Problem Set 6

Extensions to Rosenzweig-MacArthur Model

Part A

In class we studied the classic Rosenzweig-MacArthur model of a consumer-resource interaction,

$$\frac{dR}{dt} = bR(1 - \alpha R) - \frac{aRC}{1 + ahR} \tag{1}$$

$$\frac{dC}{dt} = \frac{eaRC}{1 + ahR} - dC \tag{2}$$

where the resource R has intrinsic birth rate b, and self-limitation rate α (i.e. it experiences logistic-growth in the absence of predation), and the consumer C feeds with a type II functional response with attack rate a and handling time h, converts eaten resources into consumers with efficiency e, and dies at a density-independent rate of d.

1. Use *Mathematica* to determine the zero-growth isoclines of this classic Rosenzweig-MacArthur model.
2. Now switch to `R` and plot these isoclines (using the `curve()` function) on phase portraits for the following two cases, choosing parameter values accordingly:
   (a) Dynamics converge to a point equilibrium
   (b) Dynamics converge to a stable limit cycle

   Note that after plotting the isocline of species 2 as a function of species 1, you will have to rearrange the equation for the isocline of species 1 in order to add it on the same plot.
3. Using an ODE solver (package *deSolve*), simulate the dynamics of the model and overlay them on your phase portraits. Overlay the isocline plots and dynamics on vector-fields by first using the `plotVectorField()` function of the *VectorField.R* script (that is posted on our class website).

Part B

The type III functional response, $f(R) = \frac{aR^\theta}{1+ahR^\theta}$, is often utilized as a means of describing a "switching" consumer (i.e. a consumer that switches to a different, un-modeled alternative resources when the abundance of the focal resource is low). Extend the classic Rosenzweig-MacArthur model to include such a type III response (assume $\theta = 2$) and determine and plot its isoclines on a phase portrait.

Now do the same after extending the classic Rosenzweig-MacArthur model to include either

- a physical refuge from predation for the resources in which a fixed number of resources can avoid the consumer, or
- a density-independent immigration term for the resources population

to show that one obtains a similar shaped resources-isocline in all three cases. Describe the mechanism that is common to all three extensions that leads to this similarity.