# Quiz 1 - Monod's Nightmare

Under ideal growing conditions in the laboratory, the population size of the common bacterium *Escherichia coli* can double in around 20 minutes. *E. coli*'s cells are rod-shaped and are roughly approximated by a rectangular box that is 2 $\mu m$ long, 1 $\mu m$ wide, and 1 $\mu m$ high. (1 $\mu m$ = 1 micro-meter = 1 x 10-6 meters = 0.000001 m). For comparison, a strand of human hair is roughly 100 $\mu m$ wide.

(a) Would a continuous-time model (the solution for which is $N_t = N_0 e^{rt}$) or a discrete-time model (the solution for which is $N_t = \lambda^t N_0$) be more appropriate for describing this population? Why?

Continuous. Reproduction occurs continuously, not in discrete synchronous bouts.

(b) What is *E. coli*'s growth rate $r$ (in minutes) under these ideal growing conditions. Express your answer by solving for the equation you would use to calculate $r$ to the simplest solution possible.

$$
\begin{aligned}
t = 20 \text{ minutes}; \quad N_t &= 2N_0 \\
N_t &= N_0 e^{rt} \\
ln\left(\frac{N_t}{N_0}\right) &= rt \\
r &= \frac{ln\left(\frac{N_t}{N_0}\right)}{t} = \frac{ln(2)}{20} \approx \frac{0.693}{20} \approx 0.035\%\text{per minute}
\end{aligned}
$$

(c) Let's assume that our classroom is roughly 10 $m$ long, 10 $m$ wide, and 8 $m$ high (its not, I just made up these numbers). How long it would take for an exponentially growing population of *E. coli* under ideal conditions to fill our empty classroom when starting from a single individual bacterium?

Volume of each *E.coli*:

$1\mu m \cdot 1\mu m \cdot 2\mu m = 2\mu m^3$

$1 \cdot 10^{-6} m \cdot 1 \cdot 10^{-6} m \cdot 2 \cdot 10^{-6} m = 2 \cdot 10^{-18} m^3$

Volume of room:

$10m \cdot 10m \cdot 8m = 800m^3$

Therefore, number of *E. coli* needed to fill room:

$$\frac{800m^3}{2 \cdot 10^{-18} m^3} \approx 4 \cdot 10^{20}$$

Thus:

$$
\begin{aligned}
N_0 &= 1 \\
N_t &= N_0 e^{rt} \\
ln\left(\frac{N_t}{N_0}\right) &= rt \\
t &= \frac{ln\left(\frac{4 \cdot 10^{20}}{1}\right)}{t} = \frac{ln\left(\frac{4 \cdot 10^{20}}{1}\right)}{\frac{ln(2)}{20}} \approx \frac{47.438}{0.035} \approx 1369 \text{ minutes} \approx 22.8 \text{ hours}
\end{aligned}
$$