Supplemental Appendix for "Political Corruption Traps"

Contents

A1 Proof of Proposition 3

The $\frac{\partial g(x^*,\rho x^*)}{\partial \rho}$ term expands to:

$$\frac{\partial g(x^*,\rho x^*)}{\partial \rho} = g_1(x^*,\rho x^*)\frac{\partial x^*}{\partial \rho} + \left[\rho\frac{\partial x^*}{\partial \rho} + x^*\right] g_2(x^*(\rho x^*))$$

In equilibrium $g_1(x^*,\rho^*) = 1$, so:

$$\frac{\partial d^v}{\partial \rho} = \frac{\partial x^*}{\partial \rho}(-1+b) + b\left[\rho\frac{\partial x^*}{\partial \rho} + x^*\right] g_2(x^*(\rho x^*)) \tag{1}$$

Rearranging equation 1, $d^v(\rho)$ is increasing if $b > \tilde{b}$, where

$$\tilde{b} = \frac{\frac{\partial x^*}{\partial \rho}}{\frac{\partial x^*}{\partial \rho} + \left[\rho\frac{\partial x^*}{\partial \rho} + x^*\right] g_2(x^*(\rho x^*))}.$$

Which is strictly less than one.

A2 Derivation of Probability of Winning

The number of other candidates – denote this m_{-i} – is a binomial random variable with parameters $M-1$ and $\pi(\hat{a}_0^p, \hat{a}_1^p)$, where $\pi(\cdot)$ is the unconditional probability of running:

$$\pi(\hat{a}_0^p, \hat{a}_1^p) = (1-q)(1-F(\hat{a}_0^p)) + q(1-F(\hat{a}_1^p)).$$

The probability of winning is then:

$$Pr(\text{win}|\hat{a}_0^p, \hat{a}_1^p) = \mathbb{E}[1/(m_{-i}+1)] = \sum_{x=0}^{M-1}\frac{1}{x+1}\frac{(M-1)!}{(M-x-1)!x!}\pi(\hat{a}_0^p, \hat{a}_1^p)^x(1-\pi(\hat{a}_0^p, \hat{a}_1^p))^{M-x-1} \tag{2}$$

The distribution of m_{-i} under π first order stochastic dominates the distribution of m_{-i} for any $\pi' < \pi$, and π is increasing in both $\hat{a}_0^p$ and $\hat{a}_1^p$. So, since $\mathbb{E}[1/(m_{-i}+1)]$ is decreasing in m_{-i}, it is increasing in $\hat{a}_0^p$ and $\hat{a}_1^p$

A3 Proof of Proposition 4

First, note that since $\hat{a}_1^p = \hat{a}_0^p + d^p(\rho^I)$, we can write the equilibrium probability of winning solely as a function of $\hat{a}_0^p$: $Pr(\text{win}; \hat{a}_0^p, \hat{a}_0^p - d^p(\rho^I)) = 1/M$. Further, this probability is equal to $1/N$ for sufficiently low $\hat{a}_0^p$, equal to 1 for sufficiently high $\hat{a}_0^p$, and is continuous and increasing in $\hat{a}_0^p$. Since $\hat{a}_0^p < k$, there is a unique $\hat{a}_0^p \geq 0$ and $\hat{a}_1^p \geq$ that jointly meet equations 3, 4, and 2.

For part ii, consider any $\rho^1 < \rho^2$. It cannot be the case that both thresholds are higher under ρ^2, as this would imply the marginal corrupt entrant has a higher ability and a higher probability of winning, and hence cannot be indifferent between running and not. Similarly if both thresholds go down from ρ^1 to ρ^2, the marginal non-corrupt entrant would have a lower ability and lower probability of winning than under ρ^2 than ρ^1, and hence cannot be indifferent. Combining these observations with the fact that $\hat{a}_0^p + d^p(\rho) = \hat{a}_1^p$ and d^p increasing implies that $\hat{a}_0^p$ must be weakly increasing in ρ and $\hat{a}_1^p$ weakly decreasing.

A4 Proof of Proposition 5

Finding a ρ^* that solves equation 6 is a necessary condition for an equilibrium, as if it does not hold for any ρ^* then either equation 3, 4, or 5 must be violated. when ρ^* does solve equation 6, then the $\hat{a}_0^p(\rho^*)$, $\hat{a}_1^p(\rho^*)$, and $x^*(\rho^*)$ as constructed in the main text meet the equilibrium conditions.

Since the right-hand side of equation is a 6 is continuous function from $[0, 1]$ to $[0, 1]$, it must have a fixed point, proving the existence of an equilibrium. The possibility of multiple ρ^* and hence equilibria to the entry model is demonstrated by example below.

A5 Illustrations of Multiplicity in the Entry Model

We now present some illustrations to generate the intuition that there can be a corruption trap in the entry model. For all illustrations $k = .5$, we fix the distribution of ability to be uniform on $[0, 1]$, the probability of a (potential) candidate being corrupt at $q = .5$, and consider various voter corruption differentials $d^p(\rho)$.

In Figure A1, the corruption bonus function is $d^p(\rho^I) = .2\rho$. That is, when no other politicians are corrupt, there is no benefit to being corrupt ($d^p(0) = 0$). However, as the proportion of other corrupt politicians increases, corrupt entrants value winning office more highly, up to the point where when all others are corrupt an 80th percentile corrupt politician values winning office as much as the most able non-corrupt politician.

Analogous to the plots in Section 3.3.1, we plot this bonus function in the left panel, and the corresponding equilibrium condition in the right panel. In this case, the curve corresponds to the proportion of corrupt entrants as a function of the proportion of corrupt incumbents, so an intersection between the curve and 45 degree line indicates an equal proportion of corrupt incumbents and entrants. In the first illustration, there is one intersection and hence a unique stable corruption level.

Figure A2 shows the analogous illustration where the bonus for corruption is nearly zero for low levels of corrupt politicians until it rapidly increases when the proportion corrupt is around 70%. In this case, there are three intersections of the equilibrium condition where the highest and lowest intersections correspond to stable equilibria, one with a relatively low level of corrupt politicians and one with a higher proportion of corrupt politicians. Intuitively, in the low corruption equilibrium the corruption bonus is negligible, so around half of the politicians in office are corrupt (as $q = .5$). We can think of this as a situation where

Figure A1: Illustration of equilibrium to entry model with $d^p(\rho) = .2\rho$

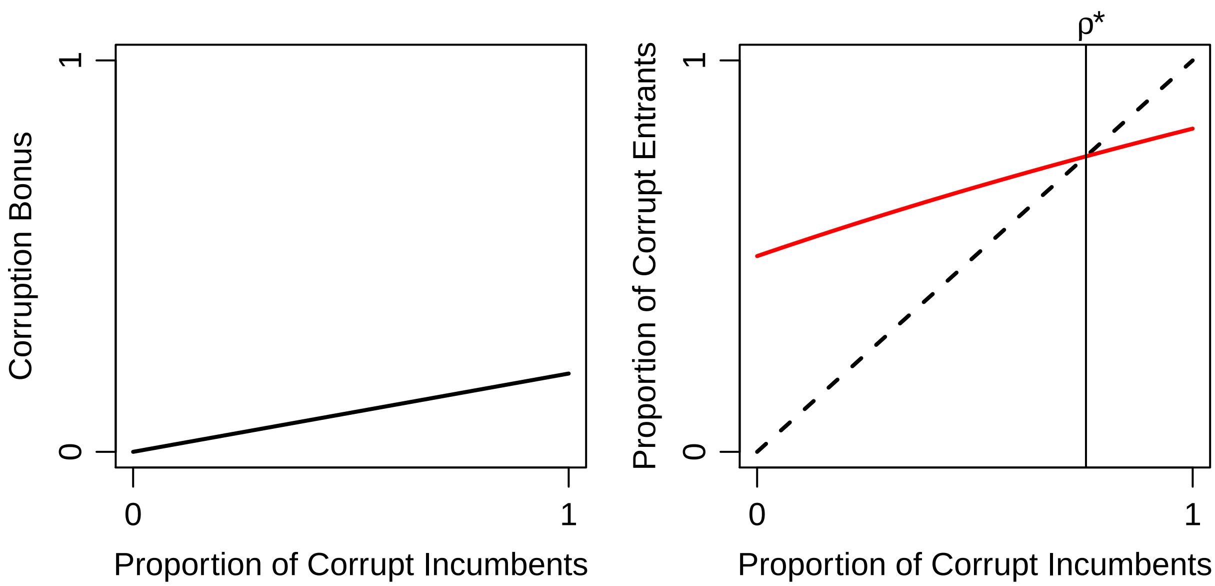
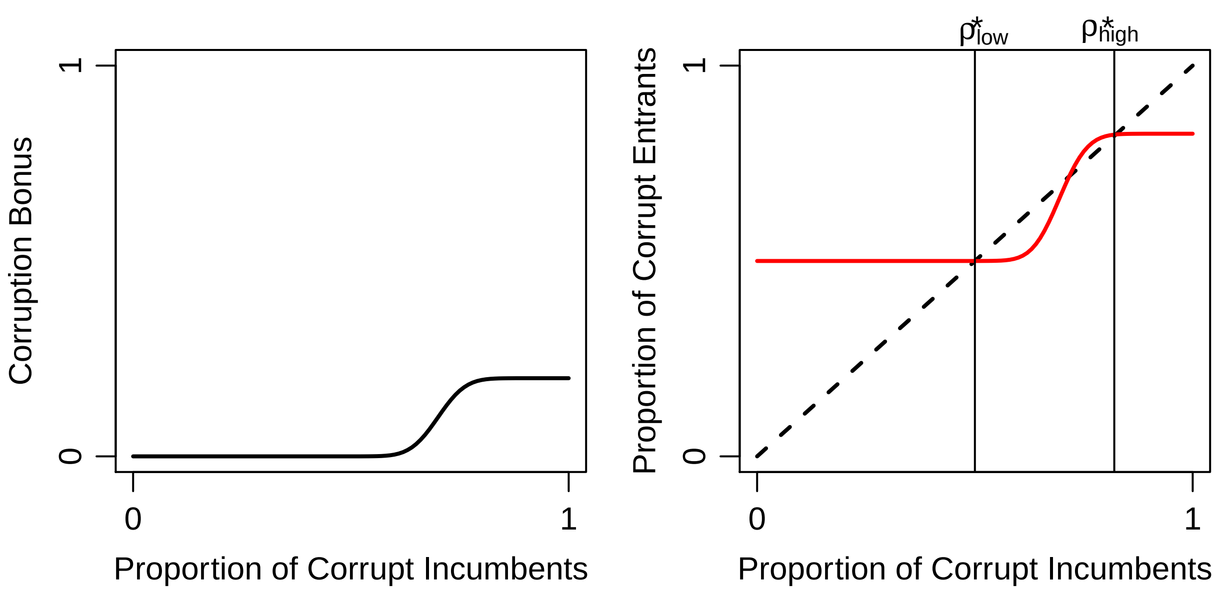


Figure A2: Illustration of equilibrium to entry model with $d^p(\rho^I)$ is the cumulative density function of a standard normal random variable with mean 0.7 and variance 0.1.



many politicians would be willing to engage in corrupt behavior, but there is not the critical mass required to make this behavior profitable. In the higher corruption equilibrium, the majority of politicians are corrupt, with this proportion above the critical mass required for corruption to be profitable. As a result, only exceptionally able clean politicians will run for office, leaving the vast majority of candidates as corrupt types and therefore perpetuating the high level of corruption.

A6 Proof of Proposition 6

The existence of of an equilibrium follows from the fact that the right-hand side of equation 12 is a continuous function from $[0, 1]$ to $[0, 1]$ and hence must have a fixed point. When $\frac{\partial d^v}{\partial \rho} < 0$ the right-hand side is decreasing, ensuring this intersection is unique. If not, the right-hand side is increasing, and as shown in the illustrations may have multiple intersections.

A7 Illustrations of Voter Model with Nonlinear d^v

The assumption that citizens prefer a corrupt politician is not too implausible (see the discussion in the main text), but it is not a necessary condition for there to be multiple stable equilibria. Here we show a case where (Figure A3) the voter corruption differential is a s-shaped function, indicating that it is bad to have a corrupt politician when nearly all others are clean, but as the proportion of corrupt politicians reaches around .3, this effect rapidly diminishes to zero. However, in no case do the voters prefer a corrupt politician. Again, there are multiple intersections and hence multiple equilibria.

While the case where the corruption effect is an non-linear, s-shaped function is more in accordance with the micro-foundations from the previous sections, it is more difficult to explore as there are more free parameters to set. For example, Figure A3 uses a penalty function of $d^v(\rho) = -2 + 2\Phi\left(\frac{\rho - .25}{.1}\right)$ where Φ is the cumulative density function of a standard normal random variable. Figure A4 indicates when there are multiple equilibria for penalty functions of the form $d^v(\rho) = -2 + 2\Phi\left(\frac{\rho - \mu}{\sigma}\right)$ for $0 \leq \mu \leq 1$ and $0 \leq \sigma \leq 0.3$. As the figure indicates, there tend to be multiple equilibria when (1) σ is low, and (2) μ is intermediate. The first part is not surprising given the analysis in Section 3.3: multiple equilibria are possible when the penalty function can sharply increase, which in this case is when σ is low. The second part indicates that the sharp part of the g function must be in the "right" place. In this case, the sharp part must be around 1/3, which, as shown in Figure A3, is near the middle intersection of the equilibrium condition. More intuitively, if having a corrupt politician is always bad, and the effects do not diminish until around 50% of politicians are corrupt, there will always be an equilibrium with low corruption.

In sum, for there to be a corruption trap solely from the voter selection effect, it must be the case that (1) the value of having a corrupt representative rapidly changes in the proportion of others that are corrupt, and (2) having a corrupt politician is good (or at least innocuous) when most other are corrupt.

Figure A3: Illustration with multiple equlibria

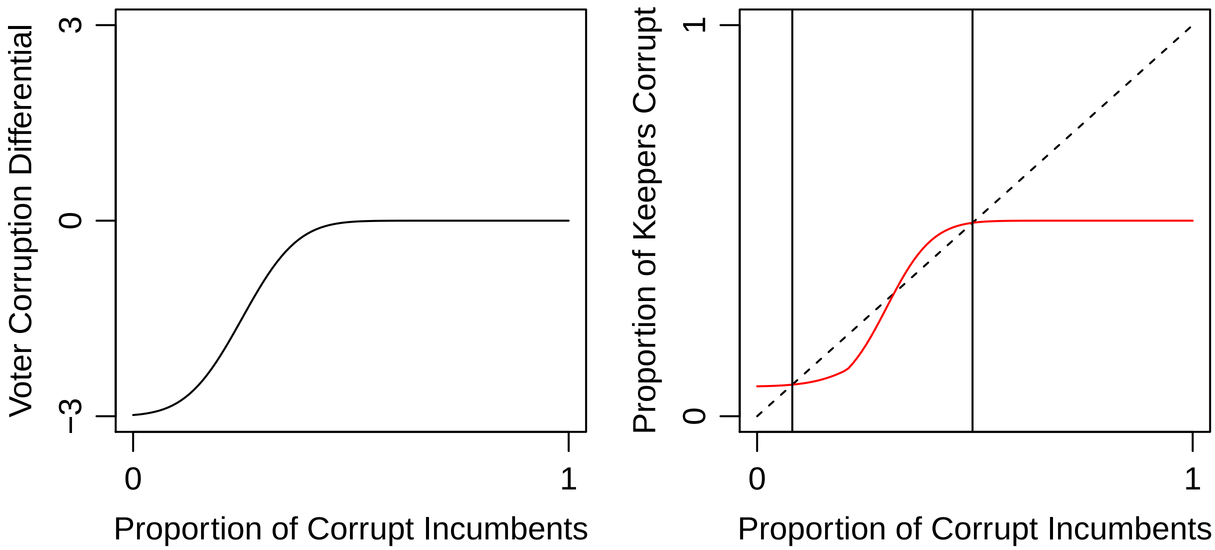
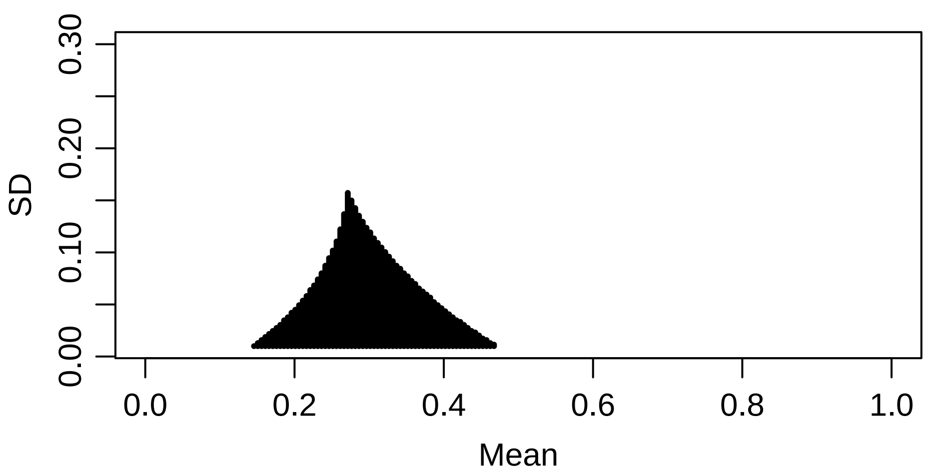


Figure A4: Range of s-shaped corruption effects with multiple equilibria



A8 Derivation of Equilibrium Condition with Non-Constant Entry

The model in the main text assumes that the types of candidates running for office are independent of the current level of corruption, i.e., q is constant. However, for the reasons discussed in the paper and formalized in this Appendix, this is unlikely to be true. Therefore, we now extend the model in the previous section to allow the probability of an entrant being corrupt to be a function of ρ.

With this modification, the probability of entering and being retained for a corrupt and non-corrupt politician is:

$$q_0^k(\rho) = (1 - q(\rho))(1 - F(\mathbb{E}[a] + q(\rho)d^v(\rho)))$$
$$q_1^k(\rho) = q(\rho)(1 - F(\mathbb{E}[a] - (1 - q(\rho))d^v(\rho)))$$

Differentiating with respect to ρ gives:

$$\frac{\partial q_0^k}{\partial \rho} = -(1 - q(\rho))(q(\rho)g'(\rho) + p_c'(\rho)d^v(\rho))f(\mathbb{E}[a] + q(\rho)d^v(\rho)) - (1 - F(\mathbb{E}[a] + q(\rho)d^v(\rho)))q'(\rho)$$
$$\frac{\partial q_1^k}{\partial \rho} = q(\rho)(1 - q(\rho))g'(\rho) - q(\rho)d^v(\rho))f(\mathbb{E}[a] - (1 - q(\rho))d^v(\rho)) + (1 - F(\mathbb{E}[a] - (1 - q(\rho))d^v(\rho)))q'(\rho)$$

If $q' = 0$, these reduce to the derivatives derived above. While it is not a completely fair comparison as any change to q' will also change q globally, increasing q' at any fixed ρ will decrease $\frac{\partial q_0^k}{\partial \rho}$ and increase $\frac{\partial q_1^k}{\partial \rho}$. Referring back to equation 11, this will tend to make the slope of $\overline{q}$ steeper, making it easier to have multiple equilibria.