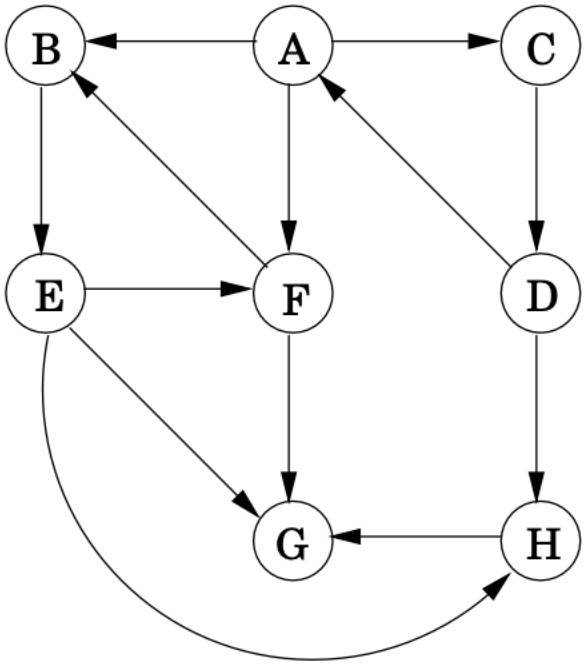
Practice Quiz 3

Problem 1 True and False with Justification

1.1 The shortest path between two nodes is necessarily a part of some MST.
1.2 Prim's algorithm can be applied on a graph with negative edges.
1.3 Kruskal's algorithm can be applied on a graph with negative edges.
1.4 There can be multiple MSTs for the same graph G.
1.5 If all edge weights in a connected graph G are distinct, then G has a unique minimum spanning tree.

Problem 2 Consider the graph below for the following subproblems:



2.1 Perform a depth-first search and provide the pre and post labels for the vertices A-H below
2.2 How many strongly connected components exist in the graph above? (Only list the number; no work needed)
2.3 List the strongly connected components in the graph above

Problem 3 Computopia

The police department in the mythical city of Computopia has made all streets one-way. The mayor contends that there is still a way to drive legally from any intersection in the city to any other intersection, but the opposition is not convinced. Furthermore, the city elections are coming up soon, and there is only enough time to run a linear time algorithm.

(a) Formulate this problem as a graph-theoretic problem, and explain why this problem can indeed be solved by a linear-time algorithm (i.e., linear in the number of intersections and streets).

(b) Suppose now that the mayor needs to show that, even though her original claim was false, something weaker does hold: If you start navigating one-way streets from town hall, there is always a way to drive legally back to the town hall. Formulate this weaker property as a graph-theoretic problem and show that this can also be solved in linear time.

Problem 4 Batman and Buzz Boi

Tanmoy is graduating soon and has decided to take up a job as Batman's new sidekick in Gotham city: Buzz Boi. New to the job, Tanmoy is trying to minimize his run-ins with the city's numerous villains.

The Batman provides Tanmoy with a list of n neighborhoods (labeled 0 to n-1) on a map (with marked distances between each neighborhood) of Gotham City. Each of the neighborhoods is connected by a two-way highway. Tanmoy is also provided with a solar-powered BuzzMobile that can travel up to 2d miles a day.

Given that every neighborhood within d miles from Tanmoy's house will be his responsibility, help him come up with an algorithm to pick the neighborhood for him to choose his house so that he has the fewest other neighborhoods to look after. Assume no two neighborhoods have the same number of other neighborhoods within distance d.

(a) Describe (in words) the algorithm for finding the required neighborhood.
(b) What is the runtime of the algorithm that you gave above?

Problem 5 MST Check

Suppose we are given an undirected graph G = (V,E) with positive weights w(e) for each edge e in E. We are also given a minimum spanning tree T. Now suppose that for every edge e in E, we replace its edge weight w(e) by w(e) + 1. Is the tree T still guaranteed to be a minimum spanning tree for this new weighted graph? Explain.