForm[VarDesigns[[All,4;;5,All]]]];
    *)
Out[\circ] = \{6, 5, 2\}
    Export "Fix" designs - varying length and constant maximum value
In[*]:= FixDesigns = Table[
        {PreyVals[i, Fibonacci[PreyMaxLevelsFix], AbundSeries],
         PredVals[j, Fibonacci[PredMaxLevelsFix], AbundSeries]},
        {i, PreyMinLevels, PreyMaxLevelsFix},
        {j, PredMinLevels, PredMaxLevelsFix}
      ];
    TableForm[FixDesigns];
    Dimensions[FixDesigns]
     (*
    Export["DesignsFix1.txt",TeXForm[FixDesigns[[All,1;;2,All]]]];
    Export["DesignsFix2.txt",TeXForm[FixDesigns[[All,3;;4,All]]]];
    *)
Out[\circ]= {6, 4, 2}
  Define "GeomComp[]" Wrapper to apply "GeomComplex[]" across
  experimental designs
In[*]:= ClearAll[GeomComp];
    GeomComp[
      ModelAbb_,
      Type_,
      AbundSeries_] :=
     Module[
```

{

```
(****** Be sure to match the following with exported designs above ******)
 PreyMinLevels = 5,
 PreyMaxLevelsVar = 10,
 PreyMaxLevelsFix = 10,
 PredMinLevels = 1,
 PredMaxLevelsVar = 5,
 PredMaxLevelsFix = 4
}
Which[
 Type == "Var",
 Flatten[
  ParallelTable[
   Flatten[{
     Max[PreyVals[i, Fibonacci[i], AbundSeries]], (* Maximum prey level *)
     Max[PredVals[j, Fibonacci[j], AbundSeries]], (* Maximum pred level *)
     Length[PreyVals[i, Fibonacci[i], AbundSeries]],
      (* Number of prey levels *)
     Length[PredVals[j, Fibonacci[j], AbundSeries]],
      (* Number of pred levels *)
     GeomComplex[
      PreyVals[i, Fibonacci[i], AbundSeries],
      PredVals[j, Fibonacci[j], AbundSeries],
      Model = ModelAbb]
    }],
   {i, PreyMinLevels, PreyMaxLevelsVar},
   {j, PredMinLevels, PredMaxLevelsVar}
  ],
  1]
 Type == "Fix",
 Flatten[
  ParallelTable[
   Flatten[{
     Max[PreyVals[i, Fibonacci[PreyMaxLevelsFix], AbundSeries]],
      (* Maximum prey level *)
     Max[PredVals[j, Fibonacci[PredMaxLevelsFix], AbundSeries]],
      (* Maximum pred level *)
     Length[PreyVals[i, Fibonacci[PreyMaxLevelsFix], AbundSeries]],
      (* Number of prey levels *)
     Length[PredVals[j, Fibonacci[PredMaxLevelsFix], AbundSeries]],
      (* Number of pred levels *)
     GeomComplex[
      PreyVals[i, Fibonacci[PreyMaxLevelsFix], AbundSeries],
```

```
PredVals[j, Fibonacci[PredMaxLevelsFix], AbundSeries],
        Model = ModelAbbl
     }],
    {i, PreyMinLevels, PreyMaxLevelsFix},
    {j, PredMinLevels, PredMaxLevelsFix}
   ],
   1]
 ]
]
```

# Apply across designs

#### k = 1 models

```
In[*]:= varH1 = GeomComp[Model = "H1", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k1varH1.txt"]; Save["k1varH1.txt", varH1]
In[ • ]:=
<code>ln[v]:= fixH1 = GeomComp[Model = "H1", Type = "Fix", AbundSeries = AbundSeries];</code>
    DeleteFile["k1fixH1.txt"]; Save["k1fixH1.txt", fixH1]
In[•]:=
In[*]:= varLR = GeomComp[Model = "LR", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k1varLR.txt"]; Save["k1varLR.txt", varLR]
In[*]:= fixLR = GeomComp[Model = "LR", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k1fixLR.txt"]; Save["k1fixLR.txt", fixLR]
In[•]:=
ln[*]: varBWL1 = GeomComp[Model = "BWL1", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k1varBWL1.txt"]; Save["k1varBWL1.txt", varBWL1]
In[•]:=
In[v]:= fixBWL1 = GeomComp[Model = "BWL1", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k1fixBWL1.txt"]; Save["k1fixBWL1.txt", fixBWL1]
In[ • ]:=
```

#### k = 2 models

```
<code>ln[*]:= varH2 = GeomComp[Model = "H2", Type = "Var", AbundSeries = AbundSeries];</code>
     DeleteFile["k2varH2.txt"]; Save["k2varH2.txt", varH2]
Out[*]= $Aborted
In[*]:= fixH2 = GeomComp[Model = "H2", Type = "Fix", AbundSeries = AbundSeries];
     DeleteFile["k2fixH2.txt"]; Save["k2fixH2.txt", fixH2]
     DeleteFile: Directory or file k2fixH2.txt not found.
In[•]:=
In[*]:= varMM = GeomComp[Model = "MM", Type = "Var", AbundSeries = AbundSeries];
     DeleteFile["k2varMM.txt"]; Save["k2varMM.txt", varMM]
     DeleteFile: Directory or file k2varMM.txt not found.
In[*]:= fixMM = GeomComp[Model = "MM", Type = "Fix", AbundSeries = AbundSeries];
     DeleteFile["k2fixMM.txt"]; Save["k2fixMM.txt", fixMM]
     --- DeleteFile: Directory or file k2fixMM.txt not found.
In[•]:=
ln[*]: varHV = GeomComp[Model = "HV", Type = "Var", AbundSeries = AbundSeries];
     DeleteFile["k2varHV.txt"]; Save["k2varHV.txt", varHV]
     DeleteFile: Directory or file k2varHV.txt not found.
ln[*]: fixHV = GeomComp[Model = "HV", Type = "Fix", AbundSeries = AbundSeries];
     DeleteFile["k2fixHV.txt"]; Save["k2fixHV.txt", fixHV]
     DeleteFile: Directory or file k2fixHV.txt not found.
In[•]:=
In[*]:= varR = GeomComp[Model = "R", Type = "Var", AbundSeries = AbundSeries];
     DeleteFile["k2varR.txt"]; Save["k2varR.txt", varR]
     DeleteFile: Directory or file k2varR.txt not found.
ln[*]:= fixR = GeomComp[Model = "R", Type = "Fix", AbundSeries = AbundSeries];
     DeleteFile["k2fixR.txt"]; Save["k2fixR.txt", fixR]
     DeleteFile: Directory or file k2fixR.txt not found.
In[ • ]:=
```

```
Infer: varAG = GeomComp[Model = "AG", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varAG.txt"]; Save["k2varAG.txt", varAG]
    --- DeleteFile: Directory or file k2varAG.txt not found.
ln[*]: fixAG = GeomComp[Model = "AG", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixAG.txt"]; Save["k2fixAG.txt", fixAG]
    --- DeleteFile: Directory or file k2fixAG.txt not found.
In[=]:=
ln[*]: varCDAO = GeomComp[Model = "CDAO", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varCDA0.txt"]; Save["k2varCDA0.txt", varCDA0]
    DeleteFile: Directory or file k2varCDAO.txt not found.
ln[*]: fixCDAO = GeomComp[Model = "CDAO", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixCDA0.txt"]; Save["k2fixCDA0.txt", fixCDA0]
    --- DeleteFile: Directory or file k2fixCDAO.txt not found.
In[•]:=
ln[@]:= varGI = GeomComp[Model = "GI", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varGI.txt"]; Save["k2varGI.txt", varGI]
    DeleteFile: Directory or file k2varGl.txt not found.
In[*]:= fixGI = GeomComp[Model = "GI", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixGI.txt"]; Save["k2fixGI.txt", fixGI]
    DeleteFile: Directory or file k2fixGl.txt not found.
In[•]:=
ln[*]:= varGIA = GeomComp[Model = "GIA", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varGIA.txt"]; Save["k2varGIA.txt", varGIA]
    DeleteFile: Directory or file k2varGIA.txt not found.
ln[*]: fixGIA = GeomComp[Model = "GIA", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixGIA.txt"]; Save["k2fixGIA.txt", fixGIA]
    DeleteFile: Directory or file k2fixGIA.txt not found.
In[•]:=
ln[=]: varGB = GeomComp[Model = "GB", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varGB.txt"]; Save["k2varGB.txt", varGB]
    DeleteFile: Directory or file k2varGB.txt not found.
```

```
In[*]:= fixGB = GeomComp[Model = "GB", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixGB.txt"]; Save["k2fixGB.txt", fixGB]
    DeleteFile: Directory or file k2fixGB.txt not found.
In[•]:=
ln[*]:= varHT = GeomComp[Model = "HT", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varHT.txt"];
    Save["k2varHT.txt", varHT]
    DeleteFile: Directory or file k2varHT.txt not found.
In[*]:= fixHT = GeomComp[Model = "HT", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixHT.txt"]; Save["k2fixHT.txt", fixHT]
    DeleteFile: Directory or file k2fixHT.txt not found.
In[•]:=
ln[*]: varHTb = GeomComp[Model = "HTb", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varHTb.txt"];
    Save["k2varHTb.txt", varHTb]
    DeleteFile: Directory or file k2varHTb.txt not found.
In[**]:= fixHTb = GeomComp[Model = "HTb", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixHTb.txt"]; Save["k2fixHTb.txt", fixHTb]
    --- DeleteFile: Directory or file k2fixHTb.txt not found.
In[=]:=
ln[*]: varH3 = GeomComp[Model = "H3", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varH3.txt"]; Save["k2varH3.txt", varH3]
    DeleteFile: Directory or file k2varH3.txt not found.
In[*]:= fixH3 = GeomComp[Model = "H3", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixH3.txt"]; Save["k2fixH3.txt", fixH3]
    --- DeleteFile: Directory or file k2fixH3.txt not found.
In[ • ]:=
ln[*]: varAGK = GeomComp[Model = "AGK", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varAGK.txt"]; Save["k2varAGK.txt", varAGK]
    DeleteFile: Directory or file k2varAGK.txt not found.
ln[*]: fixAGK = GeomComp[Model = "AGK", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixAGK.txt"]; Save["k2fixAGK.txt", fixAGK]
    --- DeleteFile: Directory or file k2fixAGK.txt not found.
```

```
ln[*]: varA0 = GeomComp[Model = "A0", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varA0.txt"]; Save["k2varA0.txt", varA0]
    DeleteFile: Directory or file k2varA0.txt not found.
<code>ln[v]:= fixA0 = GeomComp[Model = "A0", Type = "Fix", AbundSeries = AbundSeries];</code>
    DeleteFile["k2fixA0.txt"]; Save["k2fixA0.txt", fixA0]
    DeleteFile: Directory or file k2fixA0.txt not found.
In[ • ]:=
In[*]:= varA1 = GeomComp[Model = "A1", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varA1.txt"]; Save["k2varA1.txt", varA1]
    --- DeleteFile: Directory or file k2varA1.txt not found.
ln[*]: fixA1 = GeomComp[Model = "A1", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixA1.txt"]; Save["k2fixA1.txt", fixA1]
    DeleteFile: Directory or file k2fixA1.txt not found.
In[ • ]:=
ln[-]:= varA3 = GeomComp[Model = "A3", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varA3.txt"]; Save["k2varA3.txt", varA3]
    --- DeleteFile: Directory or file k2varA3.txt not found.
ln[*]: fixA3 = GeomComp[Model = "A3", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixA3.txt"]; Save["k2fixA3.txt", fixA3]
    --- DeleteFile: Directory or file k2fixA3.txt not found.
In[•]:=
In[*]:= varSH = GeomComp[Model = "SH", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varSH.txt"]; Save["k2varSH.txt", varSH]
    --- DeleteFile: Directory or file k2varSH.txt not found.
ln[*]: fixSH = GeomComp[Model = "SH", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixSH.txt"]; Save["k2fixSH.txt", fixSH]
    DeleteFile: Directory or file k2fixSH.txt not found.
```

In[ • ]:=

In[•]:=

```
Infer: varSHb = GeomComp[Model = "SHb", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k2varSHb.txt"]; Save["k2varSHb.txt", varSHb]
    --- DeleteFile: Directory or file k2varSHb.txt not found.
In[*]:= fixSHb = GeomComp[Model = "SHb", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k2fixSHb.txt"]; Save["k2fixSHb.txt", fixSHb]
    DeleteFile: Directory or file k2fixSHb.txt not found.
In[=]:=
    k = 3 \text{ models}
In[*]:= varBD = GeomComp[Model = "BD", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varBD.txt"]; Save["k3varBD.txt", varBD]
    --- DeleteFile: Directory or file k3varBD.txt not found.
ln[*]: fixBD = GeomComp[Model = "BD", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixBD.txt"]; Save["k3fixBD.txt", fixBD]
    DeleteFile: Directory or file k3fixBD.txt not found.
In[ • ]:=
ln[-]:= varCM = GeomComp[Model = "CM", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varCM.txt"]; Save["k3varCM.txt", varCM]
    DeleteFile: Directory or file k3varCM.txt not found.
In[**]: fixCM = GeomComp[Model = "CM", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixCM.txt"]; Save["k3fixCM.txt", fixCM]
    DeleteFile: Directory or file k3fixCM.txt not found.
In[•]:=
<code>ln[e]:= varAA = GeomComp[Model = "AA", Type = "Var", AbundSeries = AbundSeries];</code>
    DeleteFile["k3varAA.txt"]; Save["k3varAA.txt", varAA]
    DeleteFile: Directory or file k3varAA.txt not found.
ln[*]: fixAA = GeomComp[Model = "AA", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixAA.txt"]; Save["k3fixAA.txt", fixAA]
    DeleteFile: Directory or file k3fixAA.txt not found.
In[•]:=
ln[*]:= varBWL2 = GeomComp[Model = "BWL2", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varBWL2.txt"]; Save["k3varBWL2.txt", varBWL2]
```

```
Infer:= fixBWL2 = GeomComp[Model = "BWL2", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixBWL2.txt"]; Save["k3fixBWL2.txt", fixBWL2]
In[•]:=
In[*]:= varH3R = GeomComp[Model = "H3R", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varH3R.txt"]; Save["k3varH3R.txt", varH3R]
    DeleteFile: Directory or file k3varH3R.txt not found.
ln[=]: fixH3R = GeomComp[Model = "H3R", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixH3R.txt"]; Save["k3fixH3R.txt", fixH3R]
    --- DeleteFile: Directory or file k3fixH3R.txt not found.
In[ • ]:=
ln[*]: varA2 = GeomComp[Model = "A2", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varA2.txt"]; Save["k3varA2.txt", varA2]
    DeleteFile: Directory or file k3varA2.txt not found.
ln[*]: fixA2 = GeomComp[Model = "A2", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixA2.txt"]; Save["k3fixA2.txt", fixA2]
    --- DeleteFile: Directory or file k3fixA2.txt not found.
In[=]:=
<code>ln[v]:= varHLB = GeomComp[Model = "HLB", Type = "Var", AbundSeries = AbundSeries];</code>
    DeleteFile["k3varHLB.txt"]; Save["k3varHLB.txt", varHLB]
    --- DeleteFile: Directory or file k3varHLB.txt not found.
ln[*]: fixHLB = GeomComp[Model = "HLB", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixHLB.txt"]; Save["k3fixHLB.txt", fixHLB]
    DeleteFile: Directory or file k3fixHLB.txt not found.
In[ • ]:=
ln[*]: varHLBb = GeomComp[Model = "HLBb", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varHLBb.txt"]; Save["k3varHLBb.txt", varHLBb]
    DeleteFile: Directory or file k3varHLBb.txt not found.
ln[*]: fixHLBb = GeomComp[Model = "HLBb", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixHLBb.txt"]; Save["k3fixHLBb.txt", fixHLBb]
    --- DeleteFile: Directory or file k3fixHLBb.txt not found.
In[•]:=
```

```
Infer: varMH = GeomComp[Model = "MH", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varMH.txt"]; Save["k3varMH.txt", varMH]
    --- DeleteFile: Directory or file k3varMH.txt not found.
ln[∗]:= fixMH = GeomComp[Model = "MH", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixMH.txt"]; Save["k3fixMH.txt", fixMH]
    --- DeleteFile: Directory or file k3fixMH.txt not found.
In[=]:=
ln[*]: varMHb = GeomComp[Model = "MHb", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varMHb.txt"]; Save["k3varMHb.txt", varMHb]
    DeleteFile: Directory or file k3varMHb.txt not found.
ln[*]: fixMHb = GeomComp[Model = "MHb", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixMHb.txt"]; Save["k3fixMHb.txt", fixMHb]
    DeleteFile: Directory or file k3fixMHb.txt not found.
In[•]:=
ln[*]: varMHc = GeomComp[Model = "MHc", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varMHc.txt"]; Save["k3varMHc.txt", varMHc]
    DeleteFile: Directory or file k3varMHc.txt not found.
In[e]:= fixMHc = GeomComp[Model = "MHc", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixMHc.txt"]; Save["k3fixMHc.txt", fixMHc]
    DeleteFile: Directory or file k3fixMHc.txt not found.
In[•]:=
ln[*]:= varT = GeomComp[Model = "To", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varT.txt"]; Save["k3varT.txt", varT]
    DeleteFile: Directory or file k3varT.txt not found.
ln[*]: fixT = GeomComp[Model = "To", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixT.txt"]; Save["k3fixT.txt", fixT]
    DeleteFile: Directory or file k3fixT.txt not found.
In[ • ]:=
ln[=]: varFHM = GeomComp[Model = "FHM", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varFHM.txt"]; Save["k3varFHM.txt", varFHM]
    DeleteFile: Directory or file k3varFHM.txt not found.
```

```
Infer:= fixFHM = GeomComp[Model = "FHM", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixFHM.txt"]; Save["k3fixFHM.txt", fixFHM]
    DeleteFile: Directory or file k3fixFHM.txt not found.
In[ • ]:=
ln[*]: varW = GeomComp[Model = "W", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varW.txt"]; Save["k3varW.txt", varW]
    --- DeleteFile: Directory or file k3varW.txt not found.
ln[*]: fixW = GeomComp[Model = "W", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixW.txt"]; Save["k3fixW.txt", fixW]
    DeleteFile: Directory or file k3fixW.txt not found.
In[•]:=
<code>ln[v]:= varTTA = GeomComp[Model = "TTA", Type = "Var", AbundSeries = AbundSeries];</code>
    DeleteFile["k3varTTA.txt"]; Save["k3varTTA.txt", varTTA]
    --- DeleteFile: Directory or file k3varTTA.txt not found.
ln[*]: fixTTA = GeomComp[Model = "TTA", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixTTA.txt"]; Save["k3fixTTA.txt", fixTTA]
    DeleteFile: Directory or file k3fixTTA.txt not found.
In[ • ]:=
In[*]:= varSBB = GeomComp[Model = "SBB", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varSBB.txt"]; Save["k3varSBB.txt", varSBB]
    DeleteFile: Directory or file k3varSBB.txt not found.
In[v]:= fixSBB = GeomComp[Model = "SBB", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixSBB.txt"]; Save["k3fixSBB.txt", fixSBB]
    DeleteFile: Directory or file k3fixSBB.txt not found.
In[•]:=
ln[*]: varSSS = GeomComp[Model = "SSS", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varSSS.txt"];
    Save["k3varSSS.txt", varSSS]
    DeleteFile: Directory or file k3varSSS.txt not found.
```

```
In[*]:= fixSSS = GeomComp[Model = "SSS", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixSSS.txt"];
    Save["k3fixSSS.txt", fixSSS]
    DeleteFile: Directory or file k3fixSSS.txt not found.
In[•]:=
In[*]:= varRGD = GeomComp[Model = "RGD", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varRGD.txt"];
    Save["k3varRGD.txt", varRGD]
In[*]:= fixRGD = GeomComp[Model = "RGD", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixRGD.txt"];
    Save["k3fixRGD.txt", fixRGD]
In[•]:=
In[*]:= varS3 = GeomComp[Model = "S3", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varS3.txt"];
    Save["k3varS3.txt", varS3]
    DeleteFile: Directory or file k3varS3.txt not found.
ln[*]: fixS3 = GeomComp[Model = "S3", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixS3.txt"];
    Save["k3fixS3.txt", fixS3]
    --- DeleteFile: Directory or file k3fixS3.txt not found.
In[•]:=
In[*]:= varS3b = GeomComp[Model = "S3b", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k3varS3b.txt"];
    Save["k3varS3b.txt", varS3b]
    --- DeleteFile: Directory or file k3varS3b.txt not found.
In[*]:= fixS3b = GeomComp[Model = "S3b", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k3fixS3b.txt"];
    Save["k3fixS3b.txt", fixS3b]
    --- DeleteFile: Directory or file k3fixS3b.txt not found.
In[•]:=
```

#### k = 4 models

```
<code>ln[*]:= varBDOR = GeomComp[Model = "BDOR", Type = "Var", AbundSeries = AbundSeries];</code>
    DeleteFile["k4varBDOR.txt"]; Save["k4varBDOR.txt", varBDOR]
    DeleteFile: Directory or file k4varBDOR.txt not found.
ln[*]: fixBDOR = GeomComp[Model = "BDOR", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k4fixBDOR.txt"]; Save["k4fixBDOR.txt", fixBDOR]
    --- DeleteFile: Directory or file k4fixBDOR.txt not found.
In[•]:=
Infel:= varCMOR = GeomComp[Model = "CMOR", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k4varCMOR.txt"]; Save["k4varCMOR.txt", varCMOR]
    --- DeleteFile: Directory or file k4varCMOR.txt not found.
ln[**]: fixCMOR = GeomComp[Model = "CMOR", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k4fixCMOR.txt"]; Save["k4fixCMOR.txt", fixCMOR]
    DeleteFile: Directory or file k4fixCMOR.txt not found.
In[ • ]:=
ln[*]: varAAOR = GeomComp[Model = "AAOR", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k4varAAOR.txt"]; Save["k4varAAOR.txt", varAAOR]
    DeleteFile: Directory or file k4varAAOR.txt not found.
ln[*]: fixAAOR = GeomComp[Model = "AAOR", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k4fixAAOR.txt"]; Save["k4fixAAOR.txt", fixAAOR]
    DeleteFile: Directory or file k4fixAAOR.txt not found.
In[•]:=
In[v]:= varSN1 = GeomComp[Model = "SN1", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k4varSN1.txt"];
    Save["k4varSN1.txt", varSN1]
     --- DeleteFile: Directory or file k4varSN1.txt not found.
ln[*]: fixSN1 = GeomComp[Model = "SN1", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k4fixSN1.txt"];
    Save["k4fixSN1.txt", fixSN1]
    --- DeleteFile: Directory or file k4fixSN1.txt not found.
In[•]:=
```

```
In[*]:= varSN2 = GeomComp[Model = "SN2", Type = "Var", AbundSeries = AbundSeries];
    DeleteFile["k4varSN2.txt"];
    Save["k4varSN2.txt", varSN2]
    --- DeleteFile: Directory or file k4varSN2.txt not found.
In[*]:= fixSN2 = GeomComp[Model = "SN2", Type = "Fix", AbundSeries = AbundSeries];
    DeleteFile["k4fixSN2.txt"];
    Save["k4fixSN2.txt", fixSN2]
    --- DeleteFile: Directory or file k4fixSN2.txt not found.
```