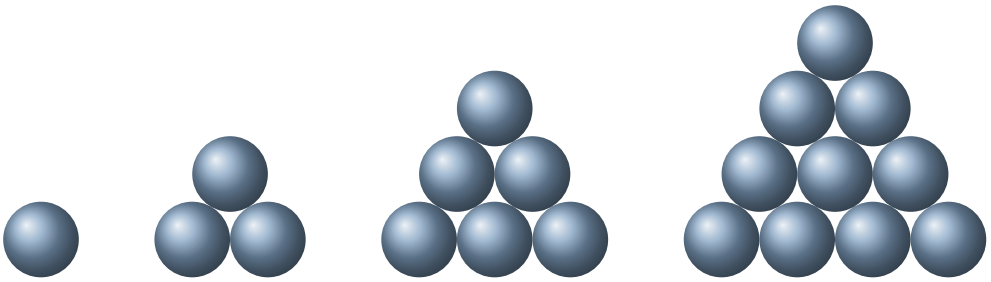
WORK∫HEET

Triangular Numbers



If we start with one ball, we can generate an equilateral triangles by adding a row of balls below the first ball such that all sides have a length of two balls. If we let the variable n represent the position of the object in a list, then ball one would have position $n = 1$ and the equilateral triangle with side length two would have position $n = 2$. To generate the triangle at position $n = 3$ in the list, we would add another row of balls to the bottom of the triangle such that each side of the triangle would have a length of three balls.

1. What is the side length of the equilateral triangle located at position $n = 4$ in the list?

2. What is the side length of the equilateral triangle located as position $n = 5$ in the list? *It might be helpful to draw the triangle.*

3. What does the value of the position, n, in the list tell us about the triangle?

4. Each object in the list that is located at position n is made up T_n balls. For example, ball at position $n = 2$ is made up of $T_2 = 3$ balls. T_n is a special list of numbers that we will call the **sequence** of triangular numbers. Complete the table below. *It might be helpful draw each triangle.*

n	1	2	3	4	5	6	7	8
T_n	1	3	6					

5. There is a formula for calculating the number of balls that make up the triangle, T_n, for any position n in the triangular number sequence:

$$T_n = \frac{n(n+1)}{2}.$$

Can you explain using a picture how this formula is working? Hint: You might want to consider the area of a triangle formula:

$$A_{\triangle} = \frac{\text{base} \times \text{height}}{2}.$$