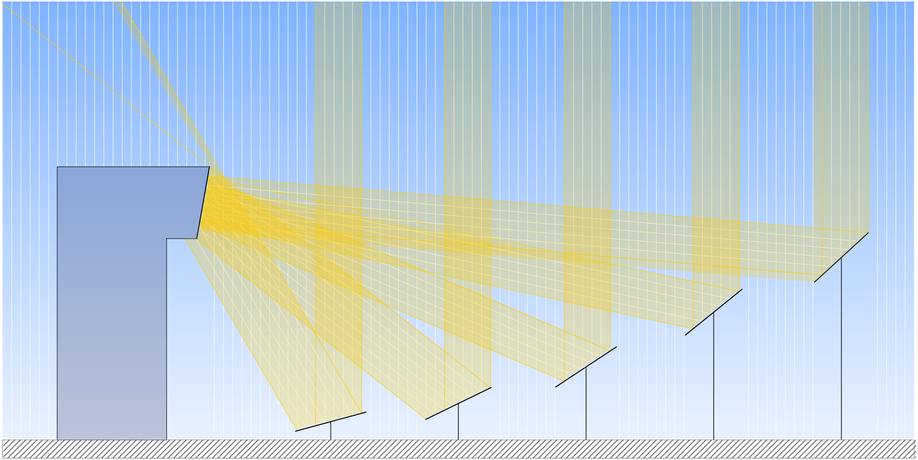
Two Dimensional Toy Model and Layout Optimization

Marko Lalovic



Toy Model

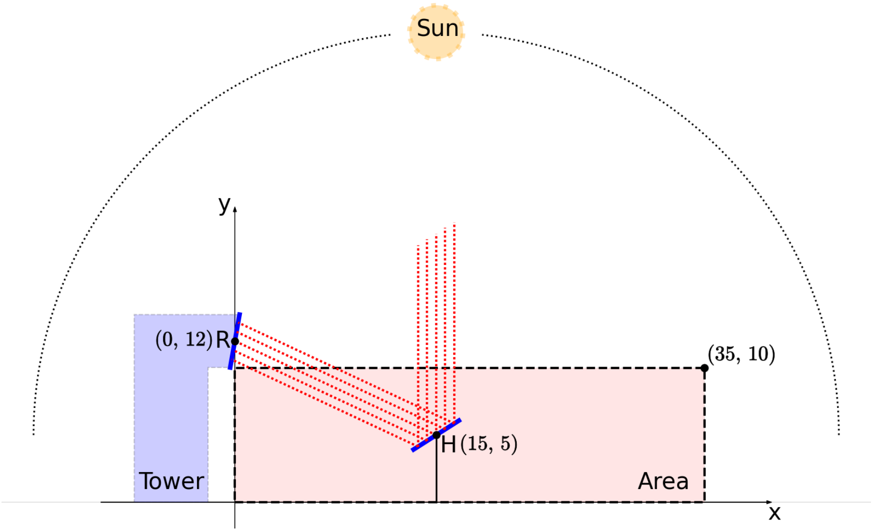


Figure 1: Toy Model and (Tiny) Power Plant, units are meters. Solar angle $\phi = 90°$.

Toy Model

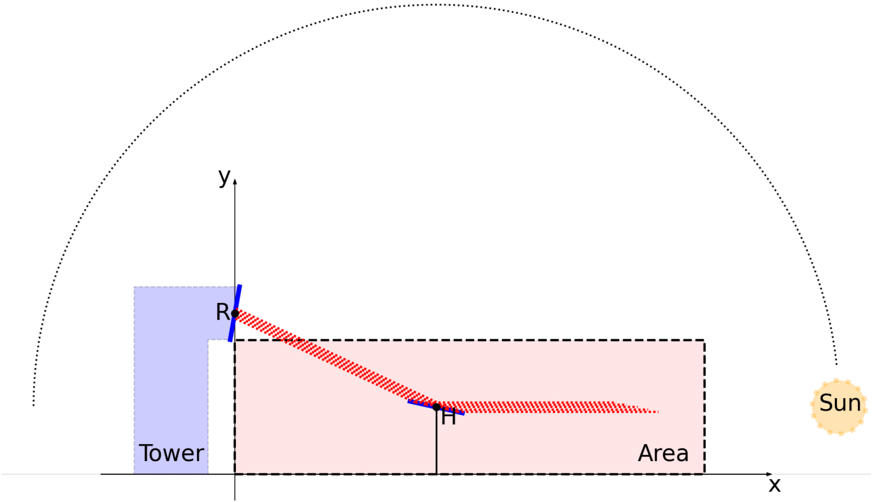


Figure 2: Solar angle $\phi = 0°$

Toy Model

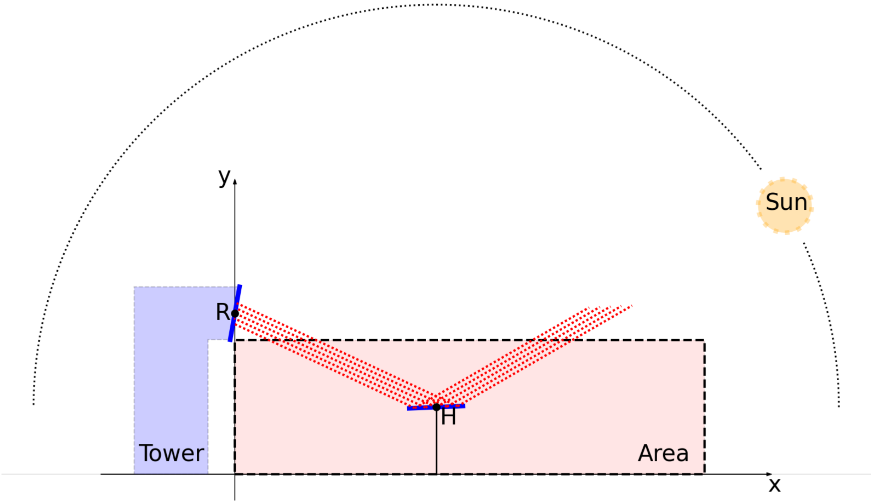


Figure 3: Solar angle $\phi = 30^\circ$

Toy Model

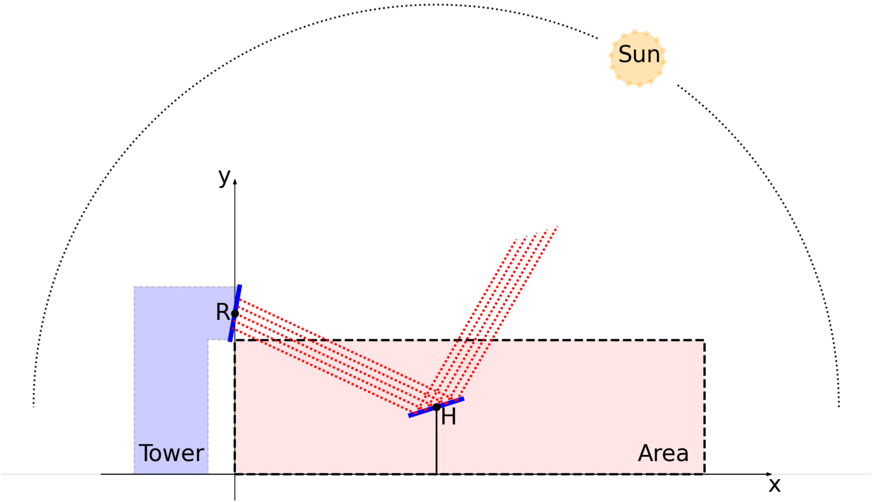


Figure 4: Solar angle $\phi = 60°$

Toy Model

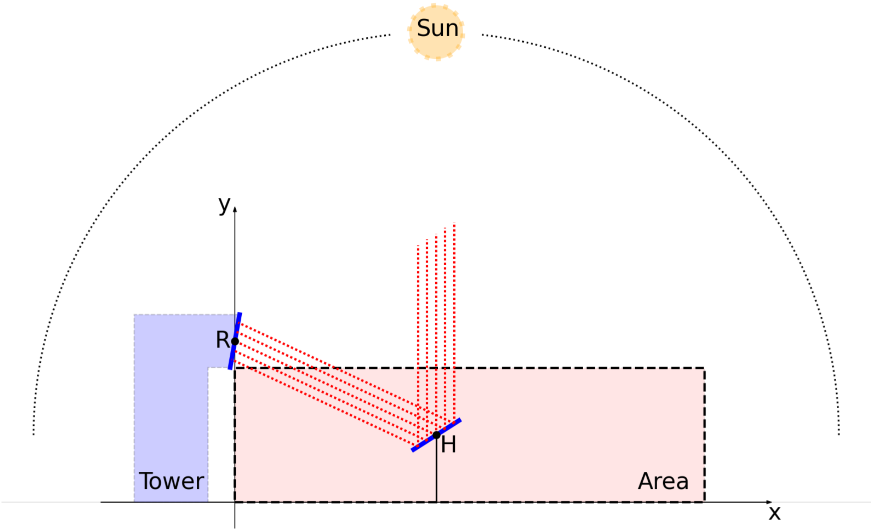


Figure 5: Solar angle $\phi = 90°$

Toy Model

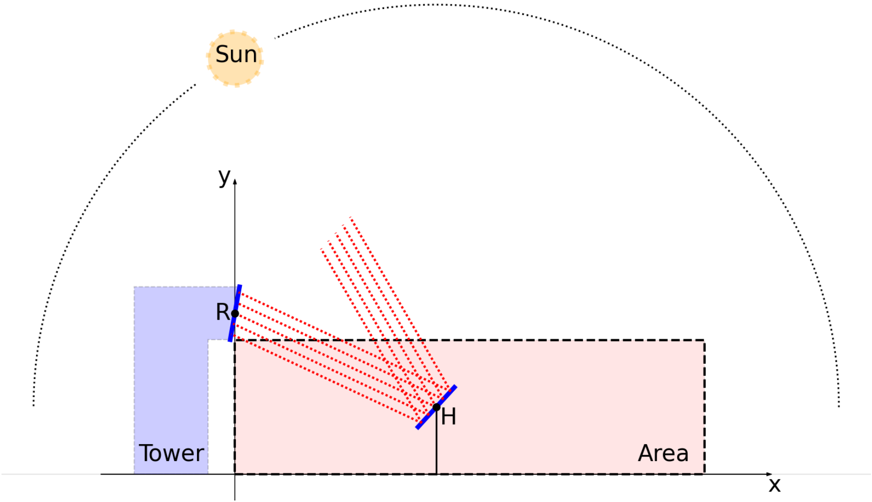


Figure 6: Solar angle $\phi = 120°$

Toy Model

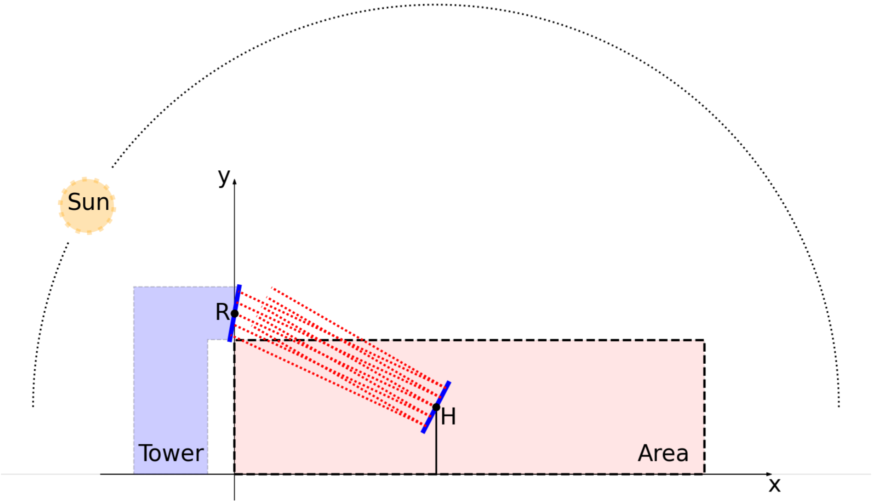


Figure 7: Solar angle at $\phi = 150^\circ$

Toy Model

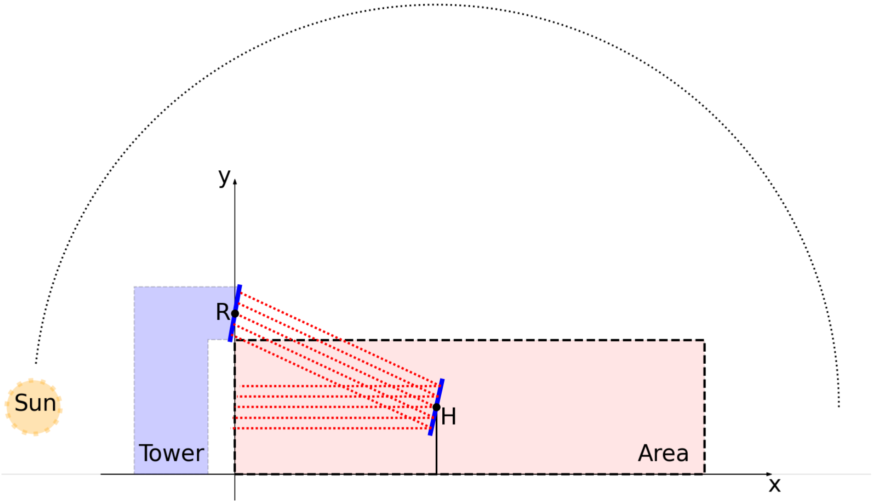


Figure 8: Solar angle at $\phi = 180°$

Definition (Layout Optimization Problem)

Find the optimal *layout*, positions H_i of heliostats $i = 1, \ldots n$, that maximizes the daily efficiency E of the Toy Model Power Plant. Subject to:

- Positions H_i are in the defined Area of the Plant
- $H_i, H_j, i \neq j$ are at the distance at least 4m
- H_i, R are at the distance at least 4m

for all $i = 1, \ldots n$.

Where we only consider the effects caused by

Cosine effect η_{cos} : Solar angle not being perpendicular to the heliostat

Spillage effect η_{spill} : Reflected rays from flat mirror missing the receiver

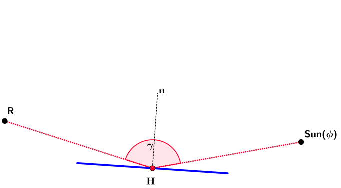
Atmospheric Attenuation η_{aa}: Sun radiation being lost due to attenuation

In case of $n > 1$ also

Shading effect η_{shade} : Solar rays being blocked by other heliostats

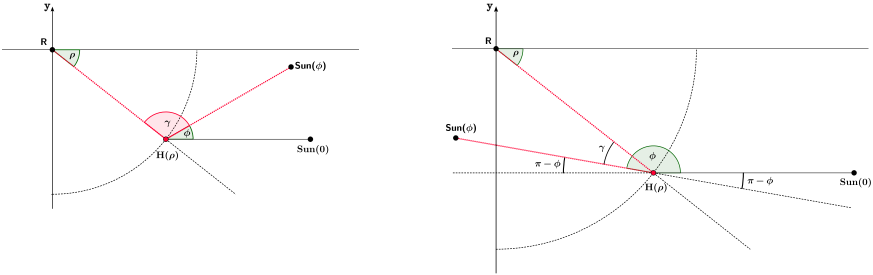
Blocking Effect η_{block} : Reflected rays being blocked by other heliostats

Single Heliostat Optimization $H = H(\rho)$



(a) $\eta_{\cos} = \cos\left(\frac{\gamma}{2}\right) \approx 0 \implies$ No power

(b) $\eta_{\cos} = \cos\left(\frac{\gamma}{2}\right) \approx 1 \implies$ Power is maximized



(c) $\phi + \rho + \gamma = \pi$

(d) $\phi + \rho - \gamma = \pi$

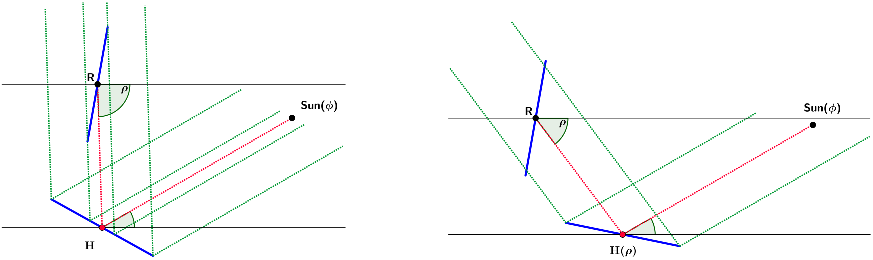
Figure 9: Cosine Effect

Single Heliostat Optimization $H = H(\rho)$

Using relations **(c)** and **(d)**

$$\eta_{cos}(\rho) = \int_0^{\pi} \cos\left(\frac{|\pi - \phi - \rho|}{2}\right) d\phi = 2\left(\sin\left(\frac{\rho}{2}\right) + \cos\left(\frac{\rho}{2}\right)\right)$$

$$\eta'_{cos}(\rho) = \cos\left(\frac{\rho}{2}\right) - \sin\left(\frac{\rho}{2}\right) \implies \rho^* = \frac{\pi}{2}$$



(a) $\eta_{\text{spill}} \approx 0 \implies$ **No power**

(b) $\eta_{\text{spill}} = 1 \implies$ **Power is maximized**

Figure 10: Spillage Effect

Single Heliostat Optimization $H = H(\rho)$

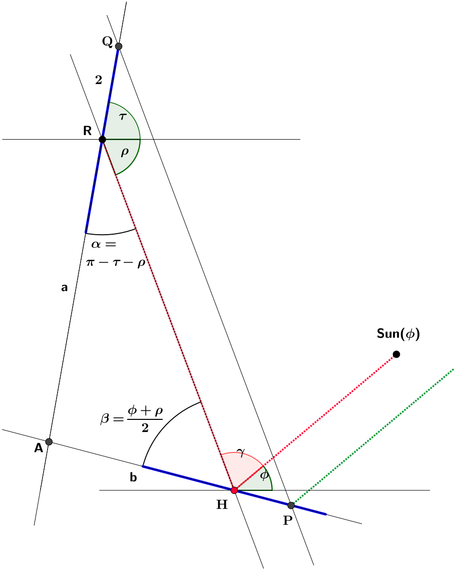


Figure 11: Spillage Effect Solution

Since $\tau = 80^\circ$ and $|RQ| = 2$, then by using

$$\frac{a+2}{a} = \frac{b+|HP|}{b} \quad \text{(Similarity)}$$

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \quad \text{(Sine Rule)}$$

we get

$$\eta_{spill} = \min\left(1, \frac{|HP|}{2}\right) \tag{1}$$

$$= \min\left(1, \frac{\sin(\pi - \tau - \rho)}{\sin\left(\frac{\phi+\rho}{2}\right)}\right) \tag{2}$$

Single Heliostat Optimization $H = H(d)$

Let $d = \|H - R\|$, then the *atmospheric attenuation* [1] is

$$\eta_{\text{aa}}(d) = \begin{cases} 0.99321 - 1.176 \cdot 10^{-4} d + 1.97 \cdot 10^{-8} d^2, & d \leq 1000m \\ \exp(-1.106 \cdot 10^{-4} d), & d > 1000m \end{cases} \tag{3}$$

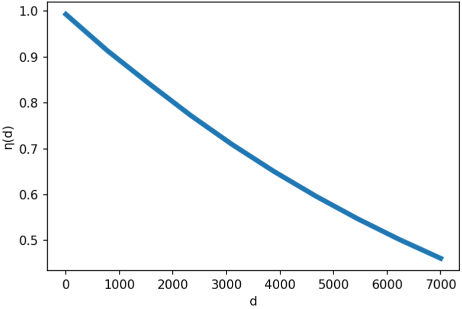


Figure 12: Atmospheric Attenuation Effect

$$\implies d^* = \arg\min_{d} \|H - R\| \qquad \text{subject to: } \|H - R\| > 4$$

[1] P. Richter, Simulation and optimization of solar thermal power plants. PhD dissertation, RWTH Aachen University, 2017.

Definition (Daily efficiency E)

$$\mathrm{E}^* = \int_0^{\pi} d\phi = \pi \qquad \text{when} \quad \eta_{aa} = \eta_{cos} = \eta_{spill} = 1 \tag{4}$$

$$\mathrm{E}(\rho, d) = \eta_{aa}(d) \cdot \frac{1}{\pi} \int_0^{\pi} \eta_{cos}(\rho, \phi) \cdot \eta_{spill}(\rho, \phi) d\phi \tag{5}$$

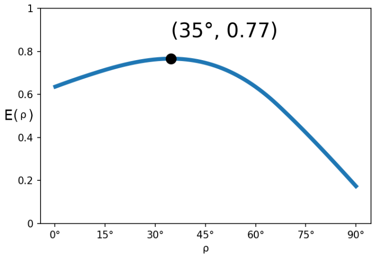


Figure 13: Plot of $\mathrm{E}(\rho) \implies \rho^* \approx 35°$

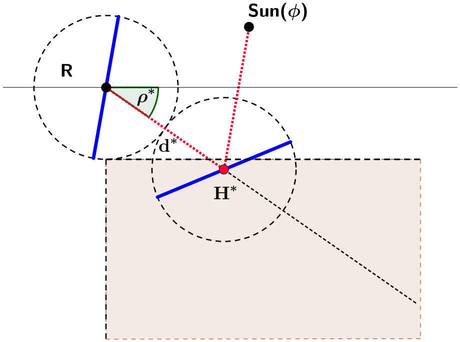


Figure 14: Solution $H^* \approx (3.29, 9.73)$.

Multiple Heliostats Optimization

For each heliostat $i = 1, \ldots n$:

$$\eta_{shade} = \text{proportion of solar rays that are not blocked by other heliostats} \tag{6}$$

$$\eta_{block} = \text{proportion of reflected rays that are not blocked by other heliostats} \tag{7}$$

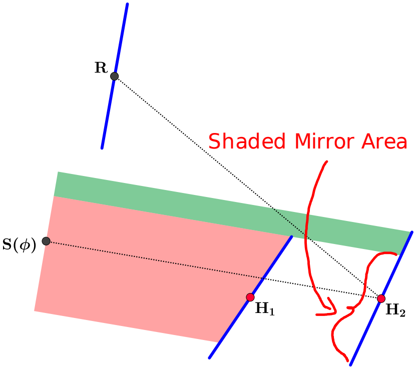


Figure 15: Shading Effect.

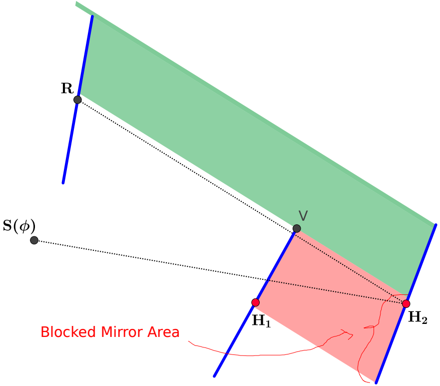
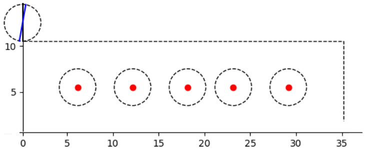


Figure 16: Blocking Effect.

Multiple Heliostats Optimization

For a given *layout* $H_1, H_2, \ldots, H_n$ we approximate daily efficiency E by

$$\widehat{\mathrm{E}}(H_1, H_2, \ldots, H_n) = \frac{1}{n \cdot m} \sum_{i=1}^{n} \eta_{aa} \cdot \sum_{k=1}^{m} \eta_{cos}(\phi_k) \cdot \eta_{ray}(\phi_k) \tag{8}$$



Example (Line Layout)

$$H_1 = (6, 5), \quad H_2 = (12, 5), \quad H_3 = (18, 5), \quad H_4 = (23, 5), \quad H_5 = (29, 5)$$

Using $m = 17$ and 5 rays per heliostat we get $\widehat{\mathrm{E}} \approx 0.53$ [a]

[a] Found one error in the code, but the results do not change significantly so I'm showing the same results as in the Report.

Multiple Heliostats Optimization

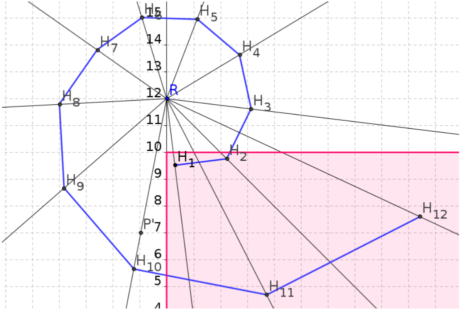


Figure 17: Golden spiral idea applied in 2D.

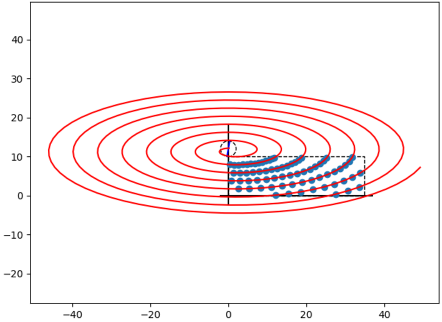


Figure 18: Valid heliostat positions.

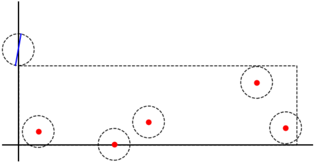


Figure 19: Spiral Layout optimized sequentially, imitating how leaves grow.

Multiple Heliostats Optimization

Placing the heliostats equidistantly some distance apart on the parabola

$$y = \frac{1}{48}x^2 \tag{9}$$

with the focal point $R = (0, 12)$

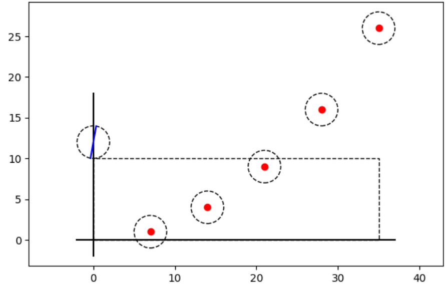


Figure 20: Parabolic Mirror Layout.

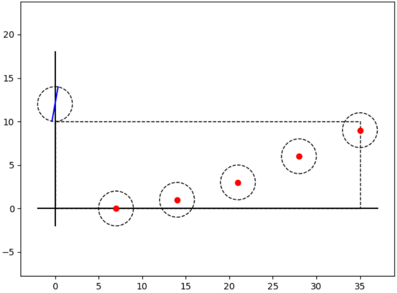


Figure 21: Valid Parabolic Layout.

Multiple Heliostats Optimization

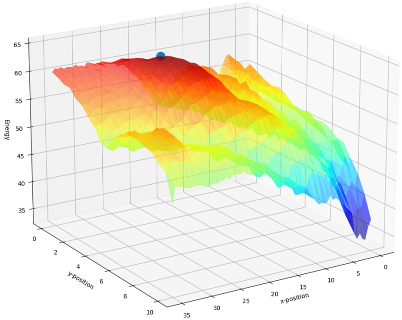


Figure 22: Landscape.

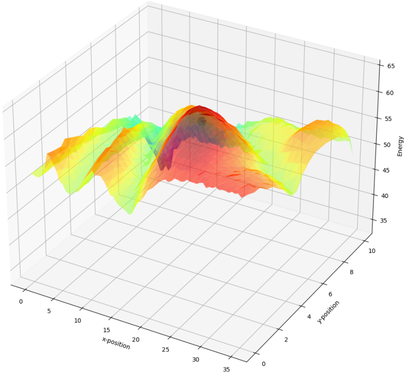


Figure 23: Same landscape rotated.

Plot of $\widehat{\mathrm{E}}(x_3, y_3)$ for the valid parabolic layout.

Multiple Heliostats Optimization

Running gradient ascent

$$x_k = x_{k-1} + \sigma \cdot g \tag{10}$$

where g is first finite difference estimate and $\sigma = 1$

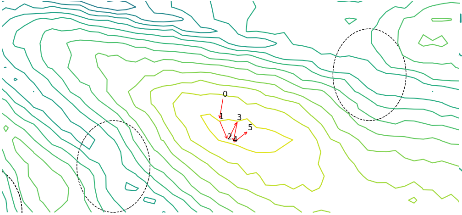


Figure 24: Starting close to the optimum.

Figure 25: Starting far from the optimum.

Multiple Heliostats Optimization

- Same story with running higher order methods and taking constraints into account
- Although, still got some interesting results
- In this case Sequential Quadratic Programming (SQP) method from a class of Lagrange-Newton methods:

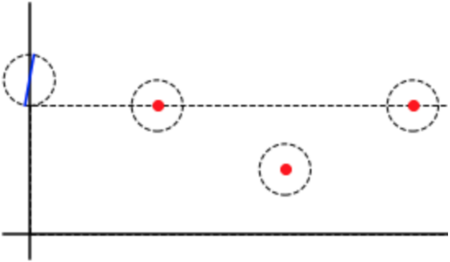


Figure 26: Initial value.

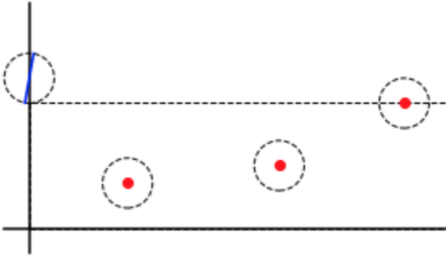
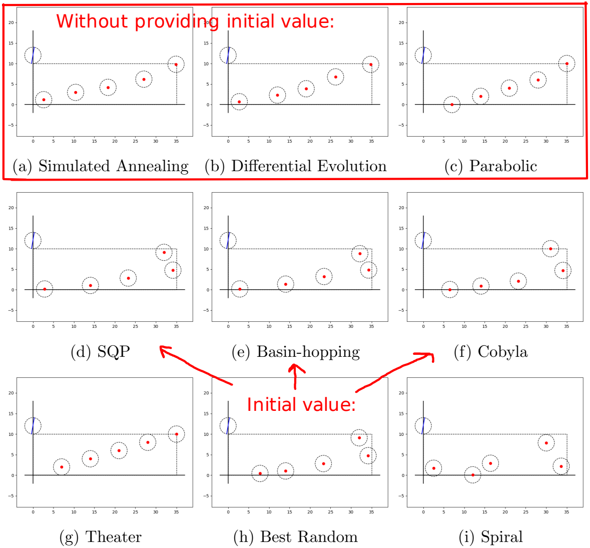


Figure 27: Converged.

Multiple Heliostats Optimization

Running a set of different methods, notice the similarity of rows!



(a) Simulated Annealing (b) Differential Evolution (c) Parabolic

(d) SQP (e) Basin-hopping (f) Cobyla

(g) Theater (h) Best Random (i) Spiral

Figure 28: Top 9 Layouts.

Numerical Results

- Increasing the number of solar angles from 17 to 180 and rays from 5 to 50
- Most layouts were only exploiting the inaccuracies in the Toy model

Layout	$\widehat{E}$
Simulated Annealing	0.77
Differential Evolution	0.77
Parabolic	0.74
Basin-hopping	0.72
SQP	0.72
Cobyla	0.72
Theater	0.71
Best Random	0.70
Spiral	0.66
Random	0.42 ± 0.09

Table 1: By using layout optimization model accuracy.

Layout	$\widehat{E}$
Parabolic	0.80
Simulated Annealing	0.77
Differential Evolution	0.77
Theater	0.77
Cobyla	0.76
Best Random	0.75
Basin-hopping	0.73
SQP	0.73
Spiral	0.66

Table 2: By using increased model accuracy.

TODOs and Open Questions

- Improve the model, test the code and repeat the optimization and evaluation
- Try showing that putting heliostats on parabola is the best layout
- Or find a better layout (github.com/markolalovic/math-mods-camp)

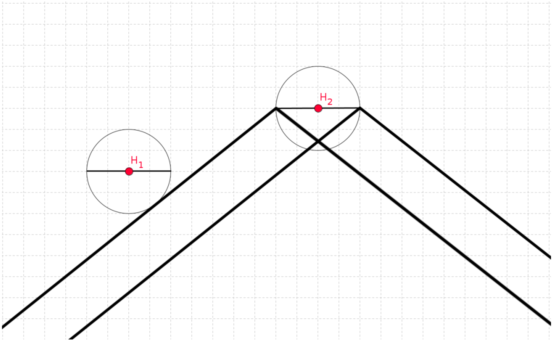


Figure 29: Minimizing the effects of shading and blocking.