Algorithms Design.
Georgia Institute of Technology.

Rudrata path.

Graph problems in the class NP-complete.

- Independent set (IS), search version.
- Clique, search version.
- Vertex Cover, search version.
- Today: Rudrata path.

Rudrata path

Definition

Given an graph $G = (V, E)$ a *Rudrata* $(s, t)-$*path* is a path starting at s and ending at t with the property that every vertex in the graph is visited exactly once. If we take $s = t$ (thus, allowing this vertex to be visited twice) this is called a *Rudrata cycle*.

What is the difference with *Hamiltonian* path and cycles?

None!

- Rudrata (9th century) poet and literary theorist.
- Sir. William Rowan Hamilton (19th century) Irish mathematician.
- Thomas Kirkman (19th century), al-Adli ar-Rumi (9th century), Abraham de Moivre, Leonard Euler (18th century)

Rudrata path to Rudrata cycle

Given an input of the Rudrata $(s, t)-$path add a new vertex v with two new edges (vs) and (tv):

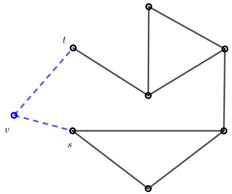

Figure: This graph has a Rudrata cycle iff the given one has a Rudrata $(s, t)-$path.

Rudrata path is NP-hard

We will outline: `Vertex-cover` $\rightarrow$ `Rudrata cycle` *in a directed graph*.

Rudrata path is NP-hard

Given an input of `Vertex-cover`, $\{G = (V, E), k \in \mathbb{N}\}$, we build a *directed graph* H in three steps:
(1) For each edge $e = (uv) \in E$ build a *gadget*:

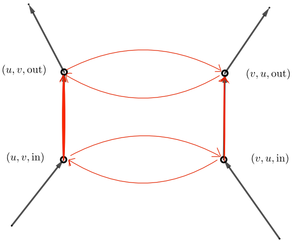

Figure: Each edge in G leads to four vertices and six edges in H.

Rudrata path is NP-hard

(2) For each vertex $u \in V$ with d neighbors build a *chain* connecting the corresponding gadgets: add edges $(u, v_i, \text{out}) \to (u, v_{i+1}, \text{in})$ for $i = 1, 2, \ldots, d-1$.

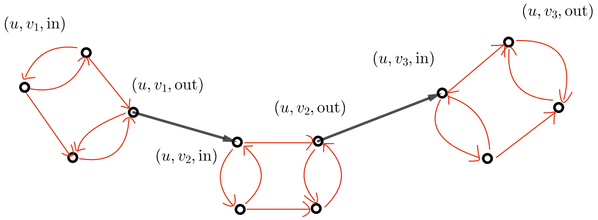

Figure: Gadgets associated to one vertex are connected by the chain.

(3) Add k new vertices, $x_1, x_2, \ldots, x_k$ with edges $x_i \rightarrow (u, v_1, \text{in})$ and $(u, v_d, \text{out}) \rightarrow x_i$, for $i = 1, 2, \ldots, k$.

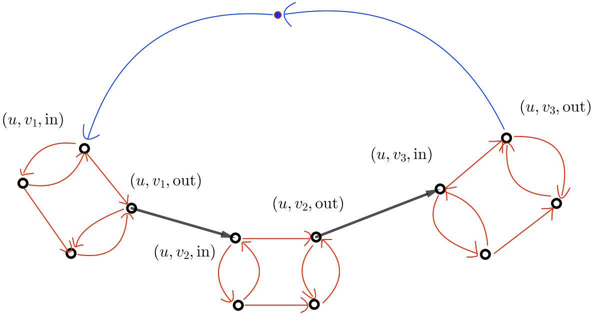

Figure: All chains are interconnected through the vertices x_i.

Rudrata path is NP-hard

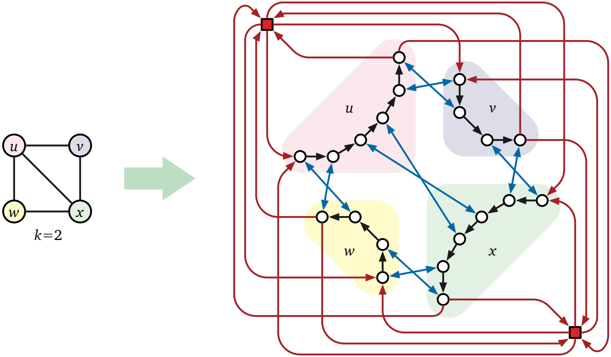

Figure: The reduction from `Vertex-Cover` to `Rudrata`.

Rudrata path is NP-hard

Given a vertex cover: $C = \{c_1, c_2, \ldots, c_k\}$ of G build a Rudrata cycle moving from x_i to the chain of the vertex c_i. When entering vertex (u, v, in) traverse the corresponding gadget as follows:

- If $v \in C$, move to (u, v, out).
- If $v \notin C$, move $(u, v, \text{in}) \to (v, u, \text{in}) \to (v, u, \text{out}) \to (u, v, \text{out})$.

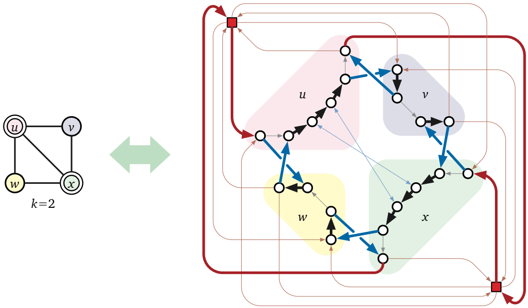

Figure: The cycle for our example.

Rudrata path is NP-hard

Given Rudrata cycle of the graph H, it can be shown that such cycle traverses exactly k chains, and the corresponding vertices form a vertex cover of G.

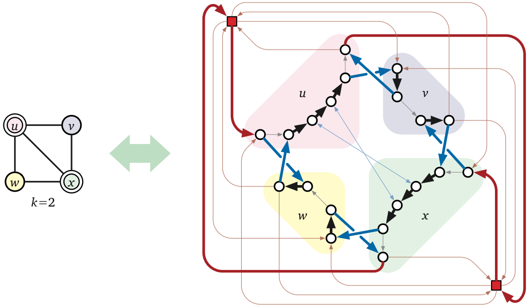

Figure: The cycle for our example.