# Investigating the effect of pickup position on the harmonic response of an electric guitar

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## Contents

# 1 Rationale

Personally, I have always been very interested in electronics. I have always been fascinated by electrical devices. Considering that I am a guitarist, I was more than happy to realize how much of a role electronics and engineering play in creating music.

For my extended essay, the choice of what I would write was not a difficult one. After deciding that I would do my extended essay in physics - my favorite subject - it did not take me long before I decided that I wanted to combine both of my hobbies - electronics and music - and do research in the field.

Considering that I've built a guitar myself, I wanted to do a bit more research on the specific effects that are at play in a guitar. It struck me how much the position of the pickup influences the sound produced, so I decided that I wanted to discover the relationship itself.

# 2 Introduction

The position of electric guitar pickups determines the way an electric guitar sounds. Depending on the position of the pickup the frequency response of a plucked string will differ largely. The goal of this research paper is to investigate the aforementioned effect and create a relationship between the frequency response of the guitar and the pickup position.

Research in this field could lead to innovations regarding electrical guitars, improvement of electrical guitar design and manufacturing and overall better music quality.

## 2.1 Background information

### 2.1.1 Historical background

Given the rise of popularity of electric instruments in the mid-20th century, music has become an art in engineering and physics as well as music itself. Around this time, due to the rising popularity of big bands, musicians were looking to incorporate guitars in bands, but they have been failing due to the fact that the classical acoustic guitar was simply not loud enough ("Early History Of Rickenbacker").

Combating this issue, in the 30s, *George Beauchamp* created the first notable electric guitar pickup, together with the first notable electric guitar - "Frying Pan" ("Early History Of Rickenbacker"), shown in figure 1.

The guitar pickup is unmistakably the most important piece of an electric guitar. It is the device which translates the mechanical oscillatory motion of the strings into an electrical signal linearly dependent on the original motion of the strings. Today, electric guitars and electric pickups come in various forms.

### 2.1.2 Electric guitar pickup types

Electric guitar pickups are either passive or active. Active guitar pickups can be thought of as passive guitar pickups with a preamp stage at the output of the passive filter.

Today, there are two widespread electric guitar pickup types: the *single-coil* electric guitar pickup and the *humbucker*.

The single-coil electric pickup was the one originally created by Beauchamp and innovators of his time. It was relatively simple to create and manufacture and was based on a simple physical phenomenon. Shown in figure 2a, the single coil pickup is the most popular solution today.

Figure 1: Beauchamp's "Frying Pan" (Museum of Making Music)


(a) A single coil pickup (Musician's Friend)

(b) A humbucker pickup (Amazon.com)

Figure 2: Examples of the two most popular types of electric guitar pickups

With amplifiers becoming louder and more powerful, musicians noticed that their single-coil pickups were picking up electromagnetic interference (due to their antenna-like properties) as well as the motion of the strings. In 1955, *Seth Lover* invented the *humbucher*. The humbucker, shown in 2b promised to solve this problem by combining two single-coil pickups which were connected in reverse polarity, effectively canceling out the electromotive force generated from external fields but not the electromotive force generated by the motion of the strings.

This paper considers only single-coil pickups, as they are the most popular, widespread and simple to use.

#### 2.1.3 Musical background

Musical instruments in general operate on musical scales. Musical scales are sets of ordered tones, ordered in ascending fundamental frequencies. The guitar's strings are $E$, $A$, $D$, $G$, $B$, $E$ whose frequencies are 82.41Hz, 110.00Hz, 146.83Hz, 196.00Hz, 246.94Hz and 329.63Hz, respectively. It is worth noting that the relationship between the linear increase of tones and the respective frequency of the tone is exponential. This is due to the fact that human ears hear pitch in a logarithmical fashion, so it is necessary to compensate for this biological factor.

#### 2.1.4 Classical guitar operation

Although the classical guitar is not explored in this research paper, the basic theory of its operation is necessary to understand the operation of the electric guitar. A model guitar is given in 3. When the string

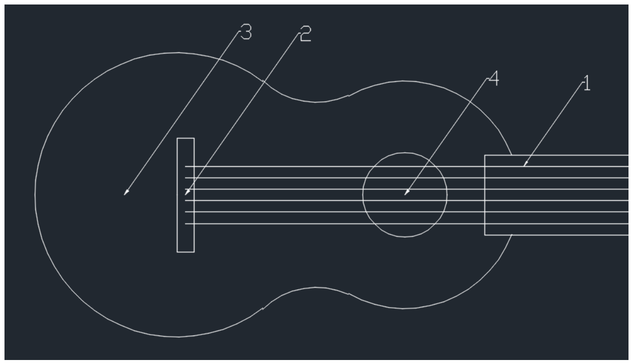


Figure 3: A model of an acoustic guitar

(1) is plucked, it resonates two main components of the guitar through the bridge (2) - the top (3)[1] and the air inside the cavity (4), effectively creating *Heimholtz resonance* [2]. The top resonates better with higher frequencies and the *Heimholtz resonance* at play is better with lower frequencies.

#### 2.1.5 Electric guitar pickup operation

The electric guitar operates similarly to the acoustic guitar, except that instead of resonance playing the role of amplifying audio, a *pickup* is used to pick the motion of the string up. The power outputted by the electric pickup is low, and is then amplified by an amplifier to be played on an electric speaker. An electric guitar pickup is given in figure 4. When a string (1) is strung on top of the permanently magnetized bobbin

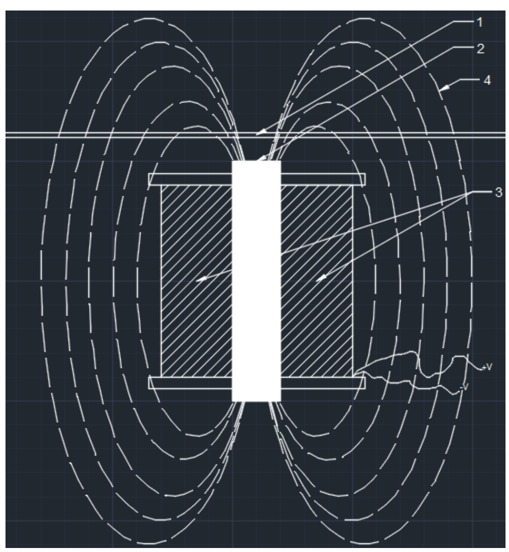


Figure 4: A model of an electric guitar pickup

(2), which is surrounded by very thin electrically insulated conducting wire (3), it becomes magnetized.

[1]The "top" is the name given to the top wooden plate on the guitar.
[2]Heimholtz resonance is a phenomenon of air resonating inside a body with a cavity (Wolfe).

When the string is plucked, the surrounding magnetic field causes the magnetic field of the bobbin (4) to be disturbed, causing the individual point charges in the wire (3) to experience a *Lorentz force* [3]. *Faraday's law* states that the electromotive force generated by the pickup is equal to the rate of change of the flux, and since there is a change of flux due to the metal string disturbing the magnetic field, an electromotive force is generated in the wire of the pickup (3) which is linearly dependent on the motion of the string (1).

### 2.1.6 String harmonic series

The string, as any oscillating body with fixed ends, creates harmonic series - sequences of sinusoidal tones whose frequencies are integer multiples of the lowest frequency, ie. the fundamental frequency. How high the amplitude of the individual harmonics is depends on a multitude of factors - the thickness of the string, the material of the string, the tension of the string, et cetera.

Because of these harmonic series an interesting effect comes to pass when an electric pickup is used. The string oscillating over a pickup at multiple harmonics is given in figure 5. As is visible from the figure,

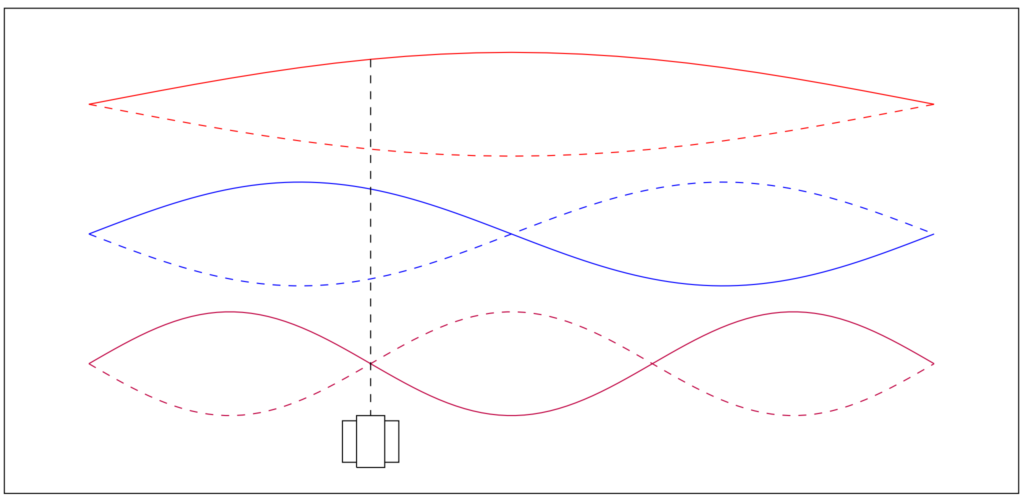

Figure 5: The string oscillating over multiple harmonics over a pickup

at the position of the pickup, the string oscillates at the fundamental frequency and the second harmonic frequency, however, the third harmonic waveform is at the node at the position of the pickup, and therefore the harmonic is not picked up. Note also, that the pickup will not pick up the first or the second harmonic maximally, as at the line parallel to the magnet intersecting the string, the string is not at its anti-node. The pickup effectively filters individual harmonics, depending on its position.

A single harmonic can be modeled as a sine wave:

$$w(t) = A\sin(2\pi n\nu_0 t), \tag{1}$$

where $A_m$ is the amplitude of the waveform, $n$ is the number of the harmonic from $n = 1$ to $n = \infty$ and $\nu_0$ is the frequency of the fundamental harmonic.

Since the maximal amplitude at the location of the pickup will not be equal to the amplitude of the waveform itself, it is necessary to derive the formula for the amplitude of the waveform at the location of the pickup. This is achieved as follows.

Consider the waveform given in figure 6. Because the shape of the string is sinusoidal, it follows that the

[3] Lorentz force is the force that is exerted on a particle moving through an electric and magnetic field (The Editors of Encyclopaedia Britannica).

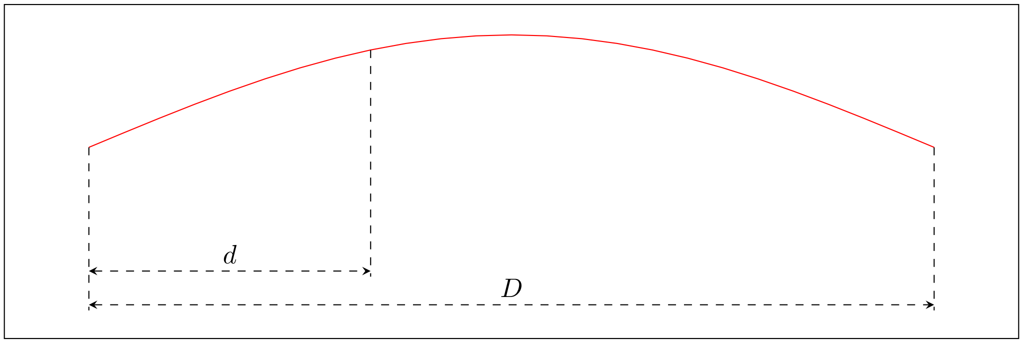


Figure 6: The relationship between the distance of the pickup from the leftmost position and the amplitude at the pickup.

relationship between the distance $d$ of the pickup from the leftmost position and the amplitude at the pickup is sinusoidal as well.

If one were to think of figure 6 as a function, it is the general function of:

$$f_A(x) = k_{h_1} \sin\left(\frac{2\pi}{2D}x\right)$$

Note the use of the constant $k_h$. The constant $k_h$ is in place due to the fact that the amplitude of a harmonic depends heavily on the way the string is plucked, the sustain of the musical instrument, the string tension, and other effects. Usually the most powerful harmonic is the first one, although this is not a rule.

It can be inferred that the amplitude $A$, when $d$ is substituted for $x$ is equal to:

$$A = k_{h_1} \sin\left(\pi\frac{d}{D}\right) \tag{2}$$

For the second harmonic, the graph is plotted in figure 7.

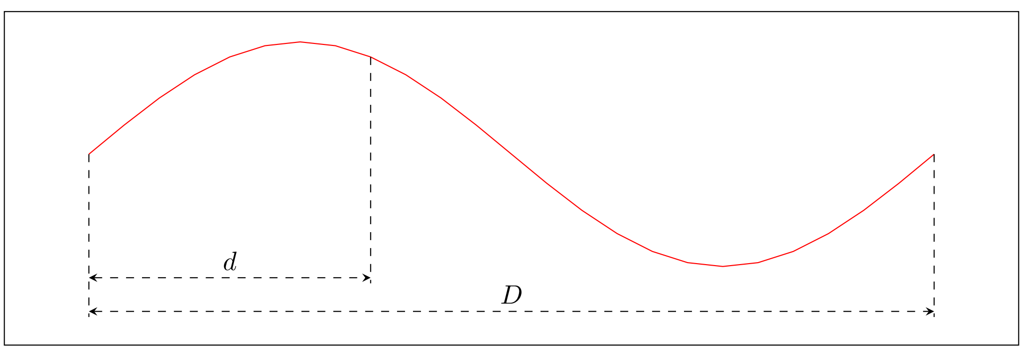


Figure 7: The relationship between the distance of the pickup from the leftmost position and the amplitude at the pickup.

One can infer from figure 7 that the value of amplitude $A$ is:

$$A = k_{h_2} \sin\left(2\pi\frac{d}{D}\right) \tag{3}$$

It follows from equations 2 and 3 that the general formula is equal to:

$$A_n = k_{h_n} \sin\left(n\pi \frac{d}{D}\right), \tag{4}$$

where $n$ is the number of the harmonic, $n = 1, 2, 3, 4\ldots$

When this derived amplitude is substituted into equation 1, the final formula is derived for a single harmonic:

$$w(t) = k_{h_n} \sin\left(n\pi \frac{d}{D}\right) \sin(2\pi n \nu_0 t) \tag{5}$$

To exemplify how the amplitude changes with increasing distance from the left-most position of the pickup, a graph is given in figure 8.

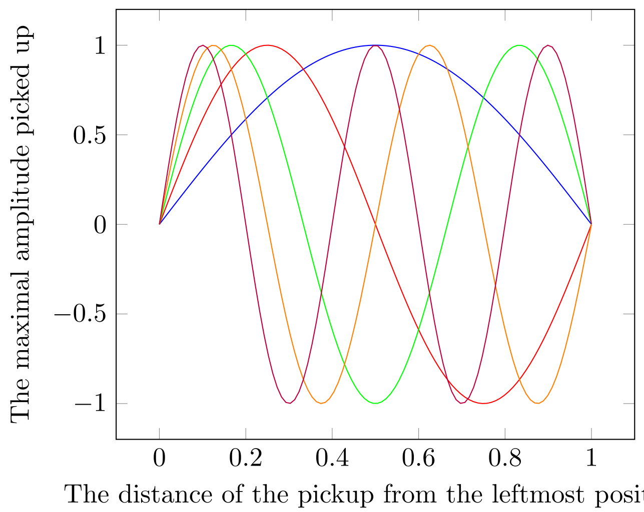


Figure 8: The relationship of the amplitude and the distance of the pickup from the leftmost position for the first five harmonics in relative units

Given that the individual harmonic waves are superimposing, the final waveform will be a sum of all individual harmonics:

$$f(t) = \sum_{n=0}^{\infty} k_{h_n} \sin\left(n\pi \frac{d}{D}\right) \sin\left(2\pi n \nu_0 t\right)$$

To differentiate these individual harmonics, it is necessary to find their amplitudes at different frequencies. The Fourier transformation is perfect for this goal.

## 2.2 The Fourier transformation

### 2.2.1 Introduction

The Fourier transformation is a function which converts an input signal given in the time domain to the frequency domain. In mathematical terms, for a signal $S(t)$, it will create a function $\hat{S}(\nu)$ where $\nu$ represents frequency.

Suppose the graph of $S(t)$ is as given in figure 9.

We shall suppose that $S(t)$ is a sum of sine and cosine waves. Now a question is posed - how can we resolve the amplitudes and the frequencies of the individual constituent sine and cosine waves?

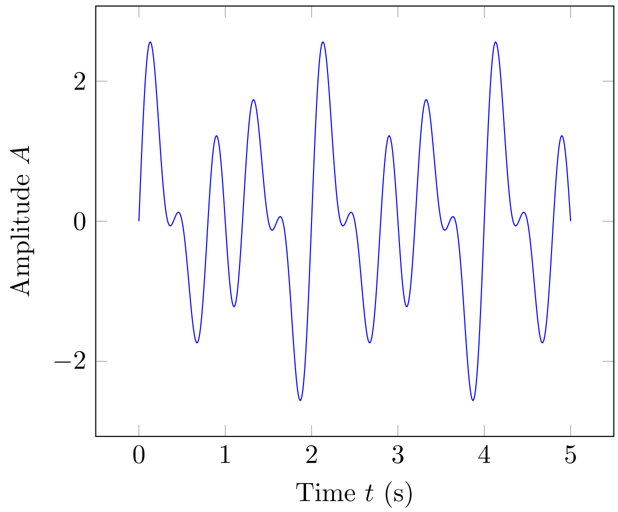


Figure 9: The graph of an unknown signal $S(t)$

The Fourier transformation, in physical values (ie. time and frequency) is defined as the following:

$$\hat{S}(\nu) = \int_{-\infty}^{\infty} S(t)e^{-2\pi it\nu}dt$$

Note the Fourier transformation is complex. Because of its complex domain, it is used for both sine and cosine waves.

The Fourier transformation creates a graph equal to the one given in figure 10[4]. Note how the graph shows peaks at values 2, 3 and 5 Hz. The graph is saying that the individual constituent frequencies were precisely that - 2Hz, 3Hz and 5Hz. The original signal was therefore:

$$S(t) = \sin(2\pi t) + \sin(3\pi t) + \sin(5\pi t)$$

### 2.2.2 Derivation

To prove the Fourier transformation, a definition of what the Fourier series is is necessary.

### 2.2.3 Fourier series

The Fourier series is a method of representing arbitrary periodic functions as a sum of simple sine and cosine functions. A method for deriving it follows.

Suppose an even function such as the one graphed in figure 11 exists.
The premise is that it is possible to express this function as a sum of cosine waves:

$$f(t) = \sum_{n=0}^{\infty} a_n \cos(2\pi nt)$$

It is necessary to calculate the values of $a_n$. This problem is done in the following manner:

[4] This is not an exact graph of the Fourier transformation of this function. Due to there being only three constituent sine waves, the computer algorithm that I've employed has created some inaccuracies. The derivative of a more accurate graph would not be as high as it is in the one I present, at the points of inflexion around the values of 2, 3 and 5. However, for the purposes of exemplifying my point, the graph is sufficiently accurate.

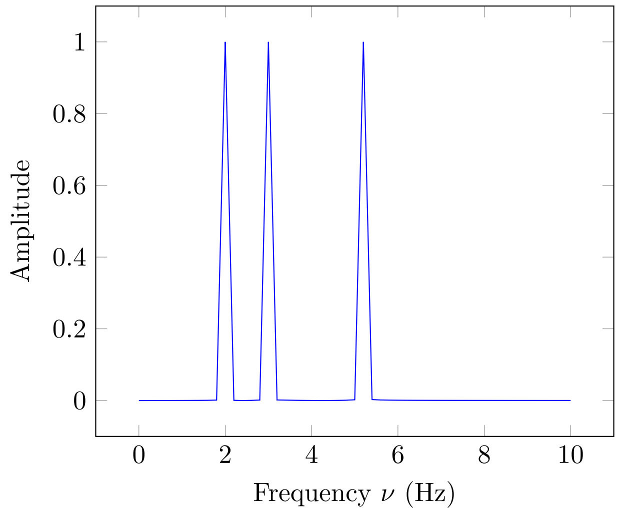


Figure 10: The Fourier transformation of $S(t)$, $\hat{S}(\nu)$

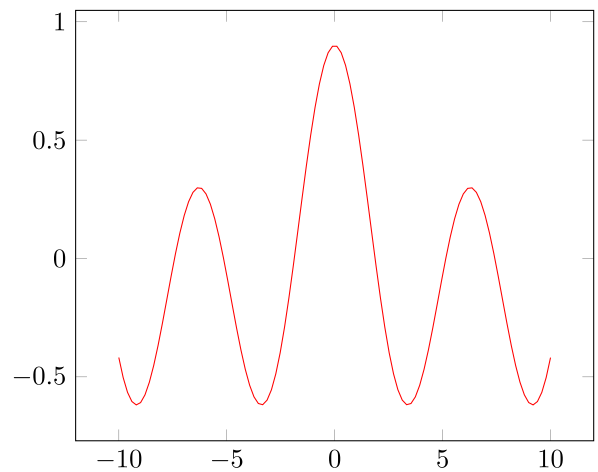


Figure 11: An arbitrary even function

Both sides are multiplied by $\cos(2\pi mt)$:

$$f(t)\cos(2\pi mt) = \sum_{n=0}^{\infty} b_n \cos(2\pi nt)\cos(2\pi mt)$$

Next, both sides are integrated over one period of the function, and the trigonometric identity of the product of cosines is employed:

$$\begin{aligned}\int_T f(t)\cos(2\pi mt)dt &= \int_T \sum_{n=0}^{\infty} a_n \cos(2\pi nt)\cos(2\pi mt)dt \\ &= \sum_{n=0}^{\infty} a_n \int_T \cos(2\pi nt)\cos(2\pi mt)dt \\ &= \frac{1}{2}\sum_{n=0}^{\infty} a_n \int_T (\cos((m+n)2\pi t) + \cos((m-n)2\pi t))dt\end{aligned} \tag{6}$$

Suppose $m \geq 0$ (this is possible to suppose because $m$ is arbitrary, and more importantly, it is physically impossible for frequency to be negative). From this it follows that $\cos((m+n)2\pi t)$ has $m+n$ oscillations in a single period $T$, and consequently, $m+n$ oscillations in the integration interval. Since this is true, when this function is integrated, the result is $0$[5]. Equation 6 is then simplified to:

$$\begin{aligned}
\int_T f(t)\cos(2\pi mt)dt &= \frac{1}{2}\sum_{n=0}^{\infty} a_n \left( \int_T \cos((m+n)2\pi t)dt + \int_T \cos((m-n)2\pi t)dt \right) \\
&= \frac{1}{2}\sum_{n=0}^{\infty} a_n \int_T \cos((m-n)2\pi t)dt
\end{aligned} \tag{7}$$

Since the cosine function is even, in a single period $T$ (the integration interval - from $-\frac{T}{2}$ to $\frac{T}{2}$) the following is true:

$$\cos((m-n)2\pi t) = \cos((n-m)2\pi t)$$

When $m=n$:

$$\cos((m-n)2\pi t) = \cos(0) = 1$$

From these statements, the following follows:

$$\int_T \cos((m-n)2\pi t) = \begin{cases} \int_T \cos((m-n)2\pi t)dt = 0 & \text{when } m \neq n \\ \int_T 1 \cdot dt = T & \text{when } m = n \end{cases}$$

It is key to note that:

$$\forall (0 < m < \infty, 0 < n < \infty \wedge m \neq n) \rightarrow \int_T cos((m-n)2\pi t)dt = 0$$

From this it follows that the whole summation in equation 7 can be reduced to the case when $m=n$:

$$\int_T = f(t)\cos(m2\pi t)dt = \frac{1}{2}a_m T$$

To get $a_m$ is trivial:

$$a_m = \frac{2}{T}\int_T f(t)\cos(m2\pi t)dt$$

Finally, replacing $m$ with $n$ we get:

$$a_n = \frac{2}{T}\int_T f(t)\cos(m2\pi t)dt \tag{8}$$

However, the case of $m=0$ is not considered. This case gives the $y$ axis offset $a_0$. Substituting with $m=0$ in equation 6, we get:

$$\begin{aligned}
\int_T f(t)\cos(2\pi mt)dt &= \sum_{n=0}^{\infty} a_n \int_T \cos(2\pi nt)\cos(2\pi mt)dt \\
\int_T f(t)\cos(0)dt &= \sum_{n=0}^{\infty} a_n \int_T \cos(2\pi nt)\cos(0)dt \\
\int_T f(t)dt &= \sum_{n=0}^{\infty} a_n \int_T \cos(2\pi nt)dt \\
\int_T f(t)dt &= Ta_0
\end{aligned}$$

This gives the final result for the $y$ axis offset:

$$a_0 = \frac{1}{T}\int_T f(t)dt \tag{9}$$

[5]The proof for this is trivial, however, it is published by a third party online and is available to the reader at the following link: `http://planetmath.org/integraloveraperiodinterval`.

This result is coincidentally intuitively the average of all of the $y$ values of the points of the function in a single period.

Odd functions are represented in a similar manner:

$$f_o(t) = \sum_{n=1}^{\infty} b_n \sin(2\pi nt)$$

The coefficients $b_n$ are, then:

$$b_n = \frac{2}{T}\int_T f_o(t)\sin(2\pi nt)dt$$

As the derivation is quite similar to the one for the even function, it has been omitted.

Since any arbitrary function $f(t)$ can be represented as a sum of odd and even functions:

$$f_o(t) = \frac{1}{2}(f(t) - f(-t))$$

$$f_e(t) = \frac{1}{2}(f(t) + f(-t))$$

$$f(t) = f_o(t) + f_e(t)$$

The Fourier series' components can be applied:

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(2\pi nt) + b_n \sin(2\pi nt))$$

with the coefficients being:

$$a_0 = \frac{1}{T}\int_T f(t)dt$$

$$a_n = \frac{2}{T}\int_T f(t)\cos(2\pi nt)dt, n \neq 0$$

$$b_n = \frac{2}{T}\int_T f(t)\sin(2\pi nt)dt$$

It is possible to express the Fourier series in its complex form, using Euler's formulas:

$$cos\phi = \frac{e^{i\phi} + e^{-i\phi}}{2}$$

$$sin\phi = \frac{e^{i\phi} - e^{-i\phi}}{2i}$$

$$\begin{aligned} f(t) &= a_0 + \sum_{n=1}^{\infty}(a_n \cos(2\pi nt) + b_n \sin(2\pi nt)) \\ &= a_0 + \sum_{n=1}^{\infty}\left(a_n \frac{e^{i2\pi nt} + e^{-i2\pi nt}}{2} + b_n \frac{e^{i2\pi nt} - e^{-i2\pi nt}}{2i}\right) \\ &= a_0 + \sum_{n=1}^{\infty}\frac{a_n - ib_n}{2}e^{2\pi nt} + \sum_{n=1}^{\infty}\frac{a_n + ib_n}{2}e^{-2\pi nt} \end{aligned}$$

Let us know define $c_n$ as

$$c_n = \frac{a_n - ib_n}{2}$$

This gives the complex Fourier series:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{i2\pi t}$$

This is referred to as the *synthesis* equation.

From this it follows that $c_n$, the complex Fourier coefficient, is:

$$\begin{aligned}
c_n &= \frac{1}{2} \cdot \left( \frac{2}{T} \int_T f(t)\cos(2\pi nt)dt - i\frac{2}{T} \int_T f(t)\sin(2\pi nt)dt \right) \\
&= \frac{1}{T} \int_T f(t)\cos(2\pi nt) - if(t)\sin(2\pi nt)dt \\
&= \frac{1}{T} \int_T f(t) \left( \frac{e^{i2\pi nt} + e^{-i2\pi nt}}{2} - i\frac{e^{i2\pi nt} - e^{-i2\pi nt}}{2i} \right) \\
&= \frac{1}{T} \int_T f(t) \frac{2e^{i2\pi nt}}{2} dt \\
c_n &= \frac{1}{T} \int_T f(t)e^{-i2\pi nt} dt
\end{aligned}$$

This is referred to as the *analysis* equation. This equation gives spectral lines for a frequency $n$. Note that it is not continuous.

### 2.2.4 The Fourier transformation derivation

The Fourier transformation can be thought of as a continuous version of the Fourier series:

$$\hat{f}(t) = \int_{-\infty}^{\infty} f(t)e^{-i2\pi\nu t} dt$$

Let the Fourier transformation be defined as the limit of the analysis formula of the Fourier series as $T$ approaches infinity:

$$\hat{f}(t) = \lim_{T\to\infty} c_n = \lim_{T\to\infty} \int_{-T/2}^{T/2} f(t)e^{-i2\pi nt} dt$$

Note the factor $\frac{1}{T}$ is completely ignored, due to the fact that $\lim_{n\to\infty} \frac{1}{n} = 0$, which would ruin the function. This is far more relevant when doing the inverse Fourier transformation, however the inverse Fourier transformation is outside of the scope of this paper and therefore is not explored. The limit makes the spectral lines come closer, creating the final equation:

$$\hat{f}(t) = \int_{-\infty}^{\infty} f(t)e^{-i2\pi\nu t} dt$$

The power of this equation lies in the fact that it gives a continuous function showing the frequencies of a *non-periodic* signal. In turn, it has practical applications in converting a signal from the time domain to the frequency domain.

## 3 Method

When a Fourier transformation is ran on a waveform it produces a continuous function of frequency, showing precisely amplitudes of frequencies. To explore the relationship between the pickup position and the harmonic effects, it is necessary to look at a three-dimensional graph where one axis is the independent variable - the position of the pickup, and two dependent variables - the frequency and the amplitude.

To measure the effect of pickup position on harmonics of the electric guitar, an electric guitar with a movable pickup was constructed, as shown in figure 12. It has several major elements to it, similar to the normal electric guitar: (1) the neck of the electric guitar, (2) the bridge of the guitar, (3) the body of the guitar, (4) the strings, and, most notably, (5) a pickup whose position can be changed.

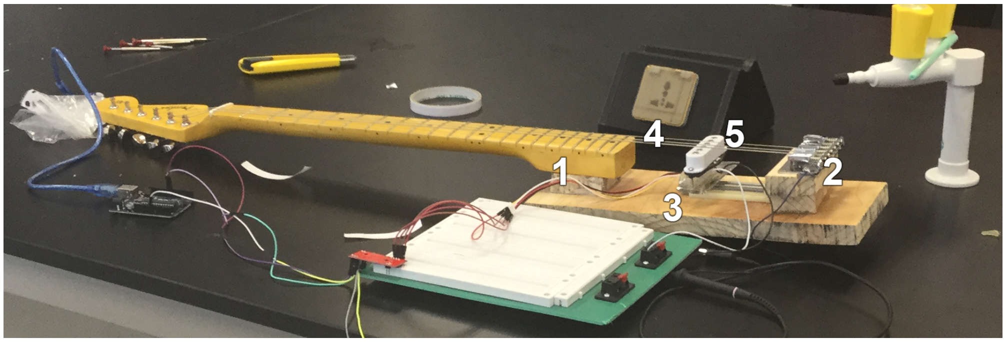


Figure 12: The constructed experimental electric guitar


Figure 13: The hatch marks of the pickup position

The body of the guitar (3) was marked with hatch marks so the pickup could be set to a desired value. These hatch marks were marked using a ruler. They are shown in figure 13. Since it was mechanically impossible to position the pickup immediately next to the bridge, the distance of the starting (left-most) position of the pickup was measured with a caliper.

The pickup wires were connected to a *GW Instek GDS-1000A-U* oscilloscope. The ground wire was also connected to the bridge of the electric guitar and the ground of the oscilloscope, to remove any potential noise that might occur[6], as given in figure 14.

The string (4) was plucked for every position of the pickup. At the same time, the oscilloscope was set to record the waveform. The waveform was recorded using the oscilloscope and stored on a computer for further data processing. Measurements were done for all full-tones, until the first octave, of the guitar for the $E$ and $A$ strings.

[6]Due to being done in a building, there was a high chance that the strings would pick up the mains hum, so grounding them was necessary.

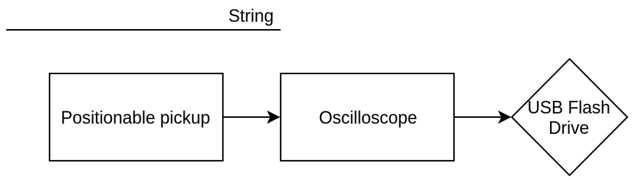


Figure 14: A schematic of how the individual devices were connected

# 4 Data

## 4.1 Raw data

The recorded data was a collection of wave-forms in the *CSV*[7] format, as the oscilloscope generated these files. As an example, the waveform for the $E$ string on the $5^{\text{rd}}$ fret is given in figure 15 in the left-most pickup position.

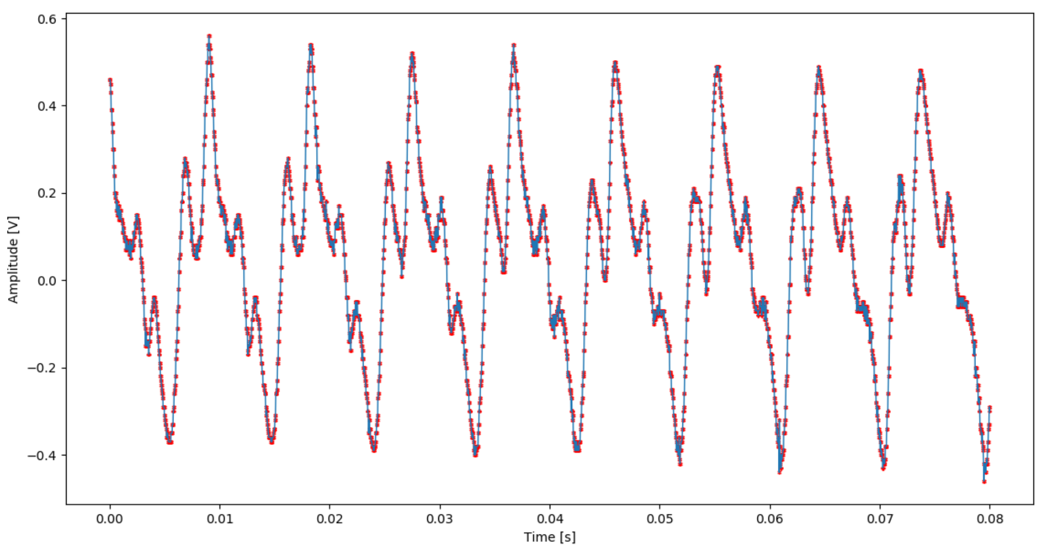


Figure 15: The waveform of the $E$ string played on the $5^{\text{rd}}$ fret in the left-most pickup position

Due to the fact that there has been a huge amount of data, it has been published online and is available at the following link: `https://github.com/markovejnovic/relationship-pickup-distance/tree/master/src/Data`

The uncertainty for the *GW Instek GDS-1000A-U* is $\pm 2$mV (Good Will Instrument Co., LTD).

The left-most position of the pickup is not at a true 0. The minimum offset of the pickup from the bridge could not be mechanically made to be 0, and has been achieved to be equal to

$$d_0 = (31.6 \pm 0.01)\text{mm}$$

Note the very low value of the uncertainty. This has been achieved by using a caliper to measure $d_0$ from the bridge of the guitar.

[7] Comma separated values

The uncertainty for the distance of the pickup from the left-most position $d$ can be calculated as follows. Since the hatch marks were copied from a ruler, the uncertainty during the copying is equal to 0.5mm. Since the pickup was positioned at the copied hatch marks, the uncertainty increases by another 0.5mm. Finally, the uncertainty increased when the distance from the bridge to the first hatch mark was measured with a caliper by the uncertainty of the caliper - 0.01mm. The final uncertainty of $d$ is then:

$$\Delta d = \pm(0.5 + 0.5 + 0.01)\text{mm} = \pm 1.01\text{mm}$$

## 4.2 Data processing

The data processing was done in the *python* programming language (Van Rossum, and L. Drake), *numpy* (E. Oliphant) and *matplotlib* (Hunter) as follows.

First the oscilloscope data was loaded into memory, denoted here as a vector $w$, with the number of elements being 4000. Next, the Fourier transformation was calculated of the waveform, showing the harmonics of the guitar:

$$f = \text{fft}(w)$$

Only the positive side of the Fourier transformation was used, due to the fact that the Fourier transformation in this case is symmetrical, as exemplified in figure 16.

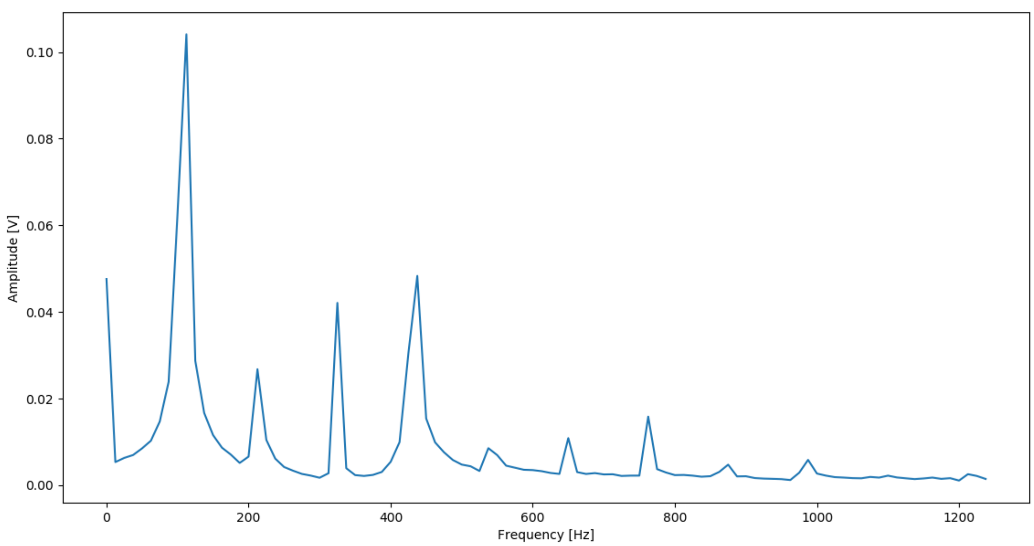


Figure 16: The Fourier transformation of the played $5^{\text{th}}$ fret of the $E$ string at the left-most pickup position

Next, the Fourier transformation was normalized, ie. mapped from 0 to 1 by multiplying the reciprocal of the highest scalar in the vector by the vector itself:

$$f' = \frac{1}{\max_{1 \leq i \leq N} w_i} \cdot f'$$

The previous example in figure 16 becomes normalized as given in figure 17.

Once this has been done, the data was plotted on a three-dimensional graph where one axis was the independent pickup position, and the other two axes were dependent frequencies and amplitudes, as shown in figure 18.

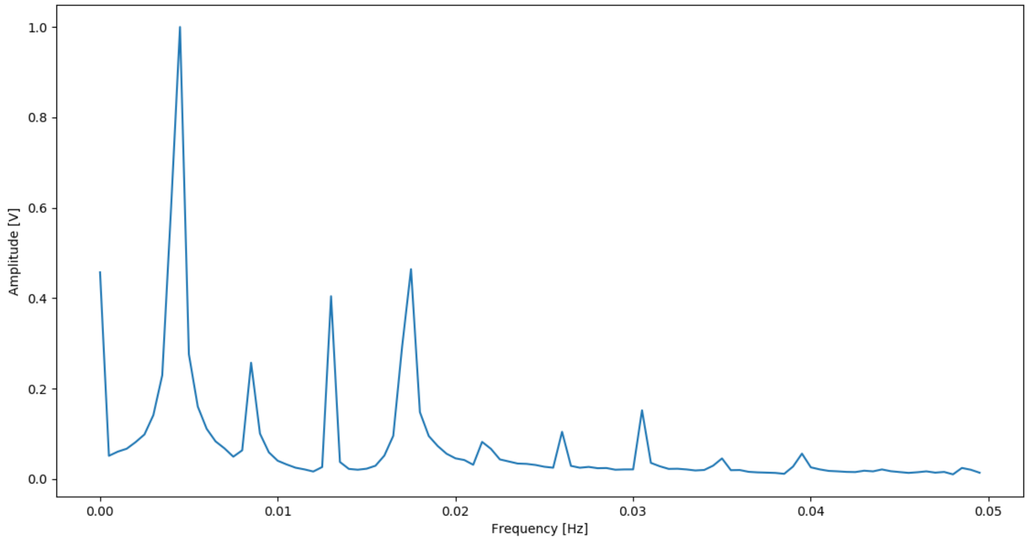


Figure 17: The normalized Fourier transformation of the played $5^{\text{th}}$ fret of the $E$ string at the left-most-pickup position.

Again, the sheer amount of data obstructs the ease of processing, so in figure 19, uncertainties have been omitted. Also, using the open-source algorithm *peakutils* (Negri), Fourier peaks have been identified. Note that the first harmonics were completely omitted from identification. This is due to the fact that the Fourier transformation was somewhat incomplete, due to the waveform recorded by the oscilloscope being temporally very short, making the fundamental frequencies cut off, and therefore imprecise.

Figure 20 shows the same graph as figure 19 with the peaks of the harmonics of different pickup positions being connected. This allows to follow how exactly, the pickup position moves.

The code used to process the data is available both online at `https://github.com/markovejnovic/relationship-pickup-distance/tree/master/src` and in the appendix.

# 5 Analysis

Considering that the scale length[8] of the *Fender Stratocaster* is 648mm ("Scale Length Explained"), and the model guitar used in this experiment was constructed according to the *Stratocaster* specification the distance from the nut to the bridge is equal to:

$$D = (648 \pm 0.5)\text{mm}$$

From this it follows that the general formula of the relationship between the harmonic amplitude and pickup position is:

$$\sin\left(n\pi\frac{d}{(648 \pm 0.5)\text{mm}}\right)$$

In figure 21, the plot for the relationship between the first 3 harmonics and the position of the pickup is given, as well as the expected values calculated using the aforementioned formula.

[8]The distance from the nut to the bridge of the guitar

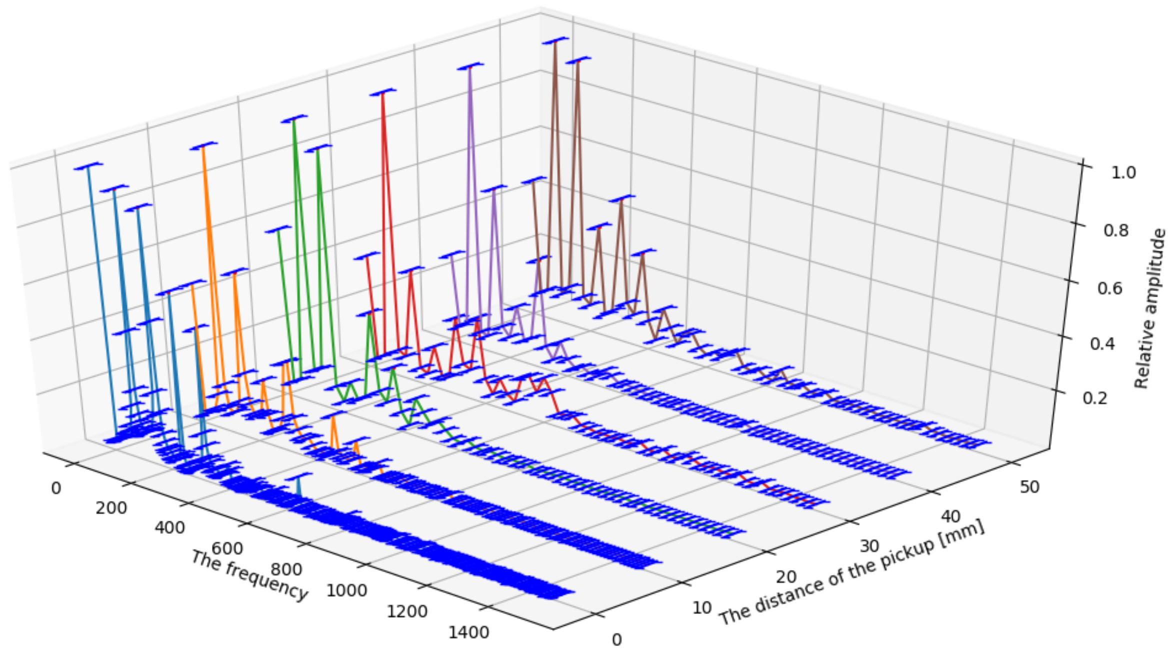


Figure 18: The graph of the dependency of the frequency and amplitudes with respect to the pickup position

The very poor fit can be attributed to two major factors, and other minor factors.

## 5.1 Poor harmonic identification

When the same graph as given in figure 21 is observed from a top-down perspective, as given in figure 22, it becomes obvious that the peaks of the Fourier transformation are not colinear. It is notable that the harmonics were poorly aligned. This poor alignment may stem from the following factor.

## 5.2 Sustain

A guitar string, when first plucked, oscillates at multiple harmonics of varying amplitudes. However, the sustain[9] of each individual harmonics is not equal. Because of this, certain harmonics will decay faster than others. This data heavily indicates that, during the experiment, the minor difference in time of recording waveforms for each pickup position affected the final Fourier transformation in a major manner.

## 5.3 Other minor effects

Other effects which could have affected the final Fourier transformation of the guitar waveform are the way the string was plucked, as plucking the string in a different manner creates a different tone; the gradual detuning of the guitar in an environment of varying temperature and mechanical impacts near the guitar which could have caused the strings to vibrate.

[9]Sustain is the time it takes for a musical note to become inaudible, ie. to decay.

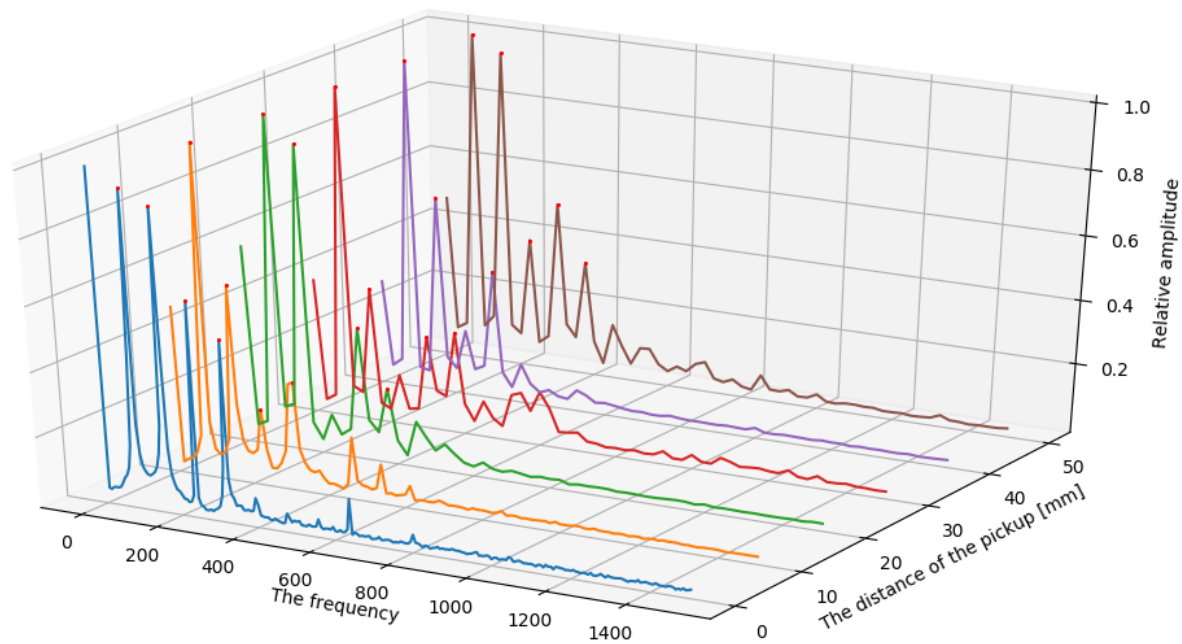


Figure 19: The graph of the dependency of the frequency and amplitudes with respect to the pickup position with omitted uncertainties and with identified Fourier peaks.

### 5.4 Qualitative analysis

Due to the poor results of the experiment, a small qualitative experiment was employed. Using a readily available *Fender Stratocaster* guitar, the same notes were played with different pickup position (bridge and neck position). It was noticed that there was a significant audible change in tone. This leads to believe that the quantitative experiment previously done was erroneous.

## 6 Conclusion

The experiment results are inconclusive. Non-experimental observations point to there being a significant change in the tone of the guitar with the change of the position of the pickup. Donald Tillman's experimental results reaffirm this observation (Donald Tillman). However, the research conducted in this paper was unsuccessful to the point of showing no notable correlation.

### 6.1 Rectifying errors

In the following, I have described possible solutions to the issues faced in this research paper.

#### 6.1.1 Sustain

To rectify the issue that was created during sustain, it would be best to record a complete waveform - from the plucking of the string to the complete decay of any sound. This would also rectify the issue of poor harmonic identification.

#### 6.1.2 Timbre from picking

To rectify the various different timbres that were encountered with different picking styles, it is necessary to construct a mechanism which can pick a string with a constant force and angle. A solenoid presents itself as an ideal solution, however, it is important for the solenoid to be far away from the pickup, so as not to interfere with the magnetic field generated by the pickup.

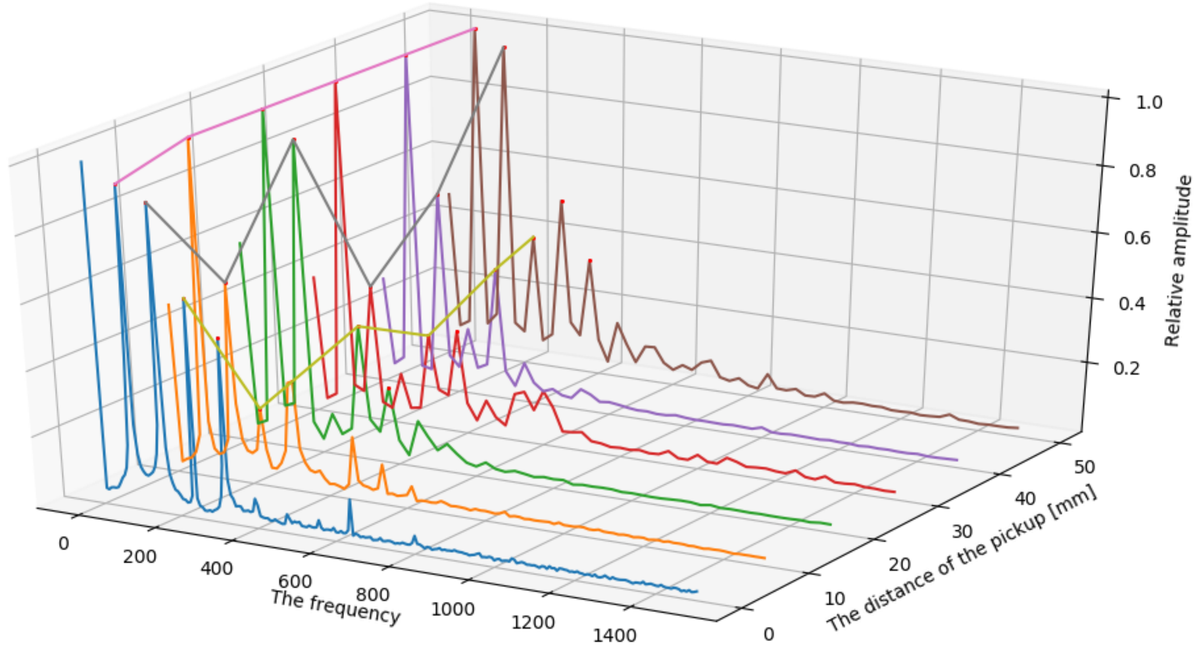


Figure 20: The graph of the relationship of the harmonic amplitudes and the pickup position with connected peaks.

Although the data is highly inconclusive, the experiment remains a valid one. As Donald Tillman's data shows, there are indeed effects on harmonics with the position of pickups. As the pickups move from the left-most position, Tillman's data shows that the frequency response moves translatory to the lower pitches, as the hypothesis suggests.

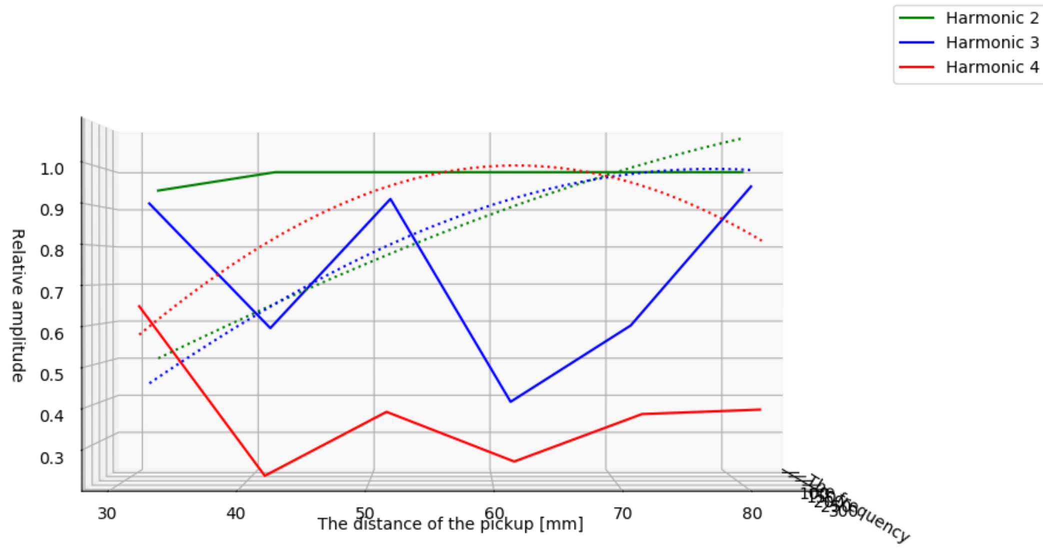


Figure 21: The relationship between the amplitudes of the second, third and fourth harmonics and the position of the pickup with the expected values

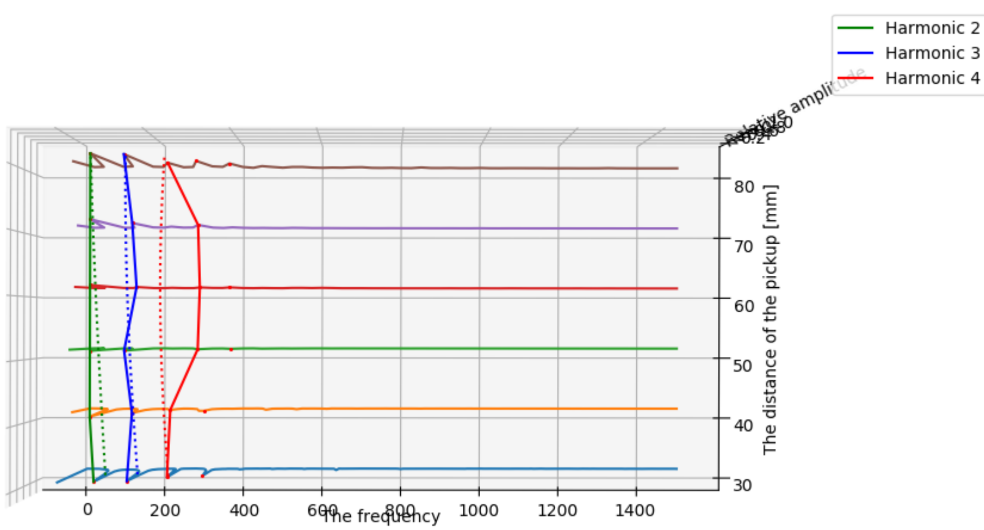


Figure 22: The relationship between the amplitudes of the second, third and fourth harmonics and the position of the pickup with the expected values as seen from a top-down perspective

## Works cited


Amazon.com. *Suhr SSV Bridge Humbucker (53Mm)*. https://www.amazon.com/Suhr-SSV-Bridge-Humbucker-53mm/dp/B008J7YLEC. Accessed 16 Oct 2018.

Autodesk. *Autocad 2014*. Version 19.1, Autodesk, 2014.

Chicago Music Exchange. *Fender Stratocaster Sunburst 1956*. https://images.reverb.com/image/upload/s--iiQ3rVKI--/a_exif,c_limit,e_unsharp_mask:80,f_auto,fl_progressive,g_south,h_1600,q_80,w_1600/v1497644319/ax58jibpcrmys6id16hl.jpg. Accessed 29 Aug 2018.

Donald Tillman, J. "Response Effects Of Guitar Pickup Position And Width". *Till.Com*, 2002, http://www.till.com/articles/PickupResponse/. Accessed 31 Aug 2018.

E. Oliphant, Travis. *A Guide To Numpy*. Trelgol Publishing, 2006.

"Early History Of Rickenbacker". *Rickenbacker.Com*, http://www.rickenbacker.com/history_early.asp. Accessed 15 Oct 2018.

Good Will Instrument Co., LTD. *GDS-1000-U Specifications*. Good Will Instrument Co., LTD, https://www.gwinstek.com/en-global/products/downloadSeriesSpec/192. Accessed 28 Aug 2018.
Guitar Center. *Fender Special Edition HH Maple Fingerboard Standard Telecaster Sea Foam Pearl*. https://www.guitarcenter.com/Fender/Special-Edition-HH-Maple-Fingerboard-Standard-Telecaster.gc. Accessed 29 Aug 2018.

Hunter, John D. "Matplotlib: A 2D Graphics Environment". *Computing In Science & Engineering*, vol 9, no. 3, 2007, pp. 90-95. *Institute Of Electrical And Electronics Engineers (IEEE)*, doi:10.1109/mcse.2007.55. Accessed 31 Aug 2018.

Museum of Making Music. *Ro-Pat-In Cast Aluminum Electric Hawaiian "Frying Pan" Guitar*. 2009, https://commons.wikimedia.org/wiki/File:Elektrofryingpan.jpg. Accessed 15 Oct 2018.

Musician's Friend. *EMG S1 Hot Passive Single Coil Pickup*. https://www.musiciansfriend.com/accessories/emg-s1-hot-passive-single-coil-pickup. Accessed 16 Oct 2018.

Negri, Lucas. *PeakUtils*. Version 1.2.0, 2018.

"Scale Length Explained". *Stewmac.Com*, 2018, https://www.stewmac.com/How-To/Online_Resources/Learn_About_Guitar_and_Instrument_Fretting_and_Fretw/Scale_Length_Explained.html. Accessed 31 Aug 2018.

The Editors of Encyclopaedia Britannica. "Lorentz Force". *Encyclopædia Britannica*, 2017, https://www.britannica.com/science/Lorentz-force. Accessed 31 Aug 2018.

Van Rossum, Guido, and Fred L. Drake. *Python 3 Reference Manual*. Paramount, CA, 2009, https://dl.acm.org/citation.cfm?id=1593511. Accessed 28 Aug 2018.

Wolfe, Joe. "Helmholtz Resonance". *Newt.Phys.Unsw.Edu.Au*, http://newt.phys.unsw.edu.au/jw/Helmholtz.html. Accessed 31 Aug 2018.

## Data processing code

waveform.py

```
import matplotlib.pyplot as plt
import numpy as np
import peakutils

class GuitarWaveform:
    """Represents a guitar waveform

    Arguments:
        array - An array containing the values of the oscilloscope read"""

    def __init__(self, array, fs, v_scale, h_scale, name):
        array = np.array(array)
        self._fs = fs
        self._samples_n = array.size
        self._times = np.linspace(0, self._samples_n / self._fs,
                self._samples_n)
        self._amplitudes = array * v_scale / 5
        self._name = name

    @property
    def name(self):
        return self._name

    @name.setter
    def name(self, name):
        self._name = name

    _dft = np.array([])
    def dft(self, force=False):
        if force or self._dft.size == 0:
            self._dft = np.fft.fft(self._amplitudes) / self._samples_n
            self._dft = self._dft[range(int(self._samples_n / 2))]
        return self._dft

    _abs_dft_freq = np.array([])
    _abs_dft = np.array([])
    def abs_dft(self, force=False):
        if force or not self._abs_dft.size or not self._abs_dft_freq.size:
            frq = np.arange(self._samples_n) * self._fs / self._samples_n
            self._abs_dft_freq = frq[range(int(self._samples_n / 2))]
            self._abs_dft = abs(self.dft())
        return self._abs_dft_freq, self._abs_dft

    _normalized_abs_dft_freq = np.array([])
    _normalized_abs_dft = np.array([])
    def normalized_abs_dft(self, force=False, cutoff_threshold=1500):
        if force or not self._normalized_abs_dft.any() or not \
                self._normalized_abs_dft.size:
            n_abs_dft = self.abs_dft()[1] / np.amax(self.abs_dft()[1])
            n_abs_dft_freq = self.abs_dft()[0]
```

```
            if cutoff_threshold:
                i = 0
                for val in n_abs_dft_freq:
                    if val > cutoff_threshold:
                        break
                    else:
                        i += 1
                self._normalized_abs_dft = n_abs_dft[0:i]
                self._normalized_abs_dft_freq = n_abs_dft_freq[0:i]

            else:
                self._normalized_abs_dft = n_abs_dft
                self._normalized_abs_dft_freq = n_abs_dft_freq

        return self._normalized_abs_dft_freq, self._normalized_abs_dft

    _peaks = np.array([])
    def ft_peaks(self, force=False, min_dist=10, thres=0.03):
        if force or not self._peaks.size:
            self._peaks = \
                    peakutils.indexes(self.normalized_abs_dft(force=force)[1],
                        min_dist=min_dist, thres=thres)
        return self._peaks

    def plot(self, x_unc=0, y_unc=0):
        if x_unc == 0 and y_unc == 0:
            plt.plot(self._times, self._amplitudes)
        else:
            plt.errorbar(self._times, self._amplitudes,
                    xerr=x_unc, yerr=y_unc, ecolor='r',
                    capsize=1.5, linewidth=1)

    def plot_ft(self, length=100, ax=None, zs=0):
        x, y = self.abs_dft()
        if not ax:
            plt.plot(x[0:length], y[0:length])
        else:
            ax.plot(x[0:length], y[0:length], zs=zs, zdir='y')

    def plot_normalized_ft(self, length=100, ax=None, zs=0):
        x, y = self.normalized_abs_dft()
        if not ax:
            plt.plot(x[0:length], y[0:length])
        else:
            ax.plot(x[0:length], y[0:length], zs=zs, zdir='y')
```

data_reader.py

```
import csv
from util import isfloat
from os import path
from waveform import GuitarWaveform

```

```
def read(f, positions=['0', '1', '2', '3', '4', '5'], strings=['E', 'A'],
        frets=['0', '3', '5', '7', '9', '12']):
    """Tries to read the data in f

    If f is a file the data will be read from the file, otherwise if f is a
    folder the program will try to read all csv files

    Arguments:
        f - folder or file
        strings - The strings to analyze

    Returns:
        An array containing the data

    TODO:
        Implement the non-recursive behavior
    """
    data = []

    if path.isfile(f):
        with open(f, newline='') as csvfile:
            values = []

            reader = csv.reader(csvfile, delimiter=',')
            for row in reader:
                if not isfloat(row[0]):
                    if 'Sampling Period' in row[0]:
                        fs = 1 / float(row[1])
                    elif 'Vertical Scale' in row[0]:
                        v_scale = float(row[1])
                    elif 'Horizontal Scale' in row[0]:
                        h_scale = float(row[1])
                    else:
                        continue
                else:
                    values.append(int(row[0]))

            return GuitarWaveform(values, fs, v_scale, h_scale, f)

    elif path.isdir(f):
        for i in positions:
            for j in frets:
                for k in strings:
                    with open(path.join(f, i, j, k, 'DS0000.CSV'),
                            newline='') as csvfile:
                        values = []

                        reader = csv.reader(csvfile, delimiter=',')
                        for row in reader:
                            if not isfloat(row[0]):
                                if 'Sampling Period' in row[0]:
                                    fs = 1 / float(row[1])
                                elif 'Vertical Scale' in row[0]:
                                    v_scale = float(row[1])
```

```
                                elif 'Horizontal_Scale' in row[0]:
                                    h_scale = float(row[1])
                                else:
                                    continue
                            else:
                                values.append(int(row[0]))

                    data.append(GuitarWaveform(values, fs, v_scale, h_scale,
                        f + i + '/' + j + '/' + k))

        return data
```

util.py

```
def isfloat(value):
    try:
        float(value)
        return True
    except ValueError:
        return False
```

boiler.py

```
#!/usr/bin/env python
from mpl_toolkits.mplot3d import Axes3D
import sys
import data_reader
import matplotlib.pyplot as plt
import numpy as np

D_MIN = 31.6 #mm
D_ERROR = 1.01 #mm
LENGTH = 100

UNC=False
CONNECT=False
CONNECT_N = 3
EXPECTED=False
PLOT=False

if __name__ == '__main__':
    if '-u' in sys.argv:
        UNC=True

    if '-c' in sys.argv:
        CONNECT=True

    if '-e' in sys.argv:
        EXPECTED=True

    if '-p' in sys.argv:
        PLOT=True

    gtr_waves = data_reader.read(sys.argv[-1], strings=['E'], frets=['0'])
    peaks_all = []
```

```python
fig = plt.figure()
ax = fig.gca(projection='3d')

if PLOT:
    i = 0
    for gtr_wave in gtr_waves:
        x_s, y_s = gtr_wave.normalized_abs_dft()
        peaks = gtr_wave.ft_peaks(min_dist=0.001, thres=0.2)
        peaks_all.append(peaks)

        ax.plot(x_s, np.full(x_s.size, i) + D_MIN, y_s)

        for peak in peaks:
            ax.plot([x_s[peak]], [i+D_MIN], [y_s[peak]], marker='o',
                    color='r',
                    markersize=1.2)

        if UNC:
            for j in range(0, x_s.size):
                ax.plot([x_s[j], x_s[j]],
                        [i - D_ERROR, i + D_ERROR],
                        [y_s[j], y_s[j]],
                        marker='_', color='b')

        i += 10

if CONNECT:
    colors = ['g', 'b', 'r']
    for i in range(0, CONNECT_N):
        ax.plot([gtr_wave.normalized_abs_dft()[0][
                    gtr_wave.ft_peaks(min_dist=0.001, thres=0.2)[i]]
                    for gtr_wave in gtr_waves],
                np.array([0, 10, 20, 30, 40, 50]) + D_MIN,
                [gtr_wave.normalized_abs_dft()[1][
                    gtr_wave.ft_peaks(min_dist=0.001, thres=0.2)[i]]
                    for gtr_wave in gtr_waves],
                color=colors[i], label=('Harmonic ' + str(i+2)))

if EXPECTED:
    colors = ['g', 'b', 'r']
    offsets = [0.2, 0, 0]
    for i in range(2, CONNECT_N + 2):
        wav = gtr_waves[0]
        ax.plot(np.full(np.linspace(D_MIN, D_MIN + 50, num=100).size,
            wav.normalized_abs_dft()[0][
                    wav.ft_peaks(min_dist=0.001, thres=0.2)[i-2]
                ]),
                np.linspace(D_MIN, D_MIN + 50, num=100),
                [np.sin(i * np.pi * (val+D_MIN) / 648) + offsets[i-2]
                    for val in np.linspace(0, D_MIN + 50, num=100)],
                linestyle=':', color=colors[i-2])

ax.set_xlabel('The frequency')
```

```
    ax.set_ylabel('The distance of the pickup [mm]')
    ax.set_zlabel('Relative amplitude')
    ax.legend()
    plt.show()
```

plot_ft.py

```
#!/usr/bin/env python

from data_reader import read
import matplotlib.pyplot as plt
import sys

gtr_w = read(sys.argv[-1])
gtr_w.plot_normalized_ft()
plt.xlabel('Frequency [Hz]')
plt.ylabel('Amplitude [V]')
plt.show()
```

plot.py

```
#!/usr/bin/env python

from data_reader import read
import matplotlib.pyplot as plt
import sys

gtr_w = read(sys.argv[-1])
gtr_w.plot(y_unc=0.002)
plt.xlabel('Time [s]')
plt.ylabel('Amplitude [V]')
plt.show()
```