Supplement to Hassell, Lawton and May 1976

Eqn. 1

Hassell, Lawton & May (1976) base their work on the model

$$N_{t+1} = \frac{\lambda N_t}{(1+\alpha N_t)^{-\beta}} \tag{1}$$

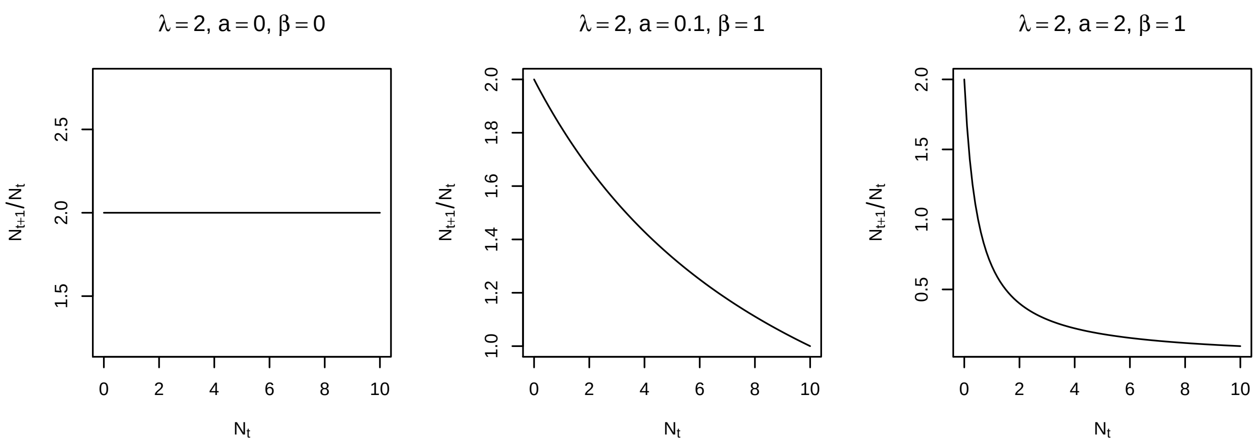
which might be easier to visualize as

$$\frac{N_{t+1}}{N_t} = \frac{\lambda}{(1+\alpha N_t)^{-\beta}}.$$

Observe that when $a = 0$ or $\beta = 0$ this reduces to density-independent growth

$$\frac{N_{t+1}}{N_t} = \lambda.$$

When $a > 0$ or $\beta > 0$ (as assumed in the paper), we can see in the figures below that the greater their values (i.e. the greater that a or β are), the more curved the function becomes. More specifically, a determines where the inflection point of the curve is (larger values create an inflection at lower values of N_t), and β determines the strength of the density-dependence (the steepness (slope) of the curve).



Eqn. 2

They also give

$$\log\frac{N_t}{N_s} = \beta\log(1 + aN_t) \tag{2}$$

where N_t is the initial number of individuals at time t, and N_s is the number of survivors at time $t + 1$ (*not* the total number of individuals, N_{t+1}).

So how is eqn. 2 related to eqn. 1? Answer: "simply", according to HLM in the line after eqn. 2!

First, let's note that

$$x^{-1} = \frac{1}{x}$$

and that

$$\log(x^a) = a\log(x).$$

So now,

$$\begin{aligned}
N_{t+1} &= \lambda N_t(1 + aN_t)^{-\beta} \qquad (3)\\
&= \lambda N_s \qquad (4)\\
\implies \frac{N_s}{N_t} &= (1 + aN_t)^{-\beta} \qquad (5)\\
\implies \frac{N_t}{N_s} &= (1 + aN_t)^{\beta} \qquad (6)\\
\implies \log\frac{N_s}{N_t} &= \beta\log(1 + aN_t) \qquad (7)
\end{aligned}$$