Supplemental Appendix for "Political Corruption Traps"

Contents

A1 Proofs of Results in the Text

A1.1 Proof of Proposition 1

Part (i) follows from the fact that corrupt politicians could always choose $x_i = 0$ and get the same payoff as a non-corrupt politician, but choose a higher level and hence must get a higher payoff. For part (ii), implicitly differentiating the equilibrium condition gives:[1]

$$\frac{\partial x^*}{\partial \rho} = -\frac{x^* g_{12}(x^*, \rho x^*)}{\frac{\partial g_1(x^*, \rho x^*)}{\partial x^*}}$$

The numerator is positive and the denominator is negative, so the expression is positive. For part (iii), the payoff for a non-corrupt politician is constant in ρ, and the envelope theorem combined with the fact that $g_2 > 0$ implies the equilibrium payoff for a corrupt politician is increasing in ρ.

A1.2 Proof of Proposition 2

Set $\hat{b} = \frac{x^*}{g(x^*, \rho x^*)}$, which is positive, and, since $d^p(\rho) > 0$, so $x^* < g(x^*, \rho x^*)$, and hence $\hat{b} < 1$.

A1.3 Proof of Proposition 3

The $\frac{\partial g(x^*, \rho x^*)}{\partial \rho}$ term expands to:

$$\frac{\partial g(x^*, \rho x^*)}{\partial \rho} = g_1(x^*, \rho x^*) \frac{\partial x^*}{\partial \rho} + \left[\rho \frac{\partial x^*}{\partial \rho} + x^*\right] g_2(x^*(\rho x^*))$$

In equilibrium $g_1(x^*, \rho^*) = 1$, so:

$$\frac{\partial d^v}{\partial \rho} = \frac{\partial x^*}{\partial \rho}(-1 + b) + b\left[\rho \frac{\partial x^*}{\partial \rho} + x^*\right] g_2(x^*(\rho x^*)) \tag{1}$$

Rearranging equation 1, $d^v(\rho)$ is increasing if $b > \tilde{b}$, where

$$\tilde{b} = \frac{\frac{\partial x^*}{\partial \rho}}{\frac{\partial x^*}{\partial \rho} + \left[\rho \frac{\partial x^*}{\partial \rho} + x^*\right] g_2(x^*(\rho x^*))}.$$

Which is strictly less than one.

[1] Technically ρ can only take on values $0, 1/N, 2/N, ...1$, but the second argument of g can be any real number, so this derivative is well-defined.

A1.4 Proof of Proposition 4

The existence of of an equilibrium follows from the fact that the right-hand side of equation 7 is a continuous function from $[0, 1]$ to $[0, 1]$ and hence must have a fixed point. When $\frac{\partial d^v}{\partial \rho} < 0$ the right-hand side is decreasing, ensuring this intersection is unique. If not, the right-hand side is increasing, and as shown in the illustrations may have multiple intersections.

A2 Entry Model

Here, we present a model where the composition of the pool of those running for office changes as a function of how corrupt they expect politics to be.

To build on the politician model presented in the main text (Section 3.1), suppose that for each of the N districts there are $M > 1$ candidates. As in Section 3.1, each candidate has an ability a_i, and is either predisposed to corruption, denoted with $c_i = 1$, or averse to corruption, denoted $c_i = 0$. Assume the prior probability of being corrupt is given by q, and that ability is independent of corruption and drawn from a cumulative distribution function F. Potential candidates do not know the type of other potential candidates.

Candidates pay a cost k to run for office. Those who do not enter office get a payoff normalized to zero, and those who win office observe the type of other winners and choose a corruption level x_i with the same utility function as in the politician model solved in Section 3.1. Let $\overline{a}$ be the highest possible ability level, and assume $k < \overline{a}$, which will imply that a non-corrupt potential candidate with the highest possible ability would run if guaranteed victory.

A strategy in the entrant model consists of a mapping from the potential candidate type (i.e., the pair (a_i, c_i)) to a decision to run, and a mapping from the type to a corruption level if elected. Our equilibrium definition (formalized below) will require that the strategies chosen are mutual best responses.

Let $Pr(\text{win}; \cdot)$ be the probability of winning, conditioned on the strategies used by others. For simplicity we assume that there is a lottery that determines which candidate wins, so the probability of winning if m_{-i} other candidates run in that district is $1/(1 + m_{-i})$.

For a fixed proportion of corrupt (winning) politicians, a candidate prefers to enter if:

$$Pr(\text{win}; \cdot)(a_i + d^p(\rho)c_i) \geq k,$$

where $d^p(\rho)$ is the difference between the equilibrium payoff for a corrupt versus clean politician as defined above. Since this payoff is increasing in a_i, any optimal entrance strategy can be characterized by a threshold for non-corrupt candidates $\hat{a}_0^p$ and corrupt candidates $\hat{a}_1^p$ such that potential candidates run if and only if $a_i \geq \hat{a}_{c_i}^p$.

The number of other candidates is a binomial random variable with parameters $M - 1$ and $\pi(\hat{a}_0^p, \hat{a}_1^p)$, where $\pi(\cdot)$ is the unconditional probability of running:

$$\pi(\hat{a}_0^p, \hat{a}_1^p) = (1 - q)(1 - F(\hat{a}_0^p)) + q(1 - F(\hat{a}_1^p)).$$

The probability of winning is then:

$$Pr(\text{win}|\hat{a}_0^p, \hat{a}_1^p) = \mathbb{E}[1/(m_{-i}+1)] = \sum_{x=0}^{M-1} \frac{1}{x+1} \frac{(M-1)!}{(M-x-1)!x!} \pi(\hat{a}_0^p, \hat{a}_1^p)^x (1-\pi(\hat{a}_0^p, \hat{a}_1^p))^{M-x-1} \tag{2}$$

The distribution of m_{-i} under π first order stochastic dominates the distribution of m_{-i} for any $\pi' < \pi$, and π is increasing in both $\hat{a}_0^p$ and $\hat{a}_1^p$. So, since $\mathbb{E}[1/(m_{-i}+1)]$ is decreasing in m_{-i}, it is increasing in $\hat{a}_0^p$ and $\hat{a}_1^p$

The winners then choose corruption levels as in the politician model. As demonstrated in the previous section, politicians with $c_i = 0$ always select $x_i = 0$ and we restrict attention to equilibria where corrupt politicians choose a symmetric corruption level x^*. Formally, our equilibrium definition is as follows:

Definition An equilibrium to the entry model comprises a corruption level x^* and entry thresholds $\hat{a}_0^p$ and $\hat{a}_1^p$ that jointly solve:

$$x^* = \arg\max_{x_i} a_i - x + g(x_i, \overline{x}) \tag{3}$$

$$\hat{a}_0^p = k/Pr(\text{win};\cdot) \tag{4}$$

$$\hat{a}_1^p + d^p(\rho) = k/Pr(\text{win};\cdot) \tag{5}$$

where

$$\rho = \frac{q(1-F(\hat{a}_1^p))}{q(1-F(\hat{a}_1^p)) + (1-q)(1-F(\hat{a}_0^p))} \tag{6}$$

And $Pr(\text{win};\cdot)$ is the expected probability of winning given the entry thresholds, derived in the Supplemental Appendix.

Condition 3 states that politicians choose a corruption level that is a mutual best response as derived in the previous section; as above this can be characterized by the level chosen by all corrupt politicians. Conditions 4-5 imply that given this behavior on the politician level, potential entrants with exactly the threshold ability level $\hat{a}_{c_i}^e$ (given their corruption disposition c_i) are indifferent between running and not. That is, those with ability level above this threshold prefer to run and those below the threshold prefer to stay out of politics. Condition 6 states that the proportion of corrupt politicians is equal to the probability of being corrupt given having a high enough ability to run.[2]

Analogous to the entry model, we solve the entry model with the following procedure:

[2]That is, potential entrants behave as if the proportion of corrupt politicians will be exactly equal to the probability of being corrupt if running, while in fact when the number of districts is finite the number of corrupt politicians is uncertain (following a binomial distribution). So, if the value of holding office is nonlinear in ρ and the number of districts is finite, this solution concept is not exactly the same as Perfect Bayesian Equilibrium (PBE), which would require the politicians to explicitly sum over the potential realized values of ρ, rendering the solution far more analytically complex. However, when N, is large, the calculation implicitly made by potential candidates approximates the PBE. A simpler justification for this assumption is a behavioral one: citizens simply make an approximation of how corrupt politics will be by equation 6.

1. Propose an equilibrium proportion of corrupt politicians $\rho^* \in [0,1]$.
2. Derive the optimal corruption level x^* and the resulting politician corruption differential $d^p(\rho^*)$.
3. Given $d^p(\rho^*)$, compute the entry rules $\hat{a}_0^p$ and $\hat{a}_1^p$ that generate a proportion of corrupt entrants by equation 6.
4. If this proportion is equal to ρ^*, then ρ^* (and the corresponding strategies derived in steps 2-3) comprise an equilibrium; if not there is no equilibrium with proportion of corrupt politicians ρ^*.

The analysis in the previous section accomplishes step 2, so what remains is steps 3-4.

First, we show how the entry behavior changes when treating the proportion of corrupt politicians (and resulting politician behavior) as exogenous:

Proposition 1. *For all ρ:*

(i) There is a unique set of thresholds meeting equations 4-5, and

(ii) $\hat{a}_0^p$ is weakly increasing in ρ and $\hat{a}_1^p$ is weakly decreasing in ρ

Proof First, note that since $\hat{a}_1^p = \hat{a}_0^p + d^p(\rho^I)$, we can write the equilibrium probability of winning solely as a function of $\hat{a}_0^p$: $Pr(\text{win}; \hat{a}_0^p, \hat{a}_0^p - d^p(\rho^I)) = 1/M$. Further, this probability is equal to $1/N$ for sufficiently low $\hat{a}_0^p$, equal to 1 for sufficiently high $\hat{a}_0^p$, and is continuous and increasing in $\hat{a}_0^p$. Since $\hat{a}_0^p < k$, there is a unique $\hat{a}_0^p \geq 0$ and $\hat{a}_1^p \geq$ that jointly meet equations 4, 5, and 2.

For part ii, consider any $\rho^1 < \rho^2$. It cannot be the case that both thresholds are higher under ρ^2, as this would imply the marginal corrupt entrant has a higher ability and a higher probability of winning, and hence cannot be indifferent between running and not. Similarly if both thresholds go down from ρ^1 to ρ^2, the marginal non-corrupt entrant would have a lower ability and lower probability of winning than under ρ^2 than ρ^1, and hence cannot be indifferent. Combining these observations with the fact that $\hat{a}_0^p + d^p(\rho) = \hat{a}_1^p$ and d^p increasing implies that $\hat{a}_0^p$ must be weakly increasing in ρ and $\hat{a}_1^p$ weakly decreasing.

These entry rules imply a proportion of corrupt entrants, and one way to describe the equilibrium condition is that the proportion of corrupt entrants is equal to the proposed equilibrium level. Formally, a ρ^* meeting

$$\rho^* = \frac{q(1 - F(\hat{a}_1^p(\rho^*)))}{q(1 - F(\hat{a}_1^p(\rho^*))) + (1-q)(1 - F(\hat{a}_0^p(\rho^*)))} \tag{7}$$

and the corresponding x^*, $\hat{a}_0^p$, and $\hat{a}_1^p$ computed by equations 3-5 comprise an equilibrium. Our main technical result in this section is as follows:

Proposition 2. *There is at least one – and potentially more than one – ρ^* meeting equation 7 and hence at least one equilibrium and potentially multiple equilibria to the entry model.*

Proof Finding a ρ^* that solves equation 7 is a necessary condition for an equilibrium, as if it does not hold for any ρ^* then either equation 4, 5, or 6 must be violated. when ρ^* does solve equation 7, then the $\hat{a}_0^p(\rho^*)$, $\hat{a}_1^p(\rho^*)$, and $x^*(\rho^*)$ as constructed in the main text meet the equilibrium conditions.

Since the right-hand side of equation is a 7 is continuous function from $[0, 1]$ to $[0, 1]$, it must have a fixed point, proving the existence of an equilibrium. The possibility of multiple ρ^* and hence equilibria to the entry model is demonstrated by example below.

The potential for multiple equilibria means it is possible that there is a stable high level of corruption – where those predisposed to be corrupt are more apt to run for office expecting others to be corrupt as well – and a stable lower level of corruption – where corrupt politicians are less apt to run under the expectation that there will be fewer gains from corrupt behavior. This is loosely analogous to the central idea in Caselli and Morelli (2004) where there can be multiple equilibria, one where most politicians are "good types" and one where most are "bad types."

It is difficult to derive clear analytic results of when such a political corruption trap arises from the entry dynamics; in the Supplemental Appendix, we provide several illustrations. Loosely speaking, there can be multiple equilibria when there value of corrupt behavior is highly contingent on the number of others who are corrupt. That is, it must be the case that returns to corruption are substantially higher when there is a large pool of others predisposed to corruption. We examine when the forces we study lead to multiple equilibria and hence a political corruption trap in the final model presented next.

A3 Illustrations of Multiplicity in the Entry Model

We now present some illustrations to generate the intuition that there can be a corruption trap in the entry model. For all illustrations $k = .5$, we fix the distribution of ability to be uniform on $[0, 1]$, the probability of a (potential) candidate being corrupt at $q = .5$, and consider various voter corruption differentials $d^p(\rho)$.

In Figure A1, the corruption bonus function is $d^p(\rho^I) = .2\rho$. That is, when no other politicians are corrupt, there is no benefit to being corrupt ($d^p(0) = 0$). However, as the proportion of other corrupt politicians increases, corrupt entrants value winning office more highly, up to the point where when all others are corrupt an 80th percentile corrupt politician values winning office as much as the most able non-corrupt politician.

We plot this bonus function in the left panel, and the corresponding equilibrium condition in the right panel. In this case, the curve corresponds to the proportion of corrupt entrants as a function of the proportion of corrupt incumbents, so an intersection between the curve and 45 degree line indicates an equal proportion of corrupt incumbents and entrants. In the first illustration, there is one intersection and hence a unique stable corruption level.

Figure A1: Illustration of equilibrium to entry model with $d^p(\rho) = .2\rho$

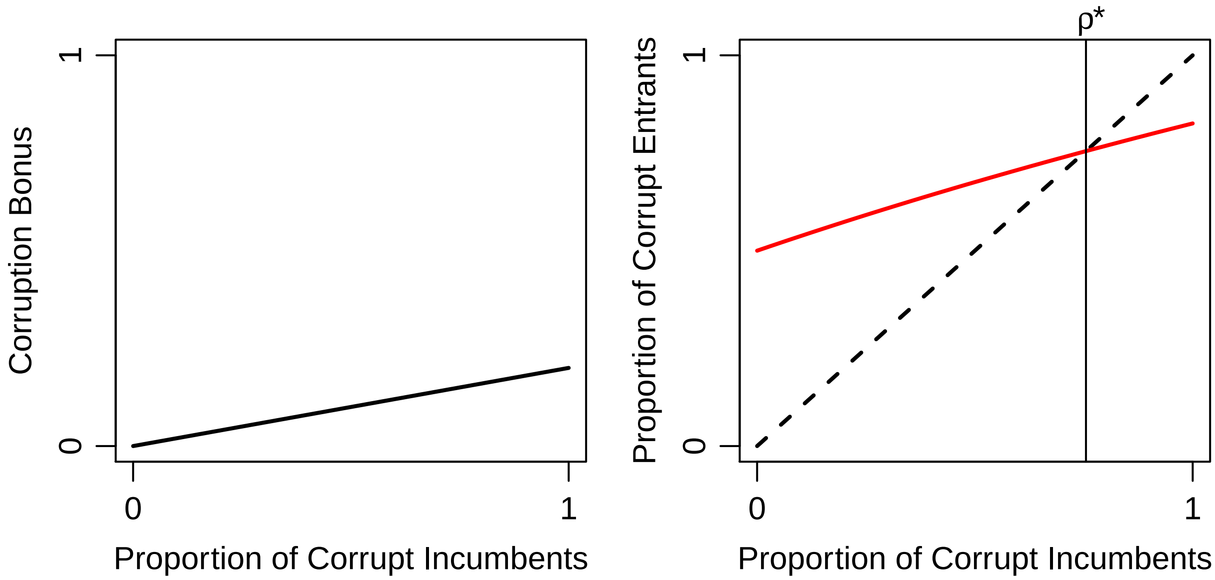

Figure A2: Illustration of equilibrium to entry model with $d^p(\rho^I)$ is the cumulative density function of a standard normal random variable with mean 0.7 and variance 0.1.

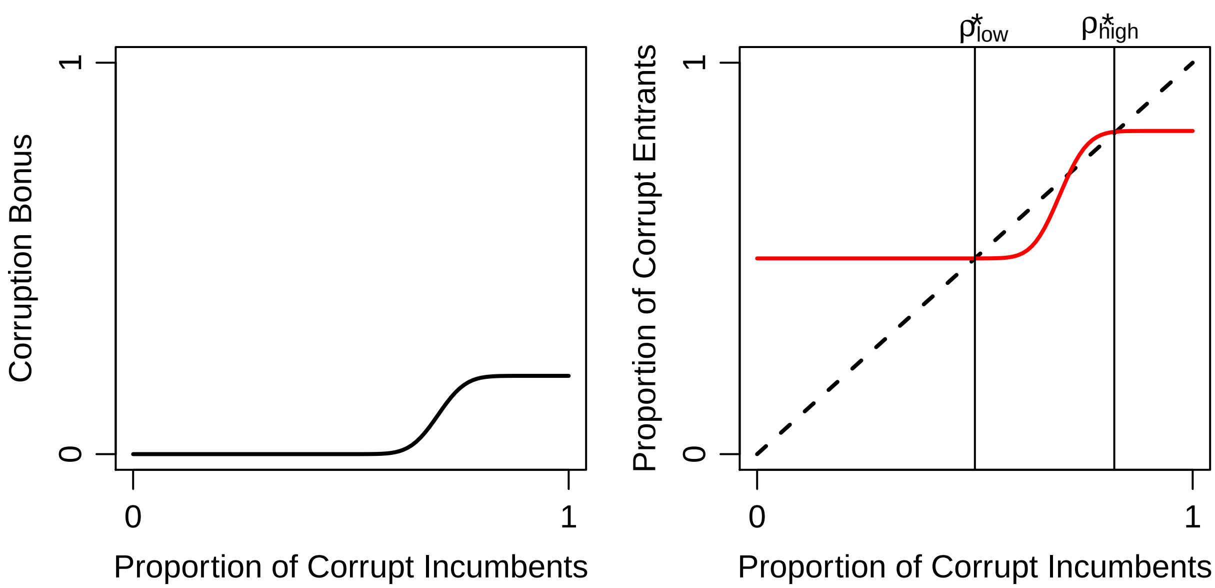

Figure A2 shows the analogous illustration where the bonus for corruption is nearly zero for low levels of corrupt politicians until it rapidly increases when the proportion corrupt is around 70%. In this case, there are three intersections of the equilibrium condition where the highest and lowest intersections correspond to stable equilibria, one with a relatively low level of corrupt politicians and one with a higher proportion of corrupt politicians. Intuitively, in the low corruption equilibrium the corruption bonus is negligible, so around half of the politicians in office are corrupt (as $q = .5$). We can think of this as a situation where many politicians would be willing to engage in corrupt behavior, but there is not the critical mass required to make this behavior profitable. In the higher corruption equilibrium, the majority of politicians are corrupt, with this proportion above the critical mass required for corruption to be profitable. As a result, only exceptionally able clean politicians will run for office, leaving the vast majority of candidates as corrupt types and therefore perpetuating the high level of corruption.

A4 Further illustration of Voter Model with Nonlinear d^v

The main text contains one example of a non-linear d^v which leads to a corruption trap. To show what kinds of s-shaped curves can lead to a corruption trap, figure A3 indicates when there are multiple equilibria for penalty functions of the form $d^v(\rho) = -2+2\Phi\left(\frac{\rho-\mu}{\sigma}\right)$ for $0 \leq \mu \leq 1$ and $0 \leq \sigma \leq 0.3$. As the figure indicates, there tend to be multiple equilibria when (1) σ is low, and (2) μ is intermediate. The first part is not surprising given the analysis in Section 3.2: multiple equilibria are possible when the penalty function can sharply increase, which in this case is when σ is low. The second part indicates that the sharp part of the g function must be in the "right" place. Intuitively, if having a corrupt politician is always bad, and the effects do not diminish until around 50% of politicians are corrupt, there will always be an equilibrium with low corruption.

In sum, for there to be a corruption trap solely from the voter selection effect, it must be the case that (1) the value of having a corrupt representative rapidly changes in the proportion of others that are corrupt, and (2) having a corrupt politician is good (or at least innocuous) when most other are corrupt.

References

Caselli, Francesco, and Massimo Morelli. 2004. "Bad Politicians." *Journal of Public Economics* 88 (3–4): 759 - 782.

Figure A3: Range of s-shaped corruption effects with multiple equilibria