CS 3510

Thursday, April 14, 2022 12:32 PM

1) T/F

T · If we solve Subset Sum in polytime, then P = NP

T · Every problem in the class P can be reduced to Vertex Cover.

2) Longest Path:

Input: $G=(V,E)$, $g \in \mathbb{N}^*$

Output: a path of len $\geq g$

The Longest Path is NP-hard because it is a generalization of:

IS

Clique

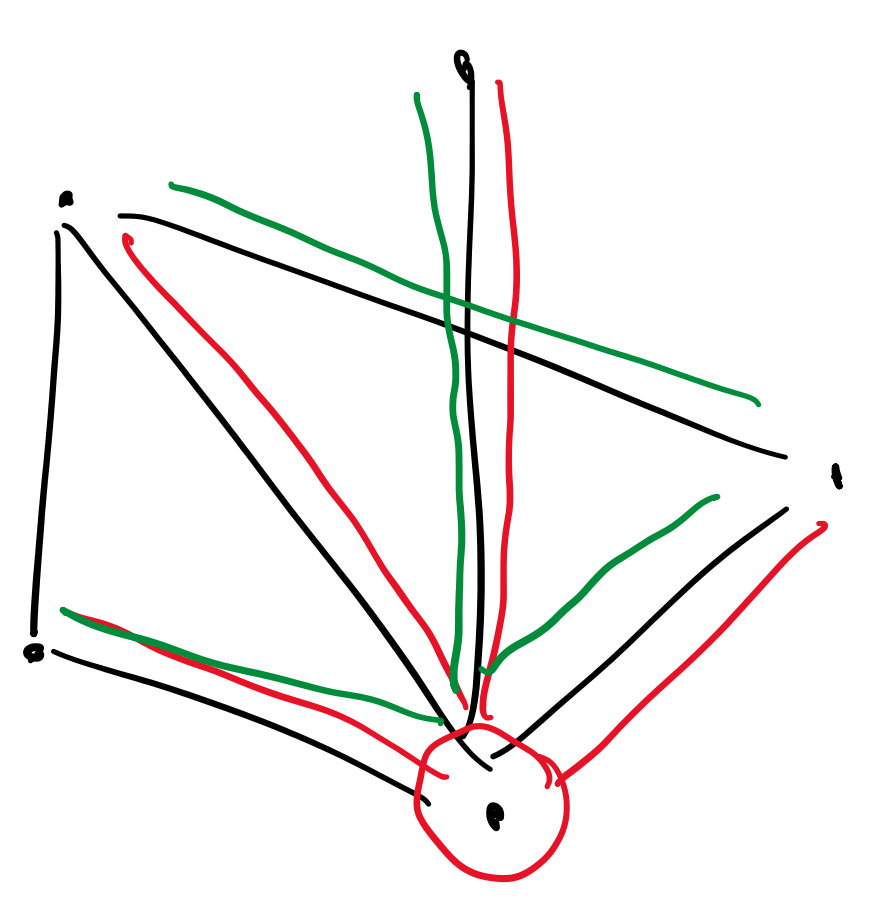
Rudrata (VC) $g=|V|$

8.12 (2006 edition)

K-spanning tree

Input: $G=(V,E)$; $K \in \mathbb{N}^*$

Output: T, subgraph of G. T is a spanning tree and $deg_T(v) \leq K$



K=3

Step 1: K-Spanning Tree is in NP.

Given $G=(V,E)$; $K \in \mathbb{N}^*$ and S, candidate sol.

- S spans G: Run Explore (v) in S, check that all vertices are marked visited.
- S has "small" deg: $\forall v \in V$: $deg_S(v) \leq K$ by check
- S is cycle-free: Run DFS, check for back-edges

Step 2

Rudrata Path ⟶ K-Spanning Tree

$G=(V,E) \longmapsto G' = G$, $K=2$.

(⟹) If S is a sol. of Rudrata, then S is also a sol. of 2-Spanning Tree for G'.

(⟸) (✱) We can recover the sol of Rudrata in polytime time.

8.14 Clique + I.S.

Input: $G=(V,E)$; $g \in \mathbb{N}^*$

Output: (S_1, S_2): $S_i \subseteq V$; $|S_1| = |S_2| = g$

S_1 is a clique in G.

S_2 is an IS in G.

Step 1 Clique + IS is in NP. $n = |V|$

- Check $|S_1| = g$ $O(n)$
- Check $|S_2| = g$ $O(n)$
- $\forall u,v \in S_1$: $(u,v) \in E$ $O(n^2)$
- $\forall u,v \in S_2$: $(u,v) \notin E$ $O(n^2)$

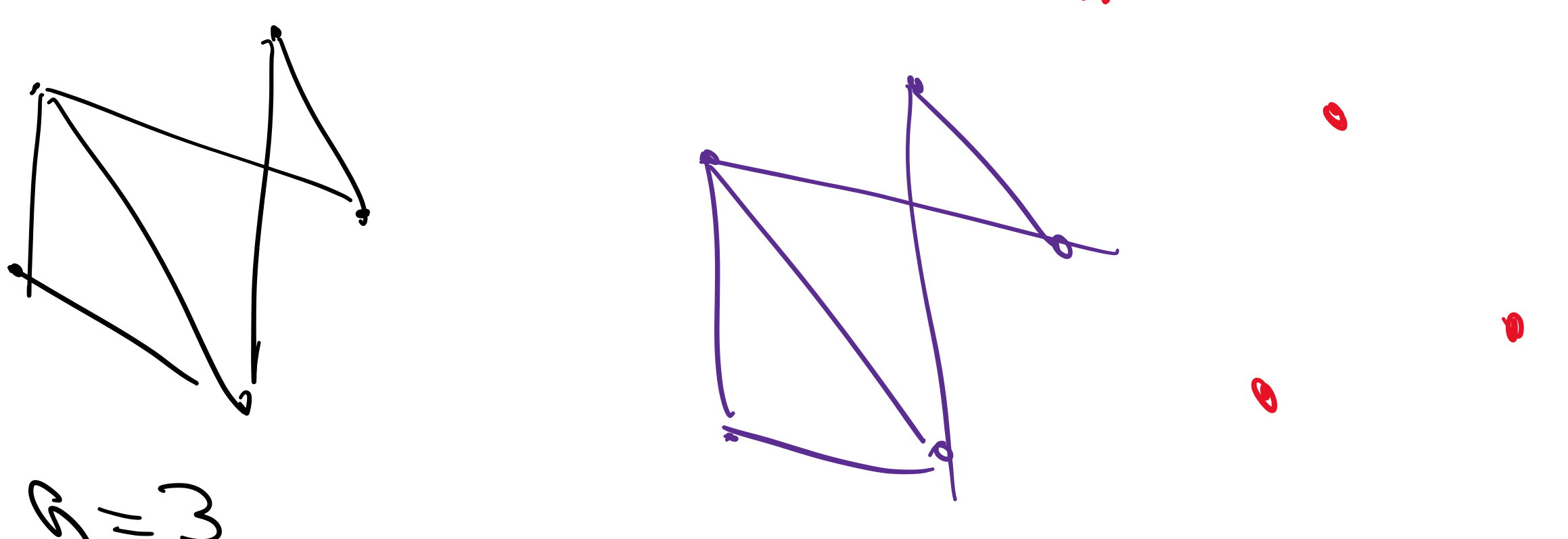
Step 2:

Clique ⟶ Clique + IS

$G=(V,E); g \longmapsto G' = G \cup H$ $g = g$

where $H = (V_H, E_H)$; $|V_H| = g$

$E_H = \emptyset$



g=3

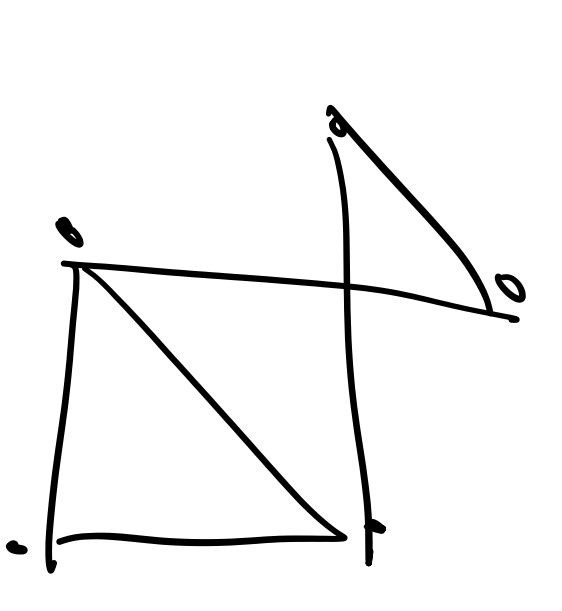
Notice: $|H| = O(|V|)$

(⟹) Given a Clique S in G, then (S, H) is a sol for Clique + IS !!!

(⟸) Given (S_1, S_2) of Clique + IS, then S_1 is a Clique in G because all vertices from H are isolated!

(✱) We recover a Clique in linear time.

IS ⟶ Clique + IS



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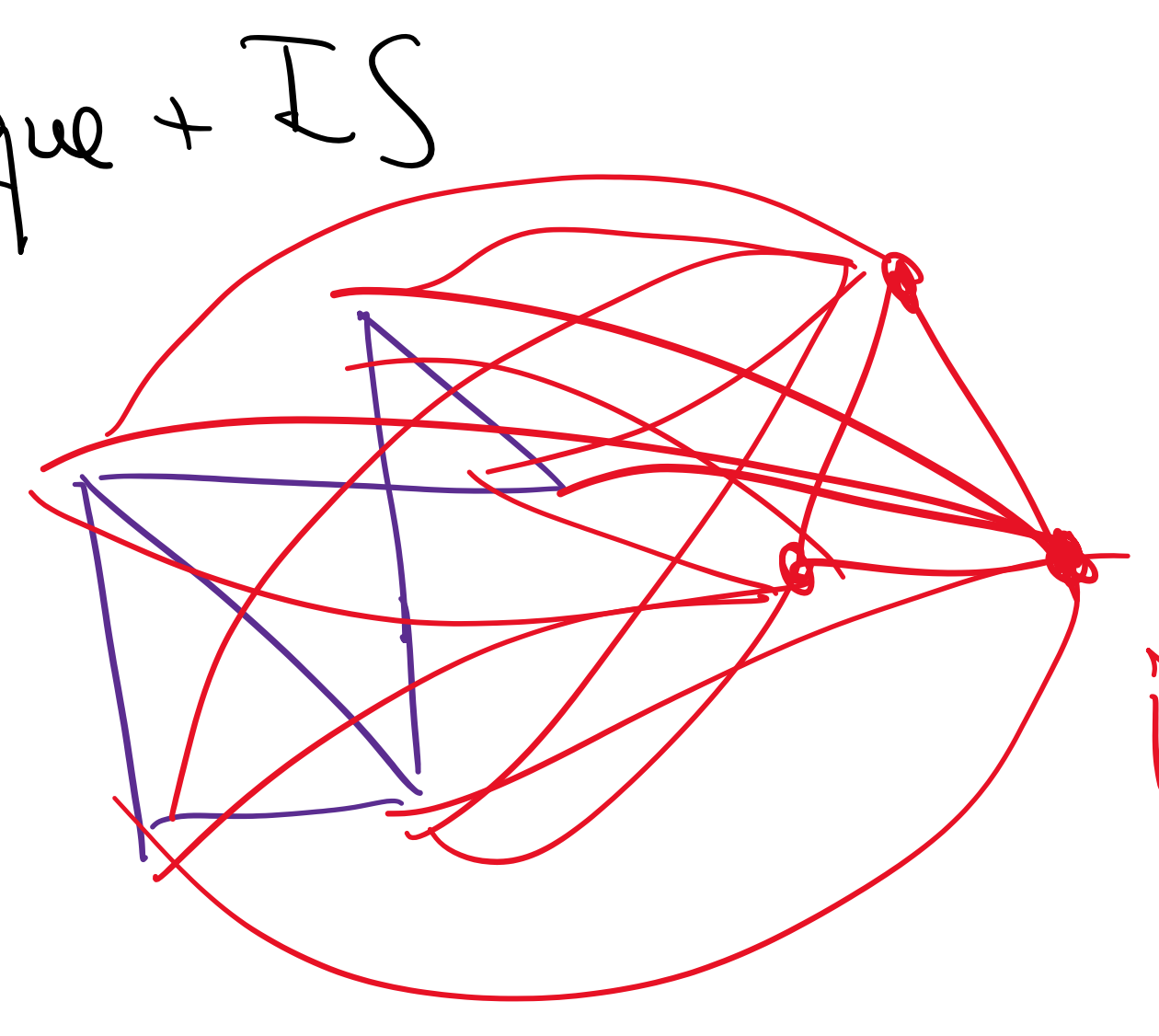

⟸ If (S_1, S_2) is a Clique + IS on $G \cup \hat{H}$ then S_2 is an IS of G because no vertex from $\hat{H}$ is on an IS of size > 1.