OPTIMIZED MATRIX MULTIPLICATION AND INVERSE

Matrix inverse

Inverse using LU factorization:

$$A * X = I$$

$$\downarrow$$

Pivoting

Rearrangement of rows

$$P * A * X = P * I$$

$$\downarrow$$

GEM

Computation of L and U

$$L * U * X = P$$

$$\downarrow$$

Forward substitution

Resolution of lower triangular systems

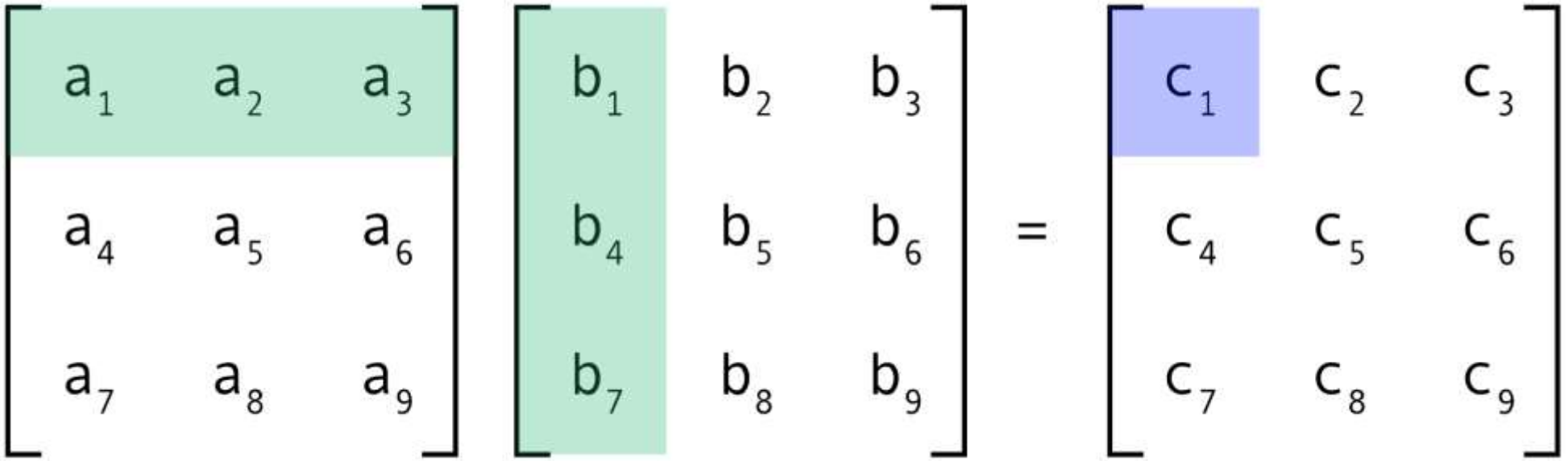
$$L * Y = P$$

$$\downarrow$$

Backward substitution

Resolution of upper triangular systems

$$U * X = Y$$



Matrix multiplication

Performance analysis

- AMDAHL'S LAW (via intel profiler)

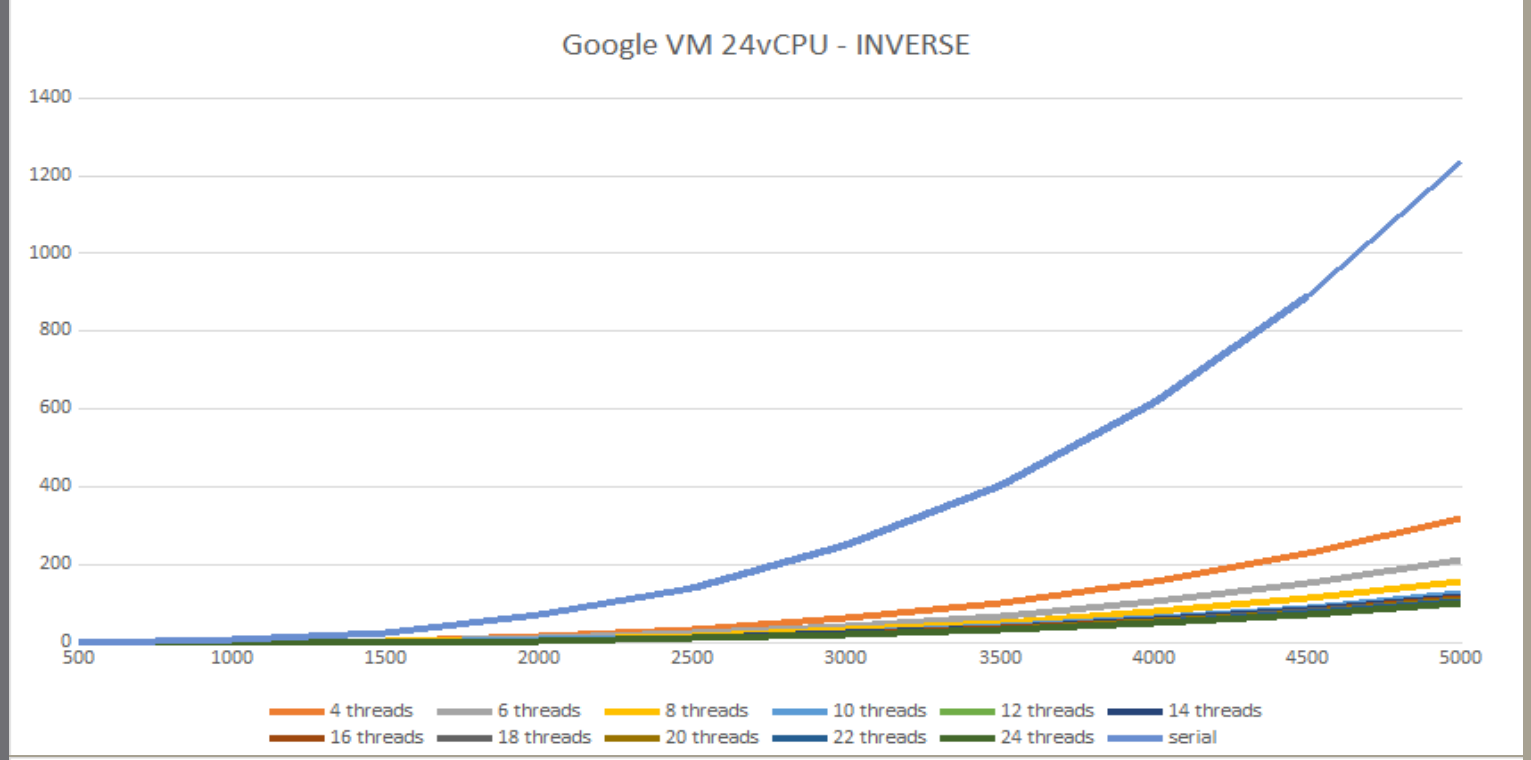
a. Speedup = 1/ ((1-P) + P/S)

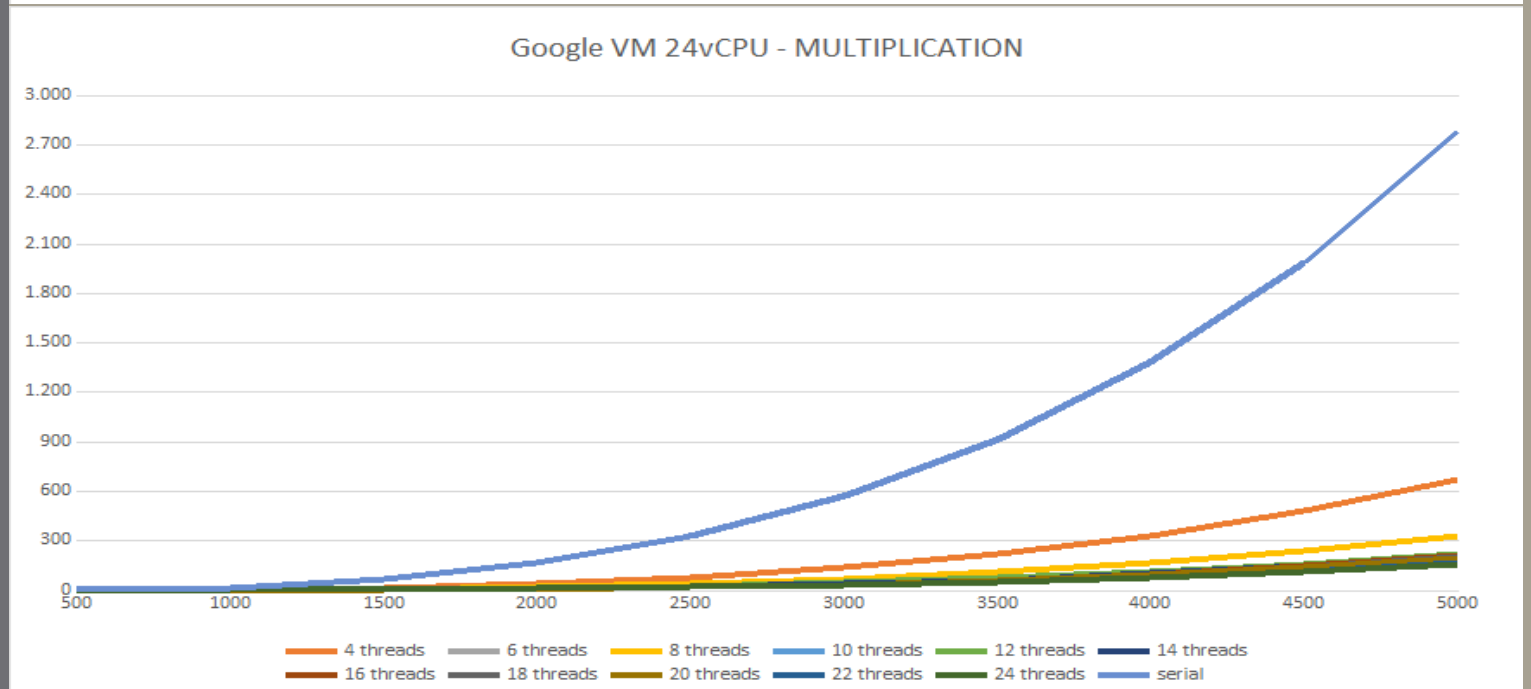
b. Speed up is 7.9

- Testing done on PC and multiple google cloud virtual machines

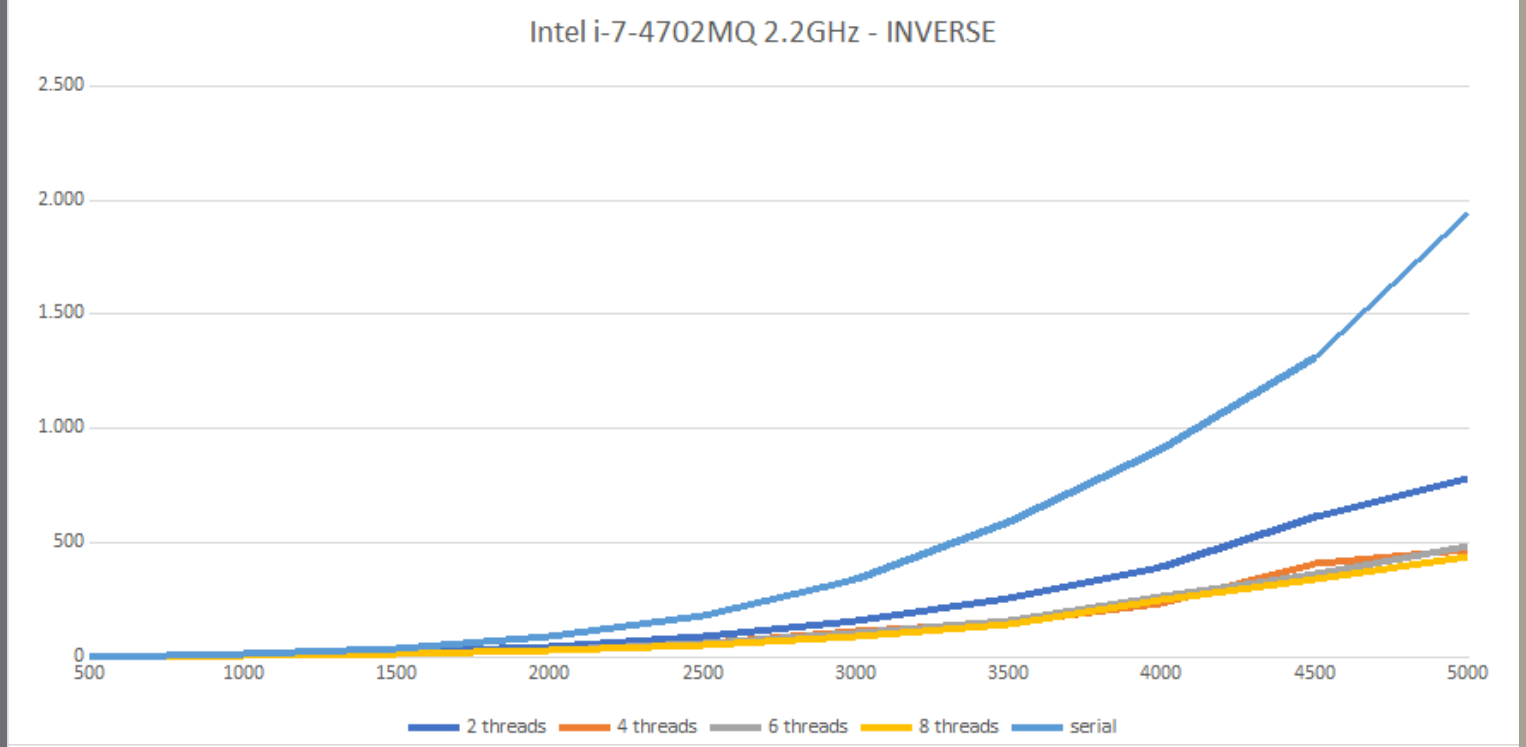
1. PC – intel i7 8 CPUs 2.4GHz
2. Google Virtual Machine – n1-highcpu-8 (8 vCPU, 7.2 GB of memory)
3. Google Virtual Machine – VM 24vCPU, 156GB of memory

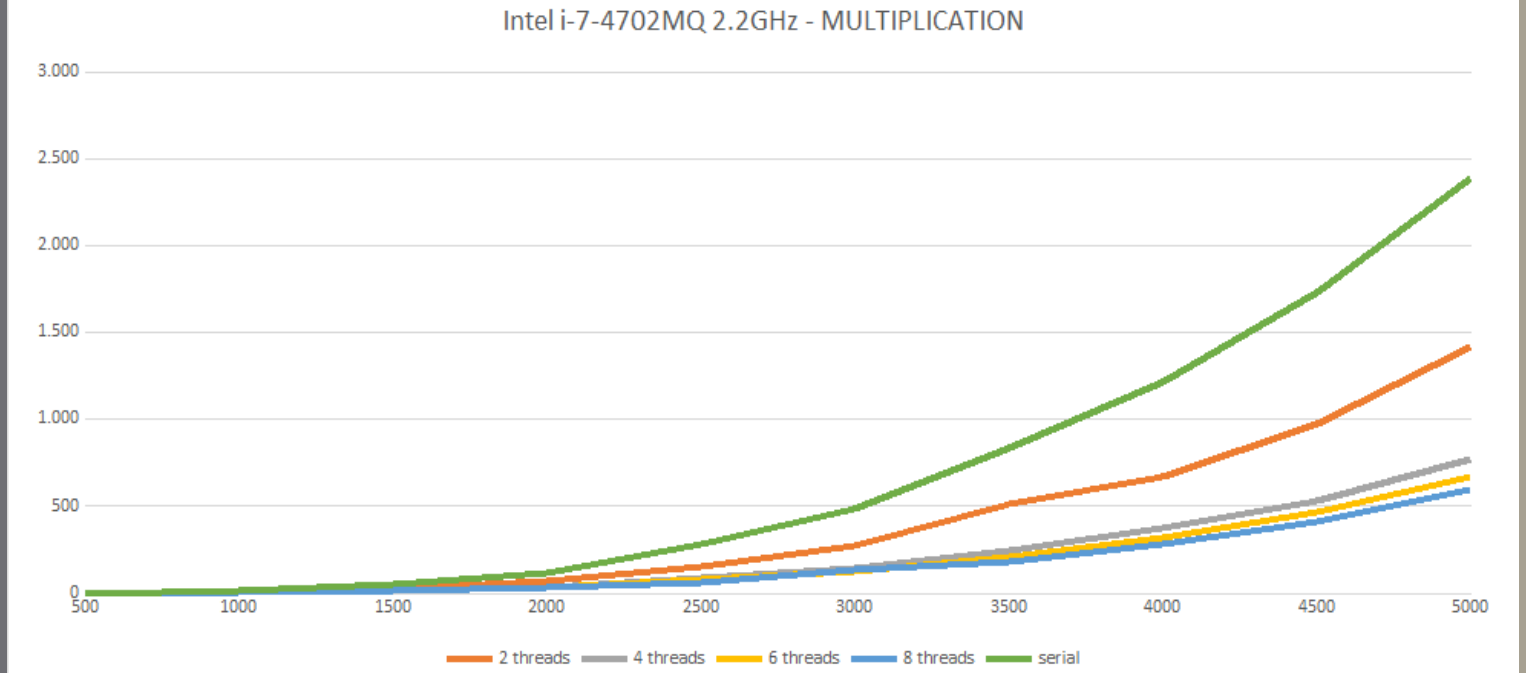
Incl.	Self	Called	Function	Location
37.35	37.35	1	backwardsub	a.out
37.29	37.29	1	forwardsub	a.out
24.91	24.91	999	gem	a.out
0.21	0.21	2 476 041	swap	a.out
0.06	0.06	2	make_diag_matrix	a.out
0.05	0.05	1 000 000	random_r	libc-2.30.so: random_r.c
25.17	0.05	1	pivoting	a.out
0.13	0.04	1	prandom	a.out
0.08	0.03	1 000 000	random	libc-2.30.so: random.c, lowlevellock.h
0.09	0.01	1 000 000	rand	libc-2.30.so: rand.c
0.09	0.00	1 000 000	0x0000000000109110	(unknown)
0.00	0.00	3 005	_int_malloc	libc-2.30.so: malloc.c
0.00	0.00	3 019	_int_free	libc-2.30.so: malloc.c
0.00	0.00	3 004	malloc	libc-2.30.so: malloc.c
0.00	0.00	2 851	systrim.isra.0.constprop.0	libc-2.30.so: malloc.c
0.00	0.00	1	_dl_addr	libc-2.30.so: dl-addr.c
0.00	0.00	3 002	unlink_chunk.isra.0	libc-2.30.so: malloc.c
0.00	0.00	3 003	free	libc-2.30.so: malloc.c
0.00	0.00	2 929	sbrk	libc-2.30.so: sbrk.c
0.00	0.00	3	allocate_matrix	a.out
0.00	0.00	3	release_mem	a.out
0.00	0.00	4	_dl_relocate_object	ld-2.30.so: dl-reloc.c, dl-machine.h, do-rel.h, ldsodefs.h
0.00	0.00	1	__GI___tunables_init	ld-2.30.so: dl-tunables.c, dl-tunables.h
0.00	0.00	98	do_lookup_x	ld-2.30.so: dl-lookup.c, ldsodefs.h
0.00	0.00	2 929	__default_morecore	libc-2.30.so: morecore.c
0.00	0.00	98	_dl_lookup_symbol_x	ld-2.30.so: dl-lookup.c
0.00	0.00	77	sysmalloc	libc-2.30.so: malloc.c, arena.c
0.00	0.00	3 003	0x00000000001090b0	(unknown)
0.00	0.00	3 003	0x0000000000109100	(unknown)
0.00	0.00	90	check_match	ld-2.30.so: dl-lookup.c
0.00	0.00	152	strcmp	ld-2.30.so: strcmp.S
0.00	0.00	2	_dl_map_object_from_fd	ld-2.30.so: dl-load.c, dl-fileid.h, dl-map-segments.h, dl-load.h, get-dynamic-info.h
0.00	0.00	4	_dl_check_map_versions	ld-2.30.so: dl-version.c
0.00	0.00	1	dl_main	ld-2.30.so: rtld.c, dl-prop.h, get-dynamic-info.h, setup-vdso.h, dl-osinfo.h
0.00	0.00	6	intel_check_word.isra.0	libc-2.30.so: cacheinfo.c, stdlib-bsearch.h
0.00	0.00	1	_dl_start	ld-2.30.so: rtld.c, dl-machine.h, get-dynamic-info.h, do-rel.h
0.00	0.00	1	__printf_fp_l	libc-2.30.so: printf_fp.c, localeinfo.h, allocalim.h, get-rounding-mode.h, rounding-mode.h
0.00	0.00	1	_dl_map_object_deps	ld-2.30.so: dl-deps.c, scratch_buffer.h
0.00	0.00	78	brk	libc-2.30.so: brk.c
0.00	0.00	1	ptmalloc_init.part.0	libc-2.30.so: arena.c, malloc.c
0.00	0.00	23	_IO_file_overflow@@GLIBC_2.2.5	libc-2.30.so: fileops.c
0.00	0.00	9	open_verify.constprop.0	ld-2.30.so: dl-load.c
0.00	0.00	1	_dl_sysdep_start	ld-2.30.so: dl-sysdep.c, dl-sysdep.c, cpu-features.c, cpu-features.c
0.00	0.00	2	__vfprintf_internal	libc-2.30.so: vfprintf-internal.c, printf-parse.h, libioP.h, lowlevellock.h
0.00	0.00	3	_dl_new_object	ld-2.30.so: dl-object.c
0.00	0.00	1	_dl_important_hwcaps	ld-2.30.so: dl-hwcaps.c, dl-hwcap.h
0.00	0.00	8	_dl_cache_libcmp	ld-2.30.so: dl-cache.c
0.00	0.00	1	open_path	ld-2.30.so: dl-load.c
0.00	0.00	22	malloc	ld-2.30.so: dl-minimal.c
0.00	0.00	13	memset	ld-2.30.so: memset-vec-unaligned-erms.S



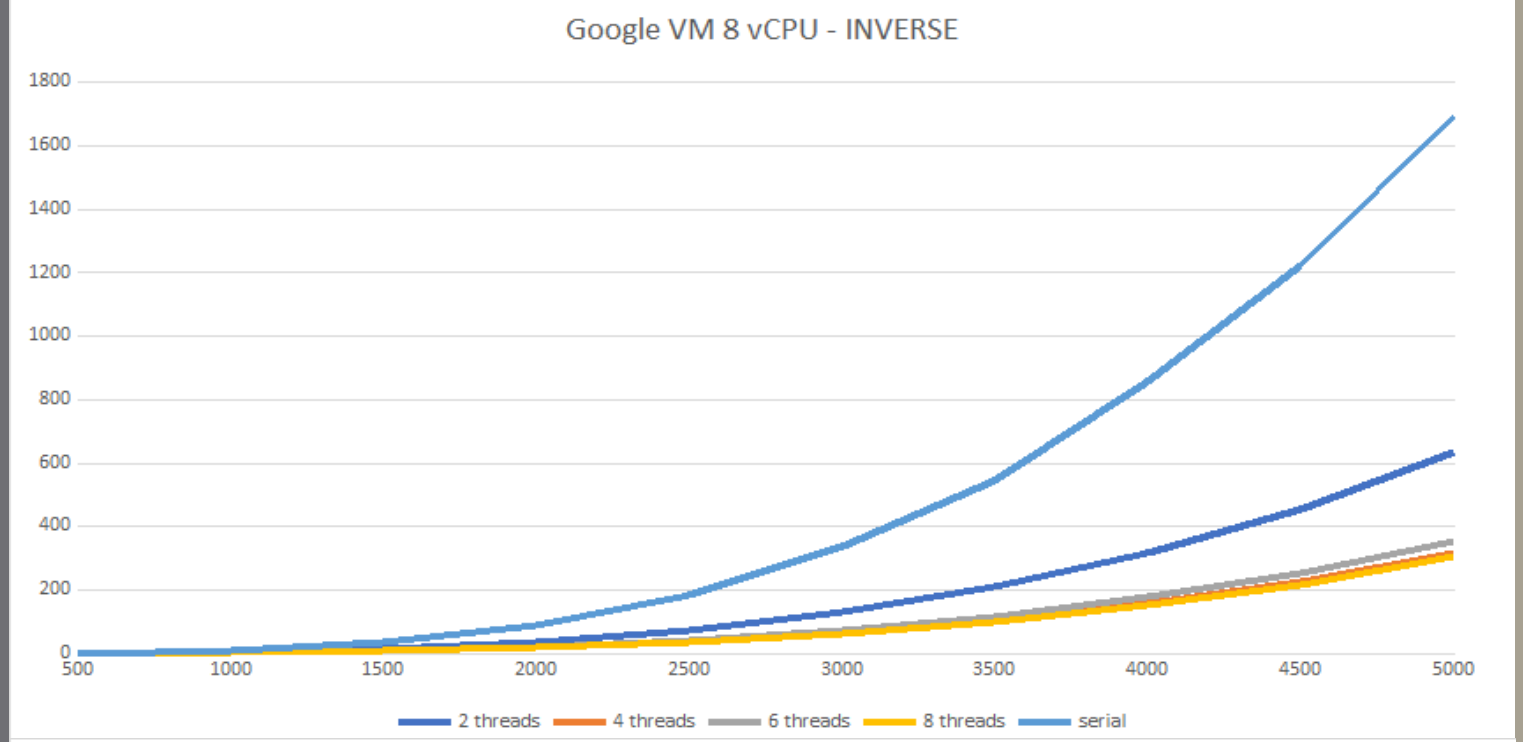


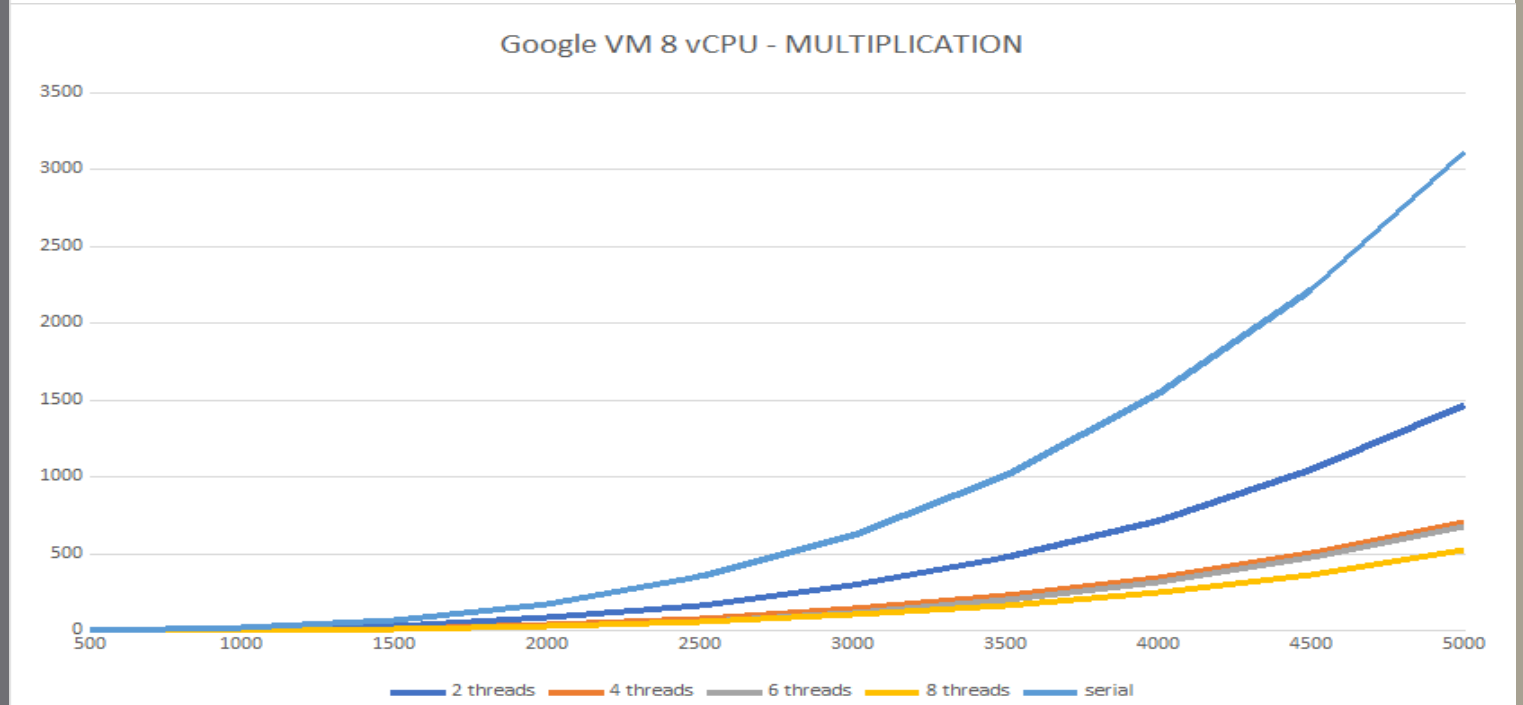
Google VM 24 CPUs





PC with 8 CPUs



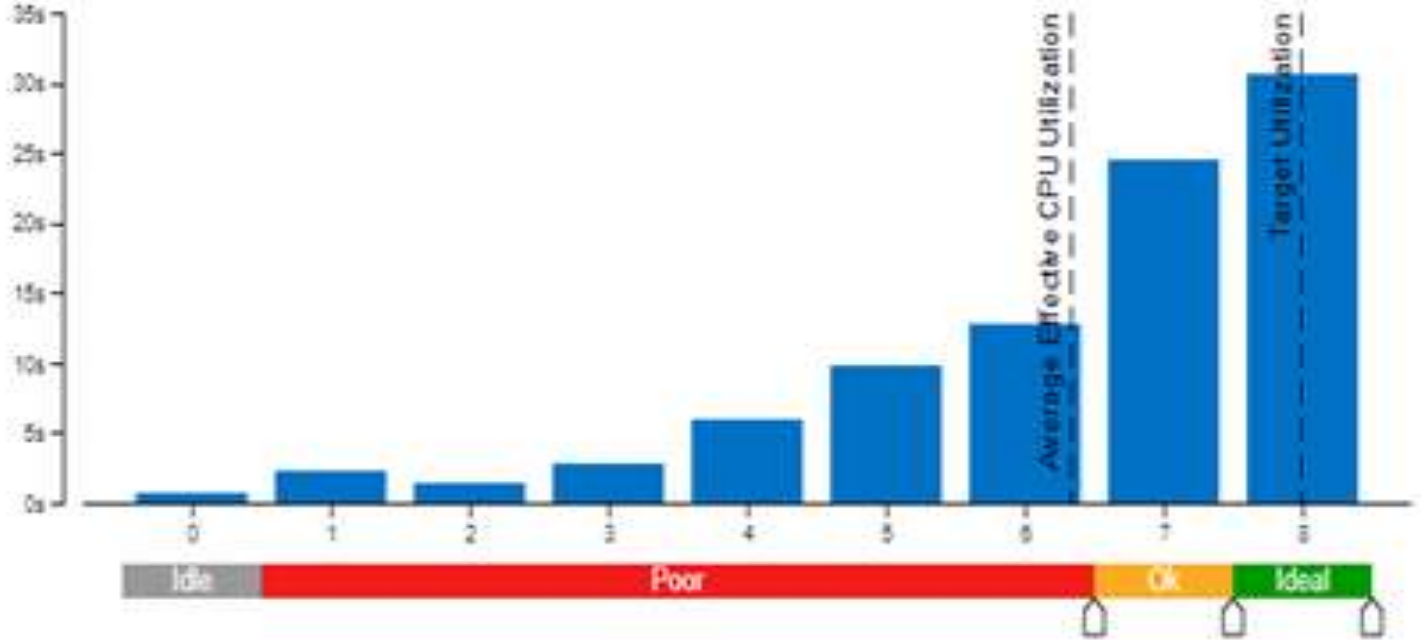


Google VM 8 CPUs

Effective CPU Utilization : 79.2% (6.332 out of 8 logical CPUs)

Effective CPU Utilization Histogram

This histogram displays a percentage of the wall time the specific number of CPUs were running simultaneously. Spin and Overhead time adds to the Idle CPU utilization value.



Results with profiler

THE END

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and

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