

CS 1332 - Week 12

Monday

Topics

I. KMP Tracing

Failure Table										
idx	0	1	2	3	4	5	6	7	8	9
pat	b	a	b	a	a	b	a	b	a	b
fail	0	0	1	2	0	1	2	3	4	3

Tracing																		
text	b	a	b	a	b	a	b	a	b	a	b	a	a	b	a	b	a	b
pat	b	a	b	a	a	b	a	b	a	b								
			b	a	b	a	a	b	a	b	a	b						
					b	a	b	a	a	b	a	b	a	b				
							b	a	b	a	a	b	a	b	a	b		
									b	a	b	a	a	b	a	b	a	b
									b	a	b	a	a	b	a	b	a	b

II. Rabin-Karp

- A. Premise - computes hashcodes for substrings and only performs character comparisons when the hashes match
- B. Calculating the hash of a string/substring
 1. Select a base number - the ideal base number is prime and fairly large, this helps eliminate hashcode collisions
 2. For each character:
 - a) Convert char to an integer - eg. use the ASCII value
 - b) Computer a power of base - to account for the position of the char in the string, we multiply the character by a power of base
 3. Add all values together to form the entire hashcode
 4. Eg. pattern "90210", size = 5, base = 10
 - a) hash(pattern)
$$= 9*10^4 + 0*10^3 + 2*10^2 + 1*10^1 + 0*10^0$$
$$= p[0]*base^{(size-1-0)} + p[1]*base^{(size-1-1)} + p[2]*base^{(size-1-2)} \dots$$
$$= (n=0, size-1) \sum p[n]*base^{(size-1-n)}$$

C. Rolling the text hash

1. $\text{newHash} = (\text{oldHash} - \text{hash of first char in oldHash}) * \text{base} + \text{next char}$
2. Eg. text "48902107" → roll hash from hash("48902") to hash("89021")
 - a) $\text{oldHash} = 48902$, $\text{base} = 10$
 - b) remove hash of first character: $48902 - 40000 = 8902$
 - c) make room for next character: $10 * 8902 = 89020$
 - d) add on the next new character: $89020 + 1 = 89021$

D. Algorithm

1. Calculate the hash of the pattern
2. Calculate the initial hash of the text (the first m characters of the text)
 - a) Recommendation: calculate the initial text hash and the pattern hash in the same loop starting from index m-1 and going to 0, this way you can calculate the power of base needed by starting a variable at 1 and multiplying it by base for each iteration
3. Compare the pattern hash to the text hash
 - a) If they match → compare the actual characters
 - b) If they don't → roll the hash and compare again

E. Hashcode Collisions

1. If we have a bad hashcode (eg. small or non-prime base), we'll have lots of places where the hashcodes are equal, but the characters are different
 - a) Eg. text "aabbcbaba", pattern "cab", base = 1, hash a to z = 0 to 26
 - (1) Pattern hash = $c + a + b = 2 + 0 + 1 = 3$
 - (2) Initial text hash = $a + a + b = 0 + 0 + 1 = 1 \rightarrow$ no match
 - (3) Rolled hash = $1 - a + b = 1 - 0 + 1 = 2 \rightarrow$ no match
 - (a) $(\text{old} - \text{oldChar}) * \text{base} + \text{newChar}$
 - (4) Rolled hash = $2 - a + c = 2 - 0 + 2 = 4 \rightarrow$ no match
 - (5) Rolled hash = $4 - b + a = 4 - 1 + 0 = 3 \rightarrow$ match
 - (a) compare t: "bca" to p: "cab" → no match
 - (6) Rolled hash = $3 - b + b = 3 - 1 + 1 = 3 \rightarrow$ match
 - (a) compare t: "cab" to p: "cab" → match!

Wednesday

Topics

I. Pattern Matching Efficiencies

	Worst	Best (first occurrence)	Best (all)
Brute Force	$O(mn)$	$O(m)$	$O(mn)$
Boyer-Moore	$O(mn)$	$O(m)$	$O(n/m)$
Knuth-Morris-Pratt	$O(m + n)$	$O(m)$	$O(m + n)$
Rabin-Karp	$O(mn)$	$O(m)$	$O(m + n)$

II. Graphs ADT

A. Graphs are made up of a set of vertices, V , and a set of edges, E

B. Terminology

1. Order(G) - number of vertices in graph G
2. Size(G) - number of edges in graph G
3. Sparse - few edges relative to the number of vertices
4. Dense - many edges relative to the number of vertices
5. Path - edges you traverse to go from one vertex to another
 - a) Simple - vertices traversed in the path are only visited once
 - b) Cycle - at least one vertex is visited more than once in a path
 - c) Length of path - number of edges traversed
6. Edge - connects two vertices, edge(a, b)
 - a) Undirected Edge - can go from a to b and b to a
 - (1) Undirected Graph - ALL edges are undirected
 - b) Directed Edge - can only go from a to b
 - (1) Directed Graph - ALL edges are directed
 - c) Weighted Edge - an edge has a "cost" associated with traversing it, eg. if vertices are locations, edge weights could be miles between two locations
 - (1) Weighted Graph - ALL edges are weighted
 - d) Self Loop - an edge that connects a vertex to itself
 - e) Parallel Edges - two edges with the same source and destination

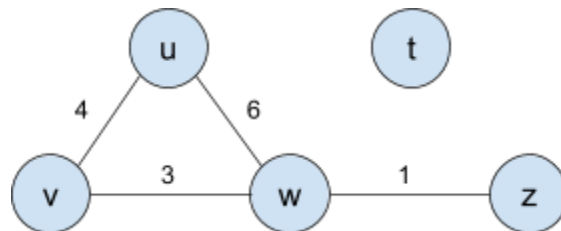
C. Information Methods

1. vertices() - returns iteration (list generated by an iterator) of all vertices
2. edges() - returns iteration of all edges
3. numV() - returns count/number of vertices
4. numE() - returns count/number of edges

Friday

Topics

I. Graph Data Structures - how do we store graphs?



A. Edge List - stores all the edges of the graph in a list

1. Edge list - [($v, u, 4$), ($u, w, 6$), ($w, v, 3$), ($w, z, 1$)]
2. Pros - easy access to all edges
3. Cons - hard to access all vertices, doesn't store disconnected vertices

B. Adjacency List - like a hashmap, the keys are the vertices and each vertex key's value is a list of the edges adjacent to it

1. Adjacency list

$w \rightarrow [(w, u, 6), (w, v, 3), (w, z, 1)]$

$u \rightarrow [(u, v, 4), (u, w, 6)]$

$v \rightarrow [(v, u, 4), (v, w, 3)]$

$z \rightarrow [(z, w, 1)]$

$t \rightarrow []$

2. Pros - easy access to all vertices and edges incident to a given vertex

3. Cons - hard to access all edges, undirected edges are stored twice

C. Adjacency Matrix - stores a 2D array where $\text{matrix}[v][w]$ would give the weight of the edge between vertices v and w if it exists

1. Adjacency matrix

	w	u	v	z	t
w		6	3	1	
u	6		4		
v	3	4			
z	1				
t					

2. Pros - easy to find an edge between two vertices ($O(1)$ array access)

3. Cons - extra memory is taken up when the graph is sparse or undirected

II. Graph Traversals

A. Depth-First Search (DFS)

1. Premise - follow one path as deep as possible before backtracking and going down another path

a) The pre-, post-, and inorder BST traversals were depth-first

2. Algorithm - can be done with a regular stack or recursive stack

a) Recursive pseudocode

```
dfs(vertex v):  
    mark v visited, output v (print or record)  
    for each neighbor w of v:  
        if w is unvisited:  
            dfs(w)
```