Exam 3 Study Guide

Partially Ordered Sets

- binary relation, partial order, poset, linear order
  - what makes a binary relation a partial order?
- graphical representations
  - cover graph, Hasse diagram, covering relation
  - comparability and incomparibility graphs
  - infer properties about the poset from these rep's
  - how to get rep. from poset
- chains, antichains, and Dilworth
  - what are subposets, chains, and antichains
  - finding a chain/antichain of a certain size
  - height/width of a poset
  - Dilworth & Dilworth Dual
    - min # of disjoint antichains = height = size of largest chain
    - min # of disjoint chains = width = size of largest antichain
    - find a decomposition into disjoint chains or antichains
      - remember alg. for antichain decomp.
      - will give something simpler in case of chain decomp.
- dual of a poset
- poset isomorphism

Generating Functions

- definition of g.f. for a sequence $\{a_n\}$
- building g.f.'s strategy; use simpler g.f.'s and take product
- rules for converting between power series form and closed form

in particular

$$\Rightarrow \frac{1}{1-\bigcirc} = \sum_{n=0}^{\infty} \bigcirc^{n}$$

$$\rightarrow \text{ antiderivative trick to reduce } \frac{1}{(1-\bigcirc)^{k}}$$

$\Rightarrow$ rewriting $\cancel{1+x+x^2+x^3+\dots}$

$$x^{m}+x^{m+1}+x^{m+2}+\dots+x^{M-2}+x^{M-1}+x^{M}$$

as

$$\frac{x^{m}-x^{M+1}}{1-x}$$

$$\left(\sum_{n=m}^{M} x^{n} = \left(\sum_{n=m}^{\infty} x^{n} - \sum_{n=M-1}^{\infty} x^{n}\right)\right)$$

- find ~~the~~ a particular coefficient of a g.f.

Recurrence Equations

- definitions: recurr. eq'n, linear, (non) homog., gen sol'n, part. sol'n, specific sol'n, initial conditions, functional equation
- dealing with hom. gen. sol'n
    - try $a_r = r^n$ substitution, find roots of polynomial
    - what about when roots have multiplicity >1?
- dealing with nonhom. gen. sol'n
    - find gen. sol'n for hom. version
    - find a particular sol'n; what to try first, how to modify if it doesn't work
- finding a specific solution
- using generating functions
    - setting up the functional equation
        - creating "alignment" of generating functions
        - then adding together, using recur. rel'n to cancel/substitute
        - plugging in initial conditions
    - solving functional equation for generating function
        - using what we learned about generating functions to manipulate
        - partial fractions, solving systems of linear equations
    - finding coeff. of x^n in g.f. to get our answer