Calculating broadband lightcurves from SuperNu spectral data

This describes how to calculate broadband light curves from `SuperNu` spectra, given a binned spectrum on a wavelength grid (see Figure 1). This is the method that is implemented in the Python script `reformat_2d.py`.
In `SuperNu` format, the spectrum is given as energy flux L_k[erg s^{-1}] with $k = 1, \ldots, N$, on a wavelength grid $\{\lambda_0, \lambda_1, \ldots, \lambda_N\}$, uniformly spaced in $\log\lambda$ between λ_0 and λ_N. The spectrum is also binned in N_Ω angular bins, set by the direction of photon propagation. These bins are uniformly distributed in $\cos\theta$ and thus provide partition of the solid angle 4π into N_Ω pieces. The filter $A_\lambda \in [0..1]$ is usually specified on a finer grid, $\{\lambda'_0, \ldots, \lambda'_M\}$.
Spectral flux at distance R_o at specific λ is given by (ignoring cosmological redshift):

$$f_\lambda = \frac{L_{k(\lambda)}}{4\pi R_\mathrm{o}^2(\lambda_{k(\lambda)} - \lambda_{k(\lambda)-1})} = \frac{L_{k(\lambda)}}{4\pi R_\mathrm{o}^2 \Delta\lambda_{k(\lambda)}} \tag{1}$$

where $k(\lambda)$ is index of the interval containing λ: $\lambda \in [\lambda_k - 1, \lambda_k)$, and $\Delta\lambda_{k(\lambda)} := \lambda_k - \lambda_{k-1}$.
Because the spectral response function A_λ represents the fraction of photons with a given wavelength that pass through the filter, it must be normalized with respect to the photon count. The number of photons in an interval $d\nu$ is $dN = hd\nu/(h\nu)$, so normalization should be:

$$\int A_\lambda \frac{d\nu}{\nu} = \int A_\lambda \frac{d\lambda}{\lambda} \to 1, \tag{2}$$

because $d\nu/\nu = -d\lambda/\lambda$.
Normalized filter is:

$$\alpha_\lambda = A_\lambda \cdot \left(\int_{\lambda'_0}^{\lambda'_M} A_\lambda \frac{d\lambda}{\lambda} \right)^{-1}. \tag{3}$$

The filtered fluxes f'_λ and f'_ν are:

$$f'_\lambda = \alpha_\lambda f_\lambda, \tag{4}$$

$$f'_\nu = \alpha_\lambda f_\lambda \cdot \frac{\lambda^2}{c} = \frac{\alpha_\lambda L_{k(\lambda)}}{4\pi R_\mathrm{o}^2 \Delta\lambda_{k(\lambda)}} \cdot \frac{\lambda^2}{c}, \tag{5}$$

where c is the speed of light.
Overall expression for the broadband flux becomes:

$$F_B = \int_{\lambda'_0}^{\lambda'_M} f'_\nu(\lambda) d\lambda = \tag{6}$$

$$= \frac{N_\Omega}{4\pi c R_\mathrm{o}^2} \sum_{i=1}^{M} \frac{A_i L_{k(i)} \lambda_i}{\Delta\lambda_{k(i)}} \Delta\lambda_i \left[\sum_{i=1}^{M} A_i \frac{\Delta\lambda_i}{\lambda_i} \right]^{-1}, \tag{7}$$

where "i" is an index on the (finer) filter grid, and $k(i) := k(\lambda_i)$ is the index of the interval containing λ_i: $\lambda_i \in [\lambda_{k-1}, \lambda_k)$. Here, we also added N_Ω – the number of angular bins, to account for the "isotropic equivalent" of a non-isotropic source. Note that $\Delta\lambda_{k(i)}$ is a segment on the (coarse) `SuperNu` spectral grid, while $\Delta\lambda_i$ is a segment of the (finer) filter grid.

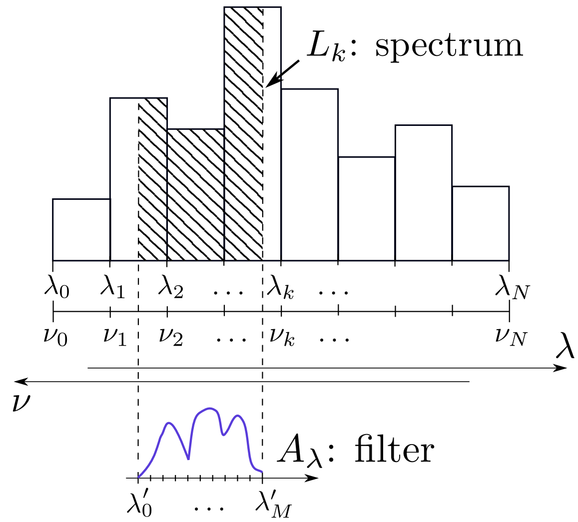

Figure 1: `SuperNu` spectral grid and the filter grid: relative size and location.

Finally, the broadband magnitude in AB system is related to the broadband flux F_B by the following simple expression:

$$m_B = -2.5 \log_{10} F_B - 48.6, \tag{8}$$

where F_B is in the units of erg s^{-1} Hz^{-1} cm^{-2}.

Files created by `reformat_2d.py`

The Python script `reformat_2d.py` reads archived `SuperNu` simulation and creates three postprocessed ASCII files:

1. `$SIMNAME_lums_DATESTAMP.dat`: luminosities;
2. `$SIMNAME_mags_DATESTAMP.dat`: magnitudes;
3. `$SIMNAME_spec_DATESTAMP.dat`: spectra.

`lums`: The first block of `lums` contains bolometric luminosities, in [erg/s], for each angular bin. To obtain the net power emitted by the source, one needs to sum bolometric luminosities over all angular bins. To obtain isotropic equivalent of the source as visible from the directional angle of a specific angular bin, one needs to multiply bolometric luminosity into that bin by N_θ.
The rest of the blocks in the `lums` contain *total broadband isotropic equivalent spectral flux with respect to frequency*, which is the quantity $4\pi R_\mathrm{o}^2 \times F_B$ from the equation (7) above. The long name of this quantity means the following:

- *total*: overall emitted by the source; not a flux through any kind of surface; doesn't contain "cm^{-2}" in its units;
- *broadband*: bandpass filter has been applied to the energy: spectrum was integrated over the band;
- *isotropic equivalent*: multiplied by N_θ;

- *spectral flux with respect to frequency*: f_ν rather than f_λ was integrated, and divided by the effective frequency width of the band; has Hz^{-1} in its units.

Overall dimension of this quantity is [erg s−1 Hz^{-1}].

`mags:` contains absolute broadband magnitudes of the source, M_B, as computed using (8) for isotropic equivalents of every angular bin. Here, "absolute" means that the flux is sampled at $R_\mathrm{o} = 10$ pc. The magnitudes are in AB system, as implied by the formula (8).

`spec:` contains time snapshots of the spectral flux at the distance $R_\mathrm{o} = 10$ pc for each angular bin, in the units of [erg s^{-1} cm^{-2} $\mathrm{\AA}^{-1}$].

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