Lecture 8 – 1-D Stability Analysis

Today: PS2 due next class!
Next class: Bring laptops again (w/ Mathematica!)
May & Hassell et al. paper discussions

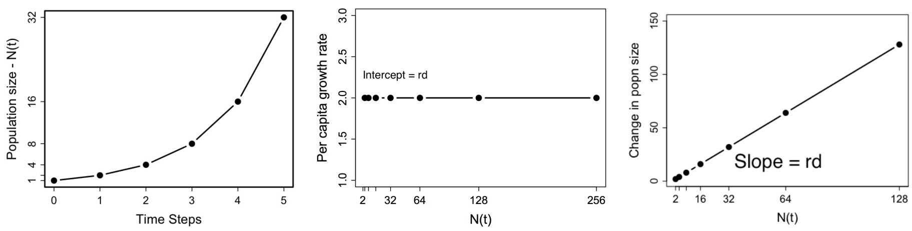
Future: Dynamics & Stability & Species-coexistence
- 1-sp. models (limit-cycles & chaos)
- 2 spp. models (linear & non-linear models)
- 3 spp. models (indirect effects)
- n spp. models

= *unstructured models* (but methods apply to structured models too!)

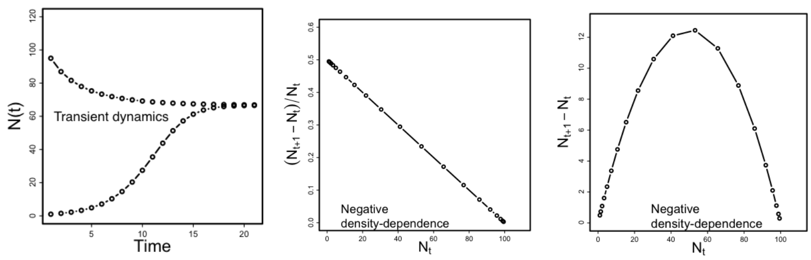
Today's concepts:
Stable point-equilibrium vs. Stable limit cycles vs. Deterministic chaos
Time-delays / Response-lags
Formal local stability analysis

Quick review: Overlay on same three graphs
Discrete-time difference equation: $N_{t+1} = F(N_t)$

$$F(N_t) = N_t + r_d N_t \rightarrow \text{ geometric growth}$$



$$F(N_t) = N_t + r_d N_t \left(1 - \frac{N_t}{K}\right) \rightarrow \text{ discrete logistic growth}$$



Notice **transient** versus **steady-state/equilibrial** dynamics.

R-code exercise - Increasing r_d
First walk through *Class-Ex-Chaos.R.*
Let class experiment with r_d (assuming $K = 10$) (look at *out* vector after transients)

$\mathbf{r_d}$	**Number of different popn sizes**
< 2	monotonic dampening & damped oscillations
2	2-point
2.449	4-point
2.544	8-point
2.564	16-point
2.5687	32-point
> 2.7	deterministic chaos

Let class experiment with $r_d < 2.7$ at different N_0.
Conclusion: → Period doubling bifurcations
→ **Stable limit cycles** (stable attractor orbits independent of initial conditions)

Deterministic chaos: Sensitivity to initial conditions

Experiment with $r_d > 2.7$ at different N_0

(look at *out* vector with $N_0 = 0.0100...001$, up to computer precision $< 10^{-18}$)

$\mathbf{N_0}$	$\mathbf{N_{t=2000}}$ $(r_d = 3)$
0.01	13.26...
0.011	12.46...
0.01001	0.37...
$0.01 + 1 \cdot 10^{-10}$	8.05...
$0.01 + 1 \cdot 10^{-18}$	2.25...

$\rightarrow$ Not stochastic! Rather, deterministic!

Bifurcation plot - Popn points as function of focal parameter.

Work through *peaks* function

Run code and explain plot

Add to table $r_d = 2.83 \rightarrow$ 3-point cycle.

Lyapunov exponent - λ (not popn growth rate!)

Measure of sensitivity to initial conditions.

$$|\Delta_t| = |\Delta_0| \cdot e^{\lambda t}$$

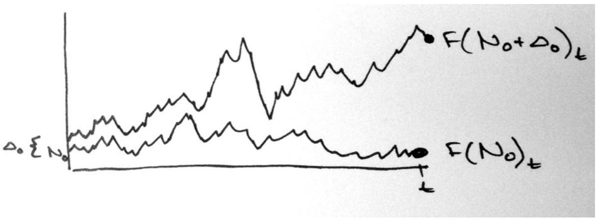
where $N_0 + \Delta_0$ is some small addition to N_0.

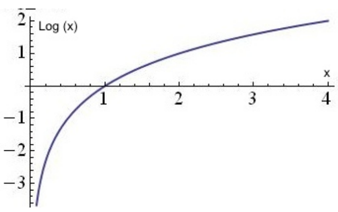
That is,

$$|\Delta_t| = |\ F(N_0 + \Delta_0)_t - F(N_0)_t\ |$$

Rearrange to:

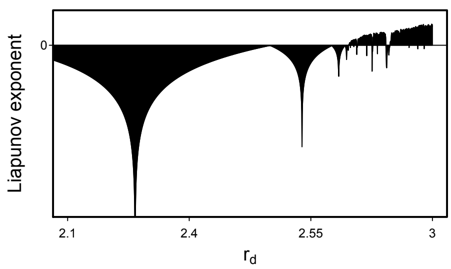
$$\lambda = \frac{log\left(\frac{|\Delta_t|}{|\Delta_0|}\right)}{t}$$





Thus, If $\lambda < 0, \rightarrow$ convergence $\rightarrow$ same dynamics $\rightarrow$ point equilibrium or limit cycle

If $\lambda > 0, \rightarrow$ diverging dynamics $\rightarrow$ chaos



Show in Keynote w/ animation

Mechanism:
$N_{t+1} = F(N_t)$ contains implicit time-lag of 1 unit time
$\rightarrow$ over- and under-compensation (like lagged thermostat)
Contrast to continuous-time model (differential eqn)

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

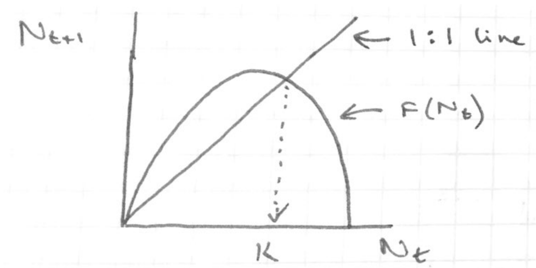
Work through and run last section of R-code.
Contrast to delay-differential model

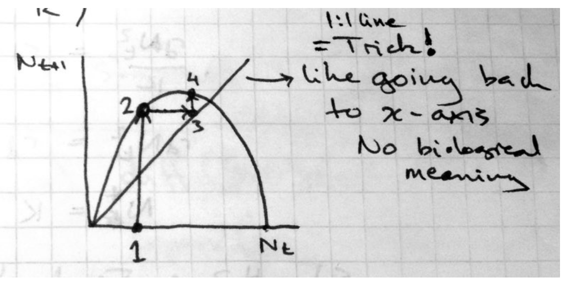
$$\frac{dN}{dt} = rN_{t-\tau}\left(1 - \frac{N_{t-\tau}}{K}\right)$$

Note: For simple delay model, only 2-point limit cycle.
Either need more delays on variables or > 2 species to get more complex dynamics.

Switch to pdf of Keynote presentation of examples
- *Daphnia* example from Case
  - Class Q: Why is Case's use as an example of period-doubling wrong!?!
  - A: *Daphnia* exhibit continuous reproduction (w/ lag)! (see slide)
- Cowpea weevils & review by Hassel and May of insect dynamics
- Other examples: Herring, Salmon, Cicadas

Cobweb (a.k.a. Ricker) Plots Class exercise - handout
To develop more intuitive notion of (implicit) time-lag and over- & under-compensation.
Note that time-lag is also implicit in interspecific species interactions.
Assume discrete-time logistic...





Class Q: Can you predict dynamics from shape of plot?
