Chapter 8

1 Section 8.2

Example 1.1. Recall the Binomial Theorem:

$$(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k.$$

Note that $(1+x)^n$ encodes all the different combinations where you choose from n objects. If you want to determine how many ways there are to choose k objects from a set of n objects, you look at the coefficient of x^k .

We thus say that $(1+x)^n$ is the **generating function** for the sequence of choosing k objects from a set of n objects.

Definition 1.2. Let $S = \{a_0, a_1, a_2, \ldots, \}$ be a sequence of numbers. We say that a power series F(x) is a **generating function** of S if the for every $k = 0, 1, 2, \ldots$ we have that a_k is the coefficient of x^k in F(x). That is,

 $S = (a_0, a_1, a_2, a_3, \dots)$ $F(x) = \sum_{k=0}^{\infty} a_k x^k.$ in c= anx 0+a, x +a2 x2+a3x3+ (1+x) = (5) x + (5) x how many ways are There to this is the generaling function for 2) chjects from a sel of S? The # of ways I go to the term whose degree is 21 to choose kobjects from a selol 5 Look alits coefficients (3), which is the answer.

Why do we like generaling functions? They preserve intermation under multiplication,

f(x) is the generaling function for the Hotways to do this g(x) is the g.f. for the # otways to do that

=> 1(x)·g(x) the a.f. for the # of ways
to do this & that
and

Exercise 1.3. You are packing for a trip. You can pick from 4 red shirts, 4 blue shirts, and 4 green shirts. Write the generating function for the number of ways to pack k shirts? (Assume that shirts of the same color are identical.) Idea: Split into task of choosing k shrits of any color into the simpler task of choosing & shirts of g. f. fer # ct pack red shirts RGJ= |xb+|x+1.x2+1.x3+1.x4+0x510x4. S'=BG) g.f.'s for picking BG)
just blue shirtspor GG)
just green shuts will so our answer will he the same Exercise 1.4. How does the above generating function change if you now can pick from up to 7 red shirts, and you have to pack at least 1 blue, shirt? 136), 660, BC Same strategy as in 13: =(1+x1x2+x3+x4)3 Compute RGD, G(x), B(x) individually, then multiply all three to gether. G(x)=1+x+x2+x3+x4 K(x)=|+x+x2+x3+x4+x2+x6+x2 I red, 2 blue, Ogreen (x . 2.1 B(x) = 0.x0+1x1+1x2+1x3+1x4 = x+x21x3+x4

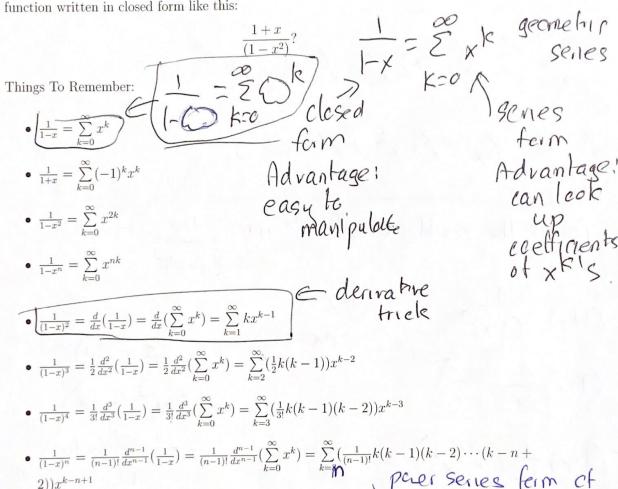
so answer

K(x) R(x)B(x)= (1+x+x2+x3+x4)(1+x2+...+x7)(x+x2+x3+x4)

Why multiply: In above answer lock at # of ways to choose 3 shirts (regardless of color)?
Many combinations of colors. ored I blue, 2 green (10x'x2) 3 red, Oblie, Ogreen (\$3.1.1) I red, I blue, I green (x, x, x)

(1+x+x2+x3+..) ((5)x+(5)x2+(5)x31..) (12+2x+3x344x1)

What about extracting the coefficients from a generating function? Especially a generating function written in closed form like this:



· Build up generating function

as a product of many
simpler g.lis. > gives us a product
of power series = (1+x) (1-x2)

· Mant to combine
the multiple power series into = (1+x) \(\frac{1}{2} \)

one big power series.
To do that, we convert to
closed ferm, multiply together,
then convert back to power series
form

Pener series ferm of

L+X

(1-x²)

=(1+x) (1-x²)

=

Contraction front.

$$\frac{2}{8}x^{2k} + \frac{2}{8}x^{2k+1} = \frac{2}{8}x^{2k+1} = \frac{2}{8}x^{2k} + \frac{2}{8}x^{2k+1} = \frac{2}{8}$$

$$x^{5}+x^{6}+x^{7}+x^{8}+...=\sum_{k=0}^{\infty}x^{k+5}=x^{5}\sum_{k=0}^{\infty}x^{k}=\frac{x^{5}}{1-x}$$

$$\frac{t \ln y}{1-x} = \frac{2}{k-20} x^{k} ?$$

$$= (1+x)(1+x+x^{2}+x^{3}+x^{4}+...) - (x+x^{2}+x^{3}+x^{4}+...)$$

$$= (1+(x-x)+(x^{2}-x^{2})+(x^{3}-x^{3})+...)$$

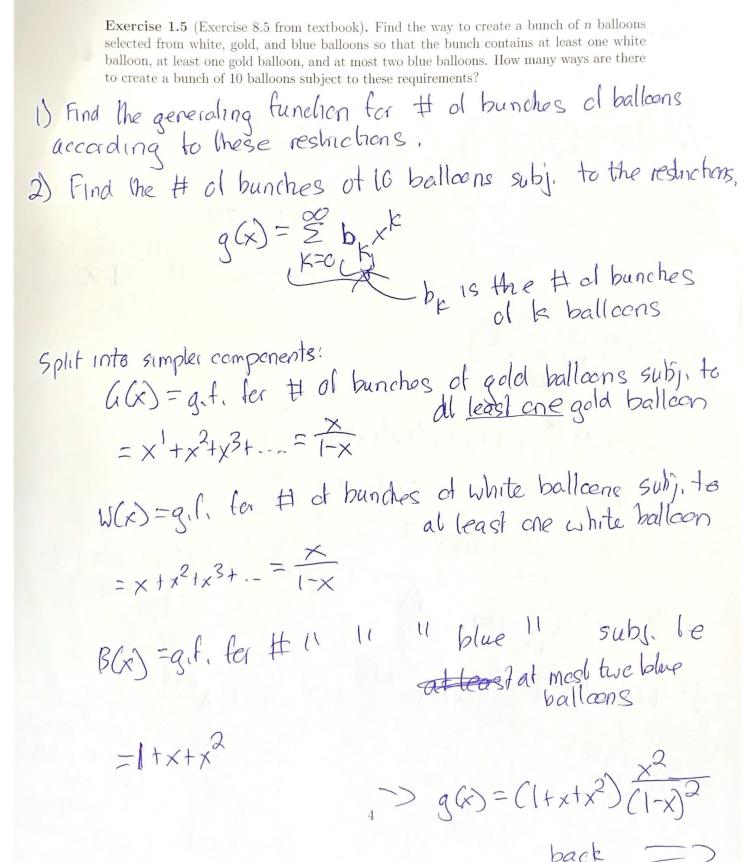
$$= (1+(x-x)+(x^{2}-x^{2})+(x^{3}-x^{3})+...)$$

$$= 1$$

$$= 1$$

$$= 2 \frac{1}{1-x} = (1+x^{2}+x^{3}+x^{4}+...)$$

$$\begin{aligned} & | 1 + x + x^{2} + x^{3} + x^{4} + ... + x^{10} \\ & = (1 + x + x^{2} + x^{3} + ... -) - (x^{11} + x^{12} + x^{13} + x^{14} + ...) \\ & = \frac{1}{1 - x} - \frac{x^{11}}{1 - x} = \frac{(1 - x^{11})}{1 - x} \end{aligned}$$



g(x)=(1+x+x2) x2 convert te power series ferm, When pick all the term x10 S(1-x) dx 1-x= 11 = (x2+x3+x4) /1-x2 -dx=du =>5/12 (-du = (x2+x3+x4) dx (1-x) =>_S u 2 du $=>(-)+\bar{u}=\bar{u}=+-x$ =(x2+2+x4) 2 (5xk) $= (x^2 + x^3 + x^4) \frac{d}{dx} \left(1 + x + x^2 + x^3 + x^4 + \dots \right)$ = (x2+x3+x4) (1+2x+3x2+4x3+...) = (2131 x4) (2 (ett) x4) (equivalent to ZKXK-1 = (x2+x3+x4) (\$ (k+1)xk) = 2 (k+1) x k+2 + 2 (k+1) x k+3 + 2 (k+1) x k+4 E extract coefficients ccelli ct x10 from allhow coeff. cl x16 coeff. of x10 kt4=10 then add k+3=10 K+2=10 together K=7 K=8 cceff. k+1=8 coeff, kt = 7 TOOK. KI1=9 9+8+7 (=(24)