

Can greedy-like heuristics be useful for solving the Weighted Orthogonal Art Gallery Problem under regular grid discretization?

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Abstract

In this paper we deal with the Weighted Orthogonal Art Gallery Problem under the regular grid discretization. We propose a novel greedy approach which is based on balancing the trade off between the total sum of guards' costs and the total number of not yet covered points from the discretization. This new approach and an existing greedy algorithm are further hybridized with the Integer Linear programming (ILP), originally formulated for the well known Minimum Set Cover problem. Experimental results show that the proposed greedy methods can achieve most of the optimal solution for a class of large area polygons, while for small area polygons, they achieve solutions of reasonable quality within lower runtime than the exact algorithms.

1. Introduction

For a given polygon P , the *Art Gallery Problem* (AGP) asks for a set of points G of minimal cardinality, such that for each point $y \in P$ there is $x \in G$ such that $xy \subset P$. We say that the point y is covered by the point x , or y is visible from x . Set G is called *guard set* of P and the points from G as *guards*. In the *Orthogonal Art Gallery Problem* (OAGP) we suppose that edges of the polygon are only horizontal and vertical w.r.t. the axes, i.e. the angles allowed between adjacent edges are 90° or 270° . The original AGP was initially stated by Victor Klee in 1973. [12]. The problem is motivated from installing the cameras inside a building (or gallery) such that the whole area of the building is covered. Orthogonality constraint naturally comes out from the orthogonality of the walls in buildings. Kahn et al. [7] formulated and proofed that $\lfloor \frac{n}{3} \rfloor$ guards are sufficient to cover an orthogonal polygon with n vertices. In this study, we are interested in the variant of the OAGP which allows only that guards are positioned at the vertices of polygon P . This restricted problem is known to be \mathcal{NP} -hard [15, 8]. When it comes to the real situations (like installing the cameras in a building), it is justified to assume that the prices of cameras are not equal and may depends on several factors, like the quality of a camera (respecting its range of spectrum of view) or installation price at some specific parts of the building (like corners or tight places). In the *Weighted Orthogonal Art Gallery Problem* (WOAGP) the task is to place guards on some vertices of the orthogonal polygon which cover all points from P , such that the total sum of prices assigned to the chosen vertices is minimal. In this paper we

consider WOAGP problem under the regular grid discretization of P , described in Subsection 1.2.

It is well known that AGP can be reduced to *the Minimum Set Cover Problem* (MSCP) by a discretization of the set of all points of the polygon P . The appropriate discretization should be performed in such a way that if each point from the discretized set $D(P)$ is covered, then the whole polygon P is covered. After the discretization is made, for each vertex of the polygon P , a set of visible points from $D(P)$ is determined. In that way, the problem of determining the minimum number of guards covering the entire polygon is reduced to determining the minimum number of subsets of points, such that each point from $D(P)$ is included in at least one of the chosen subsets, which is MSCP. Analogously, WOAGP can be reduced to the *Weighted Minimum Set Cover Problem* (WMSCP).

Concerning the exact and heuristic techniques to solve OAGP, Couto et al. [3] presented an exact and efficient exact algorithm for the OAGP based on preprocessing and refinement phases of the the discretized instance. In [4] an approximate solution of the minimum vertex guard problem, which can be computed in $O(n^4)$ time and the size of the solution is at most $O(\log n)$ times the optimal. After that, on these constructed sets Johnson’s approximation algorithm [6] for the MSCP is applied. An anytime algorithm to compute successively better approximations of the optimum to Minimum Vertex Guard is proposed in [16]. A major idea of this approach is exploring dominance of visibility regions to first detect pieces that are more difficult to guard. The same problem is solved in [17] by applying successive approximations from [16]. Tozoni et al. [18, 19] presented an exact *Integer Linear Programming* (ILP)-based algorithm, which iteratively generates upper and lower bounds through the resolution of discretized space of the AGP. Although many variants AGP are present in literature, WOAGP has not been so intensively studied, which motivated us to consider this problem. A comprehensive analysis of various greedy-like heuristics for the WMSCP was presented in [20]. More detailed overview of the extensive literature regarding SCP and AGP is out of the scope of this paper and for further reading we suggest review papers [1, 13, 5].

1.1. Main contributions

The main contributions of this paper are:

- We developed a novel greedy approach which is based on balancing the trade off between the total sum of guards’ costs and the total number of not yet covered points from the discretization.
- The greedy algorithm from [2] and the novel greedy algorithm are hybridized with the ILP.
- We considered different types of weights for our benchmarks, based on an approximation of the costs in real situations.
- In a comprehensive computational experiment, we tested, analysed and checked the efficiency of the developed algorithms. The methods are then compared to the exact approaches ILP and *Constraint Programming* (CP) w.r.t. the quality of obtained heuristic solutions as well as runtimes.

Algorithm 1 Discretization $D(P)$ of polygon P

- 1: **Input:** The set of vertices V of polygon P
 - 2: **Output:** The discretization $D(P)$ of polygon P
 - 3: $BB \leftarrow \text{bounding_box}(P)$;
 - 4: $\Delta_x, \Delta_y \leftarrow \text{resolution}(P)$;
 - 5: $D(BB) \leftarrow \text{regular_grid}(BB, \Delta_x, \Delta_y)$;
 - 6: $D(P) \leftarrow D(BB) \cap P$;
 - 7: $D(P) \leftarrow D(P) \cup V$;
-

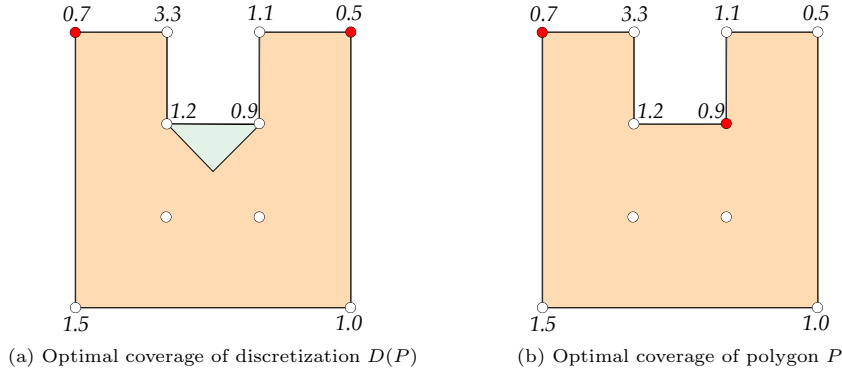


Figure 1: Optimal coverage of discretization $D(P)$ does not imply a coverage of polygon P .

1.2. The Regular Grid Discretization of Polygon

The discretization $D(P)$ of the polygon P is built as follows. Consider the regular grid with resolution $\Delta_x \times \Delta_y$ starting at the lower left corner of the bounding box of polygon P which height and weight are defined as follow:

$$\Delta_x = \min\{|u_x - v_x| \mid u_x \neq v_x\} \text{ and } \Delta_y = \min\{|u_y - v_y| \mid u_y \neq v_y\}, \quad (1)$$

where $(u_x, u_y), (v_x, v_y)$ are adjacent vertices of polygon P .

All intersection points of this regular grid with polygon P and all vertices of P are added into the discrete set $D(P)$. The pseudocode which presents the building steps of the discretizations $D(P)$ is given in Algorithm 1.

Fixme: **TODO:** Milan: Treba prevesti na engleski

Treba primjetiti da iz pokrivenosti diskretizacije $D(P)$ optimalnim skupom stražara iz Z ne mora da slijedi pokrivenost poligona P istim skupom stražara Z . Na slici 1 pod a) prikazano je optimalno pokrivanje crvenim tačkama diskretizacije $D(P)$. Crvene i bjele tačke na poligonu su tačke iz diskretnog skupa $D(P)$. Ovo optimalno rješenje ne pokriva poligon sa slike. Poligon na slici 1 pod b) je optimalno pokriven, ali ovo pokrivanje nije optimalno pokrivanje njegove diskretizacije $D(P)$.

Izabrana rezolucija $\Delta_x \times \Delta_y$ predstavlja kompromis između brzine i tačnosti. Odnosno, smanjenjem rezolucije povećava se tačnost rješenja, ali se sporije pronalazi diskretizacija. Povećanjem rezolucije smanjuje se tačnost rješenja, ali i vrijeme pronalazanja diskretizacije.

Couto et al. [3] koristili su pomenutu diskretizaciju za problem ortogonalne galerije. U njihovom radu broj pologona nad kojim je primjenjena ova

diskretizacija je 10282. Za 59.4% poligona je vazilo da iz pokrivenosti diskretnog skupa $D(P)$ slidi pokrivenost poligona P za optimalno pronadjen skup strazara koji pokrivaju $D(P)$ **FiXme: Sta je sa ostalim poligonima koji nisu bili pokriveni?** Za ostale poligone je bilo potrebno dodati prosjecno $0.003 \cdot n + 1.1$ dodatnih tacaka u diskretizaciju kako bi iz pokrivenosti diskretizacije slijedila pokrivenost poligona. Dakle nad testiranim uzorkom poligona, ili je citav poligon pokriven ili je mali broj regija nepokriveno optimalnim skupom strazara Z za $D(P)$. U sekciji 4 ce biti prikazane vrijednosti površina ovih nepokrivenih regija. **FiXme: Koliki je procenat pokrivenosti bio u tom slucaju?** **FiXme: Do ove info ne mogu da dodjem** **FiXme: Kako se ovo odnosi na druge diskretizacije, bilo je puno bolje ili?** **FiXme: Treba pregledati literaturu.** Pitanje je koliko je ljudi pokusavalo ovom metodologijom dopunjavati diskretizaciju kako bi dosli do optimalnog rjesenja - ja bih izbjegao o ovom pricati/pisati.

2. Exact methods

In this section we present the exact ILP and *Constraint Programming* (CP) models for solving WMSCP under regular grid discretization, which are used in the rest of the paper.

2.1. Integer linear programming model

Let us suppose we are given a polygon P and the discretization $D(P)$ of P . The task we consider is covering all points from $D(P)$ by some vertices $V = \{v_1, \dots, v_n\}$ of P such that the sum of their weights is minimized. The problem is related to the known MWSC problem as follows. Family \mathcal{F} of nonempty sets consists of the sets $S_i \in \mathcal{F}$ which include points $p \in D(P)$ that are visible from guard $v_i \in V$, i.e., $pv_i \subset P$. Note that set S_i includes points $p \in D(P)$ that might also be included by some other guard $v_j \in V$, $i \neq j$. For each set S_i , the cost $c(S_i) = w_i$ is assigned. In this way, our starting task is equivalent of finding a minimum cardinalty covering $\mathcal{C} \subseteq \{S_1, \dots, S_n\}$ of the set of points $D(P)$, that is

$$\bigcup_{C \in \mathcal{C}} C = D(P),$$

such that $\sum_{S_i \in \mathcal{C}} c(S_i)$ is minimized. The ILP model for the WSCP is already known from the literature, where here adapted to WOAGP as follows:

$$\sum_{i=1}^n w_i x_i \longrightarrow \min \quad (2)$$

$$\text{s.t.} \quad (3)$$

$$\sum_{j \in V} a_{ij} x_j \geq 1 \quad (\forall p_i \in D(P)) \quad (4)$$

$$x_j \in \{0, 1\}, j \in V, \quad (5)$$

$$\text{where } a_{ij} = \begin{cases} 1, & p_i \in V(j), \\ 0, & \text{otherwise} \end{cases} \quad \text{and } x_i = \begin{cases} 1, & \text{if the point } p_i \in \mathcal{C}, \\ 0, & \text{otherwise,} \end{cases}$$

where $V(j)$ is the set of all points from $D(P)$ that are visible from j -th vertex of polygon P . Set $Z = \{j \in V \mid x_i = 1\}$ represents a solution of the

Algorithm 2 Greedy Heuristic

```
1: Input: an instance of a problem
2: Output: A (feasible) non-expandable solution (or reporting that no feasible
   solution)
3:  $s^{ps} \leftarrow ()$  // partial solution set to empty solution
4: while  $\text{Extend}(s^{ps}) \neq \emptyset$  do
5:   Select solution component  $e \in \text{Extend}(s^{ps})$  // w.r.t. some criterion  $g$ 
6:   Extend  $s^{ps}$  by  $e$ 
7: end while
```

problem w.r.t. discretization $D(P)$ of polygon P . Constraint (4) enforces that any point $p_i \in D(P)$ will be visible from at least one guard from Z .

In order to solve this model, we apply a general purpose solver CPLEX [10].

2.2. Constraint programming model

An equivalent CP model was implemented and tested by IBM ILOG CP Optimizer [9]. In this case, Constraint (4) is transformed into

$$\bigvee_{j \in V} (a_{ij} \wedge x_j) = 1, \quad (6)$$

whereas the other constraints and the objective function are the same like in the above ILP model. Note that CP approach works in a branch-and-bound manner employing a constraint propagation and variable domain filtering [14].

3. Algorithmic Approaches for WOAGP

3.1. Greedy approaches for solving WOAGP

Greedy algorithms produce a solution of reasonable quality within a short interval of time and are, in essence, easy to implement. Efficiency of such heuristic is related to a greedy criterion utilized to expand current (non-complete, i.e., partial) solution to complete one. Among all candidates (solution components for expansion, that is not-yet-considered guards) to extend current partial solution, we choose one with the smallest greedy value and add it to the current solution until it becomes complete (i.e., it covers all points from $D(P)$).

A general pseudocode of Greedy heuristics is given in Algorithm 2.

3.1.1. An existing greedy method

Concerning the greedy heuristic for the literature for solving WMSC problem [2, 11], one of the most efficient greedy heuristic was based on the following criterion:

$$g_1(s^{ps}, v_i) = \frac{w_i}{f(s^{ps} \cup \{v_i\}) - f(s^{ps})}, \quad (7)$$

where

$$f(s^{ps}) = \left| \bigcup_{i \in s^{ps}} S_i \right|. \quad (8)$$

This heuristic also ensures an approximation with $O(\log(n))$ approximation factor.

3.1.2. A Novel Greedy Heuristic

In this subsection we present a novel greedy criterion. First, we introduce a term “incorrect point”. For a point from $D(P)$ we say that it is *incorrect* if it is not covered by any guard from the current partial solution s^{ps} . Let us denote by $incorrect_{total}(s^{ps})$ the total number of incorrect points from discretization $D(P)$ w.r.t. s^{ps} . Note that $incorrect_{total}(s^{ps}) = |D(P)| - f(s^{ps})$, where $f(s^{ps})$ is defined in (8) and $|D(P)|$ is the cardinality of discretization set. Let w_{total} be the total sum of all weights among all vertices. The greedy function w.r.t. partial solution s^{ps} and guard v is defined as follows.

$$g_2(s^{ps}, v) = \frac{\sum_{i \in s^{ps} \cup \{v\}} w_i}{w_{total}} + \frac{incorrect_{total}(s^{ps} \cup v)}{|D(P)|}. \quad (9)$$

In the Equation (9) both terms are normalized on order to achieve better balance between the total sum of guards’ costs and the total number of not yet covered points from the discretization. The algorithm does not ultimately prefer any of criteria for choosing next vertex. Although it cannot be a rule, it is justified to suppose that the second term has a more influence on the choice of the next vertex in earlier phases of the algorithm execution. At the beginning, more points are uncovered and the algorithm chooses such vertices which cover larger area of the polygon. At the end of algorithm, it could be expected that the first term plays more significant role, since the total number of uncovered points is small.

Ties occurred in the search are broken by using price-per-unit heuristic which is stated as follows. For each not yet considered vertex v_i , we denote the region of polygon P that is visible from v_i by $Surf(v_i)$. As the next candidate to extend s^{ps} , we choose such a guard, with the smallest ratio between the price and the visible surface area. More precisely, this criterion is given as:

$$g'(s^{ps}, v_i) = \frac{w_i}{|Surf(v_i)|}, \quad (10)$$

In our experimental studies, we found out that this heuristic does not perform well on its own, but it represents a reasonable tie-breaking mechanism which boosts quality of the aforementioned greedy heuristics.

3.1.3. Partial calculation of greedy functions

In order to enable fast calculation of greedy functions described in previous subsections, we noticed that it could be useful to allow fast updating of the number of uncovered vertices. Therefore, we introduced two useful structures and two auxiliary functions:

- structure `map<Point, int> numberOfGuards` – for each point from $D(P)$, we keep the number of guards in solution which cover that point;
- structure `set<Point> CoveredPoints` – set of points from $D(P)$ that are covered by the partial solution;
- function `updateCoveredPointsAdd (vertex v)` – does the update of structures `CoveredPoints` and `numberOfGuards` by considering all new points which are covered by vertex v , when v is added to partial solution; pseudocode of the procedure is shown in Algorithm 3.

Algorithm 3 Function `updateCoveredPointsAdd`

```
1: Input: vertex  $v$ ,  $S[v]$ : list of points covered by  $v$ 
2: Output: updated structures: coveredPoints and numberOfGuards
3: for  $p$  in  $S[v]$  do
4:   coveredPoints.insert(p)
5:   numberOfGuards.find(p).second++
6: end for
```

Algorithm 4 Function `updateCoveredPointsRemove`

```
1: Input: vertex  $v$ ,  $S[v]$ : list of points covered by  $v$ 
2: Output: updated structures coveredPoints and numberOfGuards
3: for  $p \in S[v]$  do
4:   numberOfGuards.find(p).second--
5:   if numberOfGuards.find(p).second == 0 then
6:     coveredPoints.remove(p)
7:   end if
8: end for
```

- function `updateCoveredPointsRemove(vertex v)` – does the update of structures `CoveredPoints` and `numberOfGuards` by removing all points which are covered only by guard v , when v is removed from partial solution; pseudocode is shown in Algorithm 4.

Although the time complexity of both functions is $O(|D(P)|)$, from Algorithms 3 and 4 it can be noticed that they only depends on the number of points covered by the current vertex, which is expected to be significantly lower than $|D(P)|$. These two functions allow fast calculation of the functions f and $incorrect_{total}$: a candidate vertex v is temporary added to the partial solution, new status is checked and the vertex is then removed from the solution.

3.1.4. A Hybrid of the GREEDY + CPLEX

The performance of CPLEX sooner or later degrades w.r.t. instance size due to the complexity of the problem. On the other hand, in the later stage, there is an increased chance for Greedy to worsen the obtained greedy solution. So, it makes sense to combine partial solutions generated with a Greedy procedure over a few iterations and only then to make use the CPLEX to do the completion of the partial solution. In details, our approach consists of the following steps:

1. Run a Greedy method up to K iterations (parameter) to obtain a partial solution s^{ps} (therefore, $|s^{ps}| = K$);
2. Take solution s^{ps} and make it complete by solving a corresponding sub-model via CPLEX:
 - CPLEX solves corresponding sub-model which is formed by adding constraints $x_i = 1$, for all $v_i \in s^{ps}$ into the existing WOAGP model;
 - a complete solution \bar{s}^p is obtained;
3. Return $f(\bar{s}^p)$.

4. Computational Results

We used two different kind of benchmark sets from [17]:

- *MinArea* instances: *MinArea* includes polygons with a small areas. They are specific because they have tiny interior. They presents a boundary case for the cardinality of set $D(P)$ which means that a very few points will be added in $D(P)$.
- *FAT* instances: includes polygons of large areas. They are specific because their interior is wide. In this case, a large number of points of regular grid will be included into $D(P)$.

For each benchmark set – *MinArea* and *FAT* instances – one polygon for each $n \in \{8, 10, \dots, 200\}$ has been generated, which makes 97 instances per each benchmark set. In overall, we have 194 polygons. We included two kind of weights into each of (194) instances:

- *topologically-based related weights* ($W0$): For each vertex v_i of polygon P we denote by l_i and l_{i+1} the lengths of two edges that come out of the vertex v_i . Then, we set $w_i := \frac{l_i + l_{i+1}}{2}$. This way of assigning weights is augmented by the fact that if the arithmetic length of both edges that comes out of vertex i is longer, it is expected that a guard can see a larger pieces of polygon P . This implies that the spectrum of camera installed at v_i has to be larger, which implies that it should be of a higher quality, i.e., a higher price.
- *point-based related weights* ($W1$): For each vertex j , we consider the number of points in $D(P)$ that are visible from the given vertex ($|V(j)|$). Based on this, we assign prices to each vertex on the following way:

$$w_i = n \cdot \frac{|V(i)|}{|D(P)|}, \quad i = 1, \dots, n. \quad (11)$$

Similarly as in case of $W0$, cameras which cover larger range of points are considered be of a higher quality and thus of a higher price.

4.1. Settings and the choice of the Parameters

All variants of our algorithms were implemented in C++ with g++ 7.4 and the experiments were conducted in single-threaded mode on a machine with an Intel Xeon E5-2640 processor with 2.40 GHz and a memory limit of 8GB. The maximum computation time of each of our algorithms was set to 5 *min*.

After conducting preliminary results, for GREEDY + CPLEX we decided to set up $K = \lceil 0.05 \cdot n \rceil$.

The benchmark sets and the executable file of our software for this project are provided at link.

4.2. Results and Discussion

The following algorithms are included in our computation:

- exact approaches: CP and ILP approach. The later is, henceforth, called CPLEX;

Alg.	$ s $	$\bar{t}[s]$	$ g $	optHit	$\sigma(Opt)$	#hitsMinH	$\sigma(MinH)$
Cplex	*15.93	0.152	18.93	97/97	0.00	N/A	N/A
Cp	16.04	98.215	18.96	67/97	0.02	N/A	N/A
GREEDY-1	22.63	0.002	26.28	0/97	0.79	3/97	7.61
GREEDY-2	19.97	0.002	20.14	0/97	0.44	0/97	0.20
GREEDY-1+Cplex	18.49	0.073	21.19	2/97	0.30	57/97	0.06
GREEDY-2+Cplex	18.39	0.004	19.59	0/97	0.26	53/97	0.04

Table 1: The results on the benchmark set *MinArea* for the type of weight *W0*.

Alg.	$ s $	$\bar{t}[s]$	$ g $	optHit	$\sigma(Opt)$	#hitsMinH	$\sigma(MinH)$
Cplex	*36.03	0.003	18.93	97/97	0.00	N/A	N/A
Cp	*36.03	18.953	18.93	97/97	0.00	N/A	N/A
GREEDY-1	39.28	0.001	20.15	2/97	0.35	4/97	0.24
GREEDY-2	40.92	0.001	20.96	0/97	0.51	0/97	0.40
GREEDY-1+Cplex	37.78	0.006	19.41	4/97	0.21	19/97	0.08
GREEDY-2+Cplex	37.31	0.003	19.58	2/97	0.15	81/97	0.19

Table 2: The results on the benchmark set *MinArea* for the type of weight *W1*.

- 4 heuristic approaches:
 - two pure greedy methods, guided by g_1 and g_2 , labeled as GREEDY-1, GREEDY-2, respectively;
 - two GREEDY+Cplex variants, henceforth labeled by GREEDY-1+Cplex and GREEDY-2+Cplex.

Summarized numerical results for each of our 6 variants of our algorithm are displayed in Tables 1 – 4. For each table, the average results are reported for each subset of instances grouped w.r.t. a specific kind of instances and a specific weight (97 instances per each group). Each of the tables list the considered algorithms in the first column. Starting with column two, each of the lines provide detailed statistics for respective algorithm. The statistics report: the average solution quality ($|s|$), the average time ($\bar{t}[s]$) in seconds, the average number of guards which are included in each of the solutions ($|g|$), the number of solutions which match to the optimum solutions (#optHit), average standard deviation of the obtained solutions w.r.t. optimal solutions ($\sigma(Opt)$), then the number of minimal solutions w.r.t. the four heuristics achieved by the respective algorithm (#hitsMinH) and average standard deviation w.r.t. best (minimal) solutions achieved by the four heuristic algorithms ($\sigma(MinH)$). An asterisk in front of a number means that the exact approach could prove optimality in all cases. Best values among heuristic approaches in each column are bolded.

Alg.	$ s $	$\bar{t}[s]$	$ g $	optHit	$\sigma(Opt)$	#hitsMinH	$\sigma(MinH)$
Cplex	*2.62	0.02	7.32	97/97	0.00	N/A	N/A
Cp	*2.62	8.17	7.32	97/97	0.00	N/A	N/A
GREEDY-1	3.39	0.35	7.81	0/97	0.09	23/97	0.05
GREEDY-2	3.52	0.32	6.61	2/97	0.10	28/97	0.08
GREEDY-1+Cplex	3.34	0.35	8.91	4/97	0.08	25/97	0.04
GREEDY-2+Cplex	3.32	0.33	6.81	6/97	0.08	25/97	0.04

Table 3: The results on the benchmark set *FAT* for the type of weight *W0*.

Alg.	$ s $	$\bar{t}[s]$	$ g $	optHit	$\sigma(Opt)$	#hitsMinH	$\sigma(MinH)$
Cplex	*6.72	0.02	6.44	97/97	0.00	N/A	N/A
Cp	*6.72	0.36	6.44	97/97	0.00	N/A	N/A
GREEDY-1	6.77	0.31	6.49	92/97	0.02	92/97	0.04
GREEDY-2	6.77	0.31	6.49	92/97	0.02	92/97	0.04
GREEDY-1+Cplex	6.73	0.32	6.45	96/97	0.01	96/97	0.03
GREEDY-2+Cplex	7.42	0.31	6.46	90/97	0.33	92/97	0.33

Table 4: The results on the benchmark set *FAT* for the type of weight *W1*.

From the numerical results, we observe the following conclusions concerning exact solving:

- Cplex could solve all solution to prove optimality within a fraction of a second;
- Cp was able to solve all those instances with weight *W1* but with significantly more time than the time required from Cplex;
- For the subset of instances *MinArea* which includes weight *W0*, the performance of Cp degrades significantly, which can be seen by the respective average runtime and the number of proven optimal solutions found (67/97).

From the numerical results, we observe the following conclusions concerning heuristic solving:

- In case of the instances that include small-area polygons (*Minarea*) and weight *W0* (Table (1)): A novel GREEDY-2 was able to significantly outperform the GREEDY-1 from literature. In this case, the best performing heuristic algorithms is hybrid GREEDY-2+Cplex which needs the two order of magnitude lower average runtime to finish than the pure greedy methods. The average runtimes of all four heuristic methods are an order of magnitude lower than the average runtime of Cplex. Pure greedy methods deliver solutions of reasonable quality within 20% of the quality of optimal solutions but needs the time which is 7-8 times lower than the average runtime of Cplex. Although all heuristic methods could not achieve optimal solutions (with the exception of two cases for GREEDY-1+Cplex heuristics), the average standard deviation w.r.t. optimal solution is rather small. From last two columns, it is evident that both hybrid variants are more successful, hitting more than a half best solution each, with very small average standard deviation.
- In case of the instances that include small-area polygons and weight *W1* (Table 2): the best heuristic approach is GREEDY-2+Cplex which is able to deliver solutions which are within 3% of the optimum ones. Note that in 81 instances (out of 91) GREEDY-2+Cplex is able to deliver equally good or better results than the other heuristic approaches. As it was the case of the weight *W0*, heuristic methods could not achieve many optimal solutions, but average standard deviation w.r.t. optimal solutions is rather small. The results of pure greedy method GREEDY-1 outperform the results of GREEDY-2. These two methods are effective since they deliver solutions which are within 10% of the Cplex results but need 3

times less runtime. The result of GREEDY-1 match just in two cases to the optimum.

- In case of the instances that include large-area polygons (*FAT*) and weight $W0$ (Table (3): the exact CPLEX approach is a clear winner concerning average solutions' quality as well as the average runtimes; the best heuristic algorithm w.r.t. solution quality is GREEDY-2 + CPLEX. It also matches in the most cases (on 6 instances) to the optimum results w.r.t. other heuristic approaches. Its average results are within 22% of the average of optimal solutions. The pure GREEDY-1 slightly outperforms the pure greedy GREEDY-2. It is interesting that GREEDY-1 assigns in average a higher number of guards in comparison to the average number of guards of GREEDY-2. The runtimes of our heuristic approaches are an order of magnitude lower than the time of CPLEX approach. Reason for that can be seen in the used regular discretization of the polygon. For the polygons with a larger area, more points are involved in the discrete set $D(P)$ and, therefore, iterations of greedy methods take a longer time than in case when $D(P)$ is smaller (which was the case of *Minarea* polygons).
- In case of the instances that include large-area polygons (*FAT*) and weight $W1$ (Table (4): the best heuristic algorithms w.r.t. solution quality is GREEDY-1 + CPLEX which is able to match in 96 instances the quality of optimal solutions. Slightly worse results are delivered by GREEDY-1 and GREEDY-2. The obtained (heuristic) solutions of these two approaches are within 1% of optimal solutions and they are able to reach the quality of the optimal solution for 92 instances. Unfortunately, the runtimes in comparison to the runtimes of CPLEX are significantly higher for all of our heuristic approaches. However, the average runtime for CP is a bit higher than the avg. runtimes of the heuristic approaches. Again, average standard deviation for each heuristic method is rather small w.r.t. optimal solutions, indicating the good quality of the proposed algorithms.
- Concerning the percentage of covering of polygons for the best solutions (found by CPLEX) we noticed that *FAT* instances are covered in almost all cases (see Fig.2 and 3 and the blue curve). Concerning the *Smallarea* instances, it gets harder to cover all polygon and in almost all cases the whole area of polygon P cannot be covered. For these instances, we see that the solutions cover more regions of small-area polygons when weight $W1$ is involved then when $W0$ is considered ($\approx 93\%$ vs. $\approx 85\%$). It can be argued by the fact that in weight $W0$ there exists many ties for weights of guards, and, therefore, more solutions with the minimum weight exist. Notice that this is not so often the case of the weight $W1$.

FiXme: TODO: Da li na dijagramima treba nešto dodatno napisati, smanjiti ih: Ucini da Fig. 2,3 i 4 stanu na jednu stranu. Fig 4 je pogresna, obrnuo si kategorije. Treba da grupises rezultate izvorsavanja po tipu instance, a da u legendi stoje algoritmi...

FiXme: TODO: Vrijeme izvorsavanja je skalirano logaritmaski

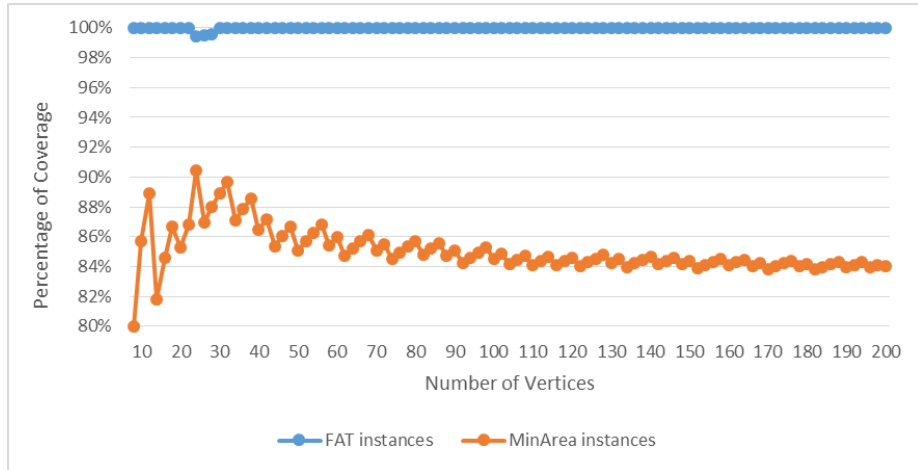


Figure 2: Polygon coverage for the type of weight W0.

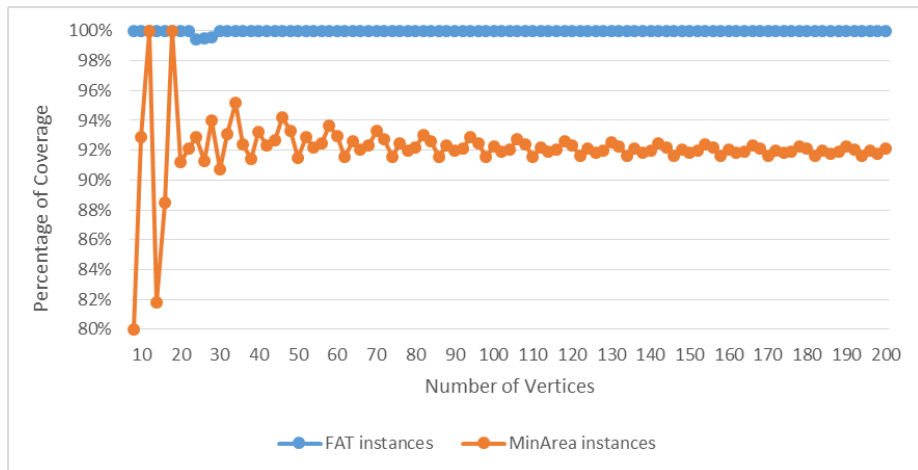


Figure 3: Polygon coverage for the type of weight W1.

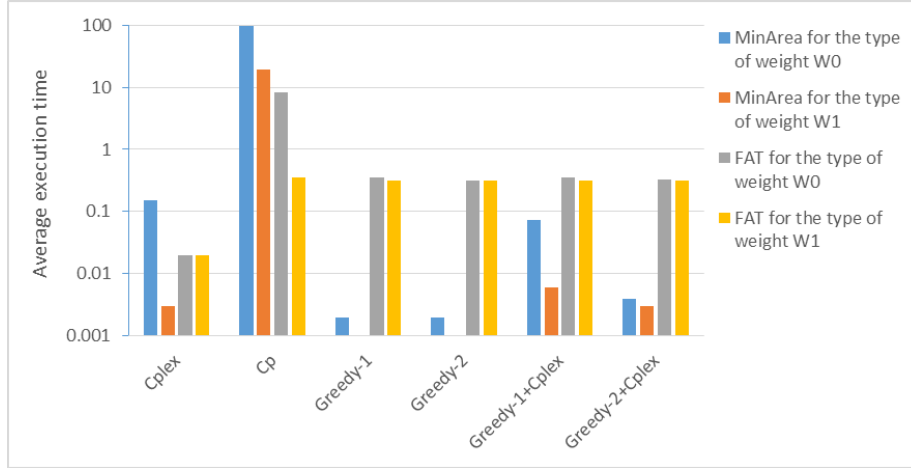


Figure 4: Polygon coverage for the type of weight W1.

5. Conclusions and Future Work

In this work we have considered the Weighted Orthogonal Art Gallery Problem under regular grid discretization. We have developed a novel greedy rule which provide a trade-off between the number of guards and the cost of guards. Moreover, a hybrid of a greedy method and CPLEX was proposed to make a maximal completion of the greedy solution obtained by the greedy method supplemented by solving respective subproblem via CPLEX. The performances of the heuristic approaches are compared to the exact ILP and CP approaches. From the computational experiments, the heuristic approaches were highly efficient in terms of obtaining solutions of reasonable quality in an order of magnitude lower runtime than the exact approaches for the small-area polygons. For the large-area polygons, heuristic approaches were able to reach the quality of optimal solutions in almost all cases but in cost of larger times than the ILP approach.

For the future work, we tend to improve the results of our greedy method as well as runtimes on the *FAT* benchmark sets by considering different discretizations of polygons, for example, the Delaunay triangulation ??.

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