Lecture 1 - Philosophical Principles

Announcements:
Today - Finish up class period with some philosophy of what it means to "model" something
Next class: Density-independent geometric deterministic single-species growth

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Write-up on board during break

"All models are wrong, but some are useful." – Box
"No model can be general, precise, and realistic." – Puccia & Levins
"Make your theory as simple as possible, but no simpler." – Einstein

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Categories by Type

What is a model?
Lots of categories; too many to list.
Many different motivations and approaches e.g., Jackson reading
$\implies$ Context for what we want to cover in course

Conceptual Models vs. **Mathematical Models**

Ideas
Hypotheses
e.g., diagrams with boxes and arrows

$$\begin{array}{ccc} & Z\ (2^\circ\ \text{consumer}) & \\ & \uparrow \gamma & \\ & Y\ (1^\circ\ \text{consumer}) & \\ \alpha \nearrow & & \nwarrow \beta \\ W & & X \end{array}$$

$$Z = \gamma Y$$
$$Y = \alpha W + \beta X$$

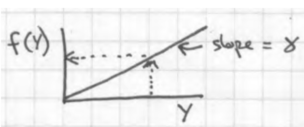
Parameters: α and β
State variables: Y, W, X, Z

Qualitative vs. **Quantitative**

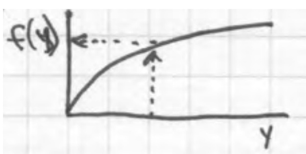
$$\Delta Z > 0 \text{ if } \begin{cases} \Delta Y > 0 \text{ and } \gamma > 0 \\ \Delta Y < 0 \text{ and } \gamma < 0 \end{cases}$$

Static Models vs. **Dynamical Models**

Feeding rate of Z on Y depends on Y's abundance
$\implies f(Y) = \gamma Y$



Could be non-linear



Iterative in time or space

$$Y_{t+1} = Y_t + (\alpha W_t + \beta X_t - \gamma Z_t) Y_t$$

Value of Y depends on the value before it.

Many other types and categories:

Individual (agent) based

spatially implicit versus spatially explicit

etc.

But categories can quickly break down:

ex1. Dynamical models contain static models

ex2. Typically interested in *qualitative* predictions from *quantitative* models

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Categories by Purpose

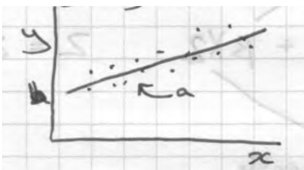
Quantitative models are tools for *evaluating* hypotheses/conceptual models
Traditionally:

Statistical Models vs. **Process Models**

Hypothesis testing & Parameter estimation

$\Longrightarrow$ Inference within bounds of data

e.g., linear regression, ANOVA, t-tests



$$y_i = ax_i + b + \epsilon_i$$

a - slope

b - intercept

ϵ - residual errors

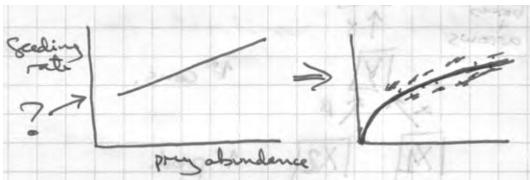
$\Longrightarrow$ Pattern

First-principals

Analytical inference

Simulation & Numerical analysis

e.g., intercept = 0 makes no sense for functional response



$\Longrightarrow$ Mechanism

The best modern-day quantitative modeling *combines* traditional Statistical and Process modeling

Deterministic (mechanistic) "core" + Stochastic (error) shell

$$\text{Feeding rate} = \underbrace{f(Prey)}_{core} + \underbrace{\epsilon}_{shell}$$

$$\epsilon \sim \mathcal{N}(0, \sigma^2)$$

$\Longrightarrow$ New world of process-model fitting & information-theoretic model comparisons

Models are *not* the same as hypotheses or theories
Multiple models can encapsulate the same hypotheses
Purpose is often to *refine* hypotheses; a form of quantitative reasoning.

e.g., "Feeding rate increases with prey abundance." = hypothesis

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Model Complexity

In Ecology: wide range of model complexities

Discrete-time Logistic: $x_{t+1} = rx_t(1 - x_t)$
  1 parameter & 1 state variable, yet chaotic dynamics possible!
EcoPath, Atlantis (end-to-end models)
  100-1000's of parameters
  10-100's of state variables

We will be dealing with the former range in this class
Common criticisms of theoretical ecology:
  "Where is reality?"
  "Too simple" "Irrelevant"
  "Real world is way more complex than just a handful of parameters and state variables."
  "Theory applies in general everywhere, but nowhere in particular. Thus useless"

Much work has shown low-dimensional model can explain most of the observed variation.
Low-dimensional models allow:
  identify & focus on most critical parameters, variables, processes
  rigorous exploration/understanding of uncertainty
  decision-making tools
  general understanding is the goal

"No model can be general, precise, and realistic." – Puccia & Levins

R demonstration: Polynomial regression - statistical model
Have population sizes of rabbits at time t – $N(t)$
Model it using polynomial regression:

$$N(t) = \sum_{n=0}^{\infty} \beta_n t^n = \beta_0 \overset{1}{\cancel{t^0}} + \beta_1 t^1 + \beta_2 t^2 + \ldots + \beta_n t^n$$

Group exercise Repeat with random numbers
What have we learned from our polynomial model? Nothing!
Yet this statistical model is a perfect fit to the data!

$\implies$ Goal of theoretical ecology is understanding (few parameters)

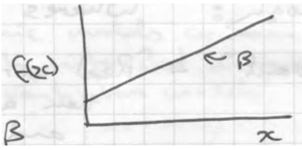
Lotka-Volterra predator-prey model with 4 parameters:

$$\frac{dN}{dt} = N(\alpha - \beta L) \qquad \frac{dL}{dt} = L(e\beta N - d)$$

[Turns out LV model is wrong for Lynx-Hare, but took a long time to realize that and provided important insights into pred-prey ecology in general.]

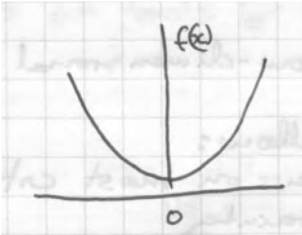
Linear vs. Nonlinear Models

$f(x) = \alpha + \beta x$
Variable: x
Parameters: α and β



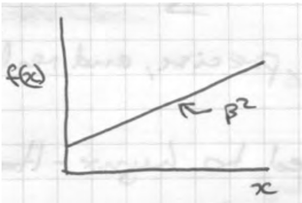
$\Longrightarrow$ Linear model

$f(x) = \alpha + \beta x^2$



$\Longrightarrow$ "Nonlinear" for ecologists & mathematicians
(nonlinear in state variable)
$\Longrightarrow$ "Linear" for statisticians
(in parameters)

$f(x) = \alpha + \beta^2 x$



$\Longrightarrow$ "Linear" for ecologists & mathematicians
$\Longrightarrow$ "Nonlinear" for statisticians
(nonlinear in parameters)

My road to Theoretical Ecology

- For some, this class will be easy & intuitive. For others, less so.
- I am not a mathematician (or statistician)! As undergrad, took usual Calculus (even remedial Calculus for biologists). But skipping equations in papers. Took authors on their word (esp. discussion section).
- Went to grad school after 3 yrs. as field ecologists to learn "theoretical ecology". (But almost failed out of first year stats class; too much calculus; over my head.)

  $\Longrightarrow$ Your ability and what you learn will come down to your motivation. Keep at it. Persevere.
- My motivation: Conceptual understanding of species interaction strengths & IGP theory.
- In 3rd year while reading a paper, realized I'd read and actually understood the equations!
- So much untested and interesting theory exists that is just waiting for empiricists to test & develop & correct
- Don't have to be a mathematician, but for Ecology to make progress we need to understand theory ("the math")
