

CS3451: L-System

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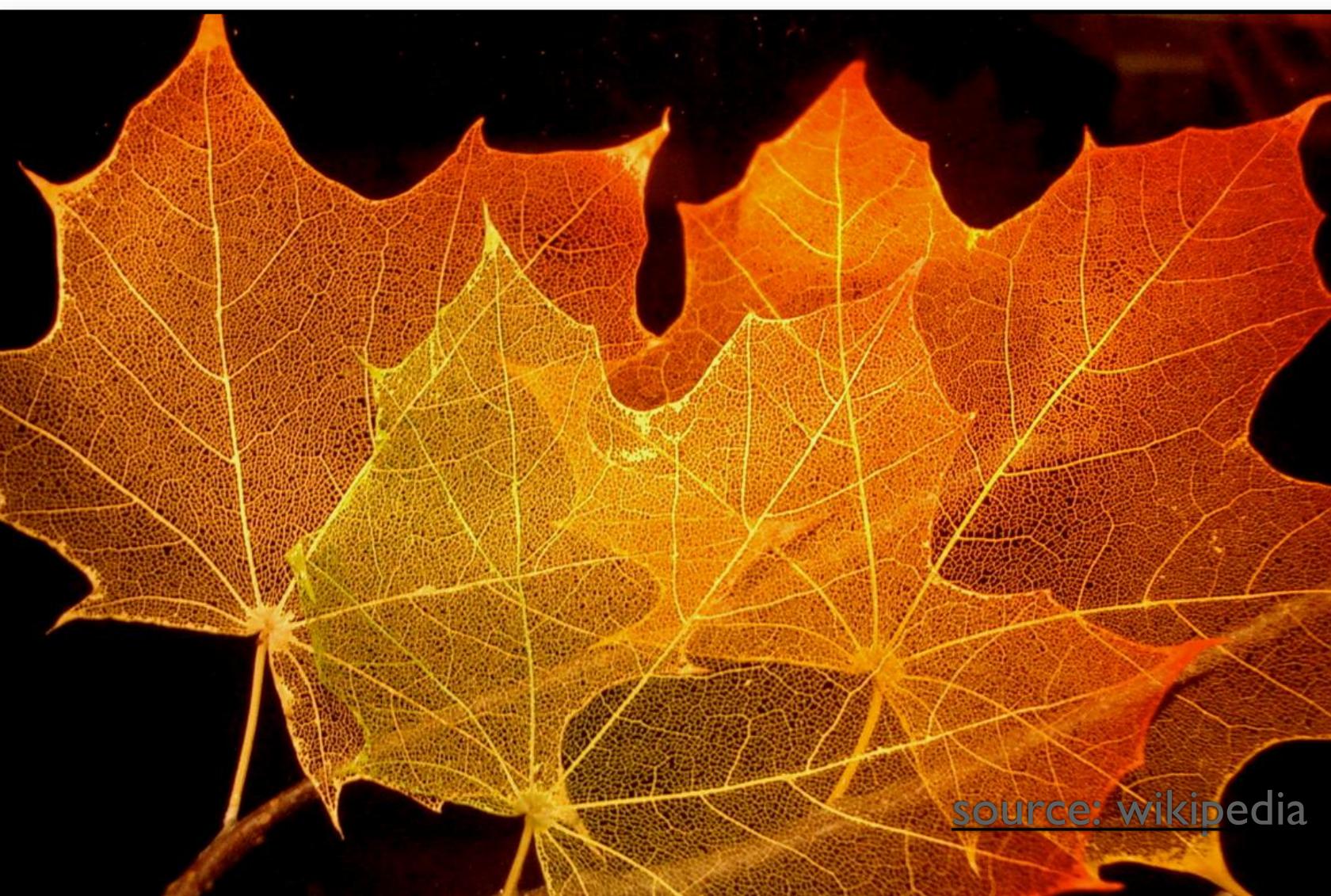
Trees



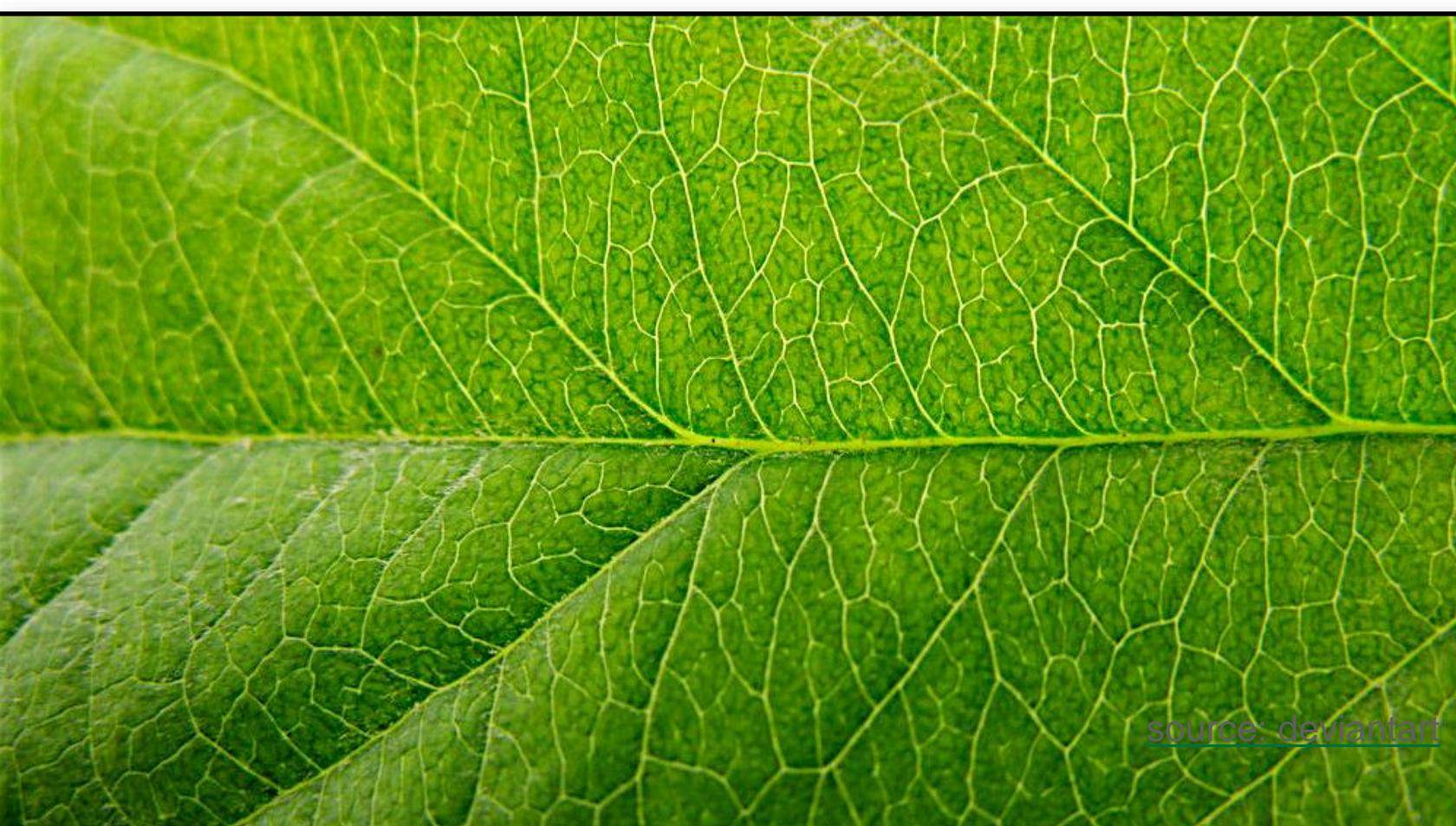
Plants



Leaves



[source: wikipedia](#)



[source: deviantart](#)

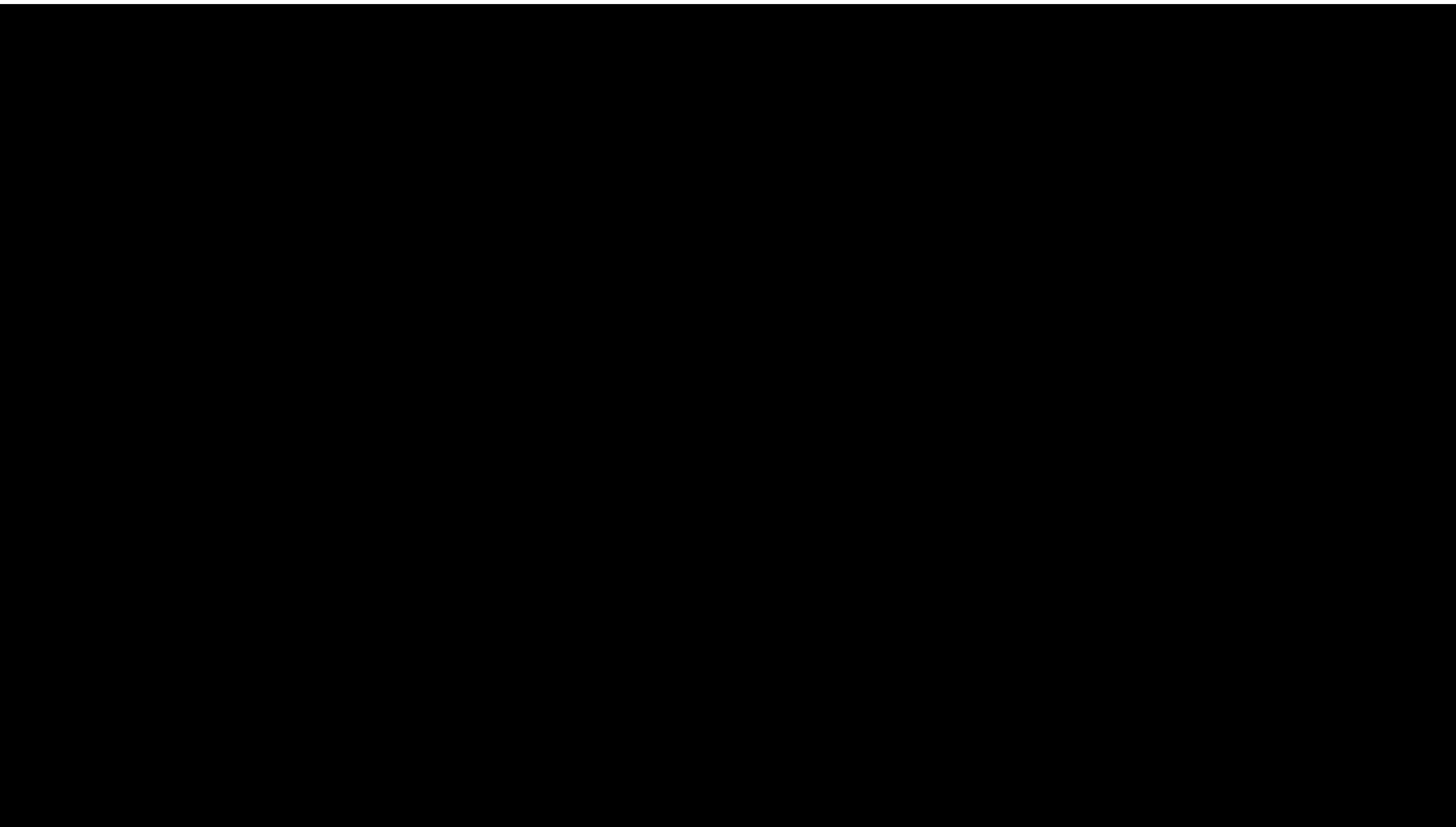


[source: ibm](#)

Fern



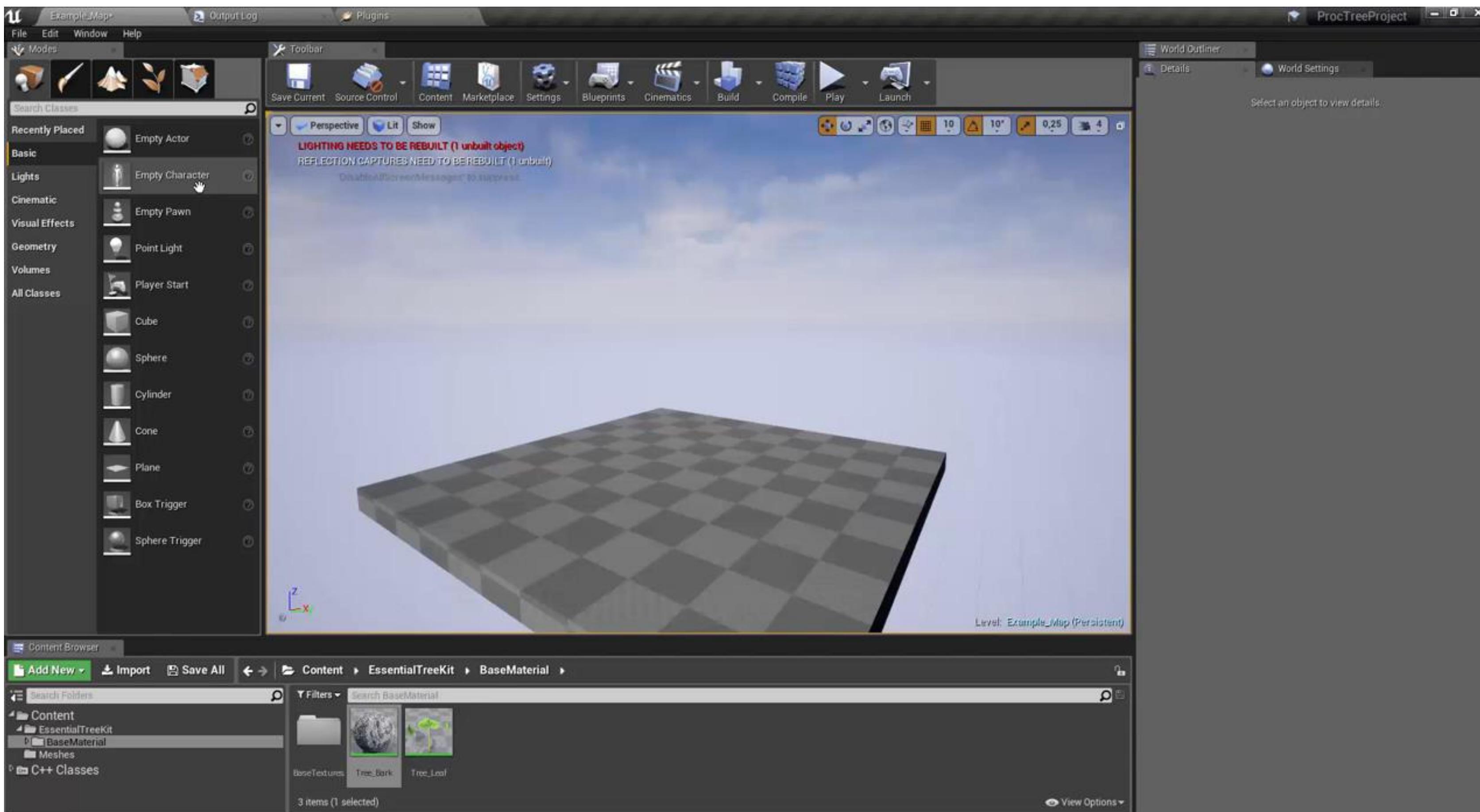
Motivational Video: Procedural Trees



<https://www.youtube.com/watch?v=fBX47KSvBIQ>



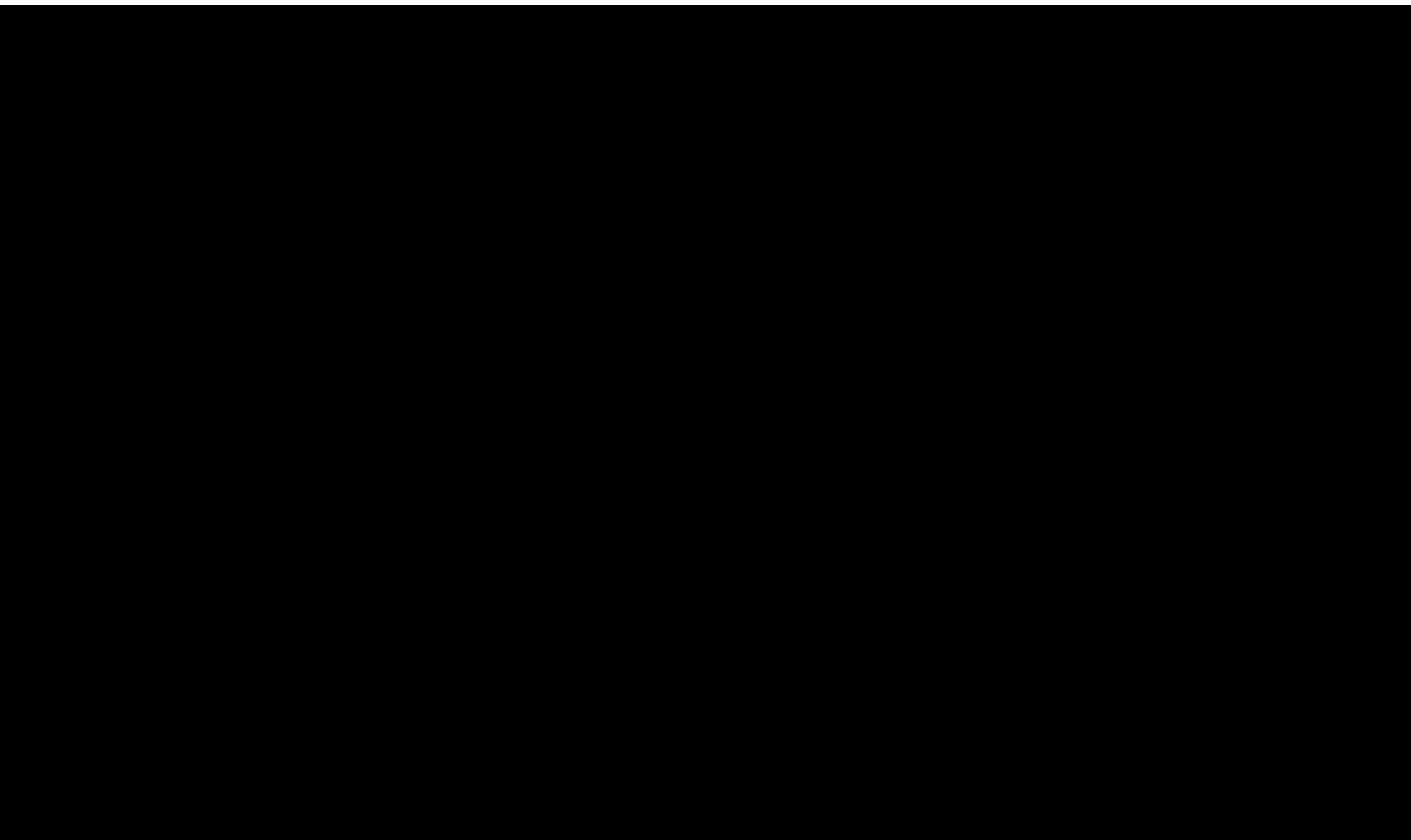
Motivational Video: Modeling a Trees in UE4



<https://www.youtube.com/watch?v=6fzcpjch5Ng>



Motivational Video: Tree Animation



My computer graphics
final project

Motivational Video: Interactive Plant Modeling

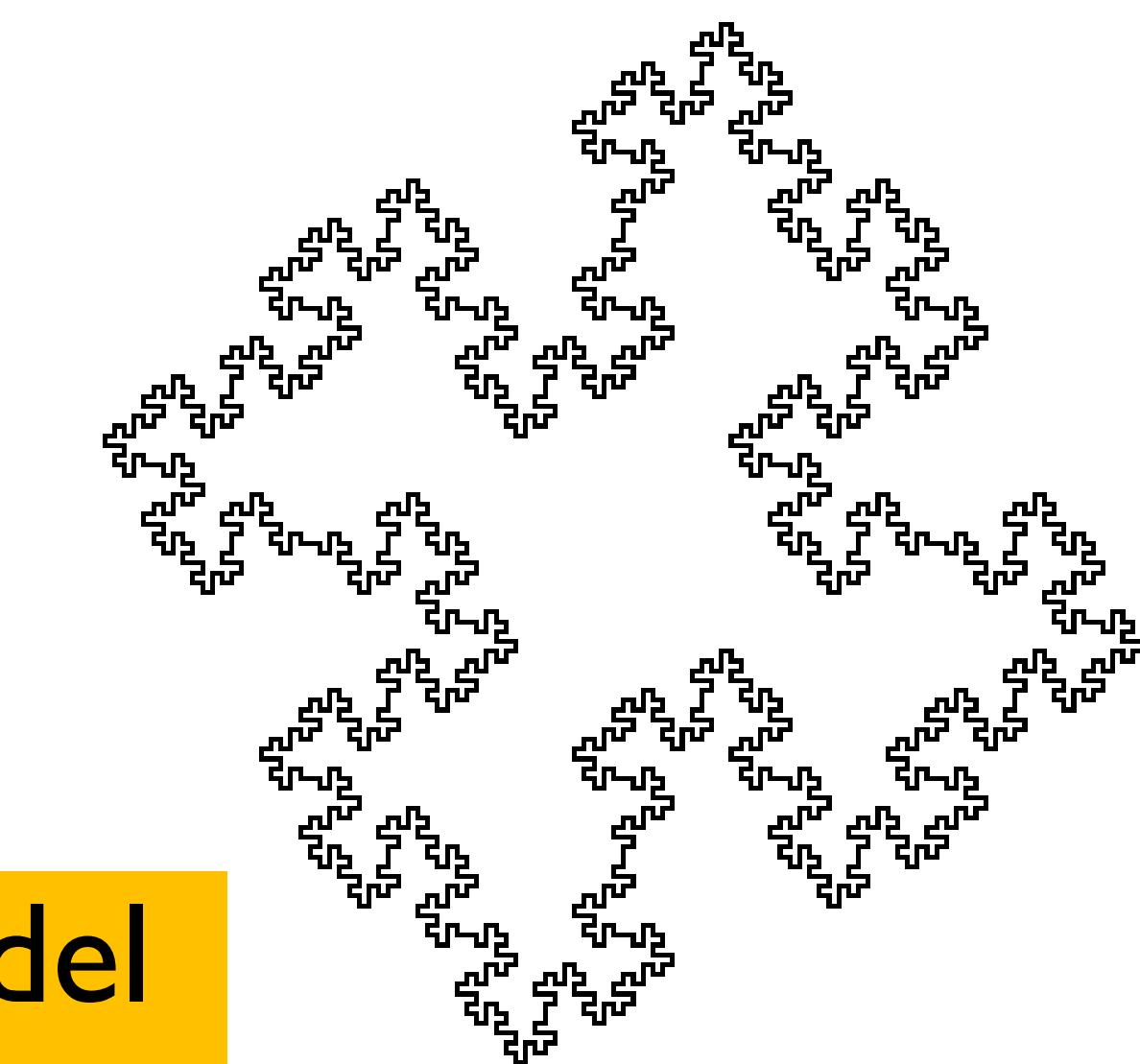
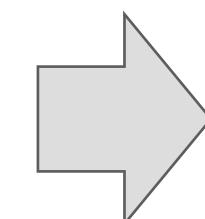


L-Systems

- Grammar on an alphabet of **symbols**, such as “F”, “+”, “-”
- **Production rules**, p , describe the replacement of a nonterminal symbol with a string of zero or more symbols
- Process is seeded with an **axiom**, ω , an initial string

$\omega: F - F - F - F$

$p : F \rightarrow F - F + F + FF - F - F + F$



convert a string to a graphical model

L-System: Grammar

- Input:

$\omega : F$

$p : F \rightarrow F+F--F+F$

- Output:

1) $F+F--F+F$

2) $F+F--F+F + F+F--F+F \dots F+F--F+F + F+F--F+F$

3) $F+F--F+F + F+F--F+F \dots F+F--F+F + F+F--F+F +$

$F+F--F+F + F+F--F+F \dots F+F--F+F + F+F--F+F \dots$

$F+F--F+F + F+F--F+F \dots F+F--F+F + F+F--F+F +$

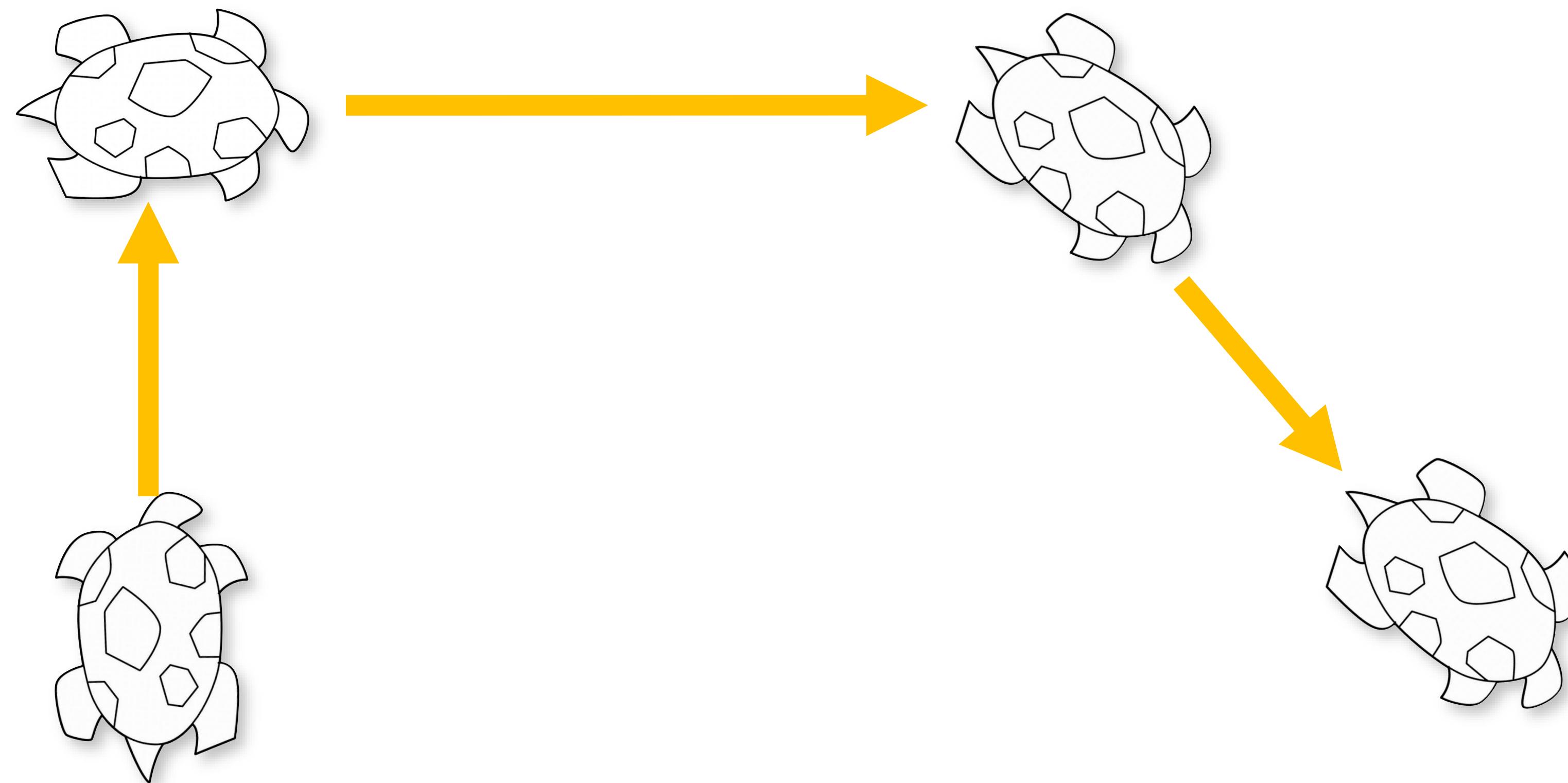
$F+F--F+F + F+F--F+F \dots F+F--F+F + F+F--F+F$



L-Systems: Graphics

- Turtle graphics interpretation of L-Systems

A state of the *turtle* is defined as a triplet (x, y, α) , where the Cartesian coordinates (x, y) represent the turtle's position, and the angle α , called the heading, is interpreted as the direction in which the turtle is facing



L-Systems: Definition of Tokens

- F: move forward a step of length d

The state of the turtle changes to (x', y', α) where

$$x' = x + d \cos(\alpha) \text{ and } y' = y + d \sin(\alpha).$$

A line segment between points (x, y) and (x', y') is drawn.

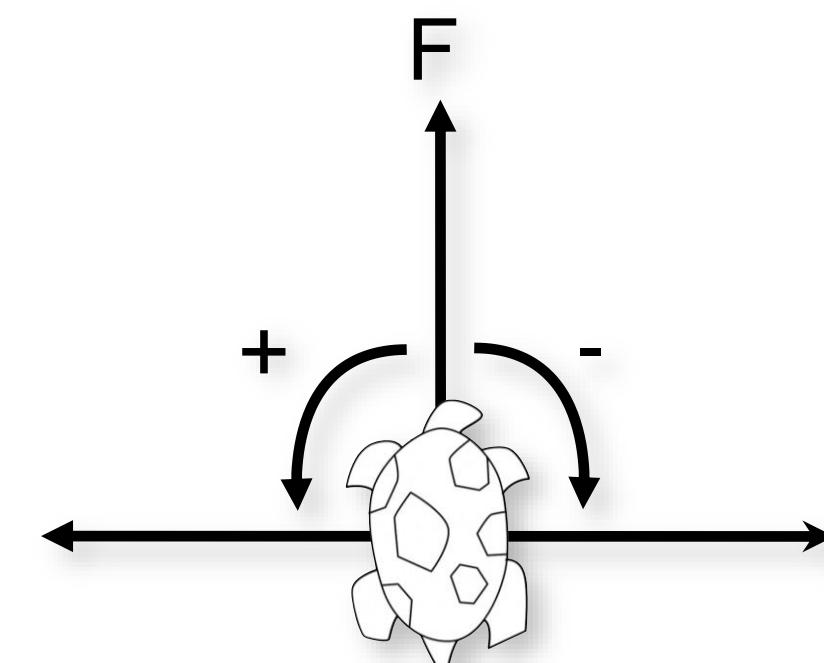
- +: Turn left by angle δ

The next state of the turtle is $(x, y, \alpha + \delta)$.

The positive orientation of angles is counterclockwise.

- -: Turn right by angle δ

The next state of the turtle is $(x, y, \alpha - \delta)$.



L-System Example

- Input:

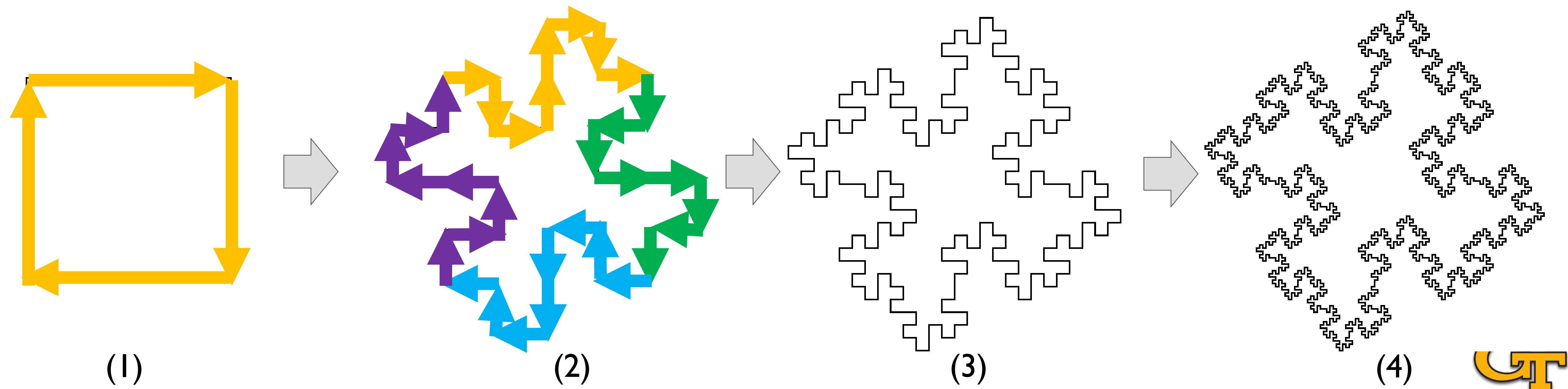
$$\omega: F - F - F - F$$
$$p : F \rightarrow F - F + F + FF - F - F + F$$

- Output:

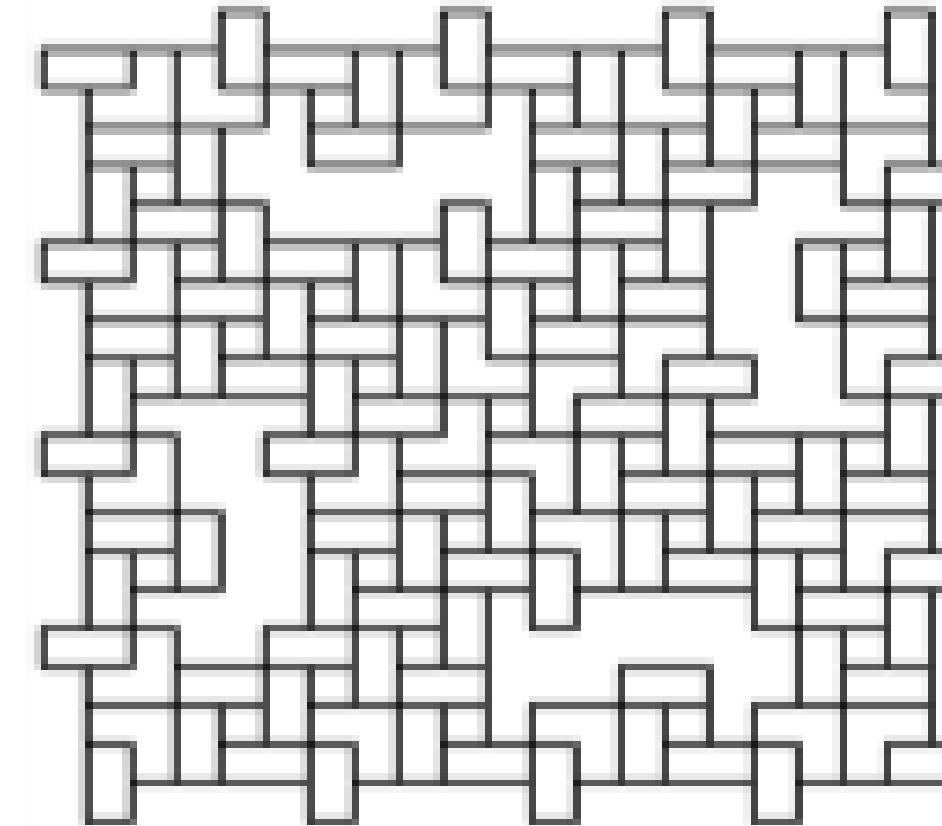
1) F - F - F - F

2) F - F + F + FF - F - F + F - F - F + F

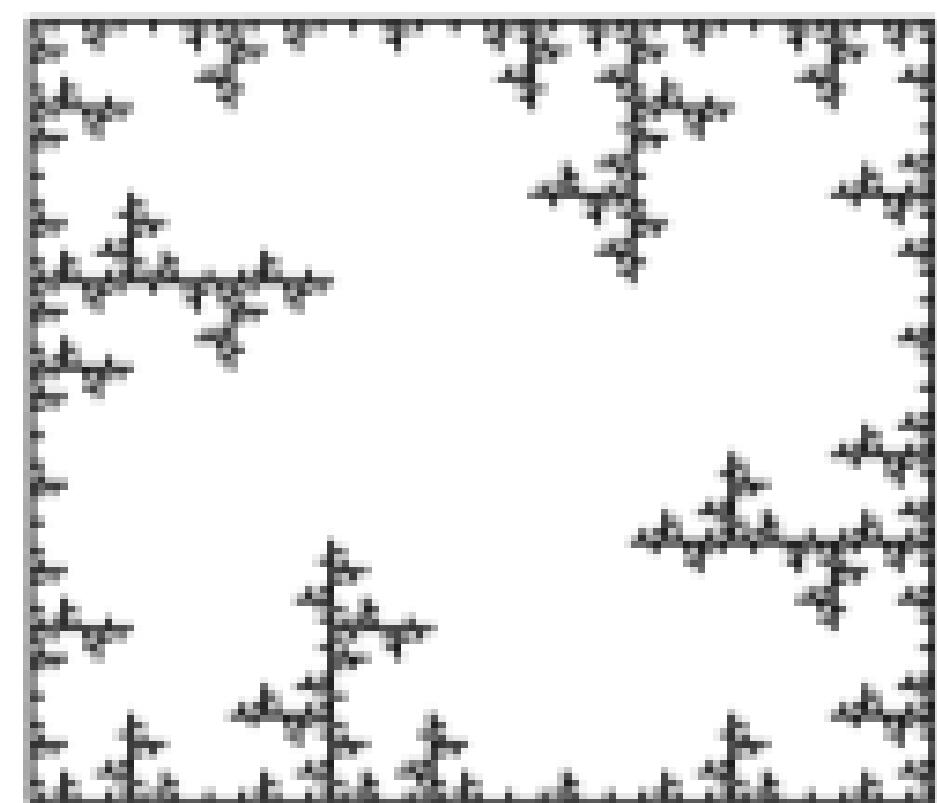
- F - F + F + FF - F - F + F - F - F + F



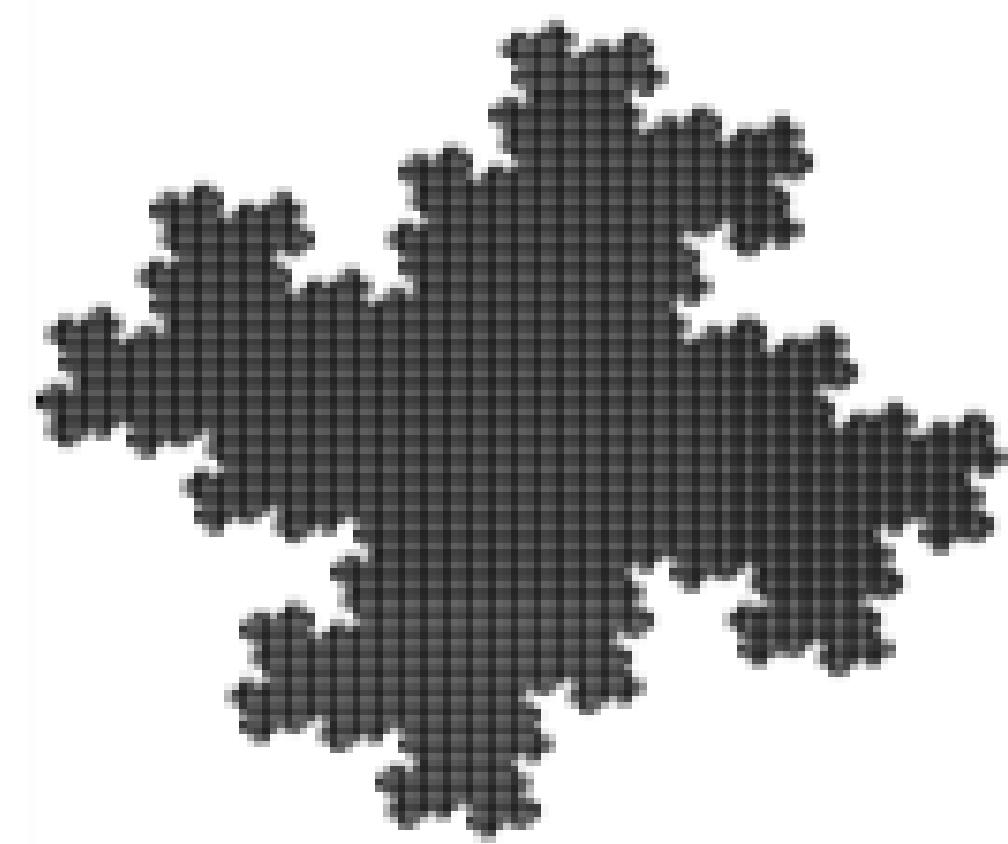
More Examples of L-System



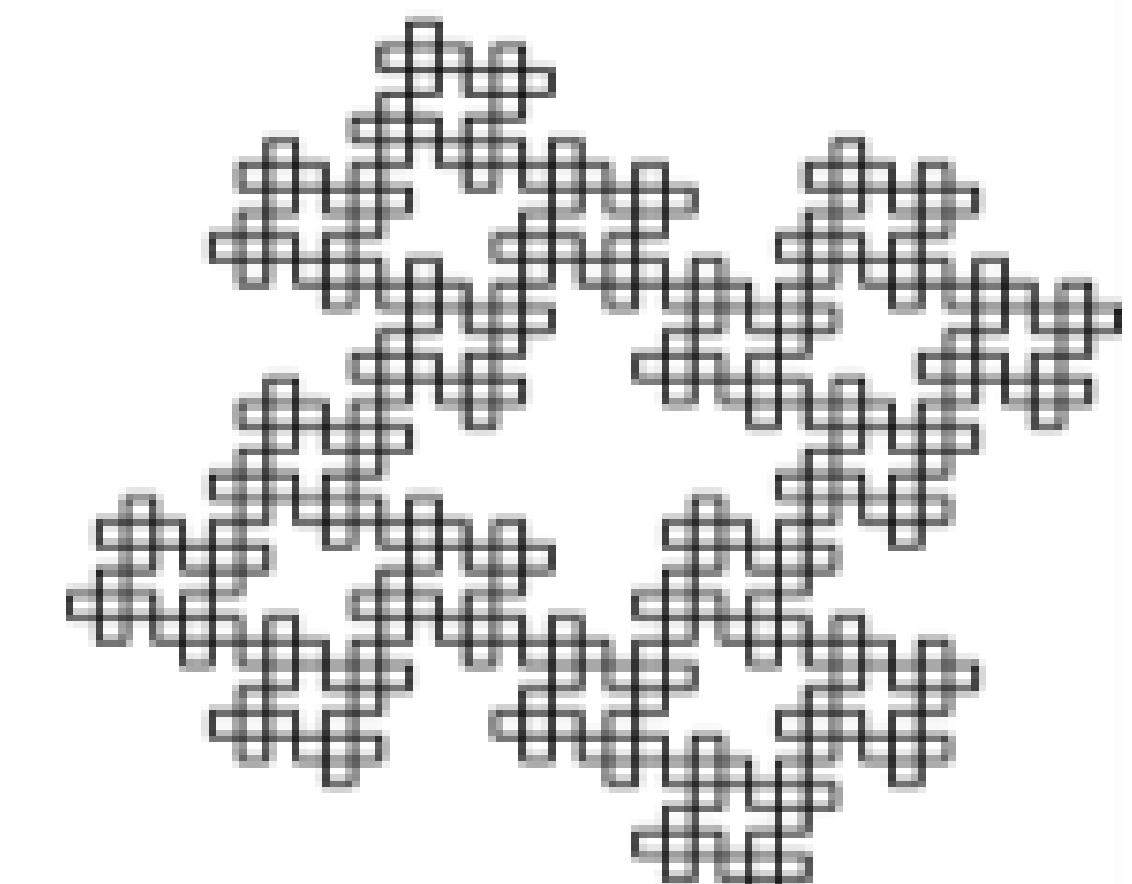
$n = 3, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow FF-F+F-F-FF$



$n = 4, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow FF-F--F-F$

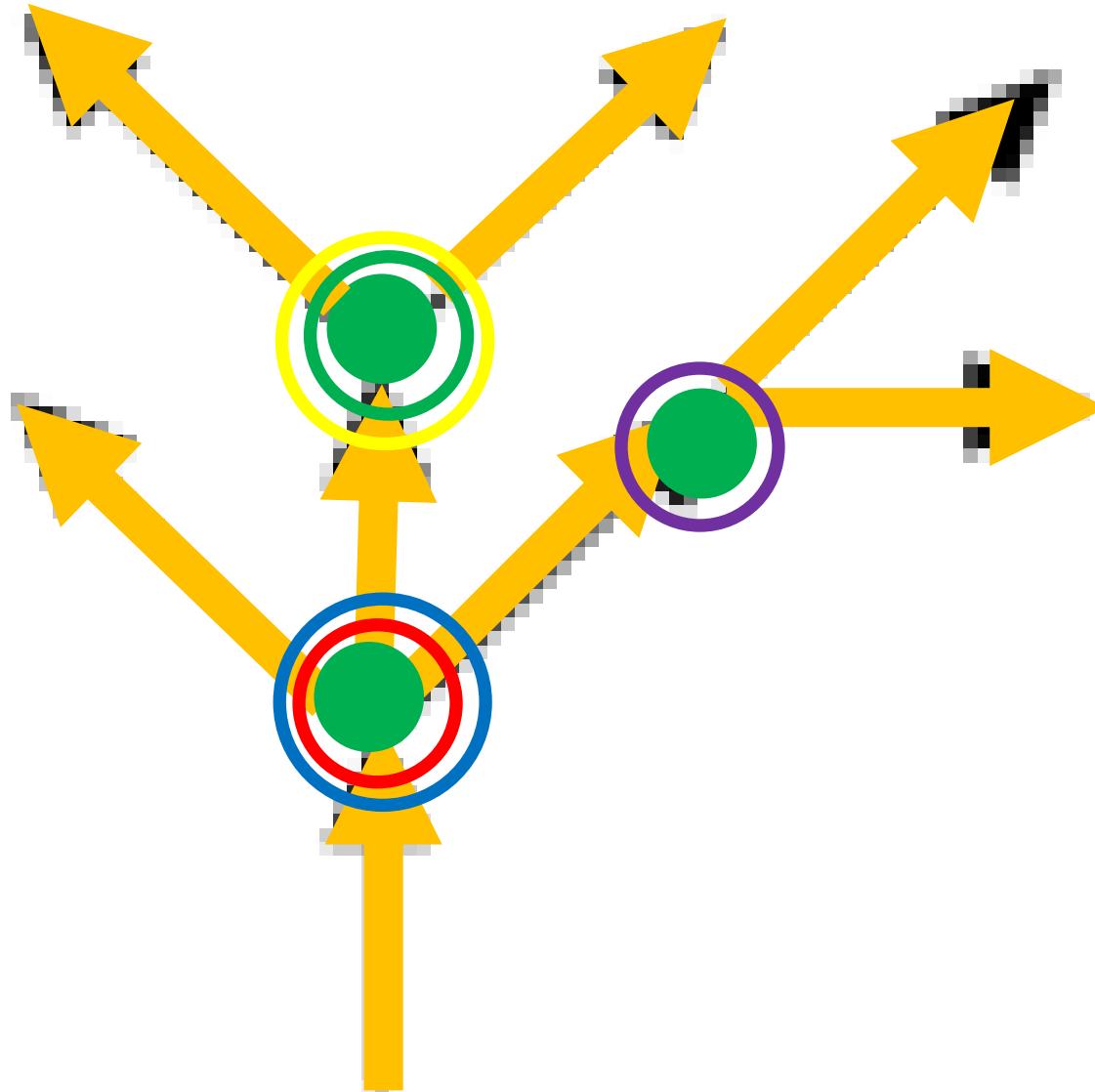


$n = 5, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow F-FF--F-F$



$n = 4, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow F-F+F-F-F$

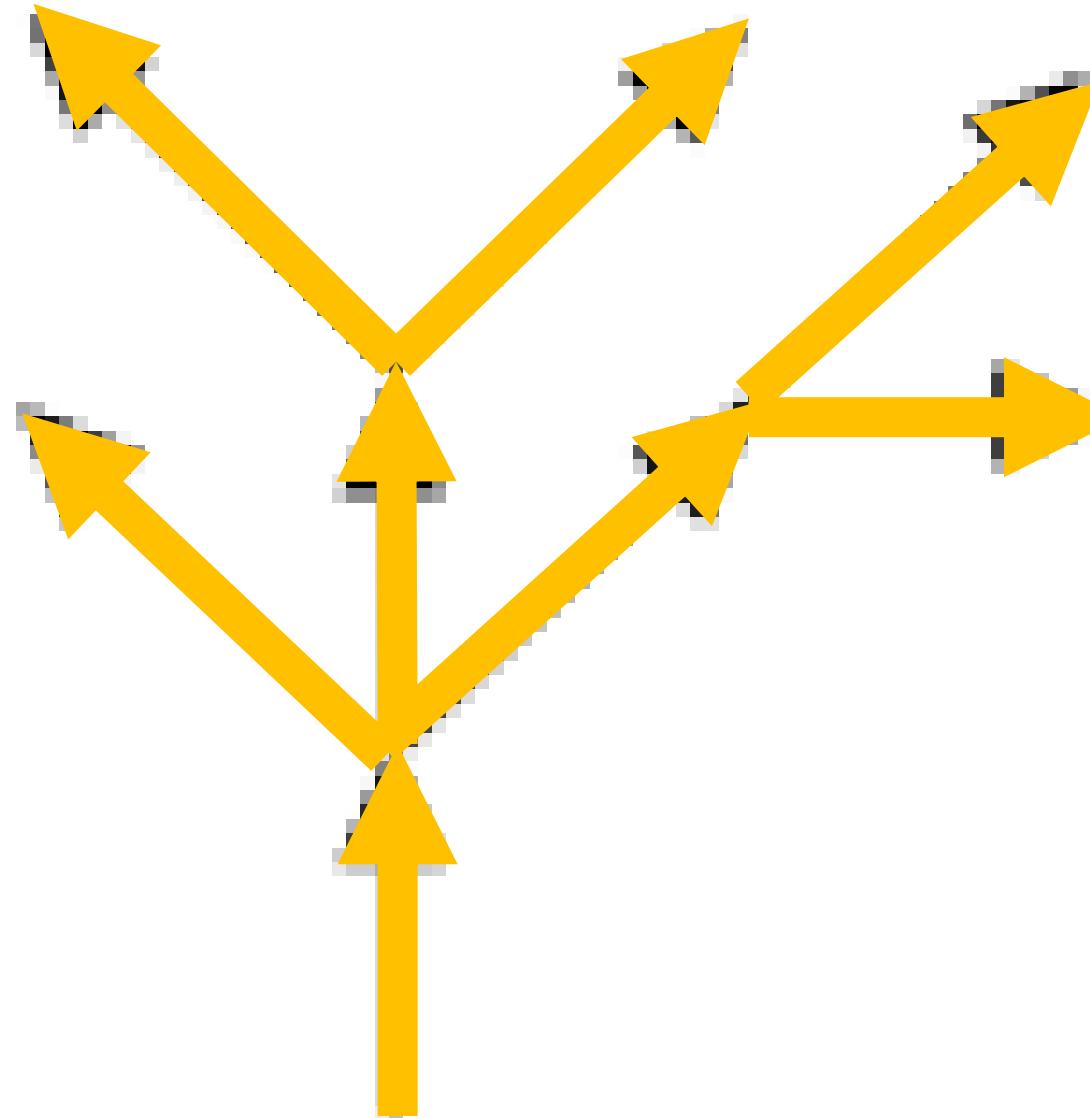
Branching Structure: Definition



F [+ F] [- F [- F] F] F [+ F] [- F]

- Branching is prevalent in plants
- Bracketed L-Systems introduce two new symbols: "[" and "]" to push and pop turtle's state

Branching Structure: A Simpler Perspective

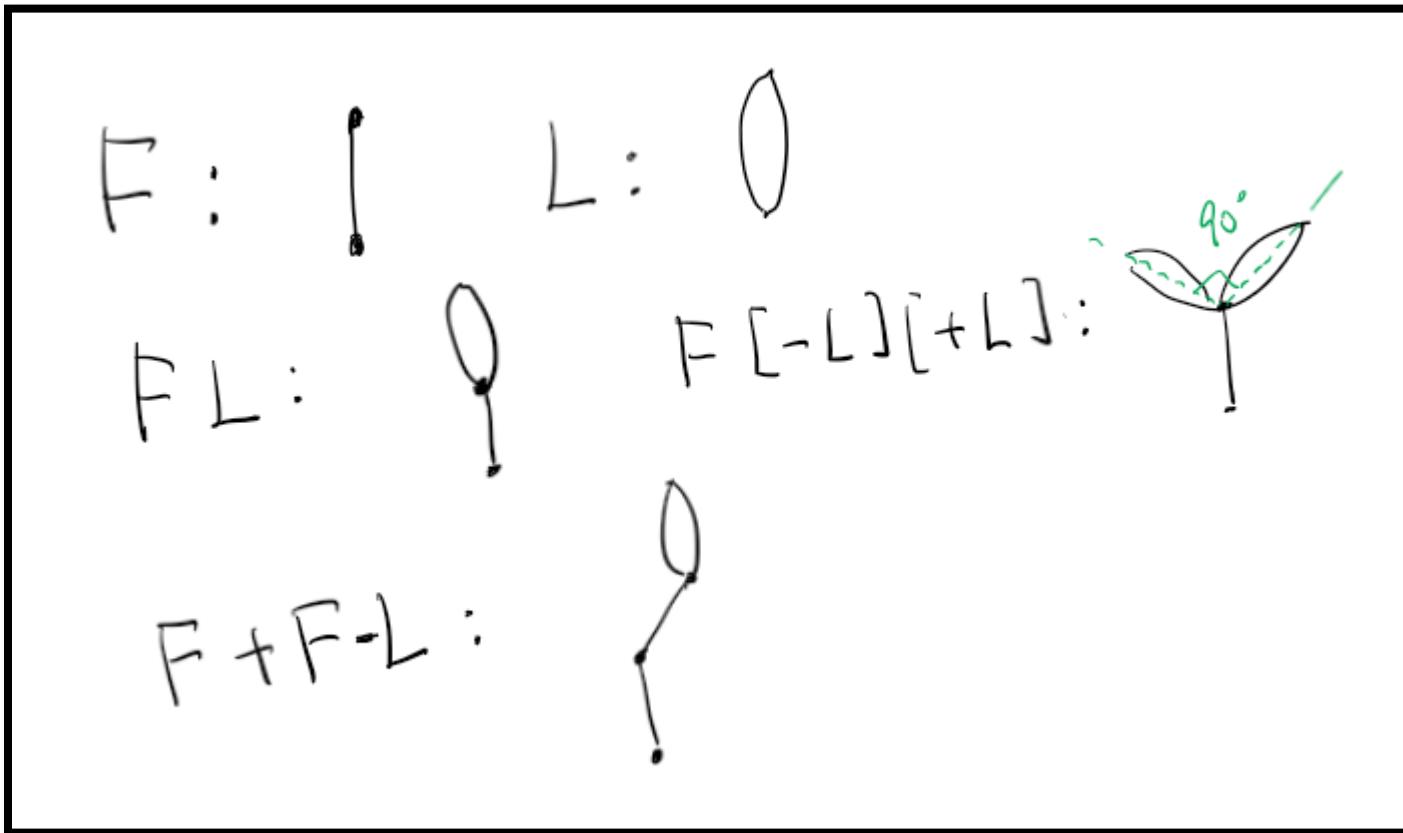


F [+ F] [- F [- F] F] F [+ F] [- F]

An easy way to check a branching L-system is to treat each part with a pair bracket as a whole

Another Example: Branching System with Leaves

In an L-system, F means moving forward by 1 unit, L means draw a leaf with the **current** position and direction, - means turn left by 45 degrees, and + means turn right by 45 degrees (See some examples in the right figure).

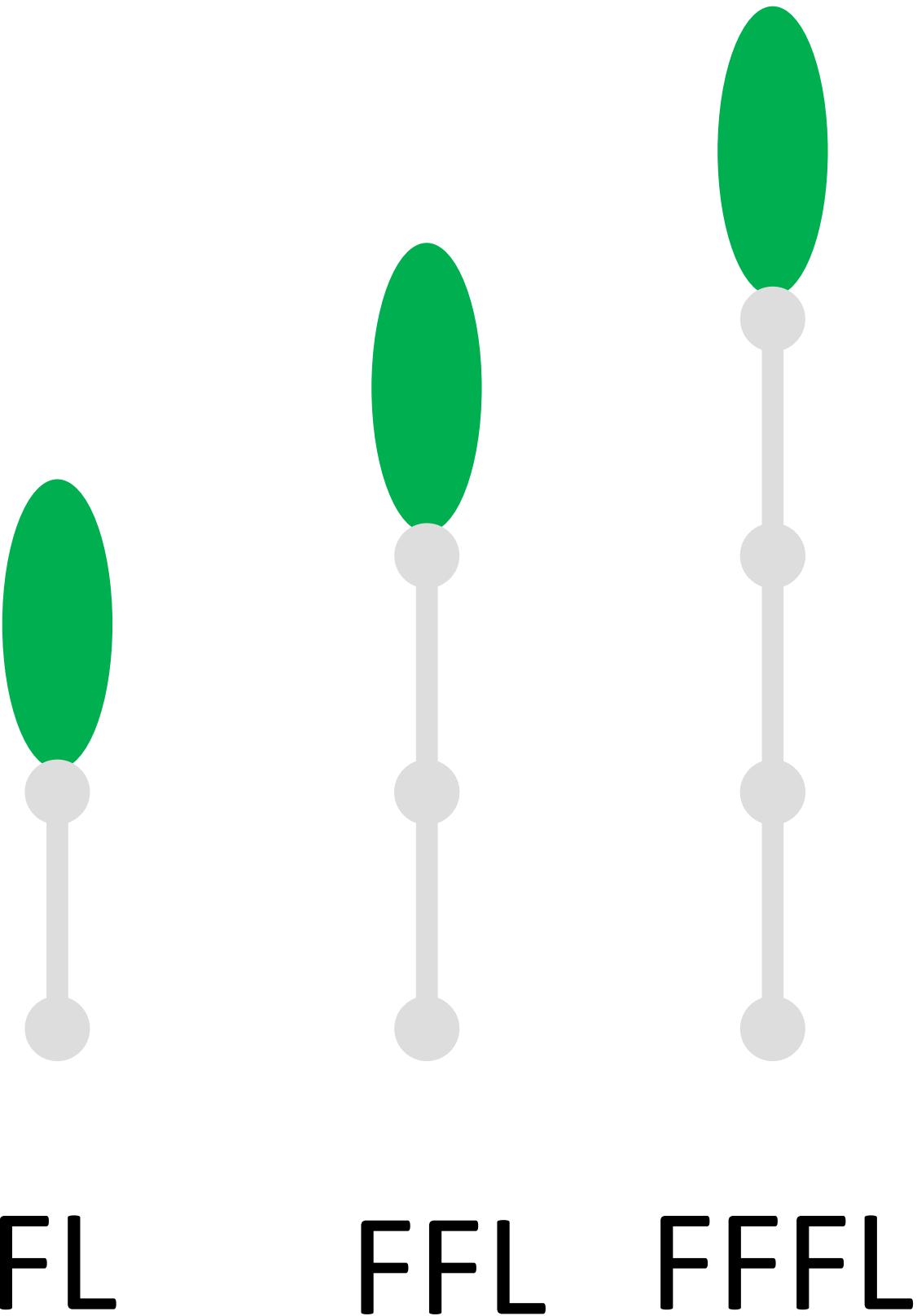


Draw the L-system **with two recursive iterations** for the following initial conditions and production rules:

- 1) initial: FL; rule: L->FL
- 2) initial: FL; rule: F->F-F
- 3) initial: L; rule: L -> F[L][-L][+L]

initial: FL; rule: L->FL

- (1) FL
- (2) FFL
- (3) FFFL

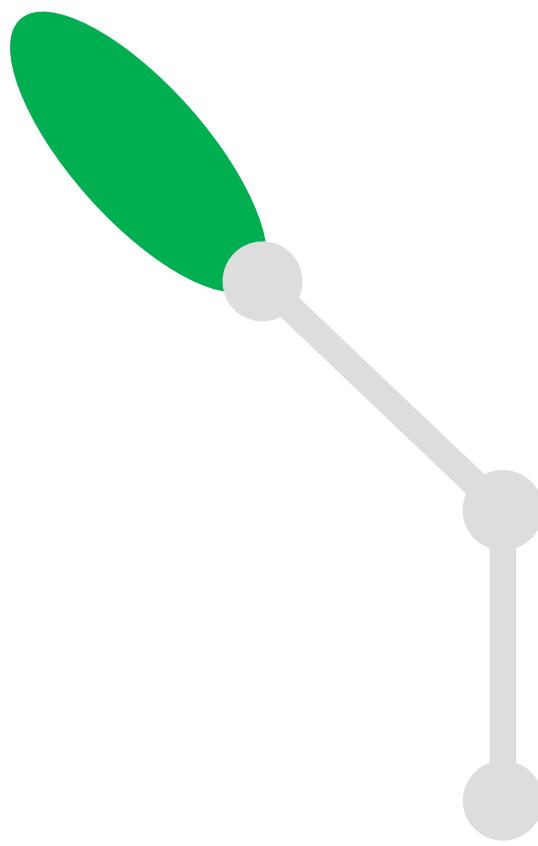


initial: FL; rule: F-> F-F

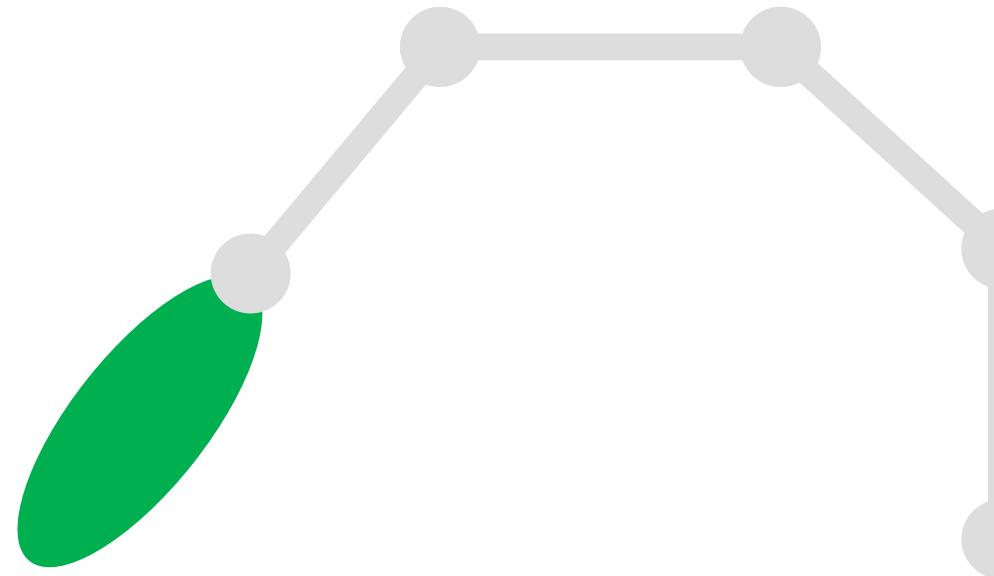
- (1) FL
- (2) F-FL
- (3) F-F-F-FL



FL



F-FL



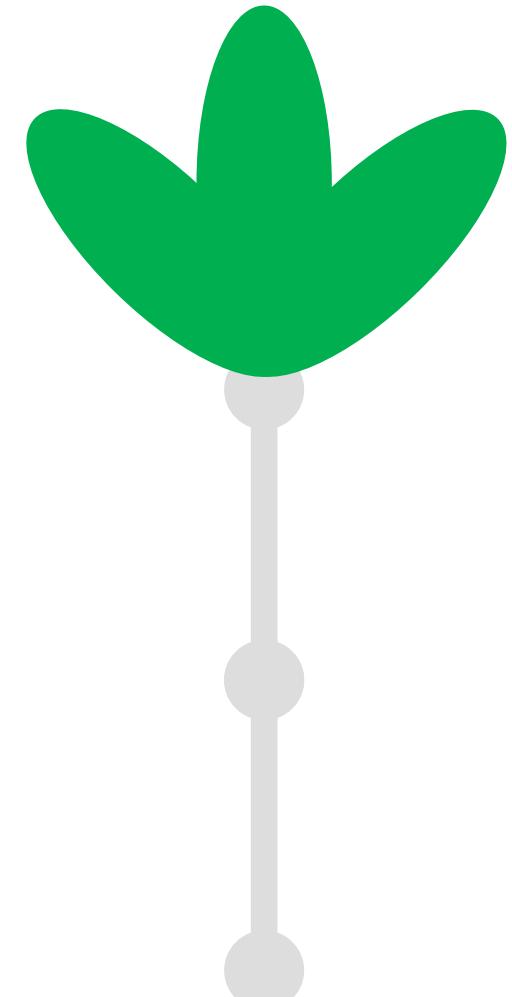
F-F-F-FL

initial: L; rule: L \rightarrow F[L][-L][+L]

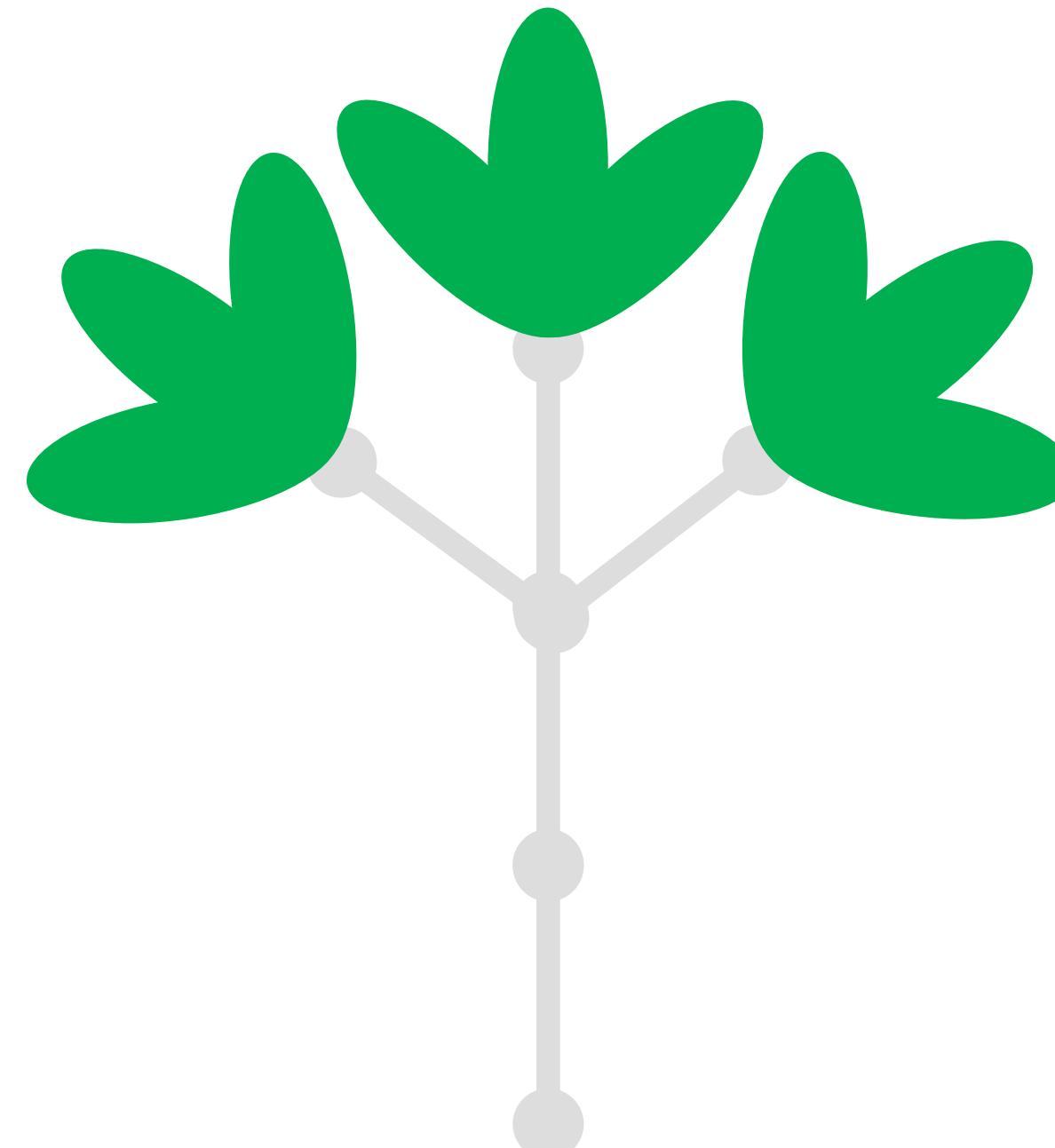
- (1) FL
- (2) FF[L][-L][+L]
- (3) FF[F[L][-L][+L]][- F[L][-L][+L]][+ F[L][-L][+L]]



FL



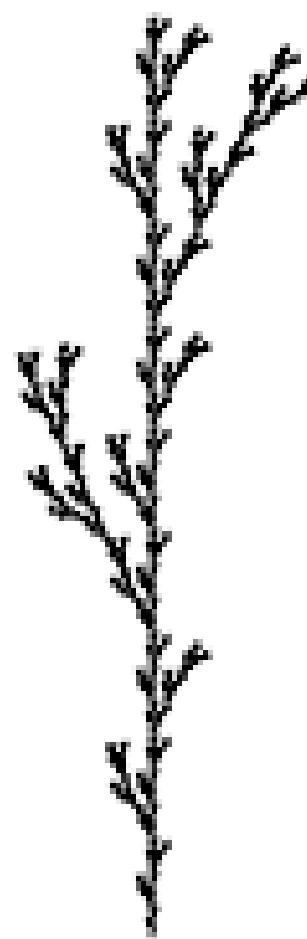
FF[L][-L][+L]



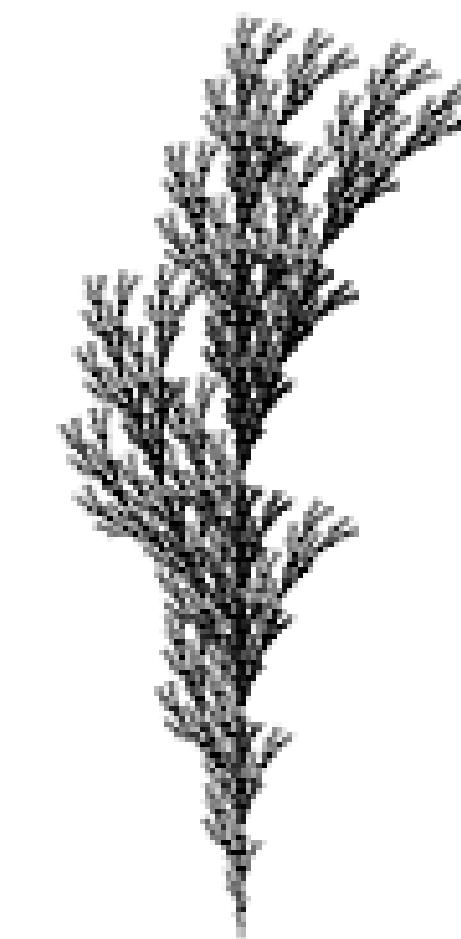
FF[F[L][-L][+L]][- F[L][-L][+L]][+ F[L][-L][+L]]



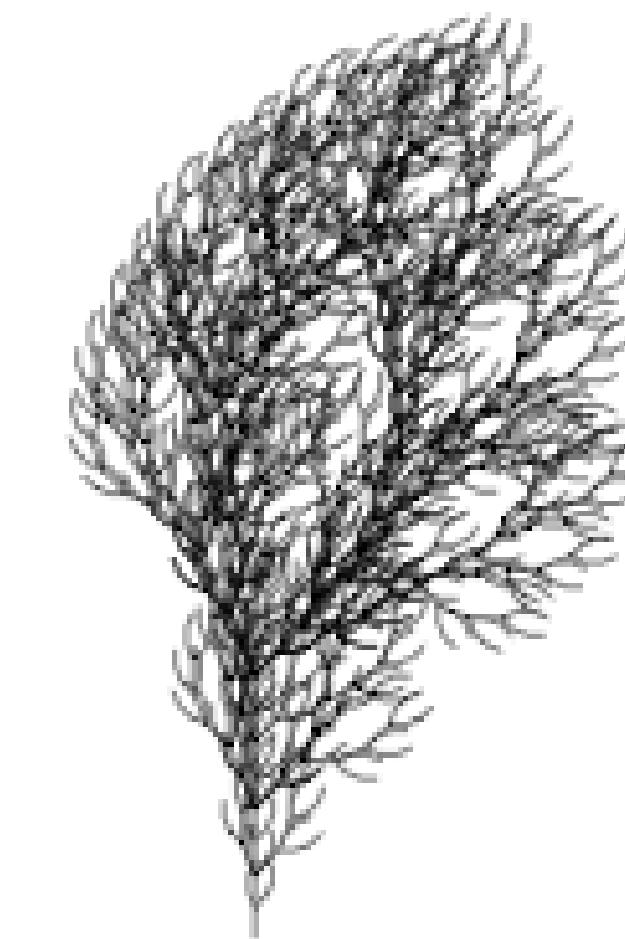
More Branching Structure Examples



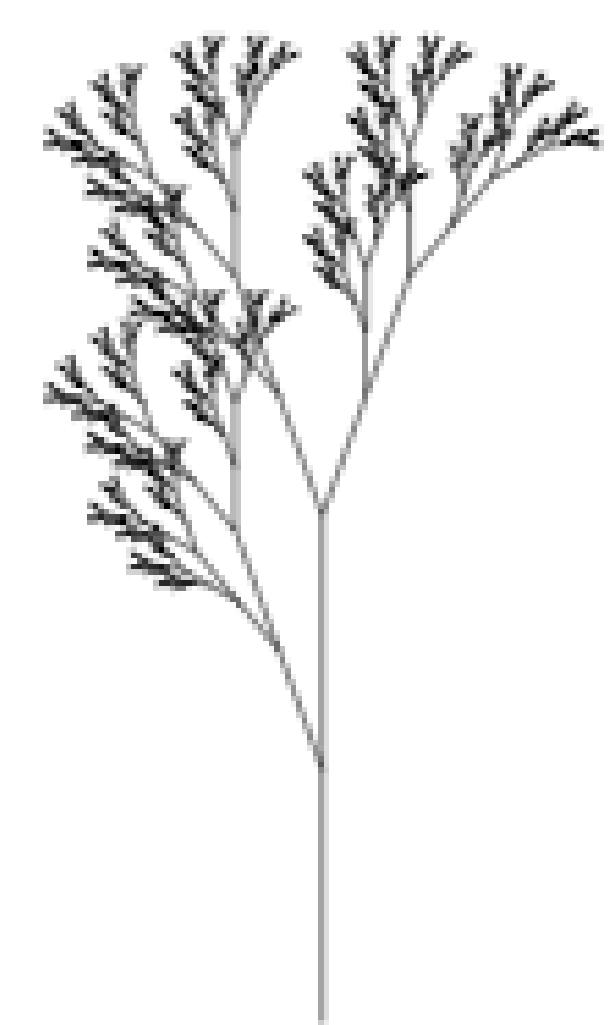
$n=5, \delta=25.7^\circ$
 F
 $F \rightarrow F [+F] F [-F] F$



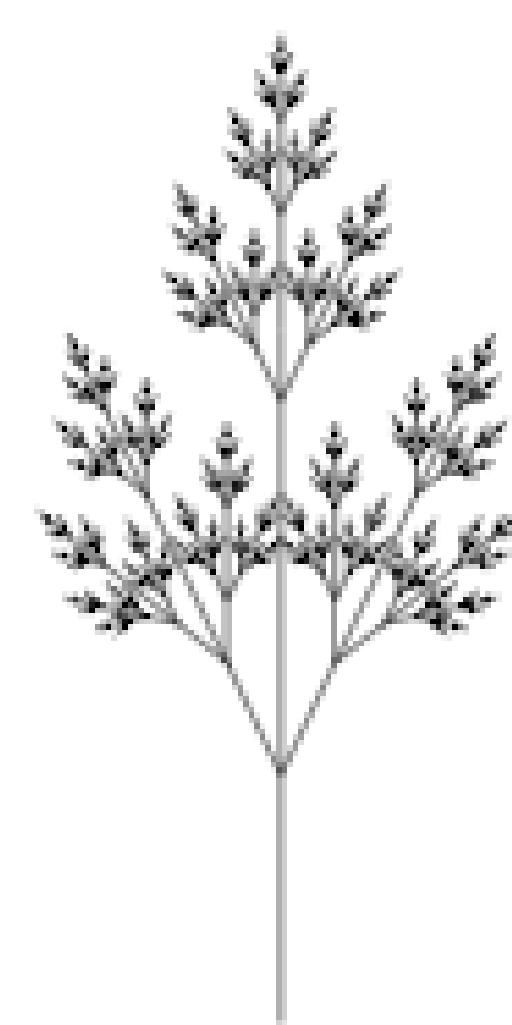
$n=5, \delta=20^\circ$
 F
 $F \rightarrow F [+F] F [-F] [F]$



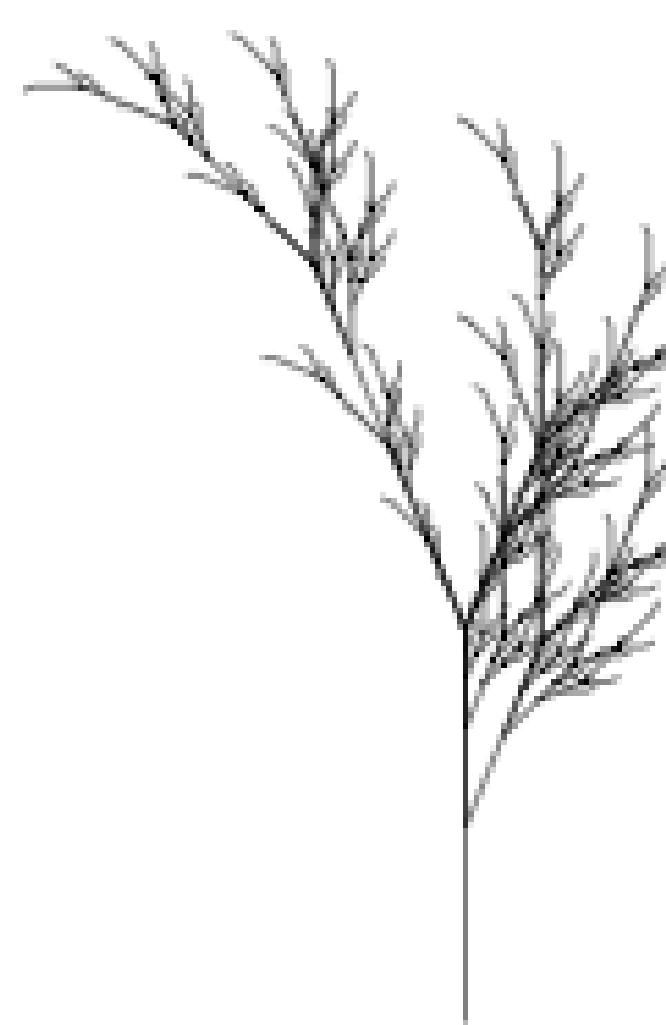
$n=4, \delta=22.5^\circ$
 F
 $F \rightarrow FF - [-F+F+F] +$
 $[+F-F-F]$



$n=7, \delta=20^\circ$
 X
 $X \rightarrow F [+X] F [-X] + X$
 $F \rightarrow FF$



$n=7, \delta=25.7^\circ$
 X
 $X \rightarrow F [+X] [-X] FX$
 $F \rightarrow FF$



$n=5, \delta=22.5^\circ$
 X
 $X \rightarrow F - [[X] + X] + F [+FX] - X$
 $F \rightarrow FF$



Stochastic L-Systems

- **Problem:** All plants generated by the same deterministic L-System are identical.
 - We want specimen-to-specimen variations that will preserve the general aspects of a plant but will modify its details.
- **Solution:** Stochastic L-Systems
 - random turn angle in some range, e.g. $\delta \in [25^\circ, 35^\circ]$; and/or
 - each production rule has some probability of being chosen

$$\begin{aligned}\omega &: F \\ p_1 &: F \xrightarrow{.33} F[+F]F[-F]F \\ p_2 &: F \xrightarrow{.33} F[+F]F \\ p_3 &: F \xrightarrow{.34} F[-F]F\end{aligned}$$



3D L-Systems

Need to handle 3D
rotations with axes

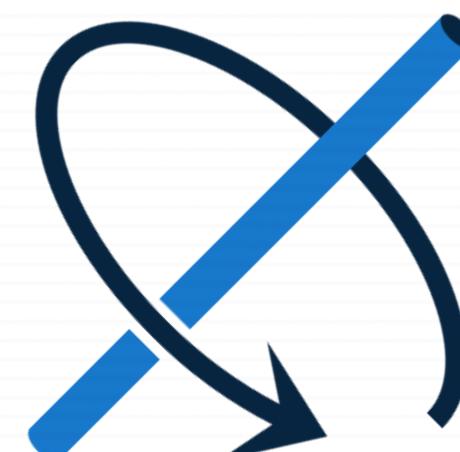


Quick Recap: 3D Rotation Matrix

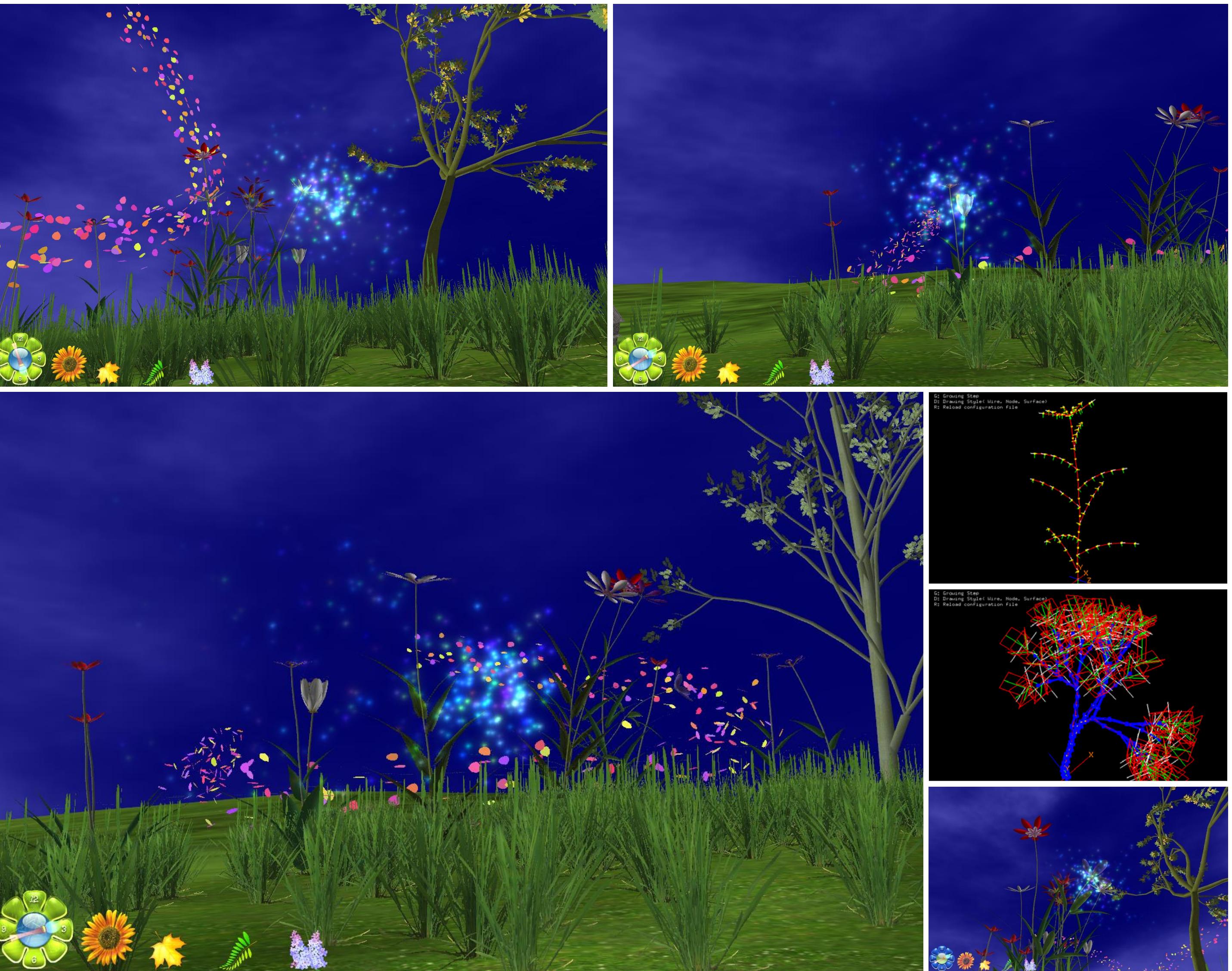
The matrix of a proper rotation R by angle ϑ around the axis $\mathbf{u} = (u_x, u_y, u_z)$, a unit vector with $u_x^2 + u_y^2 + u_z^2 = 1$, is given by:

$$R = \begin{bmatrix} \cos \theta + u_x^2 (1 - \cos \theta) & u_x u_y (1 - \cos \theta) - u_z \sin \theta & u_x u_z (1 - \cos \theta) + u_y \sin \theta \\ u_y u_x (1 - \cos \theta) + u_z \sin \theta & \cos \theta + u_y^2 (1 - \cos \theta) & u_y u_z (1 - \cos \theta) - u_x \sin \theta \\ u_z u_x (1 - \cos \theta) - u_y \sin \theta & u_z u_y (1 - \cos \theta) + u_x \sin \theta & \cos \theta + u_z^2 (1 - \cos \theta) \end{bmatrix}$$

Beyond the scope of this class: requiring some advanced math on Euler angle to derive the formula



Live Demo: Interactive L-System



Other Reading Resources

