Quiz 1 - Monod's Nightmare

My name is: ____________________

Under ideal growing conditions in the laboratory, the population size of the common bacterium *Escherichia coli* can double in around 20 minutes. *E. coli*'s cells are rod-shaped and are roughly approximated by a rectangular box that is 2 μm long, 1 μm wide, and 1 μm high. (1 μm = 1 micro-meter = 1 x 10-6 meters = 0.000001 m). For comparison, a strand of human hair is roughly 100 μm wide.

(a) Would a continuous-time model (the solution for which is $N_t = N_0 e^{rt}$) or a discrete-time model (the solution for which is $N_t = \lambda^t N_0$) be more appropriate for describing this population? Why?

Continuous. Reproduction occurs continuously, not in discrete synchronous bouts.

(b) What is *E. coli*'s growth rate r (in minutes) under these ideal growing conditions. Express your answer by solving for the equation you would use to calculate r to the simplest solution possible.

$$t = 20 \text{ minutes}; \quad N_t = 2N_0$$
$$N_t = N_0 e^{rt}$$
$$ln\left(\frac{N_t}{N_0}\right) = rt$$
$$r = \frac{ln\left(\frac{N_t}{N_0}\right)}{t} = \frac{ln(2)}{20} \approx \frac{0.693}{20} \approx 0.035\% \text{ per minute}$$

(c) Let's assume that our classroom is roughly 10 m long, 10 m wide, and 8 m high (it's not, I just made up these numbers). How long it would take for an exponentially growing population of *E. coli* under ideal conditions to fill our empty classroom when starting from a single individual bacterium?

The volume of each *E.coli* is

$1\mu m \cdot 1\mu m \cdot 2\mu m = 2\mu m^3$

$1 \cdot 10^{-6} m \cdot 1 \cdot 10^{-6} m \cdot 2 \cdot 10^{-6} m = 2 \cdot 10^{-18} m^3$

The volume of room is $10m \cdot 10m \cdot 8m = 800m^3$

Therefore, the number of *E. coli* needed to fill room is

$$\frac{800m^3}{2 \cdot 10^{-18} m^3} \approx 4 \cdot 10^{20}.$$

Thus:

$$N_0 = 1$$
$$N_t = N_0 e^{rt}$$
$$ln\left(\frac{N_t}{N_0}\right) = rt$$
$$t = \frac{ln\left(\frac{4 \cdot 10^{20}}{1}\right)}{t} = \frac{ln\left(\frac{4 \cdot 10^{20}}{1}\right)}{\frac{ln(2)}{20}} \approx \frac{47.438}{0.035} \approx 1369 \text{ minutes} \approx 22.8 \text{ hours}$$