

Lecture 8 – 1-D Stability Analysis

Today: PS2 due!

Next class: Bring laptops again (w/ Mathematica!)
May & Hassell et al. paper discussions

Future: Dynamics & Stability & Species-coexistence

- 1-sp. models (limit-cycles & chaos)
- 2 spp. models (linear & non-linear models)
- 3 spp. models (indirect effects)
- n spp. models

= *unstructured models* (but methods apply to structured models too!)

Today's concepts:

Stable point-equilibrium vs. Stable limit cycles vs. Deterministic chaos

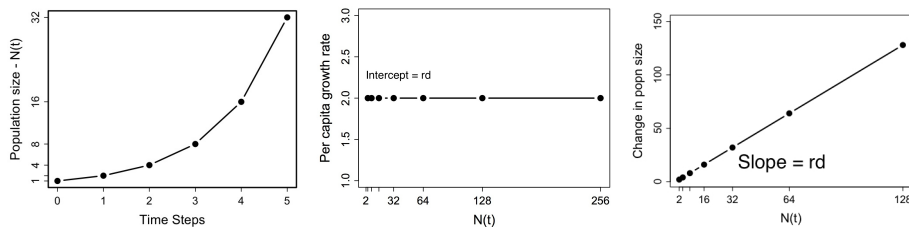
Time-delays / Response-lags & Over- and under-compensation

Formal local stability analysis

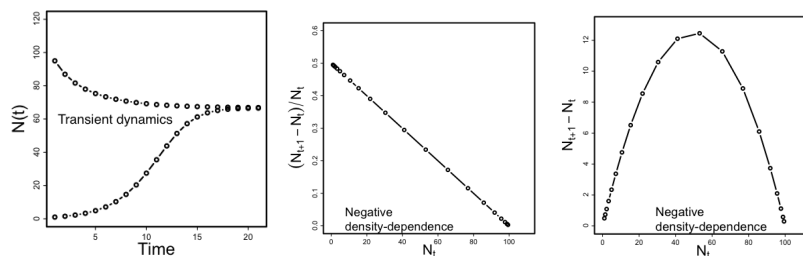
Quick review: Overlay on same three graphs

Discrete-time difference equation: $N_{t+1} = F(N_t)$

$$F(N_t) = N_t + r_d N_t \rightarrow \text{geometric growth}$$



$$F(N_t) = N_t + r_d N_t \left(1 - \frac{N_t}{K}\right) \rightarrow \text{discrete logistic growth}$$



Notice **transient** versus **steady-state/equilibrium** dynamics.

R-code exercise - Increasing r_d

First walk through *Class-Ex-Chaos.R*.

Let class experiment with r_d (assuming $K = 10$) (look at *out* vector after transients)

r_d	Number of different popn sizes
< 2	monotonic dampening & damped oscillations
2	2-point
2.449	4-point
2.544	8-point
2.564	16-point
2.5687	32-point
> 2.7	deterministic chaos

Let class experiment with $r_d < 2.7$ at different N_0 .

Conclusion: \rightarrow Period doubling bifurcations

\rightarrow **Stable limit cycles** (stable attractor orbits independent of initial conditions)

Deterministic chaos: Sensitivity to initial conditions

Experiment with $r_d > 2.7$ at different N_0

(look at *out* vector with $N_0 = 0.0100...001$, up to computer precision $< 10^{-18}$)

N_0	$N_{t=2000} \ (r_d = 3)$
0.01	13.26...
0.011	12.46...
0.01001	0.37...
$0.01 + 1 \cdot 10^{-10}$	8.05...
$0.01 + 1 \cdot 10^{-18}$	2.25...

→ Not stochastic! Rather, deterministic!

Bifurcation plot - Popn points as function of focal parameter.

Work through *peaks* function

Run code and explain plot

Add to table $r_d = 2.83 \rightarrow 3$ -point cycle.

Lyapunov exponent - λ (not popn growth rate!)

Measure of sensitivity to initial conditions.

$$|\Delta_t| = |\Delta_0| \cdot e^{\lambda t}$$

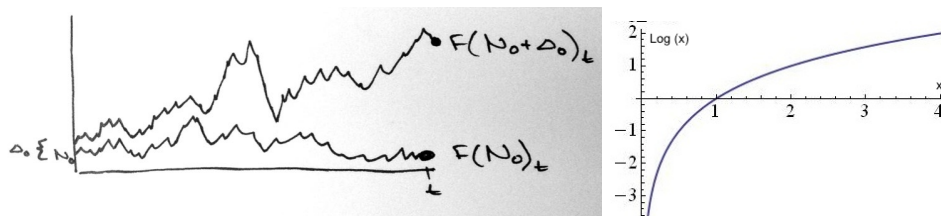
where $N_0 + \Delta_0$ is some small addition to N_0 .

That is,

$$|\Delta_t| = |F(N_0 + \Delta_0)_t - F(N_0)_t|$$

Rearrange to:

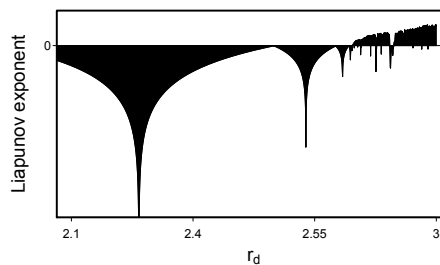
$$\lambda = \frac{\log\left(\frac{|\Delta_t|}{|\Delta_0|}\right)}{t}$$



Thus,

If $\lambda < 0$, → convergence → same dynamics → point equilibrium or limit cycle

If $\lambda > 0$, → diverging dynamics → chaos



Show in Keynote w/ animation

Mechanism:

$N_{t+1} = F(N_t)$ contains implicit time-lag of 1 unit time
 → over- and under-compensation (like lagged thermostat)

Contrast to continuous-time model (differential eqn)

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right)$$

Work through and run last section of R-code.

Contrast to delay-differential model

$$\frac{dN}{dt} = rN_{t-\tau} \left(1 - \frac{N_{t-\tau}}{K} \right)$$

Note: For simple delay model, only 2-point limit cycle.

Either need more delays on variables or > 2 species to get more complex dynamics.

Switch to pdf of Keynote presentation of examples

- *Daphnia* example from Case

Class Q: Why is Case's use as an example of period-doubling wrong!?!

A: *Daphnia* exhibit continuous reproduction (w/ lag)! (see slide)

- Cowpea weevils & review by Hassel and May of insect dynamics

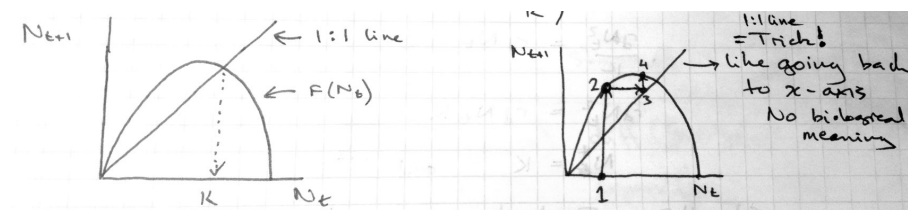
- Other examples: Herring, Salmon, Cicadas

Ricker Plots Class exercise - handout

To develop more intuitive notion of (implicit) time-lag and over- & under-compensation.

Note that time-lag is also implicit in interspecific species interactions.

Assume discrete-time logistic...



Class Q: Can you predict dynamics from shape of plot?