Thursday, February 17, 2022 12:27 PM

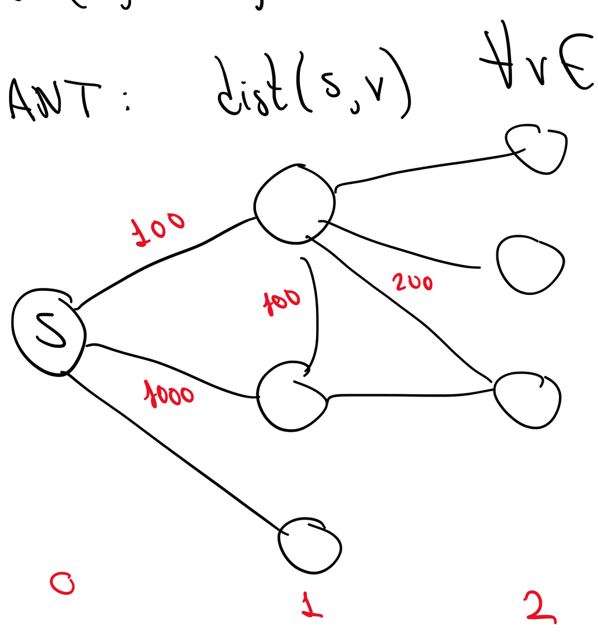
- Quiz 2 02/24 (DP)
- Practice Quiz out soon!
- HW 4 Today!

Prof Randall is back! Graph Algs.

Shortest paths on graphs using DP.

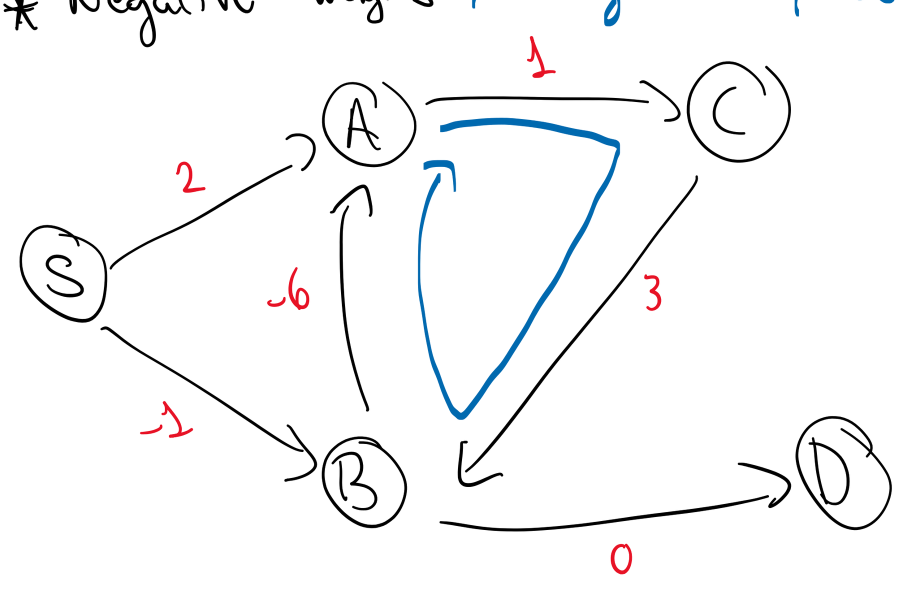
- $G=(V,E)$; $s\in V$; $\{w_e\}_{e\in E}$ ← real numbers.

WANT: $dist(s,v)$ $\forall v\in V$.



0 1 2 3 ...

* Negative weights / Negative cycles!



Input: $G=(V,E)$; $s\in V$; $\{w_e\}_{e\in E}$

Output: YES if there is a negative cycle in G.
Else $dist(s,v)$ $\forall v\in V$.

* Bellman-Ford.

- $D(v,k)$ = len of the shortest path from s to v using at most $\boxed{k}$ edges.

$v\in V$
$0\le k\le \cancel{|E|}$ $|V|-1$

$$D(v,k)=\min\left\{\min_{u\to v}\{D(u,k-1)+W_{uv}\};D(v,k-1)\right\}$$

where $u\to v$ means there is an edge from u to v.

BASE CASE(S):

$D(s,k)=0$ $\forall\ 0\le k$

$D(v,0)=\infty$ $\forall\ v\neq s$

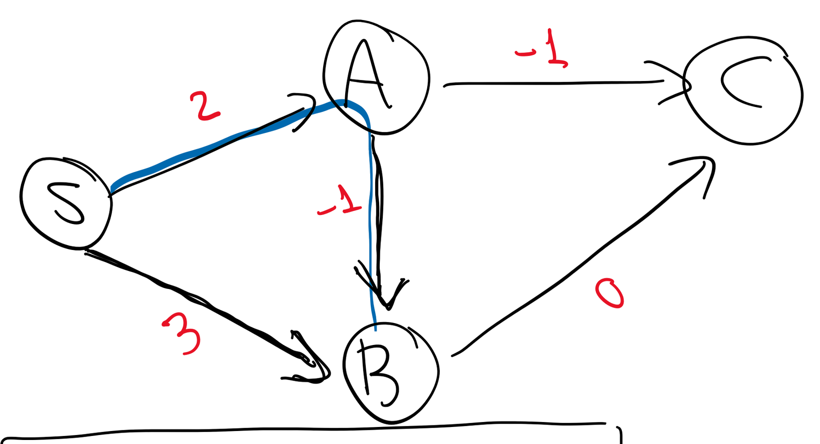
	0	1	2 ...
s	0	0	0
v_1	∞		
v_2	∞		
⋮			
v_n	∞		

"The incorrect recurrence"

$D(v,1)=\min_{u\to v}\{D(u,0)+W_{uv}\}$ % updates neig. of s

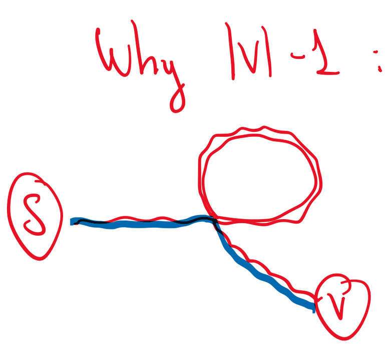
$D(v,2)=\min_{u\to v}\{D(u,1)+W_{uv}\}$ % updates grandchilds of s ;)

See remark at the end ↓



	0	1	2	3
S	0	0	0	0
A	∞	2	2	2
B	∞	3	1	1
C	∞	∞	1	1

Why $|V|-1$:

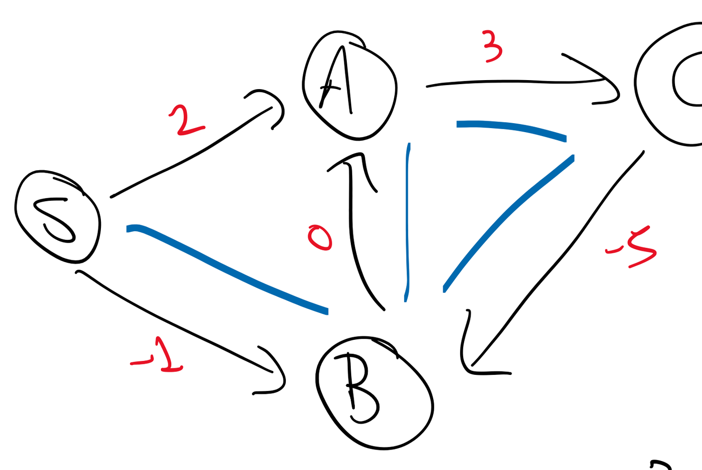


$$D(A,1)=\min\{\overset{0+2}{D(s,0)+W_{SA}};\overset{\infty}{D(A,0)}\}$$

$$D(B,1)=\min\left\{\min\left\{\begin{matrix}\overset{0+3}{D(s,0)+W_{SB}}\\ \underset{\infty+}{D(A,0)+W_{AB}}\end{matrix}\right\},\underset{\infty}{D(B,0)}\right\}$$

$$D(B,3)=\min\left\{\min\left\{\begin{matrix}\overset{0+3}{D(s,2)+W_{SB}}\\ \underset{2+(-1)}{D(A,2)+W_{AB}}\end{matrix}\right\};\underset{1}{D(B,2)}\right\}$$

Return $D(v,|V|-1)$.



	0	1	2	3	4
S	0	0	0	0	0
A	∞	2	-1	-1	-1
B	∞	-1	-1	-1	-3
C	∞	∞	5	2	2

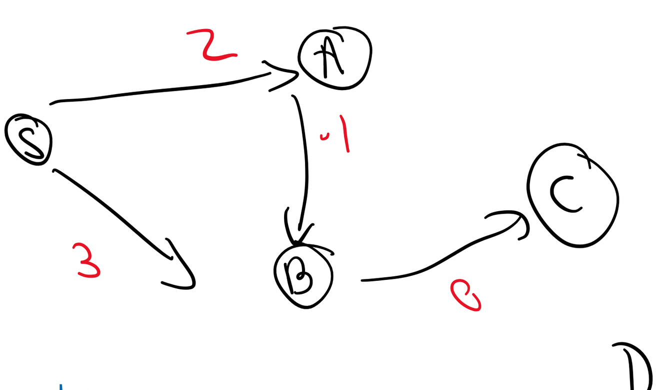
* Do $|V|$ rounds of Bellman-Ford:

If Columns $|V|-1$ and $|V|$ are the same, output $D(v,|V|)$.

Else: report "there is a neg-cycle".

Runtime: for $n=|V|$

$O(|V||E|)$ because every column is populated doing $O(|E|)$ comparisons.



Wrong recurrence: $D(v,k)=\min_{u\to v}\{D(u,k-1)+W_{uv}\}$

	0	1	2	3
S	0	0	0	0
A	∞	2	2	2
B	∞	3	1	1
C	∞	∞	3	1

- These base cases allow us to use the shortest recurrence!

$D(A,2)=\cancel{\min}\ D(s,1)+W_{SA}$