Thursday, February 10, 2022 9:44 AM

Input: $M_1, M_2, \dots, M_n$

Output: optimize the operation $M_1 M_2 M_3 \dots M_n$

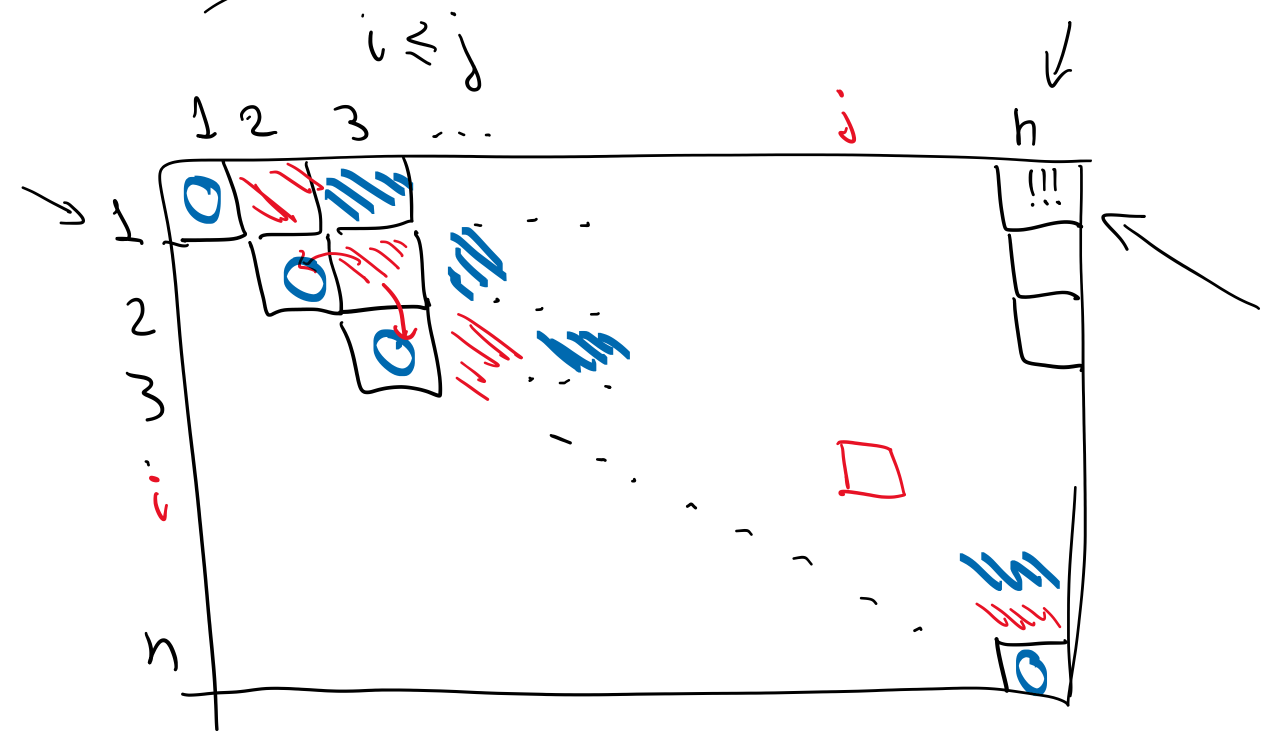
If M_i has dim (m_{i-1}, m_i)

Input: $(m_0, m_1, m_2 \dots m_n)$

(1) $T[i,j]$ = min cost of multiplying $M_i M_{i+1} \dots M_j$

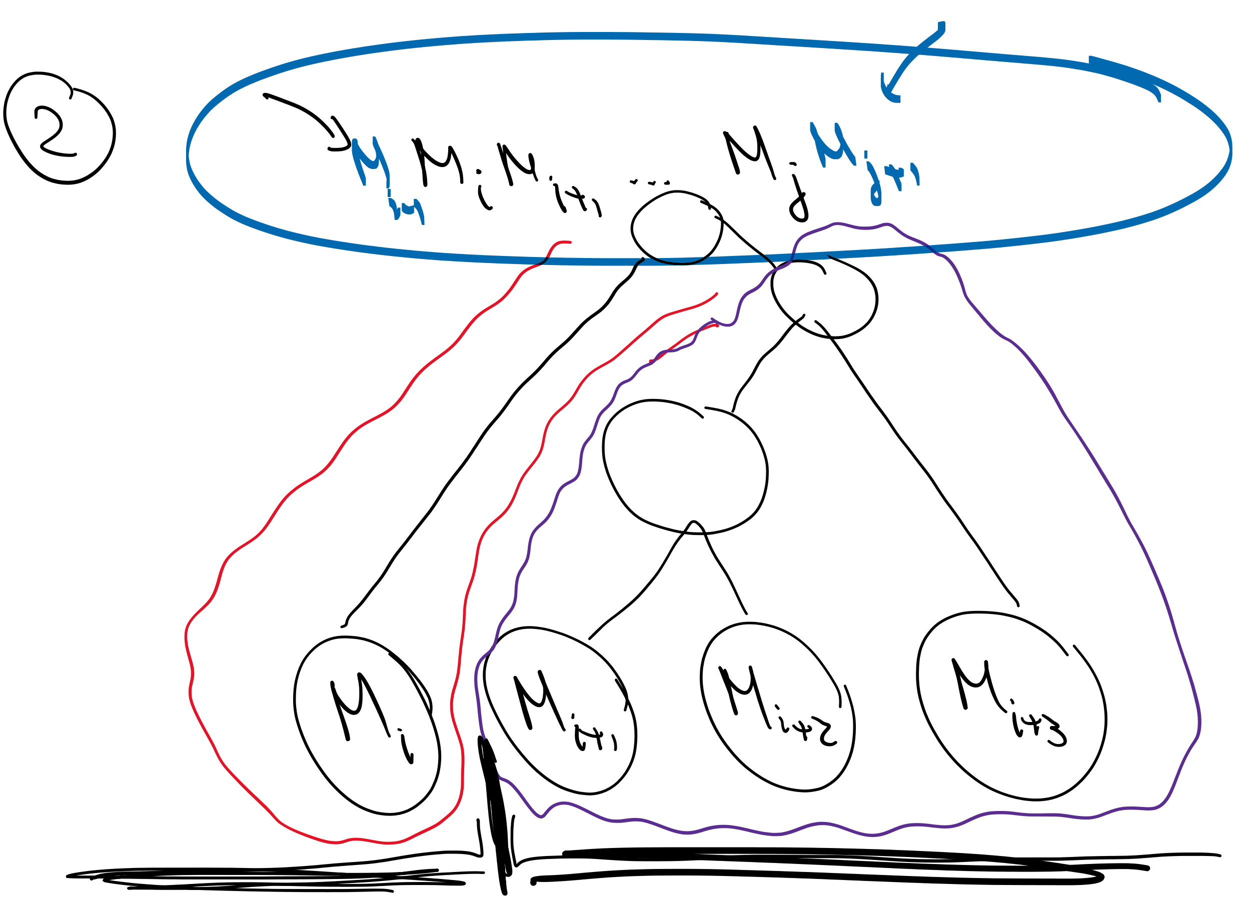
(NEED: $m_{i-1}, m_i, m_{i+1}, \dots, m_j$)

$i \le j$



$1 \le i \le j \le n$.

(2)



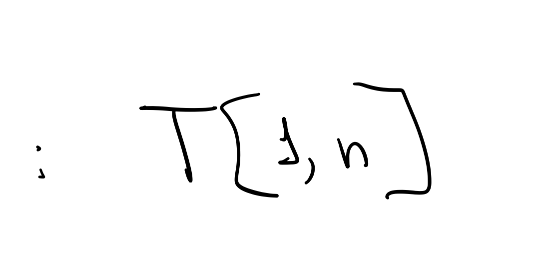
$M_i M_{i+1} \dots M_k | M_{k+1} \dots M_j$

Blue lines corresponds to all possible breaking into sub-problems. We need to chose the best (i.e.: win!)

$$T[i,j] = \min_{i \le k \le j-1} \left\{ T[i,k] + T[k+1,j] + m_{i-1} m_k m_j \right\}$$

BASE CASE(S):

$M_i M_{i+1} M_{i+2}$



$\boxed{T[i,i] = 0 \quad 1 \le i \le n}$

Return: $T[1,n]$

Runtime: $O(n^3)$

See [DPV] for pseudocode & runtime.

Coin Change.

coin denominations $C = [c_1, c_2, c_3 \dots c_n]$

$K \in \mathbb{N}$

Can we make change for K using coins of denomination C?

Ex: $C = [2, 3, 7]$

$K = 11$ ✓ $(2,2,7)$ or $(2,3,3,3)$

$K = 15$ ✓ $(3,3,3,3,3)$ or $(7,3,3,2)$

Input: $C = [1 \dots n]$, $K \in \mathbb{N}$ len $\log_2 K$

Output: YES/NO : can we make change for K?

(First attempt)

$T[i]$ = 1/0 if we can make change for K using denominations from $C = [1 \dots i]$

$T[i]$ = " in terms of previous sub-problems?

if $T[i-1] = 1$:)

Else, we need to use @ least one coin of den c_i. Now can we make change for $K - c_i$?

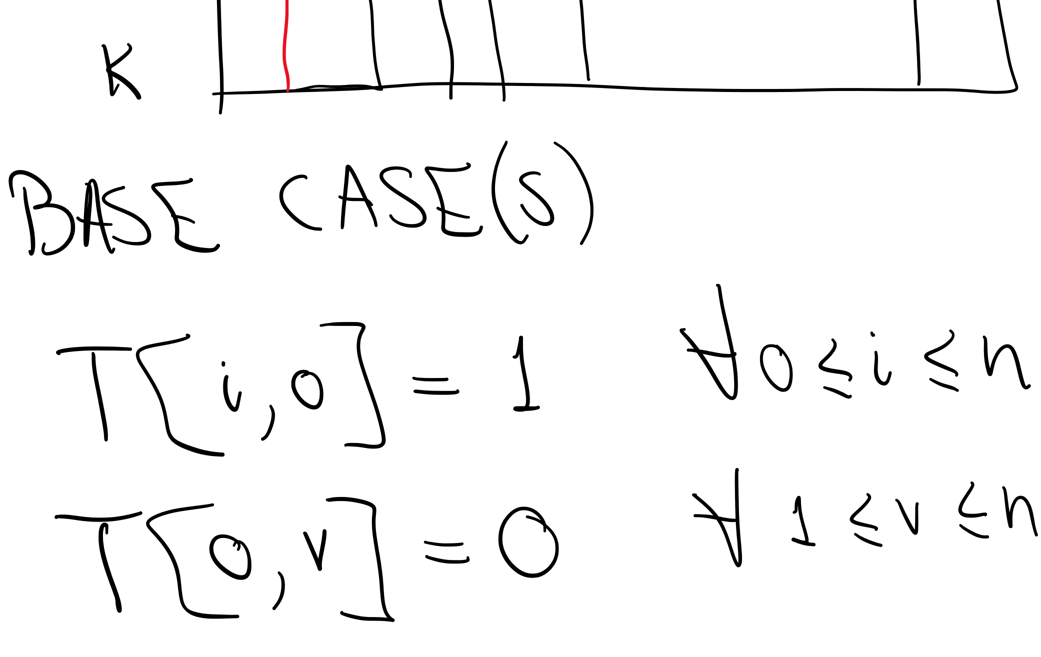
$T[i,v]$ = 1/0 if we can make change for v using denominations $C = [1 \dots i]$.

$1 \le i \le n$

$1 \le v \le K$.

Recursive relation:

$$T[i,v] = \max \left\{ T[i-1,v], T[i, v-c_i] \right\}$$



BASE CASE(S)

$T[i,0] = 1 \quad \forall 0 \le i \le n$.

$T[0,v] = 0 \quad \forall 1 \le v \le n$

Return: $T[n,K]$

Runtime: $O(nK)$ ← NOT efficient!!!

$K = 2^{\log_2 K}$