Lecture 10 – Two-species competition

Today: Graphical analysis of two-species competition.
Next: 2-D stability analysis
Concepts for the road ahead:
Coexistence, Invasibility, Priority effects, Alternative Stable States
Phase diagrams & Zero Net Growth Isoclines (ZNGI's)

Two-species competition Motivate with Crombie (1946) flour beetle experiments. 2 slides.
Extend 1 sp. logistic to 2 spp.

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1} - \alpha_{12}\frac{N_2}{K_1}\right) \qquad \frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2}{K_2} - \alpha_{21}\frac{N_1}{K_2}\right)$$

Or in slightly different general form:

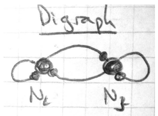
$$\frac{dN_i}{dt} = r_i N_i \left(1 - \frac{N_i}{K_i} - \alpha_{ij}\frac{N_j}{K_i}\right) \qquad \text{for } i \neq j.$$

Contrast *intra*-specific density-dependence (self-limitation) vs. *inter*-specific competition
α_{ij} - effect of 1 average j individual on 1 average i individual relative to effect that i has on self.
= per capita effect
'How much of K_i does each j individual use?'
e.g., if 10 j individuals consume equivalent to 1 i individual, then $\alpha_{ij} = \frac{1}{10}$.

This is a model of **exploitation** with *implicit* resources.
Implicit - Resource dynamics are *not* modeled
- Phenomenological model depiction of resources
- e.g., K - 'carrying capacity'

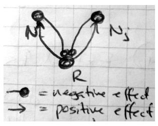
That is:

$$\frac{dN_i}{dt} = f_i(N_i, N_j)$$



Explicit - Resource dynamics *are* modeled
e.g. Consumer-resource models (next class)

$$\frac{dN_i}{dt} = f_i(N_i, R) \qquad \frac{dR}{dt} = f_R(N_i, N_j, R)$$



Exploitation - Indirect negative interaction through joint use of shared *limiting* resource(s).
Interference - Direct negative interaction preventing use of resource(s).

Today's Q:
When can two species coexist on the same shared resource(s)?
When can Sp. 1 invade a community consisting of Sp. 2?
⇒ How many equilibria are there?
⇒ Are equilibria stable or unstable?

Hand out and work through quiz as group: (a)

$$\frac{dN_i}{dt} = r_i N_i \left(1 - \frac{N_i}{K_i} - \alpha_{ij}\frac{N_j}{K_i}\right)$$

Qualitatively...

$$(N_i^*, N_j^*) = \begin{cases} 0, & 0 \\ N_i^* > 0, & 0 \\ 0, & N_j^* > 0 \\ N_i^* > 0, & N_j^* > 0 \end{cases}$$

(b)
When $N_j = 0 \Rightarrow$ reduces to 1 sp. logistic:

$$\begin{aligned} \frac{dN_i}{dt} &= r_i N_i \left(1 - \frac{N_i}{K_i}\right) \\ N_i^* &= K_i \end{aligned}$$

(c)
Replace i for j, $\Rightarrow N_j^* = K_j$ in absence of i.

(d)
Want $N_i^* > 0$ & $N_j^* > 0$
A: Solve for one as function of the other:

$$\begin{aligned} r_i N_i - \frac{r_i N_i N_i}{K_i} - \frac{r_i \alpha_{ij} N_i N_j}{K_i} = 0 \\ r_i N_i K_i &= r_i N_i N_i + r_i \alpha_{ij} N_i N_j \qquad \text{(move and multiply by } K_i\text{)} \\ K_i &= N_i + \alpha_{ij} N_j \qquad \text{(divide by } r_i \text{ and } N_i\text{)} \\ \\ N_i^* &= K_i - \alpha_{ij} N_j \quad \text{ and similarly } N_j^* = K_j - \alpha_{ji} N_i \end{aligned}$$

Note intuitive meaning of α_{ij}!
Note also that we can't solve for one species without knowing the other.

$\Rightarrow$ need to solve for N_i^* and N_j^* jointly.

Plug solution of N_j^* into solution for N_i^* and vice versa.

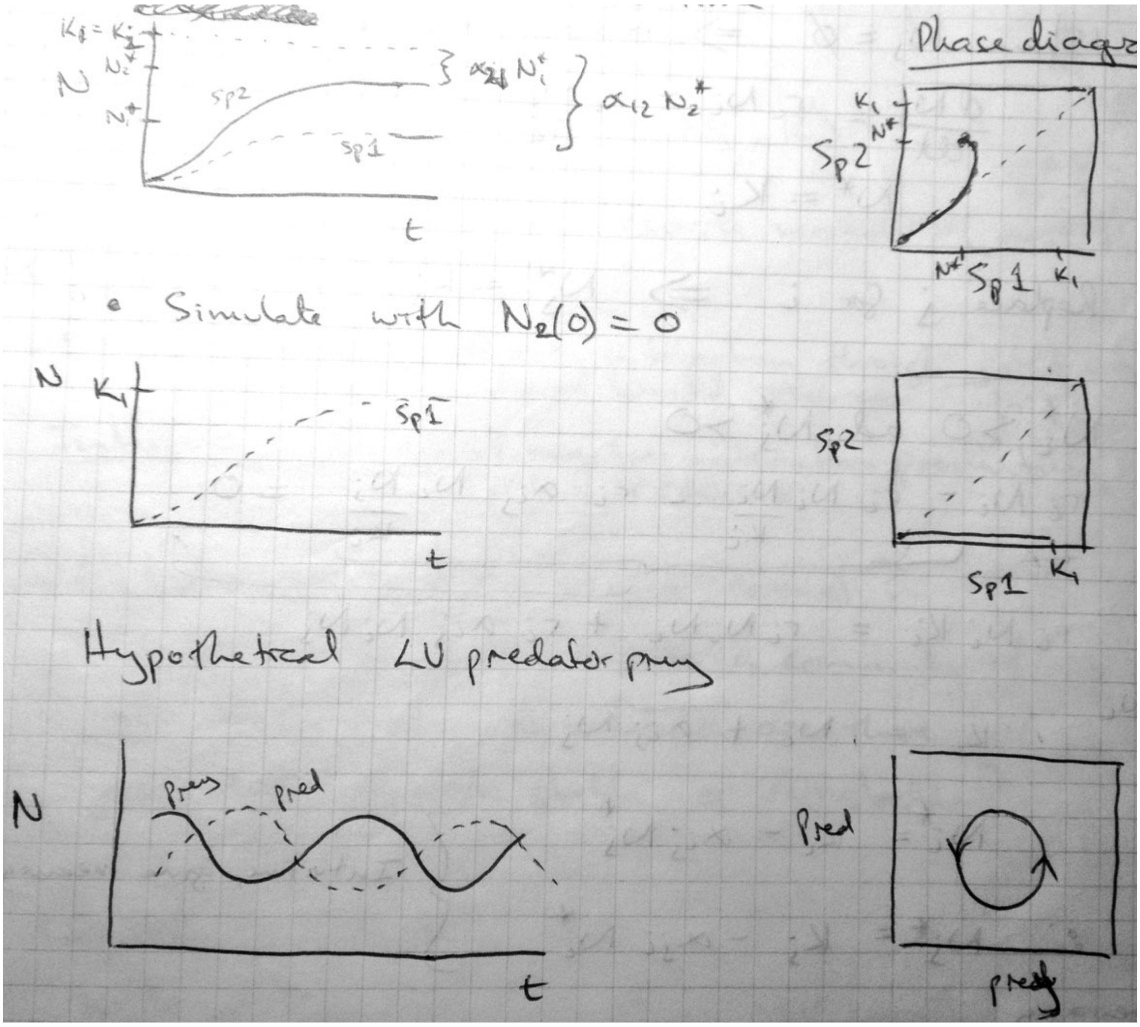
$$\begin{aligned} N_i^* &= K_i - \alpha_{ij} N_j^* \\ &= K_i - \alpha_{ij}\left(K_j - \alpha_{ji} N_i^*\right) \\ &= K_i - \alpha_{ij} K_j + \alpha_{ij}\alpha_{ji} N_i^* \\ N_i^* + \alpha_{ij} K_j - \alpha_{ij}\alpha_{ji} N_i^* &= K_i \\ N_i^* - \alpha_{ij}\alpha_{ji} N_i^* &= K_i - \alpha_{ij} K_j \\ N_i^* \left(1 - \alpha_{ij}\alpha_{ji}\right) &= K_i - \alpha_{ij} K_j \\ N_i^* &= \frac{K_i - \alpha_{ij} K_j}{1 - \alpha_{ij}\alpha_{ji}} \end{aligned}$$

Phase portraits/diagrams

R-code demonstration

Work through ode-solver code

Simulate coexistence over time



Return to main questions:

When can both spp. coexist?
When can sp 1 out-compete sp 2?
When can sp 1 invade sp 2?

Naive simulations

R-demonstration - Walk through first set of parameter values. "Naive simulations"

Then show full table before showing results with R code for other parameters.

With $r_1 = r_2 = K_1 = K_2 = 1$ and $N_1(0) = N_2(0) = 0.01$

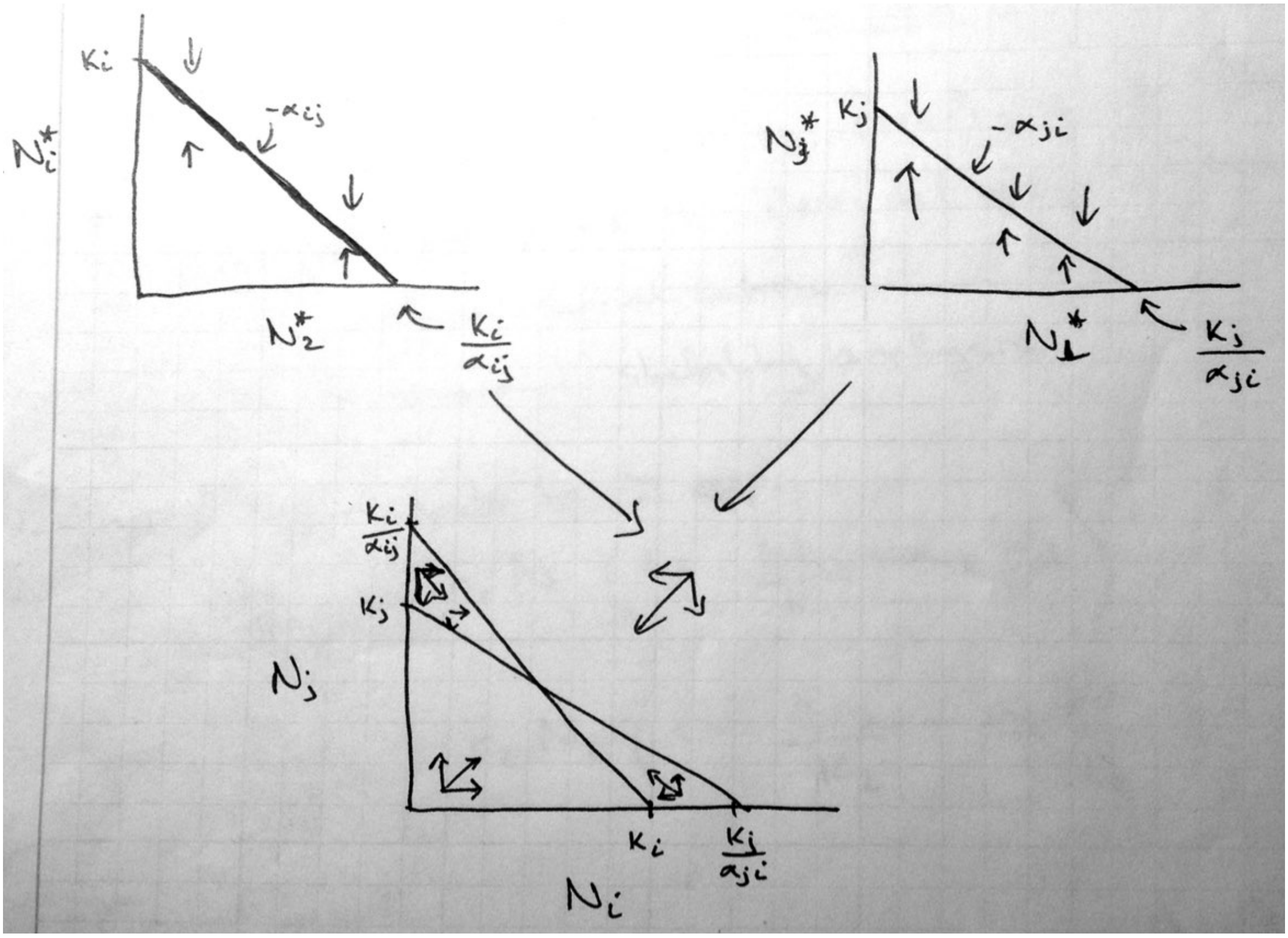
α_{21}	α_{12}	Outcome	
0.5	0.7	coexist	
1.5	0.5	sp1	
0.5	1.5	sp2	$N_1(0) = N_2(0)$
1.5	1.7	sp2	
1.5	1.7	sp1	$N_1(0) > N_2(0)$

⇒ Priority effect
⇒ Alternative stable states

Graphical Analysis
Earlier we showed that from $\frac{dN_i}{dt} = 0$ that:

$$\begin{aligned} N_i^* =& K_i - \alpha_{ij} N_j \\ \Rightarrow N_j =& \frac{K_i - N_i^*}{\alpha_{ij}} \\ \text{Thus isocline interesects x-axis at} & \\ N_j =& \frac{K_i - 0}{\alpha_{ij}} = \frac{K_i}{\alpha_{ij}} \end{aligned}$$

Same goes for 2nd species.



R-code: Work through other parameter values

Inferences/Conclusions:

Coexistence:	intra > inter for both species
	(i.e. $\frac{K_i}{\alpha_{ii}} = \frac{K_i}{1} < \frac{K_j}{\alpha_{ji}} \Rightarrow \alpha_{ii} > \alpha_{ji}$)
Sp i dominance:	intra > inter for i & intra < inter for j
Priority effect:	intra < inter for both
	(i.e. $\frac{K_i}{\alpha_{ii}} = \frac{K_i}{1} > \frac{K_j}{\alpha_{ji}} \Rightarrow \alpha_{ii} < \alpha_{ji}$)