Out[*]= /Users/marknovak/Git/aaaManuscripts/GeometricComplexity/GeometricComplexity/figs

$$SC_2 = -Log_e f(x \mid \hat{\Theta}_x) + \frac{k}{2} Log_e \left(\frac{n}{2\pi}\right) + Log_e \int_{\Theta} \sqrt{\det I(\Theta)} \ dI\Theta + o(1);$$

The first term is the likelihood. The second term

is the parameter and sample size dependent component.

The third term containing $I(\Theta)$, the Expected Fisher Information Matrix, reflects the model's flexibility.

Second Term dependence on k and n

```
In[*]:= nmax = 80;
       p1 =
        Plot
          Evaluate@Table \left[\frac{k}{2} \operatorname{Log}\left[\frac{n}{2\pi}\right], \{k, 1, 4\}\right], \{n, 1, n \}
          PlotRange \rightarrow {{0, nmax}, {0, 4}},
          \label{eq:plotLegends} \begin{split} & \texttt{Placed}[\{"k=1",\;"k=2",\;"k=3",\;"k=4"\}\,,\; \texttt{Above}]\,, \end{split}
          PlotRangeClipping → False,
          ImagePadding \rightarrow \{\{50, 5\}, \{30, 5\}\},\
          Epilog → {
             Style \left[ \text{Text} \left[ \frac{k}{2} \ln \left[ \frac{n}{2\pi} \right], \{15, 3.5\} \right], 12 \right],
             Text[Style["Sample size (n)", 13], Scaled[\{0.5, -0.2\}]], Rotate[
               Text[Style["Parametric\ncomplexity", 13], Scaled[{-0.2, 0.5}]], 90 Degree]
          ImageSize → Large
       Export["ParamComp_2ndTerm.pdf", Show[p1, ImageSize → 8 cm]];
                                          — k=1 — k=2 — k=3 — k=4
                                \frac{k}{2} \ln \left( \frac{n}{2\pi} \right)
Out[ • ]=
              0
0
```

For reference, the sample size of the largest dataset in Novak & Stouffer 2021 is n = 528.

```
The smallest is n = 10.
And the median is n = 80.
```

```
In[*]:= PC = Round[
            N[{Table[k/2Log[n/(2\pi)]/.n \rightarrow 10, \{k, 1, 4\}]},
               Table[k/2 Log[n/(2\pi)]/.n \rightarrow 80, \{k, 1, 4\}],
               Table[k/2 Log[n/(2\pi)]/.n \rightarrow 528, \{k, 1, 4\}]}]
             , 1 / 10.] // Transpose // TableForm
Out[ • ]//TableForm=
               1.3
                       2.2
       0.2
       0.5
               2.5
                       4.4
       0.7
               3.8
                       6.6
       0.9
               5.1
                       8.9
```