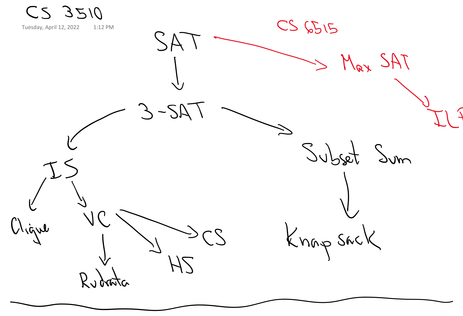
CS 3510

Tuesday, April 12, 2022 1:12 PM



Subset-Sum

Input: $S = \{s_1, s_2, \dots, s_n\}$, $T \in \mathbb{R}$

Output: $S' \subseteq S$: $\sum_{s \in S'} s = T$

Knapsack - search

$V = \{v_1 \dots v_n\}$ $\quad v_i = w_i$

$W = \{w_1 \dots w_n\}$

$B = T$

$g = T$

$S \subseteq [n]$:

$$\sum_{i \in S} w_i \overset{=}{\leq} T = T \overset{\leq}{=} \sum_{i \in S} v_i$$

$$\Downarrow$$

$$\sum v_i = T$$

Vertex Cover

$G = (V, E)$ $\quad b \in \mathbb{N}$

Output : $S \subseteq V$: $|S| \leq b$

$\forall e \in E \quad e = (uv)$: $u \in S$ OR $v \in S$.

VC $\in$ NP-hard

Hitting Set

Input : $S_1, S_2 \dots, S_n \subseteq R$, $b \in \mathbb{N}$

Output : $H \subseteq R$: $|H| \leq b$. $S_i \cap H \neq \emptyset$ $\forall i \leq i \leq n$

$\{1,2,3\}$

$\{4,5,6\}$

$\{1,3,5\}$

$\{2,4,6\}$

$b = 6$ ...

$b = 2$ $\longrightarrow$ $H = \{1, 4\}$

Vertex-Cover $\longrightarrow$ Hitting-Set.

$G = (V, E)$; b $\longmapsto$ $S_e = \{u, v\}$ $\forall e = (uv)$

$b_{HS} = b_{VC}$

$S \subseteq V$ is a VC for G $\iff$ $\forall e \in E \; e = (uv)$: $u \in S$ OR $v \in S$

$H \subseteq V$ is a HS for $\{S_e\}_{e \in E}$ $\iff$ $\forall S_e = \{u, v\}$ $S_e \cap H \neq \emptyset$

Cover Set

Input : $S_1, S_2 \dots S_n, U$: $S_i \subseteq U$, $b \in \mathbb{N}$

Output : $S'_1, S'_2 \dots S'_b$: $\bigcup S'_i = U$

$\times$ $\{1,2,3\}$

$\times$ $\{4,5,6\}$

$\{1,3,5\}$

$\{2,4,6\}$

$U = \{1,2,3,4,5,6\}$

$b = 6$ ...

$b = 2$ $\longrightarrow$ $\{\{1,2,3\}, \{4,5,6\}\}$

Vertex-Cover $\longrightarrow$ Cover Set

$G = (V, E)$; b_{VC}

$U = E$

$S_v = \{e \in E \mid e = (vx)\}$

$b = b_{VC}$

$S \subseteq V$ is a VC(...) $\iff$ $\{S_v\}_{v \in S}$ is a Cover Set