
AN ERGODIC APPROACH TO OCCAM’S RAZOR

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ABSTRACT

An algorithmic Occam’s razor may be derived using the Expected Kolmogorov Complexity of a discrete ergodic variable. This derivation is based upon a combination of Bayesian and ergodic perspectives that are both necessary and sufficient in the context of the scientific method as it is applied in the natural sciences.

1 A Bayesian perspective on computation

As computations are observer-dependent, computation is fundamentally Bayesian. In particular, the uncertainty associated with a discrete ergodic variable is defined with respect to a predictive model. It follows that Occam’s razor for a discrete ergodic variable X is a measure of epistemic uncertainty or the memory requirements of the ideal model for predicting the behaviour of X .

In this setting, the minimum description length of X is given by the most parsimonious model Ω for predicting the behaviour of X :

$$\mathbb{E}[K(X)] = \mathbb{E}[-\ln P(X|\Omega)P(\Omega)] = H(X|\Omega) + H(\Omega) \quad (1)$$

where $H(\Omega)$ is the complexity of the model Ω and $H(X|\Omega)$ is the expected information gained by Ω from observing X .

The reason why the expression in (1) has a probabilistic representation is that the behaviour of a discrete ergodic variable is described by its probability distribution. From an ergodic perspective, these probabilities also have a natural frequentist interpretation.

2 An ergodic analysis

Given that an event that occurs with frequency p generally requires modelling a sequence of length $\sim \frac{1}{p}$, in order to encode the structure of such an event, a machine would generally need a number of bits proportional to:

$$\ln\left(\frac{1}{p}\right) = -\ln(p) \quad (2)$$

But, how should we define the constant of proportionality? If we assume that the memory of the machine is finite and that the data-generating process is ergodic, an optimal encoding would use the expected number of bits:

$$-p \cdot \ln(p) \quad (3)$$

in order to encode an event that occurs with frequency p .

Regarding the assumptions, I may make a couple remarks. First, the ergodic assumption is equivalent to the premise that scientific experiments are repeatable in the natural sciences. Second, any realisable digital computer has finite memory.

3 Application: Ockcam's razor and asymptotic incompressibility

Given a binary sequence $X_N = \{x_i\}_{i=1}^N$, we say that X_N is *asymptotically incompressible* if given the subsequence X_k and $N \gg k$ on average we would not profit by gambling on the $N - k$ terms in X_N based on the partial knowledge provided by X_k . If X_N satisfies these assumptions then Occam's razor applied to X_N scales as follows:

$$\mathbb{E}[K(X_N)] \sim N \quad (4)$$

which means that the average size(in bits) of the smallest approximately correct predictive model, found using machine learning methods, scales with N . It follows that an effective betting strategy has infinite sample complexity and therefore such a strategy is not learnable.

4 Discussion

I would like to point out that mainstream Algorithmic Information Theory insists that the Expected Kolmogorov Complexity of a discrete ergodic variable equals its Shannon entropy:

$$\mathbb{E}[K(X)] = H(X) \quad (5)$$

by appealing to Solomonoff's Universal distribution which may address the No Free Lunch problem [1,2]. What we may infer from this is that the mainstream theory is both incomplete.

The implicit error in (5) is to assume that local computations are observer-independent. Since we can't remove the observer in (1), we have:

$$X = \Omega \implies \mathbb{E}[K(\Omega)] = H(\Omega) \quad (6)$$

So information that is truly incompressible is self-referential.

References

- [1] Peter Grünwald and Paul Vitányi. Shannon Information and Kolmogorov Complexity. 2010.
- [2] Tom Everitt, Tor Lattimore, Marcus Hutter. Free Lunch for Optimisation under the Universal Distribution. 2016.
- [3] Schnorr, C. P. (1971). "A unified approach to the definition of a random sequence". Mathematical Systems Theory.
- [4] Shai Shalev-Shwartz and Shai Ben-David. Understanding Machine Learning: From Theory to Algorithms. Cambridge University Press. 2014.