Quiz 4 - Lotka-Volterra Competition

My name is: _____

Two species (N_1 and N_2) are locked in a deadly battle over a set of shared resources. They thereby compete with one another such that an individual of species N_2 utilizes α_{12} amount of the resources utilized by an individual of species N_1, and an individual of species N_1 utilizes α_{21} amount of the resources utilized by an individual of species N_2. By a simple extension of the single-species logistic model, we can include such competition between two species by writing

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1} - \frac{\alpha_{12} N_2}{K_1}\right)$$

to describe the population growth rate of species N_1, and

$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{\alpha_{21} N_1}{K_2} - \frac{N_2}{K_2}\right)$$

to describe the population growth rate of species N_2.

(a) How many equilibria does this model have? Describe each of them qualitatively in terms of the population sizes of N_1 and N_2.

(b) Use the above equations to show that species N_1 will reach its equilibrium carrying capacity K_1 in the absence of species N_2.

(c) What is the equilibrium population size of N_2 in the absence of N_1?

(d) Use the equations to solve for the equilibrium population size of N_1 in the presence of N_2.