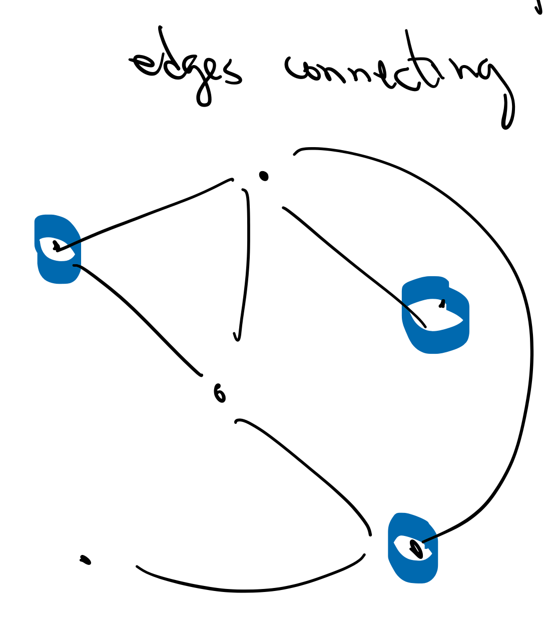
CS 3510

Thursday, April 7, 2022 12:33 PM

Def: Given a graph $G=(V,E)$ a set $S \subseteq V$ is said to be an Independent Set if there are no edges connecting any two vertices in S.



IS - search

Input: $G=(V,E)$; $\boxed{g \geq 0}$

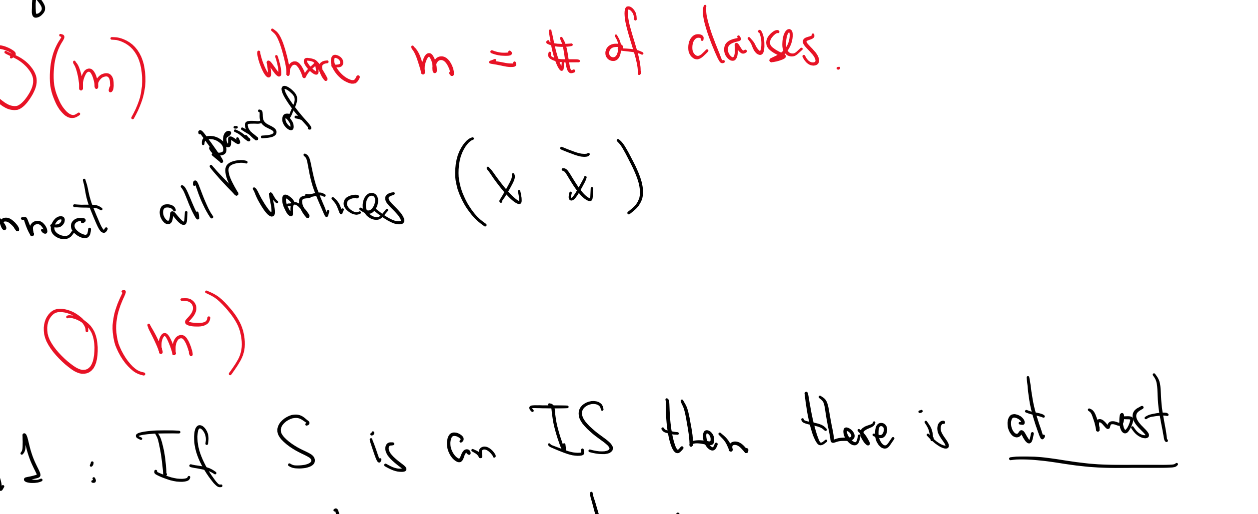
Output: an IS of ~~maximal~~ size $\geq g$.

IS- search $\in$ NP

- Check the candidate sol. is IS $O(|V|^2)$
- Check |candidate sol.| $\geq g$ $O(|V|)$.

3 SAT $\longrightarrow$ IS- search

$$f = (\bar{x}_1) \wedge (x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_2 \vee x_4) \wedge (x_1 \vee \bar{x}_4 \vee \bar{x}_5)$$



$\boxed{g = m}$

- Every clause gives us ≤ 3 vertices fully connected
  $O(m)$ where m = # of clauses.
- Connect all pairs of vertices $(x\ \bar{x})$
  $O(m^2)$

Claim 1: If S is an IS then there is at most one vertex per clause.
$\Rightarrow |S| \leq m$.

Claim 2: If S is an IS of size $\geq m$ then $|S| = m$ and there exactly one vertex associated to each clause!

Recovering a sol of 3 SAT:

Given an IS $|S| = m$. Make the literals associated to the vertices in S TRUE.
$O(n)$ n is # of variables!

If the input f for 3-SAT has @ least one sol. then IS-search also has a sol.

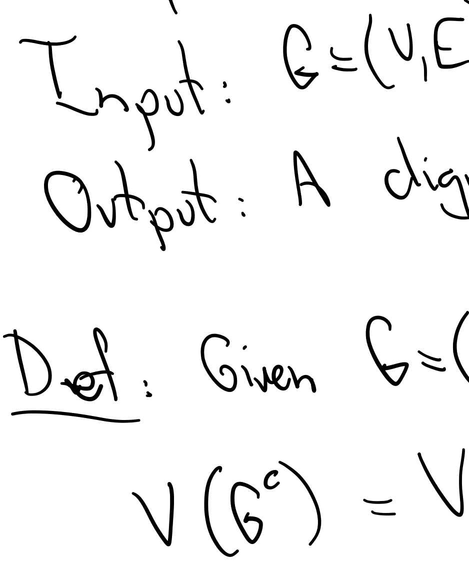
- Take a valid assignment for f. Pick one True literal in each clause. The set of corresponding vertices is IS !!! :)

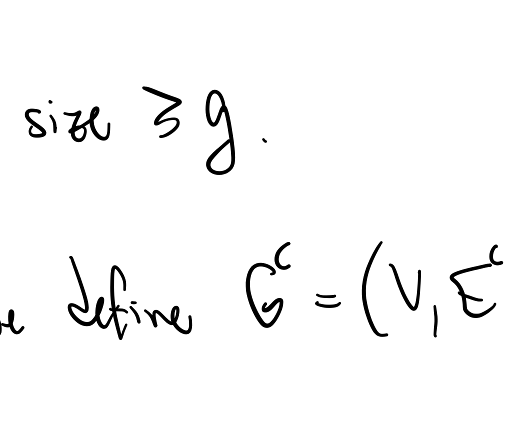
2 -SAT $\longrightarrow$ IS - search

IS-search $\longrightarrow$ 2-SAT (?)

Def: Given a graph $G=(V,E)$. A set $S \subseteq V$ is said to be a clique if:

$$\forall u,v \in S : (uv) \in E.$$





Clique - search

Input: $G=(V,E)$; $g \geq 0$

Output: A clique S of size $\geq g$.

Def: Given $G=(V,E)$, we define $G^c = (V, E^c)$

$$V(G^c) = V(G)$$

$$(uv) \in E^c \iff (uv) \notin E.$$

Claim: S is a clique in G iff S is an IS on G^c.

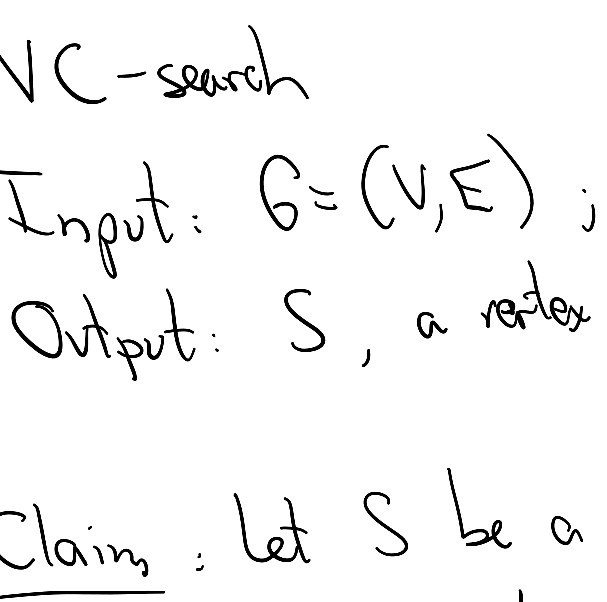
IS-search $\longrightarrow$ Clique-search

$G=(V,E)$; $g \rightsquigarrow G^c=(V,E)$; g

(*) Details left to the reader. :)

Def: Given a graph $G=(V,E)$. A set $S \subseteq V$ is called a Vertex Cover (VC) if

$$\forall e=(uv) \in E : \{u \in S\} \text{ OR } \{v \in S\}$$



VC - search

Input: $G=(V,E)$; b

Output: S, a vertex cover of size $\leq b$.

Claim: Let S be a VC. Then $V-S$ is an IS!

\proof: If $u,v \in V-S$ and $(uv) \in E$ then S is not "covering" this edge, which contradicts that S is a VC. ∎

IS - search $\longrightarrow$ VC - search

$G=(V,E)$; $g \longrightarrow G=(V,E)$; $b = |V| - g$

Read (*) :)

SAT

↓

3 SAT

↙

IS - search

↙ ↘

Clique-search VC-search

- *Reducing* IS-search to IS.
  The max-size IS is a sol. of IS-search if $\geq g$.
  Otherwise, there is no sol. for IS-search!!!
- Solving IS from a black-box for IS-search
  Linear (or binary!) search!
