CS 3510: Design & Analysis of Algorithms (Sections A & B)

Practice Quiz 4 – Section A and B

Professors Dana Randall and Gerandy Brito

1.) (20 points total) For each of the following, select the most appropriate choice.

(a.) (4 points) If A is NP-Complete and B is reducible to A then we can conclude that B is NP-Hard:

◯ **A.** False

(b.) (4 points) If we find a polynomial time algorithm for 3SAT, this gives a poly-time algorithm for SAT is a statement that is:

◯ **A.** True

(c.) (4 points) The problem 3-Clique, where you must find a clique of size 3 in G, is NP-Hard.:

◯ **A.** False

(d.) (4 points) If a problem is NP-complete then it is necessarily NP-Hard:

◯ **A.** True

(e.) (4 points) A CNF-satisfiability problem is best described as belonging to class:

◯ **A.** NP Complete

2.) (40 points)

(a.) (15 points) Select all answer choices guaranteed to have a valid assignment if there exists such an assignment for $x_1 \vee x_2 \vee x_3 \vee \bar{x_4}$:

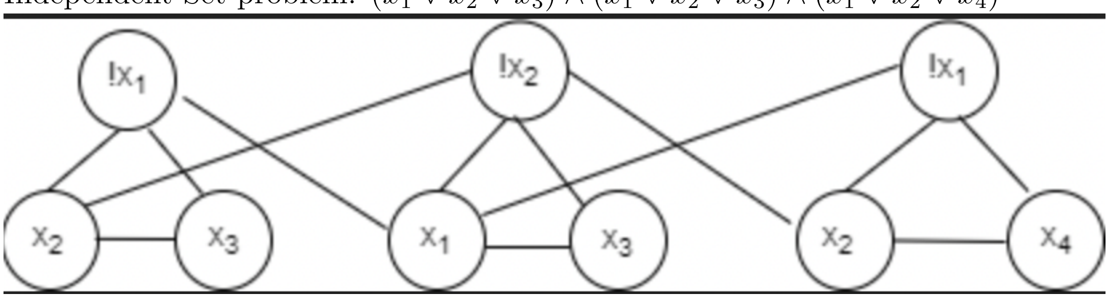
(A.) $(x_1 \vee x_2 \vee \bar{y}) \wedge (x_3 \vee y \vee \bar{x_4})$

(B.) $(x_1 \vee x_2 \vee y) \wedge (x_3 \vee y \vee \bar{x_4})$

(C.) $(x_1 \vee z) \wedge (\bar{z} \vee x_2 \vee y) \wedge (\bar{y} \vee x_3 \vee \bar{x_4})$

solution: A,C

(b.) (10 points) Draw a graph to represent the following 3CNF so it can be reduced to an Independent-Set problem: $(\bar{x_1} \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x_2} \vee x_3) \wedge (\bar{x_1} \vee x_2 \vee x_4)$



(c.) (5 points) Convert the following CNF to a 3CNF: $(x_1 \vee \bar{x_2} \vee x_3 \vee x_5) \wedge (\bar{x_1} \vee \bar{x_3} \vee x_4 \vee x_6 \vee x_7)$

Solution: (x1 v !x2 v y1) (!y1 v x3 v x5) (!x1 v !x3 v y2) (!y2 v x4 v y3) (!y3 v x6 v x7)

(d.) (10 points) What is the minimal number of variables needed to transform clauses of length k: $(x_1 \vee x_2 \vee x_3 \vee ... \vee x_k)$ to 3-SAT?
Solution: k - 3

3.) (40 points) In the HITTING SET problem, we are given a family of sets S1, S2, . . . , Sn and a budget b, and we wish to find a set H of size b which intersects every S_i if such an H exists. In other words, we want H intersection S_i ! = empty for all i. Show that HITTING SET is NP-complete.

Solution:

Logic: Like Vertex Cover

Proof:

1. Hitting Set is in NP since checking the intersection of two sets takes quadratic time [O(mn) for sets of size m and n] which is polynomial time.

2. Consider an input where you represent have a graph whose edges for each each u-v is a set containing u and v; call it S_i. And consider an instance of vertex cover with graph G and b as inputs. There will be $|E|$ sets of S_i and let's say an integer b' = b. G has a vertex

cover of size at b iff the mentioned sets S1, S2, ..., Sn have a hitting set X of size at most b'. Therefore, the problems are equivalent, and Hitting Set is NP Hard, and consequently NP Complete

Correctness and Time:

1. If a graph has a vertex cover at most b, then for any edge, the vertices connected by the edge belong to vertex cover. Therefore, the intersection of the vertices and vertex cover can't be null. Which is equivalent to saying that the hitting set is a vertex cover size at most b. Conversely, for a hitting set of size b', since for collection of sets the intersection of each set and the hitting set is not null, this means that every edge is covered by an element of the hitting set. Effectively the hitting set is a vertex cover of size b.

2. To take the intersection, we take time O(mn) and reduction is also polynomial time.