
MIMESIS AS RANDOM GRAPH COLORING

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ABSTRACT

Inspired by the thought-provoking masterpiece by René Girard, *Le Bouc Emissaire*, a simple and tractable model for mimetic behaviour occurred to me. When we change our beliefs, we do so not because of their intrinsic value. Our desire to switch from belief A to belief B is proportional to the number of adherents of belief B that we know.

1 Mimesis as a decentralised process

In this article, I propose that when we change our beliefs, we do so not because of their intrinsic value. Our desire to switch from belief A to belief B is proportional to the number of adherents of belief B that we know. Technically, I modelled the problem of two conflicting beliefs that propagate through a network with N nodes in a decentralised manner. These beliefs are in some sense competing for adherents.

Using vertex notation, two individuals v_i and v_j with identical beliefs are connected with probability q , and $1 - q$ otherwise. v_i changes its belief with a probability proportional to the number of nodes connected to v_i that have opposing views.

Two key motivating questions are:

1. Under what circumstances does a belief get completely wiped out?
2. Under what circumstances does a belief completely dominate(i.e. wipe out) all other beliefs?

In the scenario where there are only two possible beliefs these two questions are equivalent and I show that on average it's sufficient that $q > 1 - q$ and that initially, one belief has a greater number of adherents than the other.

Nodes carrying the first belief were assigned to the set of red vertices, R , and nodes carrying the second belief were assigned to the set of blue vertices, B . After further reflection, I chose $+1$ and -1 as labels. The reason being that a change of belief using this representation would be equivalent to multiplication by -1 . As a result, the N vertices could be represented by an N-dimensional vector:

$$\vec{v} \in \{-1, 1\}^N \tag{1}$$

where $N = |v_i \in R| + |v_j \in B|$.

2 A random graph model

Using this representation, between each pair of vertices we may define a virtual weight matrix W :

$$w_{ij} = v_i \cdot v_j \quad (2)$$

where $w_{ij} = +1$ implies identical beliefs and we have $w_{ij} = -1$ otherwise.

Now, we note that W may be conveniently decomposed as follows:

$$W = W^+ + W^- \quad (3)$$

where W^- denotes potential connections between nodes of different color and W^+ denotes potential connections between nodes of identical colors.

In order to simulate variations in connectivity we may assume that nodes of the same color are connected with probability $\frac{1}{2} < q < 1$ and nodes of different color are connected with probability $1 - q$. Given W we may therefore construct the adjacency matrix A by sampling random matrices:

$$M_1, M_2 \sim \mathcal{U}([0, 1])^{N \times N} \quad (4)$$

$$M^+ = 1_{[0, q)} \circ M_1 \quad (5)$$

$$M^- = 1_{(1-q, 1]} \circ M_2 \quad (6)$$

where $1_{[0, q)}$ denotes the characteristic function over the set $[0, q)$ and then we compute the Hadamard products:

$$A^+ = M^+ \cdot W^+ \quad (7)$$

$$A^- = M^- \cdot W^- \quad (8)$$

so the adjacency matrix is given by $A = A^+ + A^-$.

3 Stochastic dynamics for computer simulation

Now, in order to simulate stochastic dynamics we simply use majority vote:

$$p(v_i^{n+1} = v_i^n) = \frac{\bar{N}_i}{N_i} \quad (9)$$

$$p(v_i^{n+1} = -1 \cdot v_i^n) = 1 - \frac{\bar{N}_i}{N_i} \quad (10)$$

$$\bar{N}_i = |A(i, -) > 0| - 1 \quad (11)$$

$$N_i = \bar{N}_i + |A(i, -) < 0| \quad (12)$$

where $|A(i, -) > 0| - 1$ denotes the number of connections between v_i and nodes sharing the same belief without counting a connection to itself.

4 Analysis

If we denote the number of red vertices at instant n by α_n and the number of blue vertices by β_n we may observe that the expected number of neighbors is given by:

$$\langle N(v_i \in R) \rangle = q \cdot (\alpha_n - 1) + (1 - q) \cdot \beta_n \quad (13)$$

$$\langle N(v_i \in B) \rangle = q \cdot (\beta_n - 1) + (1 - q) \cdot \alpha_n \quad (14)$$

Using the above equations we may define:

$$\langle \alpha_{n+1} \rangle = \alpha_n \left(\frac{q \cdot (\alpha_n - 1)}{q \cdot (\alpha_n - 1) + (1 - q) \cdot \beta_n} \right) + \beta_n \left(\frac{(1 - q) \cdot \alpha_n}{q \cdot (\beta_n - 1) + (1 - q) \cdot \alpha_n} \right) \quad (15)$$

and we may deduce that $\langle \beta_{n+1} \rangle = N - \langle \alpha_{n+1} \rangle$. It must be noted that this is an approximation that works particularly well when $0 < q < 0.2$ or when $0.8 \leq q < 1.0$.

4.1 $\alpha_n > \beta_n$ implies that $\lim_{n \rightarrow \infty} \langle \alpha_n \rangle = N$

Assuming that $q > 1 - q$, a simple calculation shows that:

$$\langle \alpha_{n+1} \rangle - \alpha_n \geq 0 \iff \beta_n \geq \alpha_n \quad (16)$$

and since:

$$\langle \alpha_{n+1} \rangle - \alpha_n = 0 \iff \alpha_n = \beta_n \quad (17)$$

we may deduce that:

$$\lim_{n \rightarrow \infty} \langle \alpha_n \rangle = N \quad (18)$$

4.2 Analysis of $\Delta\alpha$

Using the fact that $\beta_n = N - \alpha_n$ we may derive the following continuous-space variant of $\Delta\alpha_n = \langle \alpha_{n+1} \rangle - \alpha_n$:

$$\Delta\alpha(\alpha, \gamma) = \frac{\alpha \cdot (N - \alpha)}{\gamma \cdot (\alpha - 1) + (N - \alpha)} - \frac{\alpha \cdot (N - \alpha)}{\gamma \cdot (N - \alpha - 1) + \alpha} \quad (19)$$

where $\gamma = \frac{q}{1-q}$.

References

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