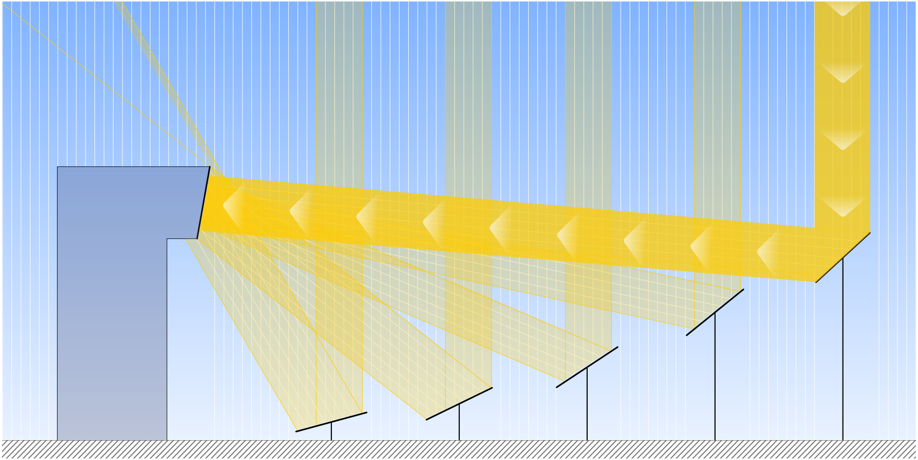
Two Dimensional Toy Model and Layout Optimization

Marko Lalovic



Toy Model

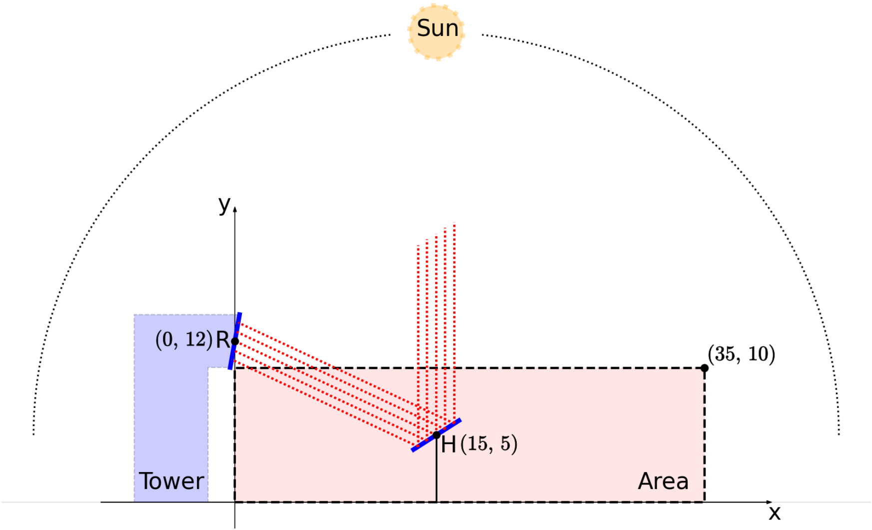


Figure 1: Model of a solar tower power plant in two dimensions

Toy Model

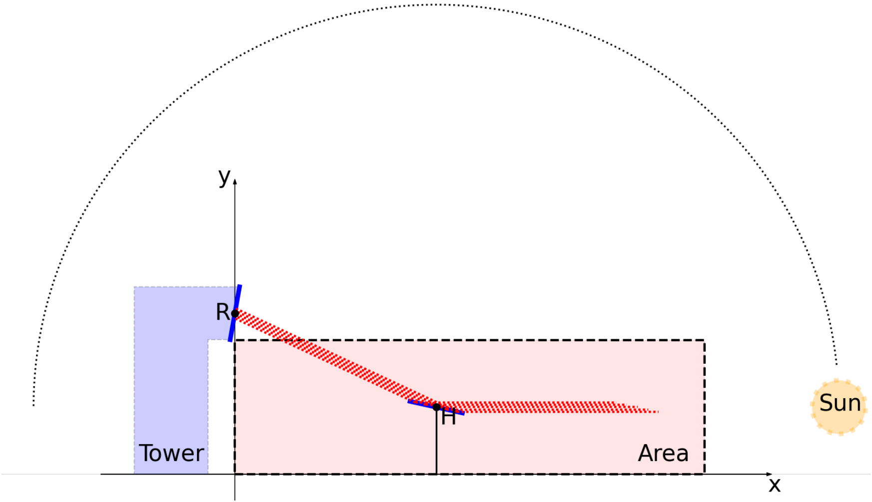


Figure 2: Sun at 0°

Toy Model

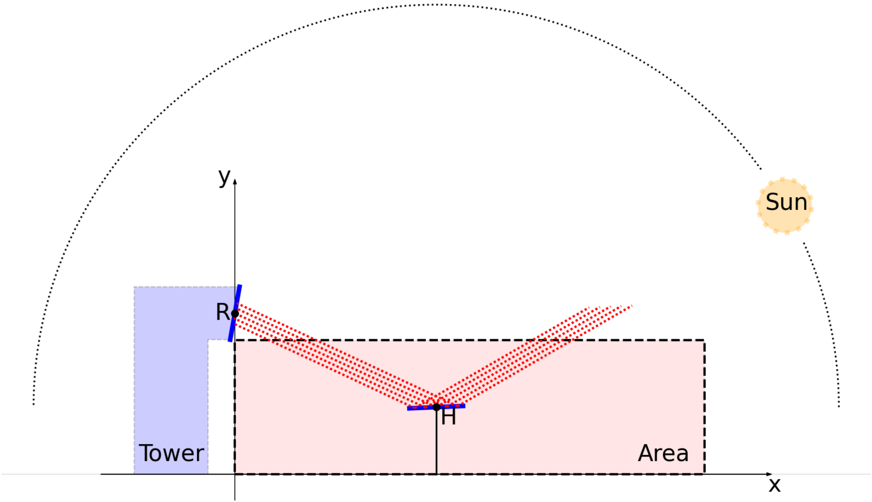


Figure 3: Sun at 30°

Toy Model

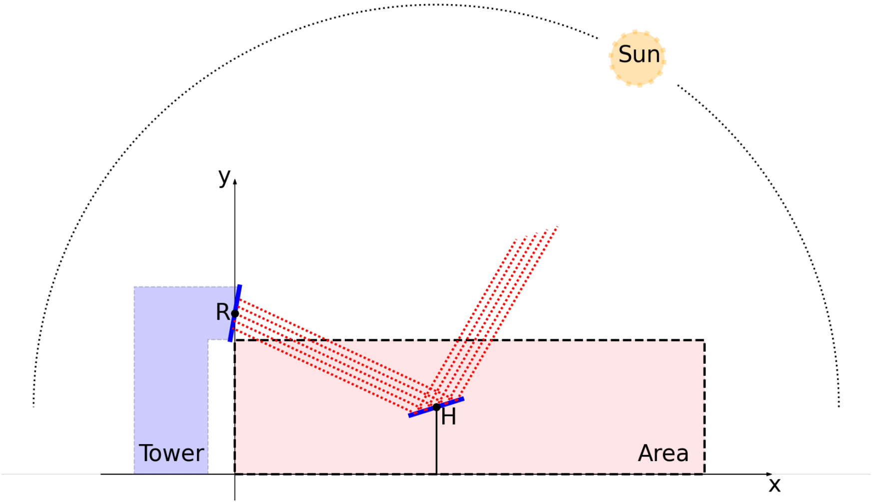


Figure 4: Sun at 60°

Toy Model

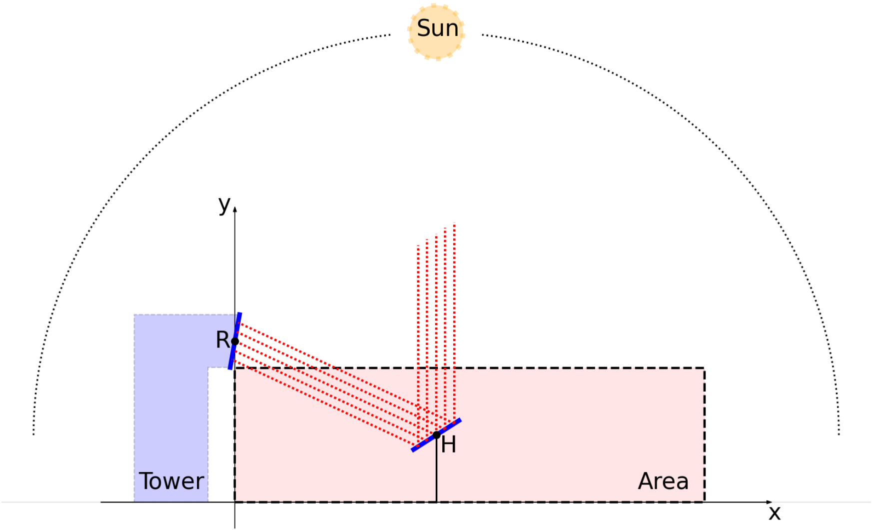


Figure 5: Sun at 90°

Toy Model

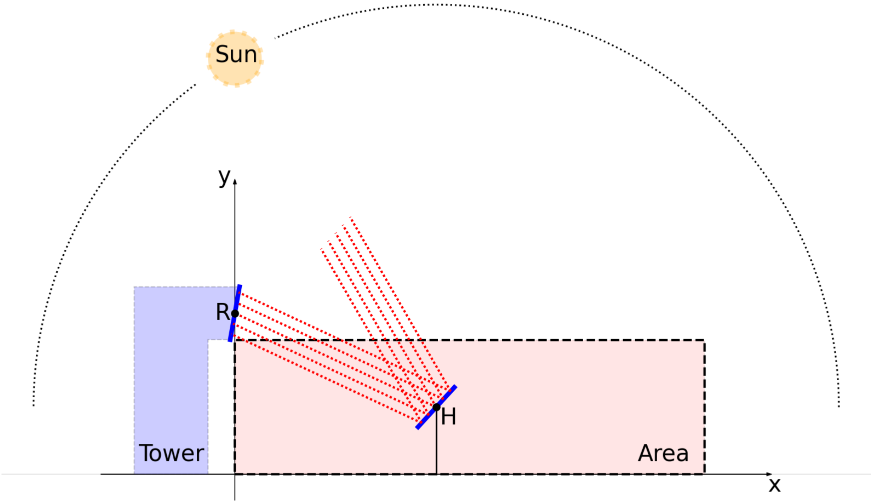


Figure 6: Sun at 120°

Toy Model

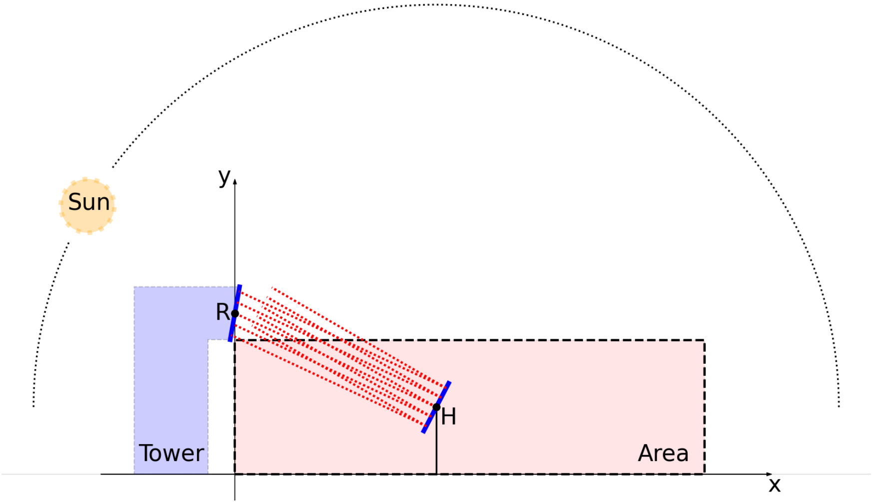


Figure 7: Sun at 150°

Toy Model

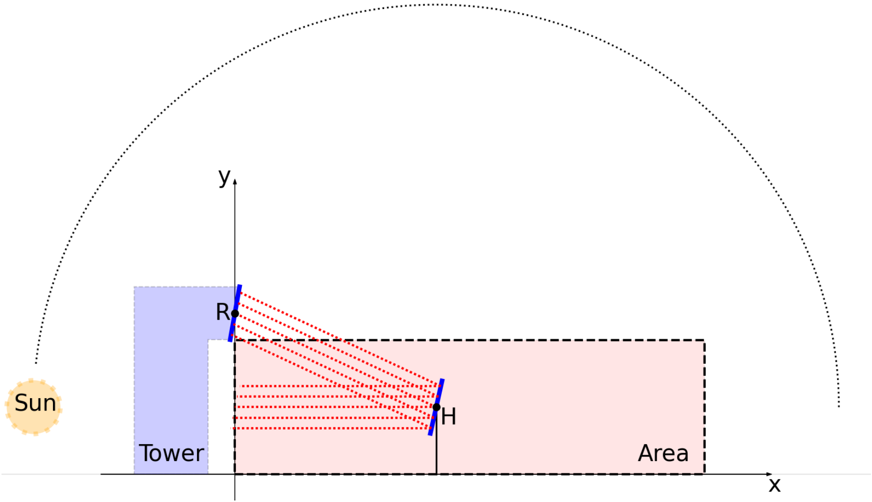
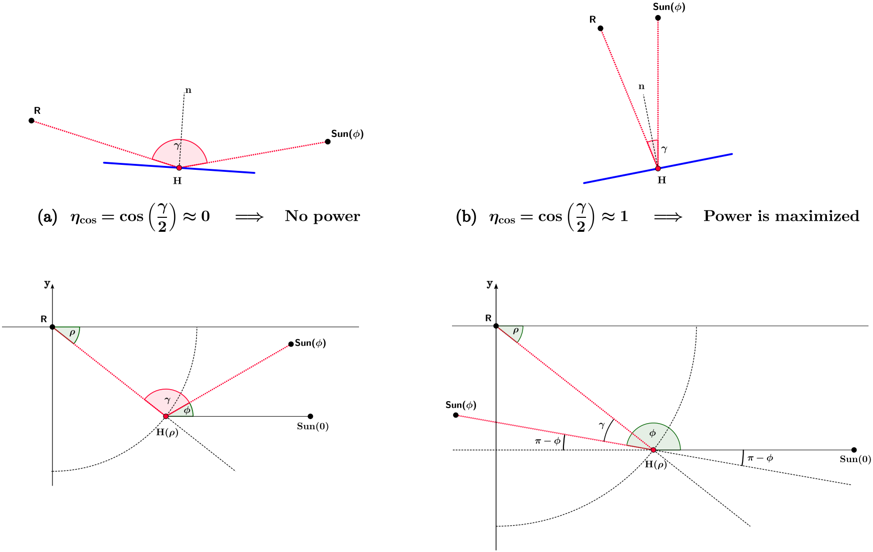


Figure 8: Sun at 180°

Layout Optimization



(a) $\eta_{\cos} = \cos\left(\frac{\gamma}{2}\right) \approx 0 \quad \Longrightarrow \quad$ No power

(b) $\eta_{\cos} = \cos\left(\frac{\gamma}{2}\right) \approx 1 \quad \Longrightarrow \quad$ Power is maximized

(c) $\phi + \rho + \gamma = \pi$

(d) $\phi + \rho - \gamma = \pi$

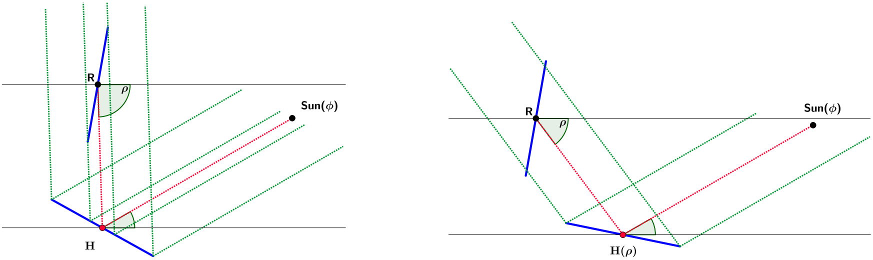
Figure 9: Cosine Effect

Layout Optimization

Using relations **(c)** and **(d)**

$$\eta_{cos}(\rho) = \int_0^{\pi} \cos\left(\frac{|\pi - \phi - \rho|}{2}\right) d\phi = 2\left(\sin\left(\frac{\rho}{2}\right) + \cos\left(\frac{\rho}{2}\right)\right)$$

$$\eta'_{cos}(\rho) = \cos\left(\frac{\rho}{2}\right) - \sin\left(\frac{\rho}{2}\right) \implies \rho^* = \frac{\pi}{2}$$



(a) $\eta_{\text{spill}} \approx 0 \implies$ **No power**

(b) $\eta_{\text{spill}} = 1 \implies$ **Power is maximized**

Figure 10: Spillage Effect

Layout Optimization

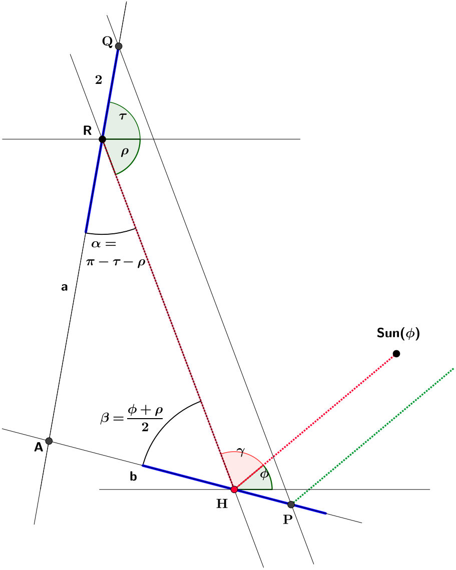


Figure 11: Spillage Effect Solution

Using

$$\frac{a + |RQ|}{a} = \frac{b + |HP|}{b} \quad \text{(Similarity)}$$

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \quad \text{(Sine Rule)}$$

we get

$$\eta_{spill} = \min\left(1, \frac{|HP|}{2}\right) \tag{1}$$

$$= \min\left(1, \frac{\sin(\pi - \tau - \rho)}{\sin\left(\frac{\phi+\rho}{2}\right)}\right) \tag{2}$$

Layout Optimization

Let $d = \|H - R\|$, then the *atmospheric attenuation* [1] is

$$\eta_{\mathrm{aa}}(d_i) = \begin{cases} 0.99321 - 1.176 \cdot 10^{-4} d + 1.97 \cdot 10^{-8} d^2, & d \leq 1000m \\ \exp(-1.106 \cdot 10^{-4} d), & d > 1000m \end{cases} \tag{3}$$

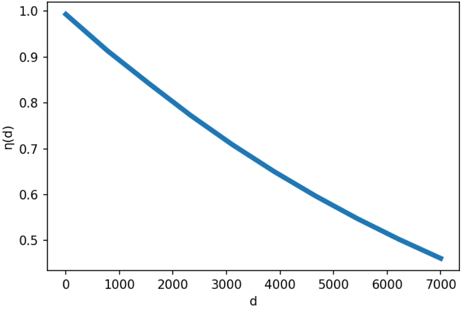


Figure 12: Atmospheric Attenuation Effect

[1] P. Richter, Simulation and optimization of solar thermal power plants. PhD dissertation, RWTH Aachen University, 2017.

Layout Optimization

Let *Daily Energy Production* (DEP) be

$$\text{DEP}^* = \int_0^\pi d\phi = \pi \tag{4}$$

$$\text{DEP}(\rho, d) = \eta_{aa}(d) \cdot \frac{1}{\pi} \int_0^\pi \eta_{cos}(\rho, \phi) \cdot \eta_{spill}(\rho, \phi) d\phi \tag{5}$$

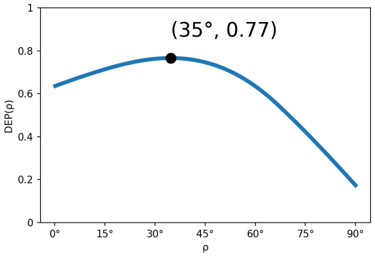


Figure 13: $\text{DEP}(\rho, d) = \text{DEP}(\rho)$.

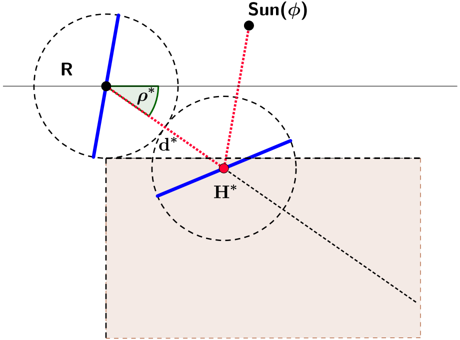


Figure 14: Singleton Solution.

Layout Optimization

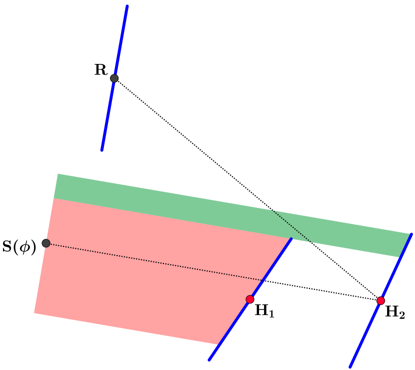


Figure 15: Shading Effect.

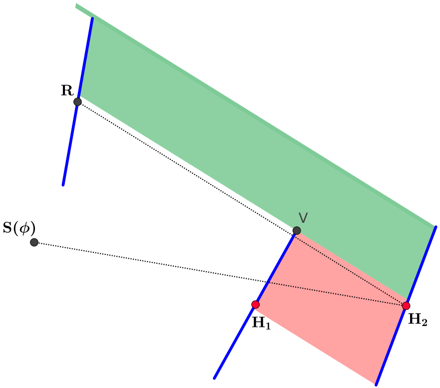
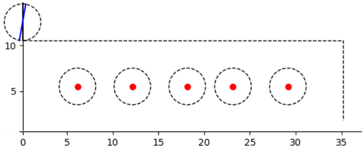


Figure 16: Blocking Effect.

Layout Optimization

For a given *layout* $H_1, H_2, \dots, H_n$

$$\widehat{\text{DAP}}(H_1, H_2, \dots, H_n) := \frac{1}{n \cdot m} \sum_{i=1}^{n} \eta_{aa} \cdot \sum_{k=1}^{m} \eta_{cos}(\phi_k) \cdot \eta_{ray}(\phi_k) \tag{6}$$



Example (Tiny Layout)

$$H_1 = (6,5), \quad H_2 = (12,5), \quad H_3 = (18,5), \quad H_4 = (23,5), \quad H_5 = (29,5)$$

Using $m = 17$ and 5 rays per heliostat we get $\widehat{\text{DAP}} \approx 0.53$ [a]

[a] Found one error in the code, but the results do not change significantly so I'm showing the same results as in the Report.

Layout Optimization

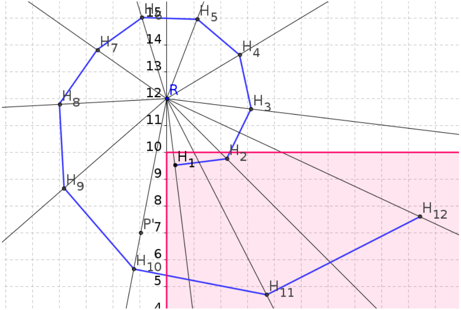


Figure 17: Spiral idea.

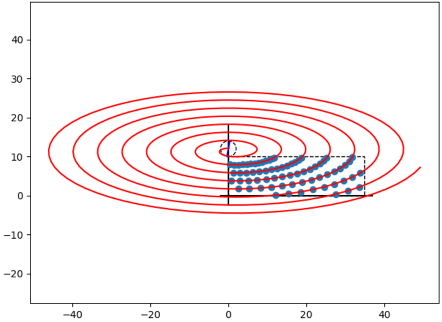


Figure 18: Possible heliostat positions.

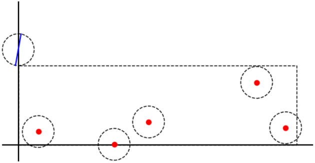


Figure 19: Spiral Layout that "grew up on the spiral".

Layout Optimization

Placing the heliostats equidistantly some distance apart on the parabola

$$y = \frac{1}{48}x^2 \tag{7}$$

with the focal point $R = (0, 12)$

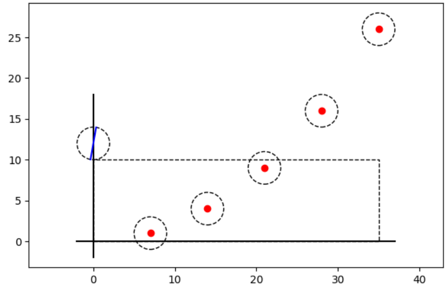


Figure 20: Parabolic Mirror Layout.

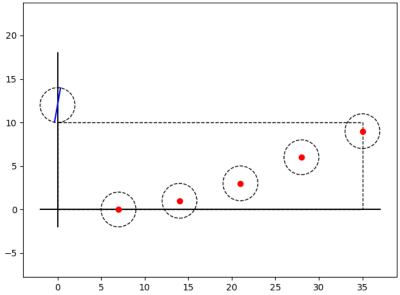


Figure 21: Valid Parabolic Layout.

Layout Optimization

Plots of $\widehat{\mathrm{DAP}}(x_3, y_3)$

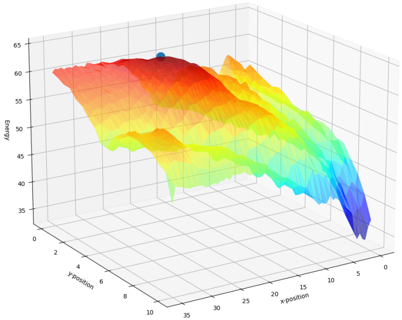


Figure 22: Landscape.

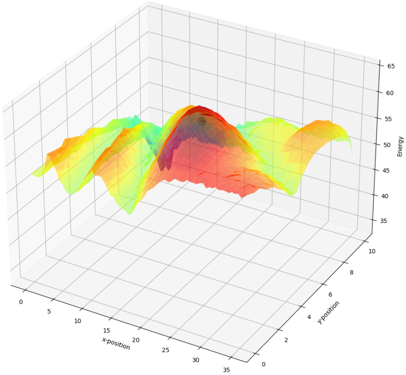


Figure 23: Landscape rotated.

Layout Optimization

Running gradient ascent

$$x_k = x_{k-1} + \sigma \cdot g \tag{8}$$

where g is first finite difference estimate and $\sigma = 1$

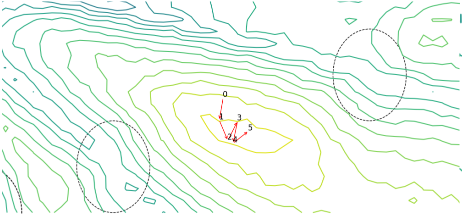


Figure 24: Starting close to the optimum.

Figure 25: Starting far from the optimum.

Layout Optimization

Running gradient ascent

$$x_k = x_{k-1} + \sigma \cdot g \tag{9}$$

where g is first finite difference estimate and $\sigma = 1$

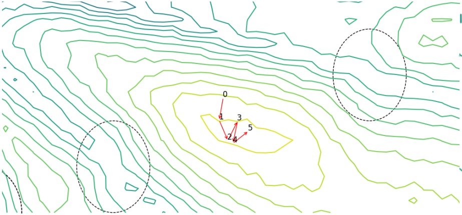


Figure 26: Starting close to the optimum.

Figure 27: Starting far from the optimum.

Layout Optimization

Running Sequential Quadratic Programming (SQP) method, from a class of Lagrange-Newton methods

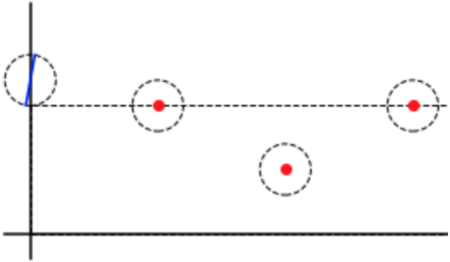


Figure 28: Initial value.

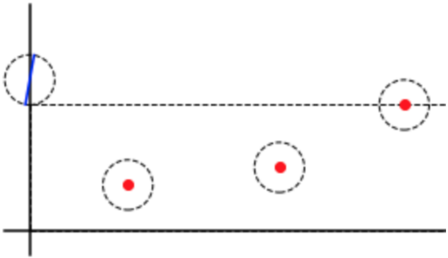
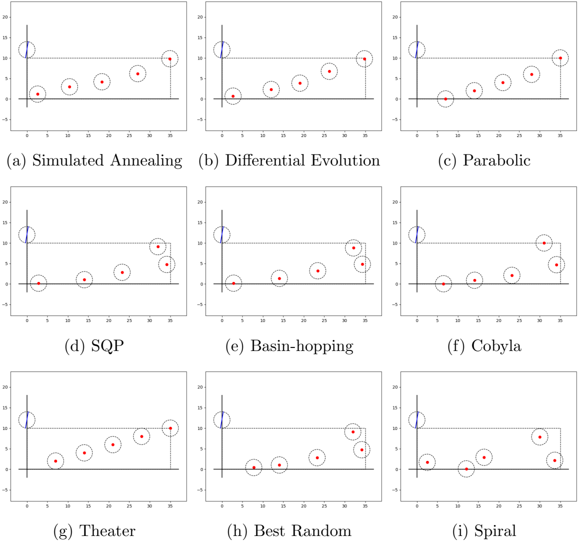


Figure 29: Converged.

Layout Optimization

Running a set of different methods



(a) Simulated Annealing (b) Differential Evolution (c) Parabolic

(d) SQP (e) Basin-hopping (f) Cobyla

(g) Theater (h) Best Random (i) Spiral

Figure 30: Top 9 Layouts.

Layout Optimization

- Most layouts were only exploiting the inaccuracies in the Toy model
- Increasing the number of angles from 17 to 180 and rays from 5 to 50:

	$\widehat{\text{DAP}}$
annealing-layout	0.77
de-layout	0.77
parabolic-layout	0.74
basinhopping-layout	0.72
sqp-layout	0.72
cobyla-layout	0.72
theater-layout	0.71
random-layout	0.70
spiral-layout	0.66

Table 1: Layout optimization accuracy.

	$\widehat{\text{DAP}}$
parabolic-layout	0.80
annealing-layout	0.77
de-layout	0.77
theater-layout	0.77
cobyla-layout	0.76
random-layout	0.75
basinhopping-layout	0.73
sqp-layout	0.73
spiral-layout	0.66

Table 2: Increased model accuracy.

TODOs and Open Questions

- Improve the model and repeat the optimization and evaluation
- Try showing that putting heliostats on parabola is the best layout
- Contribute github.com/markolalovic/math-mods-camp

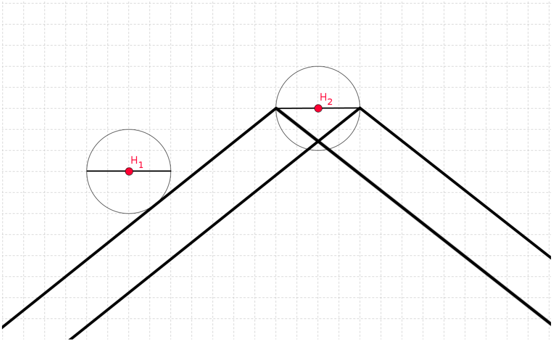


Figure 31: Minimizing the effects of shading and blocking.