



BENSOLVE - VLP Solver

Reference Manual

for Version 2.1.x

July 25, 2017

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1 Introduction

BENSOLVE is a solver for vector linear programs (VLP), in particular, for the subclass of multiple objective linear programs (MOLP). The present version is based on **Benson's algorithm** and its extensions, see e.g. [3, 4, 1, 2, 10, 8, 5, 6, 9] and the references therein. For the theoretical background of this program, the reader is referred to [8, 6].

The present version utilizes the GNU Linear Programming Kit (GLPK). BENSOLVE (from version 2.0.0) is written in C programming language (C99 standard).

1.1 Multiple Objective Linear Program

An important special case of a vector linear program (VLP) is a multiple objective linear program (MOLP), that is, a linear program (LP) with multiple linear objective functions. BENSOLVE assumes the following formulation of a MOLP:

$$\text{minimize (or maximize)} \begin{pmatrix} P_{11}x_1 + P_{12}x_2 + \cdots + P_{1n}x_n \\ P_{21}x_1 + P_{22}x_2 + \cdots + P_{2n}x_n \\ \vdots \\ P_{q1}x_1 + P_{q2}x_2 + \cdots + P_{qn}x_n \end{pmatrix} \quad (1)$$

subject to the constraints

$$\begin{aligned} a_1 &\leq B_{11}x_1 + B_{12}x_2 + \cdots + B_{1n}x_n \leq b_1 \\ a_2 &\leq B_{21}x_1 + B_{22}x_2 + \cdots + B_{2n}x_n \leq b_2 \\ &\vdots \\ a_m &\leq B_{m1}x_1 + B_{m2}x_2 + \cdots + B_{mn}x_n \leq b_m \end{aligned} \quad (2)$$

$$\begin{aligned} l_1 &\leq x_1 \leq s_1 \\ l_2 &\leq x_2 \leq s_2 \\ &\vdots \\ l_n &\leq x_n \leq s_n. \end{aligned} \quad (3)$$

1.2 Vector Linear Program

In this more general setting, the minimization (or maximization) in (1) is defined with respect to a partial ordering \leq_C induced by a polyhedral cone $C \subseteq \mathbb{R}^q$, that is

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_q \end{pmatrix} \leq_C \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_q \end{pmatrix} \iff \begin{pmatrix} z_1 - y_1 \\ z_2 - y_2 \\ \vdots \\ z_q - y_q \end{pmatrix} \in C. \quad (4)$$

BENSOLVE assumes that C has a non-empty interior and contains no lines. The polyhedral cone C is assumed to be given by one of the following two representations:

The **CONE** representation is given by a matrix Y with q rows and o columns. A vector (y_1, \dots, y_q) belongs to C if and only if there are nonnegative real numbers $v_1, v_2, \dots, v_o \geq 0$ such that

$$\begin{aligned} y_1 &= Y_{11}v_1 + Y_{12}v_2 + \cdots + Y_{1o}v_o \\ y_2 &= Y_{21}v_1 + Y_{22}v_2 + \cdots + Y_{2o}v_o \\ &\dots \\ y_q &= Y_{q1}v_1 + Y_{q2}v_2 + \cdots + Y_{qo}v_o. \end{aligned} \tag{5}$$

The columns of the matrix Y are generating vectors of the polyhedral cone C .

The **DUALCONE** representation is given by a matrix Z with q rows and p columns. A vector (y_1, \dots, y_q) belongs to C if and only if the following inequalities are satisfied:

$$\begin{aligned} Z_{11}y_1 + Z_{21}y_2 + \cdots + Z_{q1}y_q &\geq 0 \\ Z_{12}y_1 + Z_{22}y_2 + \cdots + Z_{q2}y_q &\geq 0 \\ &\dots \\ Z_{1p}y_1 + Z_{2p}y_2 + \cdots + Z_{qp}y_q &\geq 0. \end{aligned} \tag{6}$$

The columns of the matrix Z are generating vectors of the dual cone of the polyhedral cone C .

1.3 Upper and lower images

The q -dimensional space of objective values is called *image space* and the n -dimensional space of variables is called the *pre-image space*. BENSOLVE computes the finite set of all vertices and extreme directions of the *upper image* in case of minimization and the *lower image* in case of maximization. This finite set can be considered as a “solution in the image space”.

For MOLP, the *upper image* is the set of all vectors $y = (y_1, \dots, y_q)$ such that

$$\begin{aligned} y_1 &\geq P_{11}x_1 + P_{12}x_2 + \cdots + P_{1n}x_n \\ y_2 &\geq P_{21}x_1 + P_{22}x_2 + \cdots + P_{2n}x_n \\ &\dots \\ y_q &\geq P_{q1}x_1 + P_{q2}x_2 + \cdots + P_{qn}x_n \end{aligned} \tag{7}$$

for $x = (x_1, \dots, x_n)$ satisfying the constraints (2) and (3). The set of vertices of the upper image is exactly the set of of *minimal* (see e.g. [8], *minimal* is also called *efficient* or *non-dominated*) vertices of the *image*, where the *image* is the set of vectors $y = (y_1, \dots, y_q)$ such that

$$\begin{aligned} y_1 &= P_{11}x_1 + P_{12}x_2 + \cdots + P_{1n}x_n \\ y_2 &= P_{21}x_1 + P_{22}x_2 + \cdots + P_{2n}x_n \\ &\dots \\ y_q &= P_{q1}x_1 + P_{q2}x_2 + \cdots + P_{qn}x_n \end{aligned} \tag{8}$$

for $x = (x_1, \dots, x_n)$ satisfying the constraints (2) and (3).

The *lower image* of a maximization problem is defined likewise by replacing \geq by \leq in (7). In the general case of a vector linear program (VLP), the component-wise ordering \geq in (7) has to be replaced by the ordering \geq_C , see (4).

1.4 Solution concept

BENSOLVE computes a *solution* of the vector linear program [8, 6]. Such a *solution* is a finite set of points and directions $x = (x_1, \dots, x_n)$ of the feasible set (i.e. the set of vectors satisfying the constraints (2) and (3)) such that the corresponding images $y = (y_1, \dots, y_q)$ (according to (8)) are the vertices and extreme direction of the upper or lower image.

A solution to the given vector linear program is also called *primal solution*, whereas a solution of the dual program (see Section 1.6) is called *dual solution*.

1.5 Duality parameter vector

BENSOLVE utilizes vector linear programming duality [7]. It computes primal and dual solutions. The duality parameter vector c is a q -dimensional vector that must belong to the interior of the ordering cone C and must have a non-zero last component $c_q \neq 0$. The duality parameter vector $c = (c_1, \dots, c_q)$ is scaled by BENSOLVE with a positive factor such that either $c_q = 1$ or $c_q = -1$. If not specified or invalid, it will be computed by the program. Note that the dual problem, in particular, the dual solution of VLP depends on this parameter. For a description of the dual program see Section 1.5. For details on the theory, see e.g. [6].

1.6 Dual program

1.6.1 Special case of MOLP

Assume that the multiple objective linear program (MOLP) is a minimization problem and has only the constraints (2) (but not (3)). Assume further that $b_i = \infty$ ($i = 1, \dots, m$) in (2), i.e. there are only lower bounds. Then the dual problem is the following vector linear program (VLP) with ordering cone $K := \{y \in \mathbb{R}^q \mid y_1 = 0, \dots, y_{q-1} = 0, y_q \geq 0\}$:

$$K\text{-maximize} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_{q-1} \\ a_1 u_1 + \dots + a_m u_m \end{pmatrix} \quad (9)$$

subject to the constraints

$$\begin{aligned} B_{11}u_1 + B_{21}u_2 + \dots + B_{m1}u_m &= P_{11}w_1 + P_{21}w_2 + \dots + P_{q1}w_q \\ B_{12}u_1 + B_{22}u_2 + \dots + B_{m2}u_m &= P_{12}w_1 + P_{22}w_2 + \dots + P_{q2}w_q \\ &\vdots \\ B_{1n}u_1 + B_{2n}u_2 + \dots + B_{mn}u_m &= P_{1n}w_1 + P_{2n}w_2 + \dots + P_{qn}w_q \end{aligned} \quad (10)$$

$$u_1 \geq 0, \dots, u_m \geq 0, w_1 \geq 0, \dots, w_q \geq 0 \quad (11)$$

$$w_1 + w_2 + \dots + w_q = 1. \quad (12)$$

In this case, the duality parameter vector is $(c_1, \dots, c_q) = (1, \dots, 1)$ and constraint (12) can be written as

$$c_1 w_1 + c_2 w_2 + \dots + c_q w_q = 1.$$

1.6.2 General case of VLP for minimization

Starting with a minimization problem, the dual problem is the following vector linear program with ordering cone $K := \{y \in \mathbb{R}^q \mid y_1 = 0, \dots, y_{q-1} = 0, y_q \geq 0\}$:

$$K\text{-maximize} \begin{pmatrix} \frac{c_q}{|c_q|} w_1 \\ \frac{c_q}{|c_q|} w_2 \\ \dots \\ \frac{c_q}{|c_q|} w_{q-1} \\ d(u_1, \dots, u_m, v_1, \dots, v_n) \end{pmatrix} \quad (13)$$

subject to the constraints

$$\begin{aligned} B_{11}u_1 + B_{21}u_2 + \dots + B_{m1}u_m &= P_{11}w_1 + P_{21}w_2 + \dots + P_{q1}w_q + v_1 \\ B_{12}u_1 + B_{22}u_2 + \dots + B_{m2}u_m &= P_{12}w_1 + P_{22}w_2 + \dots + P_{q2}w_q + v_2 \\ &\dots \end{aligned} \quad (14)$$

$$\begin{aligned} B_{1n}u_1 + B_{2n}u_2 + \dots + B_{mn}u_m &= P_{1n}w_1 + P_{2n}w_2 + \dots + P_{qn}w_q + v_n \\ Y_{11}w_1 + Y_{12}w_2 + \dots + Y_{1q}w_q &\geq 0 \\ Y_{21}w_1 + Y_{22}w_2 + \dots + Y_{2q}w_q &\geq 0 \\ &\dots \\ Y_{o1}w_1 + Y_{o2}w_2 + \dots + Y_{oq}w_q &\geq 0 \end{aligned} \quad (15)$$

$$c_1 w_1 + c_2 w_2 + \dots + c_q w_q = 1, \quad (16)$$

where, for $\alpha^+ := \max(0, \alpha)$ and $\alpha^- := \max(-\alpha, 0)$, we set

$$\begin{aligned} d(u_1, \dots, u_m, v_1, \dots, v_n) &= a_1 u_1^+ + \dots + a_m u_m^+ - b_1 u_1^- - \dots - b_m u_m^- \\ &+ l_1 v_1^- + \dots + l_n v_n^- - s_1 v_1^+ - \dots - s_n v_n^+. \end{aligned} \quad (17)$$

1.6.3 General case of VLP for maximization

Starting with a maximization problem, the dual problem is the following vector linear program with ordering cone $K := \{y \in \mathbb{R}^q \mid y_1 = 0, \dots, y_{q-1} = 0, y_q \geq 0\}$:

$$K\text{-minimize} \begin{pmatrix} \frac{c_q}{|c_q|} w_1 \\ \frac{c_q}{|c_q|} w_2 \\ \dots \\ \frac{c_q}{|c_q|} w_{q-1} \\ \bar{d}(u_1, \dots, u_m, v_1, \dots, v_n) \end{pmatrix} \quad (18)$$

subject to the constraints (14), (15), (16), where

$$\begin{aligned} \bar{d}(u_1, \dots, u_m, v_1, \dots, v_n) &= b_1 u_1^+ + \dots + b_m u_m^+ - a_1 u_1^- - \dots - a_m u_m^- \\ &+ s_1 v_1^+ + \dots + s_n v_n^+ - l_1 v_1^- - \dots - l_n v_n^-. \end{aligned} \quad (19)$$

1.7 Dual solution

BENSOLVE computes all vertices and extreme directions of the lower or upper image \mathcal{D} of the dual problem. The lower image \mathcal{D} for the dual problem in Section 1.6.2 is the set of all vectors $y^* = (y_1^*, \dots, y_q^*)$ such that

$$\begin{aligned} y_1^* &= \frac{c_q}{|c_q|} w_1 \\ &\dots \\ y_{q-1}^* &= \frac{c_q}{|c_q|} w_{q-1} \\ y_q^* &\leq d(u_1, \dots, u_m, v_1, \dots, v_n) \end{aligned} \tag{20}$$

for some $(u, w, v) = (u_1, \dots, u_m, w_1, \dots, w_q, v_1, \dots, v_n)$ satisfying the constraints (14), (15) and (16). A *dual solution* is understood to be a finite set of points and directions $(u, w, v) = (u_1, \dots, u_m, w_1, \dots, w_q, v_1, \dots, v_n)$ of the feasible set (i.e. the set of vectors satisfying the constraints (14), (15), (16)) such that the corresponding images according to

$$\begin{aligned} y_1^* &= \frac{c_q}{|c_q|} w_1 \\ &\dots \\ y_{q-1}^* &= \frac{c_q}{|c_q|} w_{q-1} \\ y_q^* &= d(u_1, \dots, u_m, v_1, \dots, v_n) \end{aligned} \tag{21}$$

are, respectively, the vertices and extreme direction of \mathcal{D} .

The upper image \mathcal{D} of the dual problem in Section 1.6.3 is defined likewise by replacing \leq and d by \geq and \bar{d} in (20).

1.8 Duality

The vertices of the lower image (or upper image) \mathcal{D} of the dual problem can be used to provide an inequality representation of the upper image (or lower image) \mathcal{P} of the given vector linear program. If $y^* = (y_1^*, \dots, y_q^*)$ is a vertex of the lower image \mathcal{D} of the dual problem in Section 1.6.2, the corresponding inequality is

$$c_q(y_1^*y_1 + \dots + y_{q-1}^*y_{q-1}) + \left(\frac{|c_q|}{c_q} - c_1y_1^* - \dots - c_{q-1}y_{q-1}^* \right) y_q \geq |c_q|y_q^*, \tag{22}$$

where $c = (c_1, \dots, c_q)$ is the duality parameter vector, see Section 1.5. Every vector $y = (y_1, \dots, y_q)$ of the upper image \mathcal{P} satisfies (22). Moreover, (22) holds with equality on some facet (that is, a $(q-1)$ -dimensional face) of \mathcal{P} . In particular, this means that the inequality representation obtained in this way does not contain any redundant inequality.

For the upper image of the dual problem in Section 1.6.3, the corresponding inequality is

$$c_q(y_1^*y_1 + \dots + y_{q-1}^*y_{q-1}) + \left(\frac{|c_q|}{c_q} - c_1y_1^* - \dots - c_{q-1}y_{q-1}^* \right) y_q \leq |c_q|y_q^*, \tag{23}$$

For further details on duality the reader is referred to [7, 8].

2 License and citation policy

BENSOLVE is free software: you can redistribute it and/or modify it under the terms of the GNU General Public License, see Appendix. This program is distributed in the hope that it will be useful, but without any warranty; without even the implied warranty of merchantability or fitness for a particular purpose. See Appendix for more details.

If you are using this version of BENSOLVE for scientific papers, please cite it as:

- [A] Löhne, A, Weïbing, B.: BENSOLVE - VLP solver, version 2.1.x, www.bensolve.org
- [B] Löhne, A, Weïbing, B.: The vector linear program solver Bensolve – notes on theoretical background, *European J. Oper. Res.*, 260(3):807–813, 2017

3 Installation

BENSOLVE is provided as source code. It is required to compile (build) an executable file using a C compiler. We recommend to use the GNU Compiler Collection (GCC).

3.1 Linux and Mac OS

Before you can install (build) BENSOLVE on your computer, you need to install (build) the GLPK library. Unless you are using a package manager, download the latest version of GLPK at

<http://ftp.gnu.org/gnu/glpk/>

and proceed with the installation instructions (see e.g. Appendix A in the GLPK manual for details). The GLPK library is a single file `libglpk.a` together with a header file `glpk.h`. Their default locations are `/usr/local/lib` and `/usr/local/include`, respectively. In order to prepare BENSOLVE for installation

1. copy the BENSOLVE distribution file `bensolve-X.Y.Z.tgz` to a working directory,
2. unarchive the distribution file with the command:

```
tar -xz < bensolve-X.Y.Z.tgz
```

A subdirectory `bensolve-X.Y.Z` will be created. Enter this directory and type `ls`, which should produce the following output:

```
bslv_algs.c      bslv_lp.c       bslv_poly.c     doc
bslv_algs.h      bslv_lp.h       bslv_poly.h     ex
bslv_lists.c     bslv_main.c    bslv_vlp.c
bslv_lists.h     bslv_main.h    bslv_vlp.h
```

If the GLPK library is installed at the default location, type

```
gcc -std=c99 -O3 -o bensolve *.c -lglpk -lm
```

(note: `-O3` like “Optimize3”) to compile (build) the program. Alternatively, you can copy the files `glpk.h` and `libglpk.a` into your `BENSOLVE` directory `bensolve-X.Y.Z` and type:

```
gcc -std=c99 -O3 -o bensolve *.c libglpk.a
```

To test the installation type:

```
./bensolve ex/ex01.vlp
```

3.2 Windows

Install CYGWIN, see <https://www.cygwin.com>, which provides functionality similar to a Linux distribution on Windows. In addition to the default CYGWIN packages, install:

```
gcc-core
```

```
libglpk-devel
```

by using the CYGWIN setup program. After CYGWIN has been installed, launch the CYGWIN terminal and type

```
pwd
```

to display your current working directory. This produces an output similar to

```
/home/my_user_name
```

Copy the BENSOLVE distribution file `bensolve-X.Y.Z.tgz` to the Windows directory (similar to)

```
C:\cygwin\home\my_user_name
```

Unarchive the distribution file by typing the following into the CYGWIN terminal:

```
tar -xz < bensolve-X.Y.Z.tgz
```

This creates a subdirectory `bensolve-X.Y.Z`. Enter this subdirectory by typing

```
cd bensolve-X.Y.Z
```

Type

```
ls
```

which should produce the following output:

```

bslv_algs.c      bslv_lp.c       bslv_poly.c    doc
bslv_algs.h      bslv_lp.h       bslv_poly.h    ex
bslv_lists.c     bslv_main.c    bslv_vlp.c
bslv_lists.h     bslv_main.h    bslv_vlp.h

```

Type

```
gcc -std=c99 -O3 -o bensolve *.c -lglpk -lm
```

(note: `-O3` like “Optimize3”) to compile (build) the program. To test the installation type:

```
./bensolve.exe ex/ex01.vlp
```

4 Using the program

4.1 Running test examples

The subdirectory `ex` contains several test examples. Larger problem instances can be found at <http://moplib.uni-jena.de>. Type

```
./bensolve --help
```

(on Windows, replace `bensolve` by `bensolve.exe`), to display all options. Here are some examples: The command

```
./bensolve ex/ex05.vlp -b
```

solves problem `ex05.vlp` in the subdirectory `ex` assuming that the problem is bounded (which is the case for this example). This means that phases 0 and 1 of the algorithm are skipped. The command

```
./bensolve ex/ex05.vlp -m 0
```

solves problem `ex/ex05.vlp` suppressing any output. Increase the number to get more output. The command

```
./bensolve ex/ex05.vlp -s
```

solves problem `ex/ex05.vlp` and writes the primal and dual solution, see Sections 1.4 and 1.7, to the files `ex/ex05_pre_img_p.sol` and `ex/ex05_pre_img_d.sol`. To run the larger example number 7, type:

```
./bensolve ex/ex07.vlp -l primal_simplex -e 0.05 -p
```

Here BENSOLVE uses the primal simplex method (the dual is default) and computes an approximate solution with $\varepsilon = 0.05$. Moreover, graphics files to visualize the upper and lower image are generated, see Section 4.4.5.

4.2 Problem generation from OCTAVE (or MATLAB)

The VLP input format is explained in the next section. One can easily generate problem files in VLP format using OCTAVE. To do this, enter the subdirectory `ex`. Modify and run with OCTAVE the files named like `exampleXX.m`.

4.3 VLP input format

The VLP input format is an extension of the GLPK LP format to the case of multiple objective linear programs (MOLP) and vector linear programs (VLP). It is a DIMACS-like format (see <http://dimacs.rutgers.edu/Challenges/>). A problem instance in VLP format is stored as a plain ASCII text file containing lines of several types. Lines are terminated by the end-of-line character. Each line begins with a one-character designator to identify the line type. Valid line designators are:

c	comment line
p	program line
i	row descriptor line
j	column descriptor line
a	constraint coefficient descriptor line
o	objective coefficient descriptor line
k	cone generator coefficient descriptor line (for VLP) or duality parameter descriptor line (for VLP)
e	end of file

The line designator is followed by several fields which are separated by at least one blank space.

A **comment line** begins with the lower-case character c:

```
c This is a comment line
```

The first line which is not a comment line must be the **program line**. In case of a MOLP, it begins with the lower-case character p followed by 7 fields:

```
p CLASS DIR ROWS COLS NZ OBJ OBJNZ
```

The **CLASS** field defines the problem class and must contain the keyword `vlp`. The **DIR** field contains the optimization direction and must contain either `min` (for minimization) or `max` (for maximization). The **ROWS** and **COLS** fields represent the number of rows and columns, that is, the integers m and n in (2), respectively. The **NZ** field contains the number of non-zero constraint coefficients, that is, the number of non-zero entries of the coefficient matrix B in (2). The **OBJ** field contains the number of objectives, that is, the integer q in (1). The field **OBJNZ** contains the number of non-zero objective coefficients, that is, the number of non-zero entries of the $q \times n$ -matrix P in (1).

A **row descriptor line** specifies the type of a constraint in (2). Such a line has one of the following formats:

```

i ROW f
i ROW l VAL1
i ROW u VAL1
i ROW d VAL1 VAL2
i ROW s VAL1

```

`ROW` contains the index of the constraint, which is an integer between 1 and m . The next character (`f`, `l`, `u`, `d`, `s`) specifies the type of the constraint. `VAL1` and `VAL2` contain the floating-point constraint values. For a constraint in (2), say the k -th constraint

$$a_k \leq B_{k1}x_1 + B_{k2}x_2 + \cdots + B_{kn}x_n \leq b_k,$$

the following types are possible:

<code>f</code>	no bound	$a_k = -\infty$	$b_k = +\infty$
<code>l</code>	lower bound	$a_k = \text{VAL1}$	$b_k = +\infty$
<code>u</code>	upper bound	$a_k = -\infty$	$b_k = \text{VAL1}$
<code>d</code>	double-sided bound	$a_k = \text{VAL1}$	$b_k = \text{VAL2}$
<code>s</code>	equation	$a_k = \text{VAL1}$	$b_k = \text{VAL1}$

A line of the form

```
i ROW f
```

is a default row descriptor line and can be omitted.

A **column descriptor line** specifies the type of a variable in (3). Such a line has one of the following formats:

```

j COL f
j COL l VAL1
j COL u VAL1
j COL d VAL1 VAL2
j COL s VAL1

```

`COL` contains the index of the variable, which is an integer between 1 and n . The next character (`f`, `l`, `u`, `d`, `s`) specifies the type of the variable. `VAL1` and `VAL2` contain the floating-point values of the variable bounds. For the variable x_k in (3), we have $l_k \leq x_k \leq u_k$. The following variable types are possible:

<code>f</code>	free variable	$l_k = -\infty$	$s_k = +\infty$
<code>l</code>	variable with lower bound	$l_k = \text{VAL1}$	$s_k = +\infty$
<code>u</code>	variable with upper bound	$l_k = -\infty$	$s_k = \text{VAL1}$
<code>d</code>	double-bounded variable	$l_k = \text{VAL1}$	$s_k = \text{VAL2}$
<code>s</code>	fixed variable	$l_k = \text{VAL1}$	$s_k = \text{VAL1}$

The default column descriptor line

```
j COL s 0
```

can be omitted.

A **constraint coefficient descriptor line** has the format:

```
a ROW COL VAL
```

For every non-zero constraint coefficient B_{ij} in (2), exactly one coefficient descriptor line must be specified. **ROW** and **COL** contain the row number i and the column number j of the coefficient B_{ij} , respectively. **VAL** contains the floating-point coefficient B_{ij} . A coefficient descriptor line where **VAL** equals zero is allowed. The number of constraint coefficient descriptor lines must be exactly the same as specified in the field **NZ** of the problem line.

An **objective coefficient descriptor line** has the format:

```
o ROW COL VAL
```

For every non-zero objective coefficient P_{ij} in (1), exactly one coefficient descriptor line must be specified. **ROW** and **COL** contain the row and column numbers i and j of P_{ij} and **VAL** contains the floating-point value of P_{ij} . A coefficient descriptor line where **VAL** equals zero is allowed. The number of constraint coefficient descriptor lines must be exactly the same as specified in the field **OBJNZ** of the problem line.

In case of a vector linear program (beyond the special case of MOLP), the program line has 10 fields (i.e. three additional fields):

```
p CLASS DIR ROWS COLS NZ OBJ OBJNZ CTYPE GEN GENNZ
```

The **CTYPE** field specifies the type of cone representation, compare Section 1.2. Valid entries are **cone** and **dualcone**. If **CTYPE** is specified, the fields **GEN** and **GENNZ** must be specified, too. The **GEN** field is the number of generating vectors, i.e., either o in (5) or p in (6). The **GENNZ** field contains the number of non-zero cone coefficients, that is, the number of non-zero entries of either the matrix Y or the matrix Z , dependent of which one is given.

A **cone coefficient descriptor line** has the format:

```
k ROW COL VAL
```

For every non-zero cone coefficient Y_{ij} in (5), respectively, Z_{ij} in (6), exactly one cone descriptor line must be specified. If **CTYPE=cone**, **ROW** and **COL** contain the row and column numbers $1 \leq i \leq q$ and $1 \leq j \leq o$ and **VAL** contains the floating-point value of Y_{ij} . If **CTYPE=dualcone**, **ROW** and **COL** contain the row and column numbers $1 \leq i \leq q$ and $1 \leq j \leq p$ and **VAL** contains the floating-point value of Z_{ij} . A cone descriptor line where **VAL** equals zero is allowed. The number of cone coefficient descriptor lines must be exactly the same as specified in the field **GENNZ** of the problem line.

A **duality parameter descriptor line** has the format:

```
k ROW 0 VAL
```

Do not count duality parameter descriptor lines in the field **GENNZ** of the problem line.

4.4 Output format

BENSOLVE writes computational results to different files. Part of the results is displayed on the screen. To explain the format, we assume that we solve a minimization problem, which is stored in `ex01.vlp`.

4.4.1 Upper and lower images

The vertices and extreme directions of the upper image \mathcal{P} and lower image \mathcal{D} (see Sections 1.3 and 1.7) are written line-wise to the files `ex01_img_p.sol` and `ex01_img_d.sol`.

The first column consists of either 1 or 0 indicating that a line represents a vertex or a direction. For instance, the line

1 1.5 2.5

means that $y = (1.5, 2.5)$ is a vertex of \mathcal{P} , whereas

0 1 0

means that $y = (1, 0)$ is an extreme direction of \mathcal{P} .

An adjacency list for \mathcal{P} is written to `ex01_adj_p.sol`. Line i in this file corresponds to vertex (or extreme direction) i of \mathcal{P} stored in line i of `ex01_img_p.sol`. Line i of the adjacency list consists of the line numbers of vertices or extreme directions adjacent to vertex (or extreme direction) i . Line numbering starts with 0.

A “facet-vertex” incidence list of \mathcal{P} is written to `ex01_inc_p.sol`. Line i in this file refers to vertex (extreme direction) i of \mathcal{D} , which refers to a (ideal) facet of \mathcal{P} , compare Section 1.8. Line i contains the line numbers of vertices and extreme directions of \mathcal{P} , which are incident to this facet.

Likewise `ex01_adj_d.sol` and `ex01_inc_d.sol` contain an adjacency list and a “facet-vertex” incidence list of \mathcal{D} .

4.4.2 Duality parameter vector

The duality parameter vector c , which is either computed or scaled by BENSOLVE, is stored in `ex01_c.sol`.

4.4.3 Primal and dual solution

Solutions are not stored by default. Use option `-s` to store primal and dual solutions. The command

```
./bensolve ex/ex01.vlp -s
```

generates the files `ex01_pre_img_p.sol` and `ex01_pre_img_d.sol`, where the first file contains a primal solution and the second one a dual solution. The finitely many vectors of a solution are stored line-wise. They appear in the same order than the corresponding

image points, see Section 4.4.1. There is no leading 0 or 1. To decide whether a vector is a point or a direction, the leading 0 or 1 of the corresponding image point can be used.

For dual solutions, only the u and w components (but not the v components) are stored, that is, every line has the format $u_1, \dots, u_m, w_1, \dots, w_q$.

Use option `-f short`, to obtain a short and easier to read format.

4.4.4 Ordering cone

The extreme directions (together with the one and only vertex, the origin) of the ordering cone and the dual of the ordering cone are written to the files `ex01_p.cone` and `ex01_d.cone`. As for the upper and lower images, adjacency and incidence lists are written to files. Input data of the ordering cone may contain redundant vectors, but the results does not.

4.4.5 Graphical output

BENSOLVE supports the generation of graphics files for problems with 3 objectives. Using option `-p`, for instance,

```
./bensolve ex/ex10.vlp -p
```

BENSOLVE produces two graphics files in OFF format, which contain a visualization of the upper (lower) image of the primal problem and the lower (upper) image of the dual problem. In this example, the files `ex10_p.off` and `ex10_d.off` are generated. They are located in the subdirectory `ex`.

OFF files can be displayed, for instance, with GEOMVIEW, see <http://geomview.org>, or JAVAVIEW, see <http://javaview.de>.

Option `-p` also generates the files `ex10_p.inst` and `ex10_d.inst` to plot a scaled version of the upper and lower images with GEOMVIEW. To scale with JAVAVIEW, load the `.off` files and use the menu “Inspector → Camara → Box Ratio”.

The above example produces the *bensolvehedron*, which is displayed on the front cover of this manual.

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If you develop a new program, and you want it to be of the greatest possible use to the public, the best way to achieve this is to make it free software which everyone can redistribute and change under these terms.

To do so, attach the following notices to the program. It is safest to attach them to the start of each source file to most effectively state the exclusion of warranty; and each file should have at least the “copyright” line and a pointer to where the full notice is found.

<one line to give the program's name and a brief idea of what it does.>

Copyright (C) <textyear> <name of author>

This program is free software: you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation, either version 3 of the License, or (at your option) any later version.

This program is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public License for more details.

You should have received a copy of the GNU General Public License along with this program. If not, see <<http://www.gnu.org/licenses/>>.

Also add information on how to contact you by electronic and paper mail.

If the program does terminal interaction, make it output a short notice like this when it starts in an interactive mode:

```
<program> Copyright (C) <year> <name of author>
```

```
This program comes with ABSOLUTELY NO WARRANTY; for details type 'show w'.
This is free software, and you are welcome to redistribute it
under certain conditions; type 'show c' for details.
```

The hypothetical commands `show w` and `show c` should show the appropriate parts of the General Public License. Of course, your program's commands might be different; for a GUI interface, you would use an “about box”.

You should also get your employer (if you work as a programmer) or school, if any, to sign a “copyright disclaimer” for the program, if necessary. For more information on this, and how to apply and follow the GNU GPL, see <http://www.gnu.org/licenses/>.

The GNU General Public License does not permit incorporating your program into proprietary programs. If your program is a subroutine library, you may consider it more useful to permit linking proprietary applications with the library. If this is what you want to do, use the GNU Lesser General Public License instead of this License. But first, please read <http://www.gnu.org/philosophy/why-not-lgpl.html>.