Diffusion Models for Audio Generation

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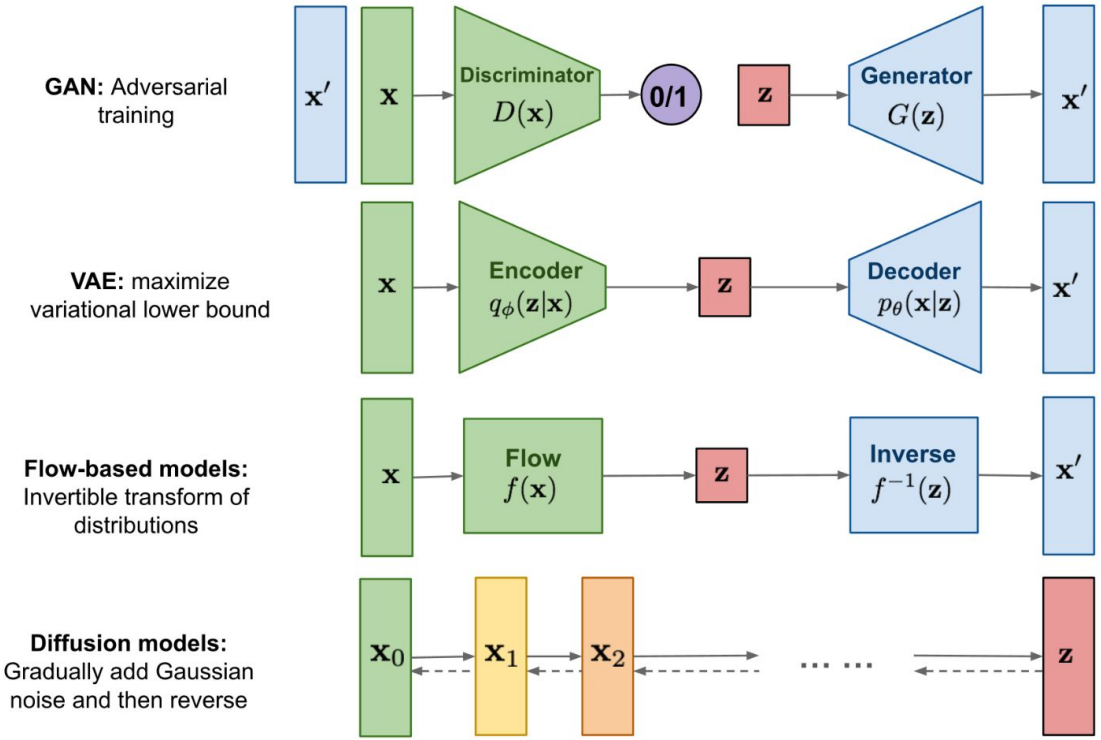
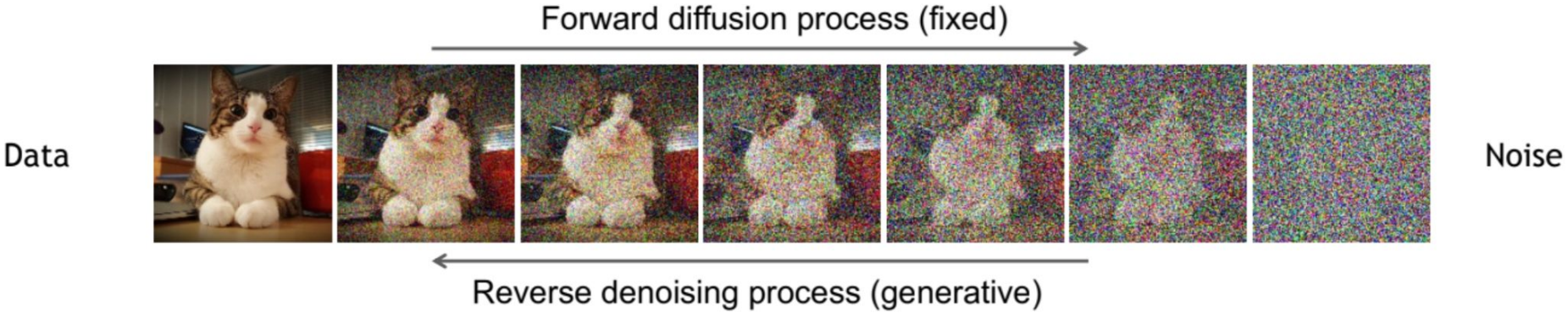
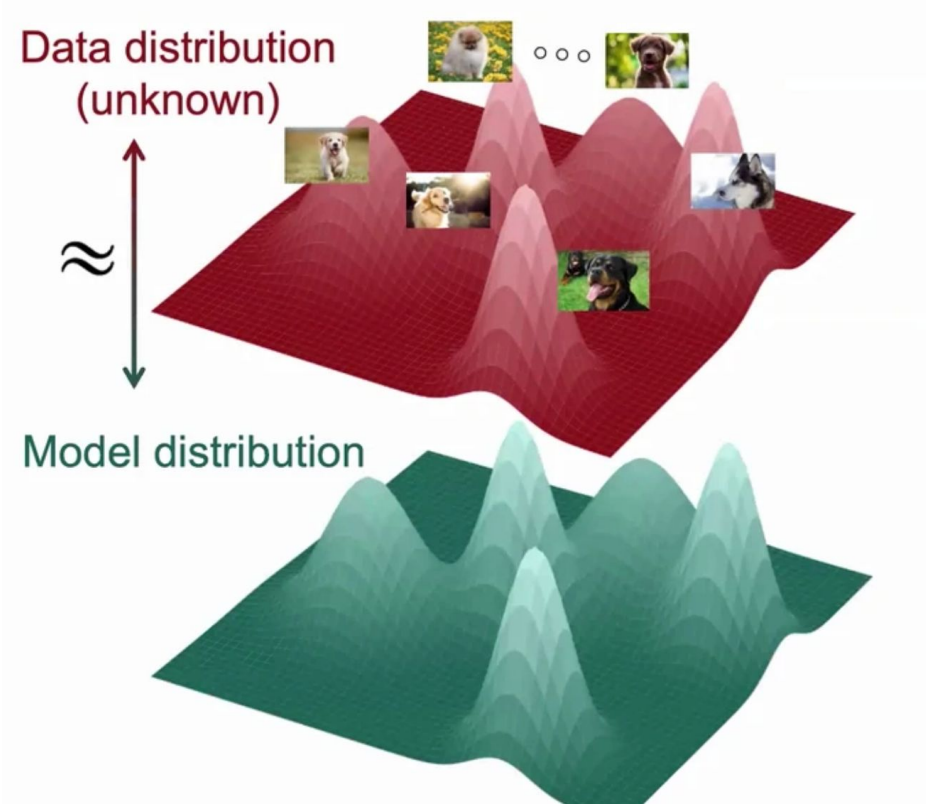
Generative models



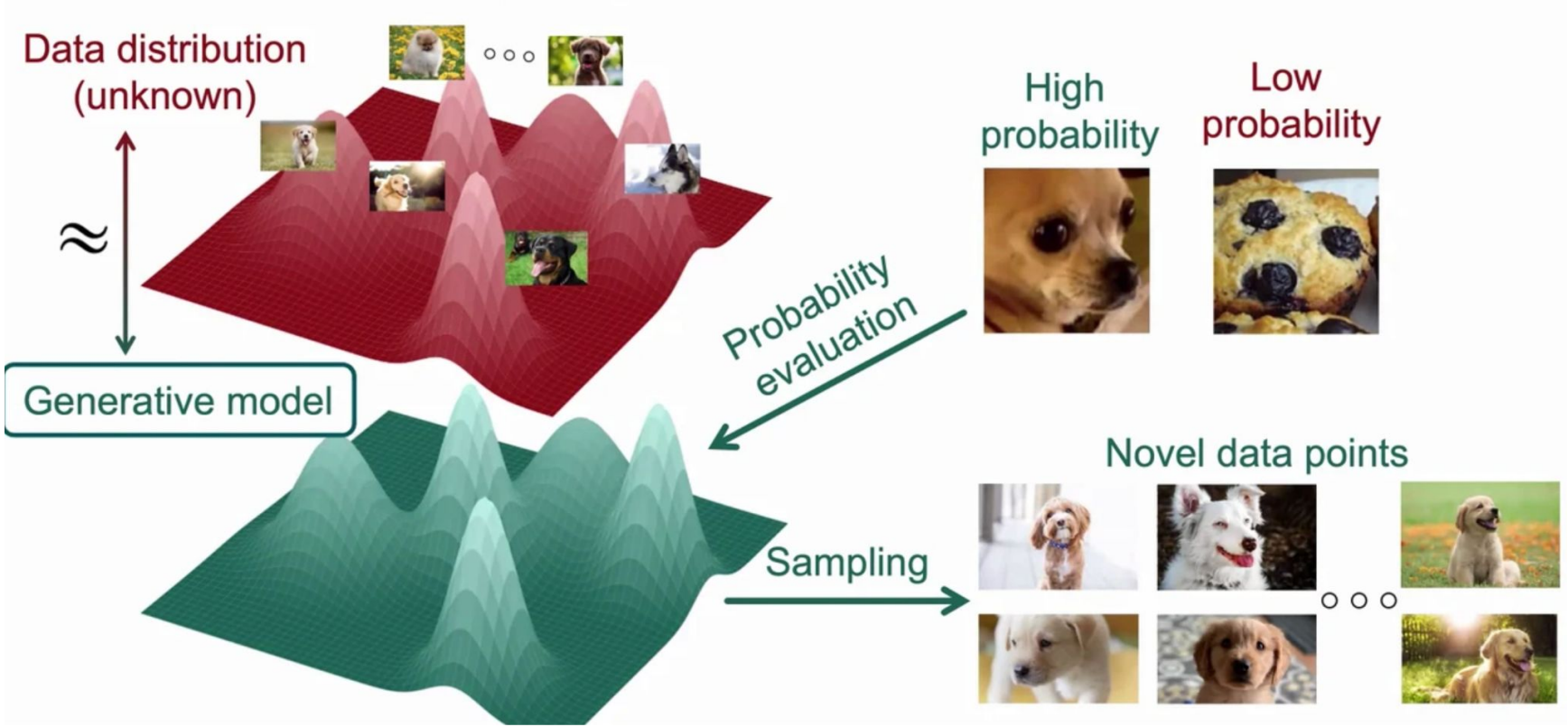
Diffusion models



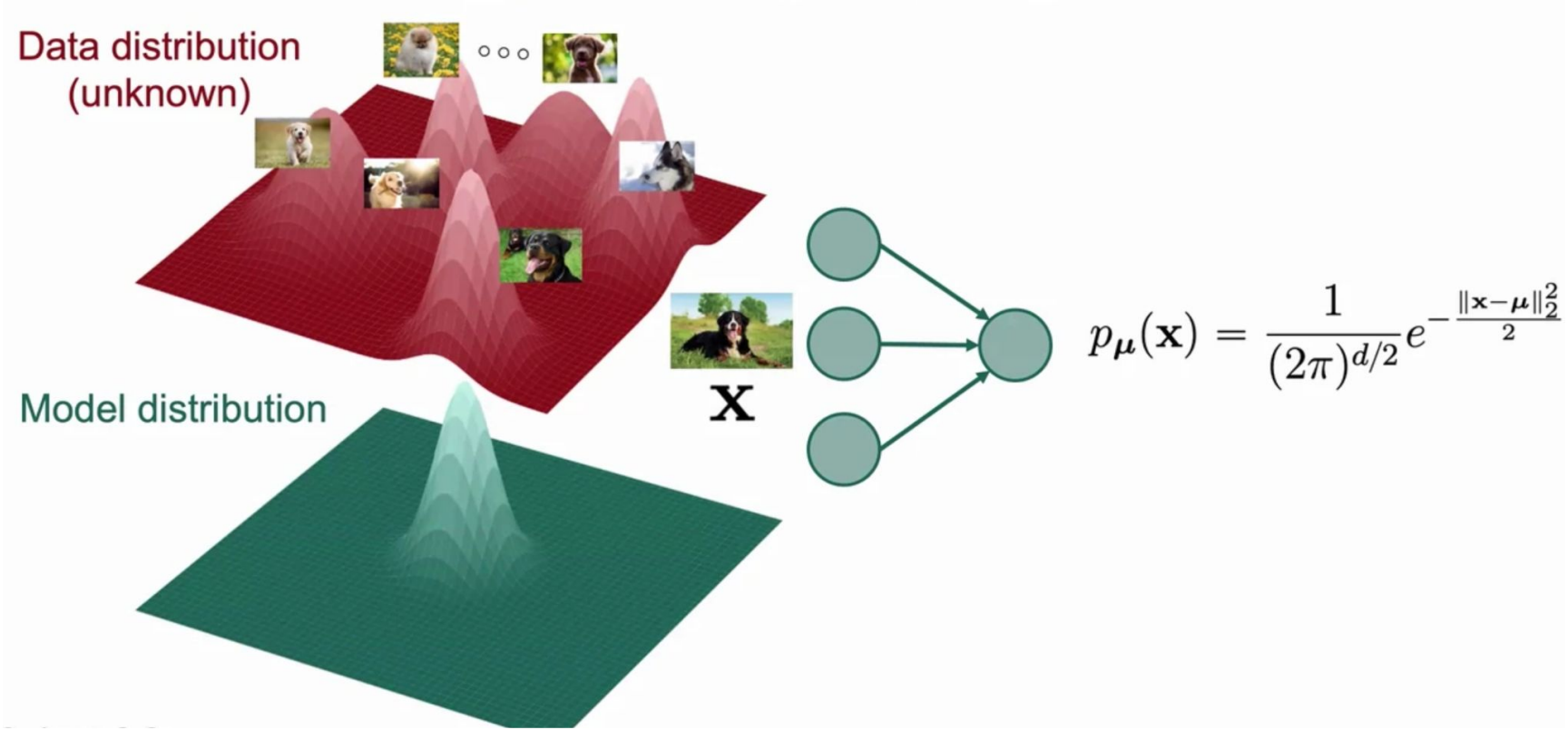
Estimating the probability distribution of data



Estimating the probability distribution of data

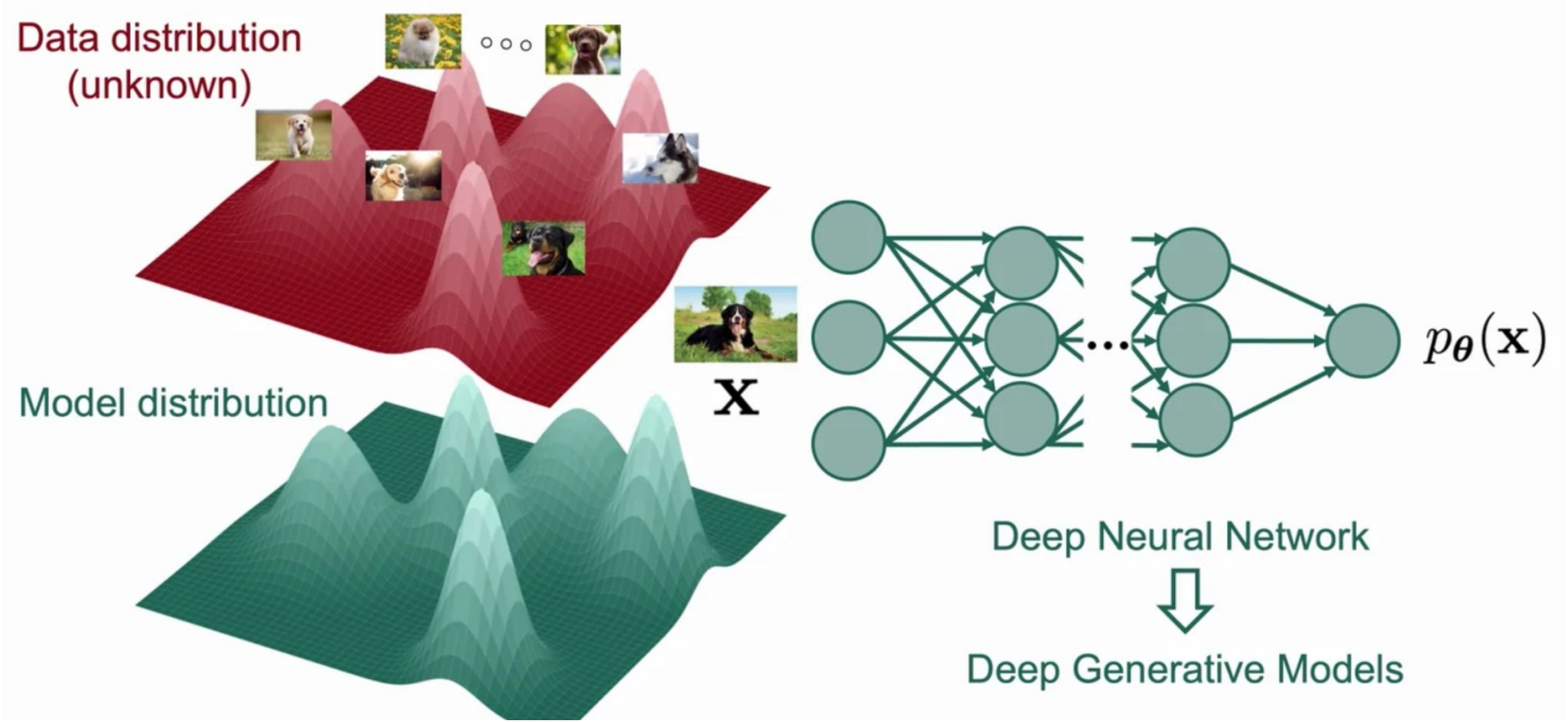


Estimating the probability distribution of data



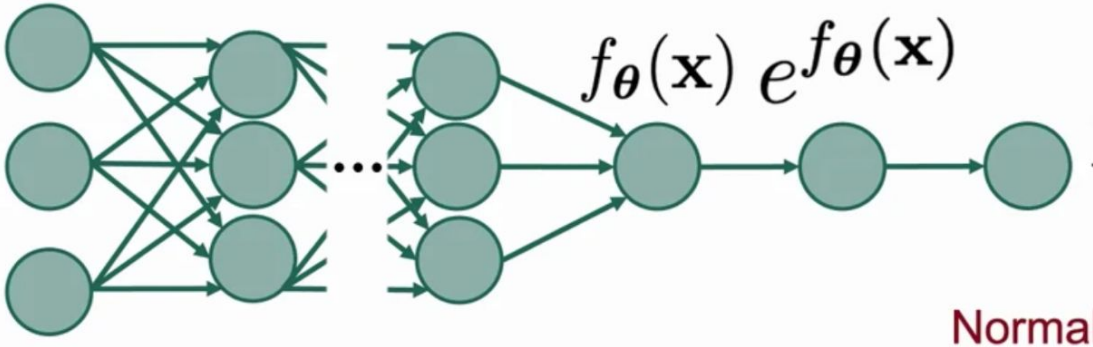
$$p_{\mu}(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}} e^{-\frac{\|\mathbf{x}-\mu\|_2^2}{2}}$$

Estimating the probability distribution of data



Estimating the probability distribution of data

$\mathbf{x}$

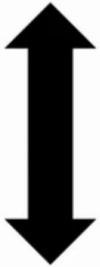


$$Z_{\boldsymbol{\theta}} = \int e^{f_{\boldsymbol{\theta}}(\mathbf{x})} \, \mathrm{d}\mathbf{x}$$

Estimating the score function of data

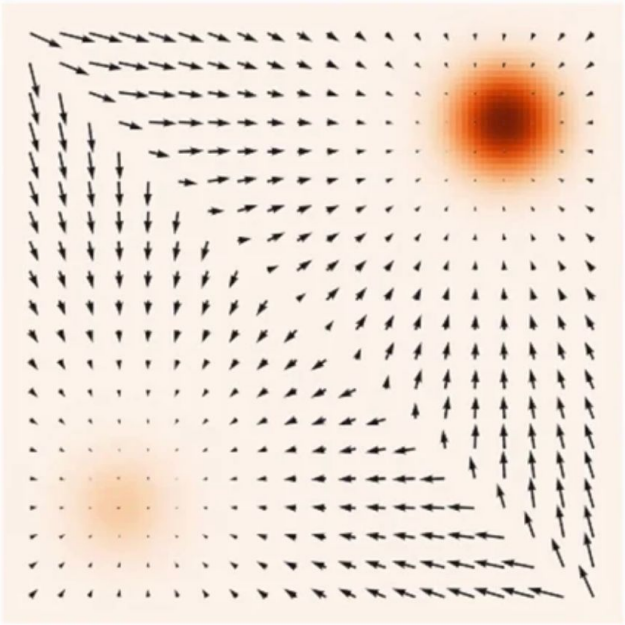
$p(\mathbf{x})$

Probability density function



$\nabla_{\mathbf{x}} \log p(\mathbf{x})$

(Stein) score function

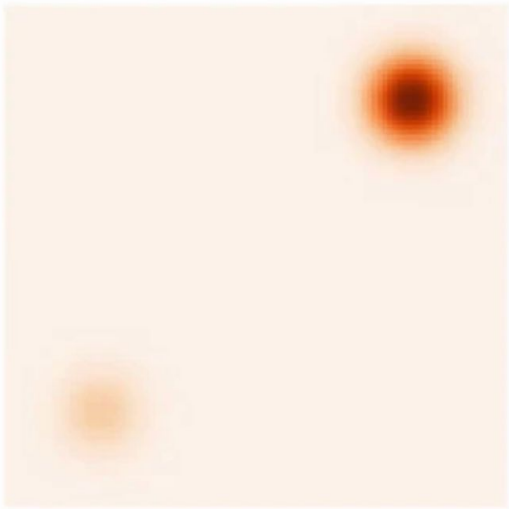


Score vs. density function

Estimating the score function of data

Training data

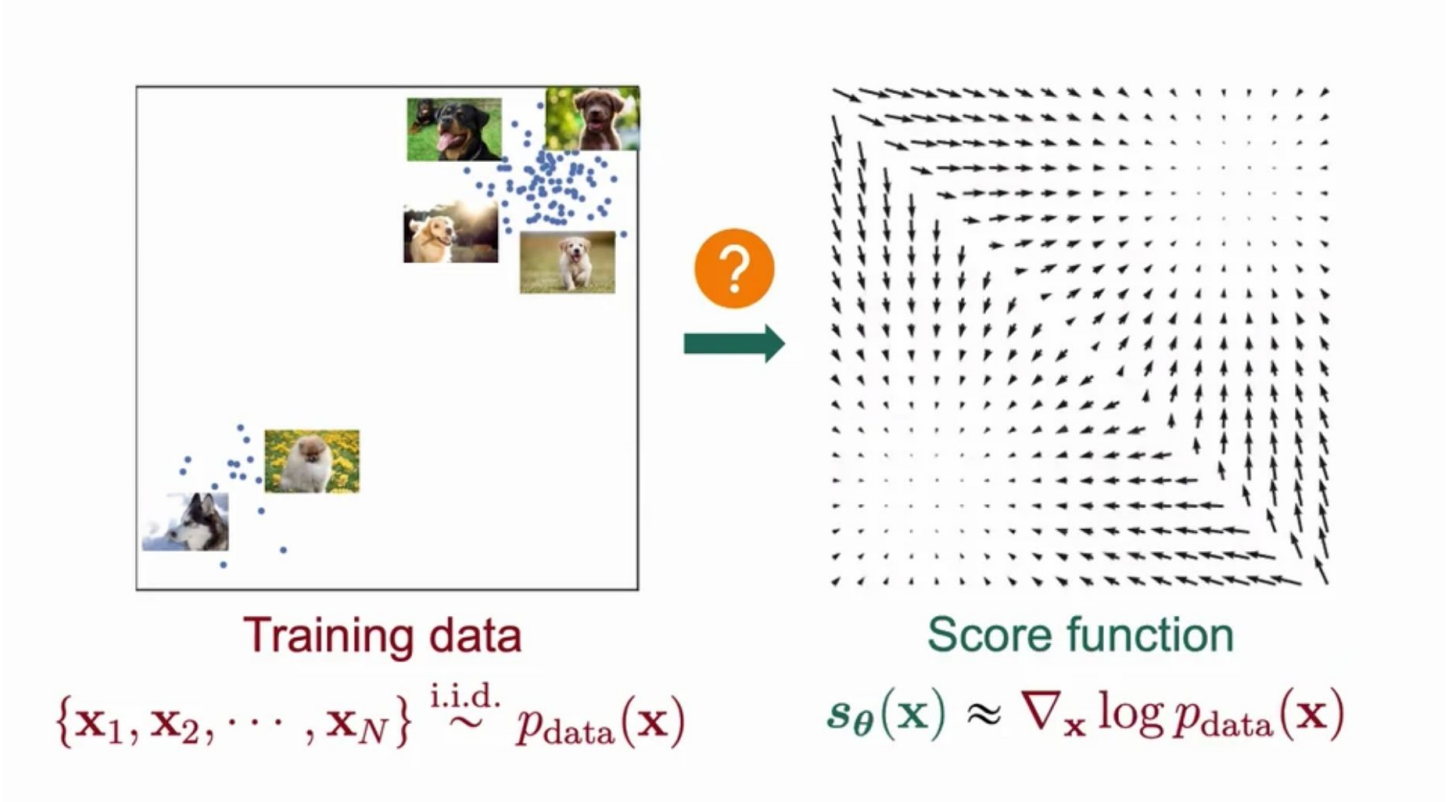
$$\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N\} \overset{\text{i.i.d.}}{\sim} p_{\text{data}}(\mathbf{x})$$



Probability density function

$$p_{\boldsymbol{\theta}}(\mathbf{x}) \approx p_{\text{data}}(\mathbf{x})$$

Estimating the score function of data



Estimating the score function of data

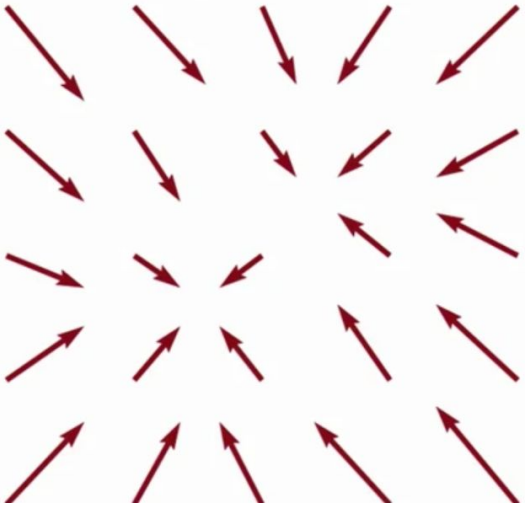
Given: $\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N\} \overset{\text{i.i.d.}}{\sim} p_{\text{data}}(\mathbf{x})$

Goal: $\nabla_{\mathbf{x}} \log p_{\text{data}}(\mathbf{x})$

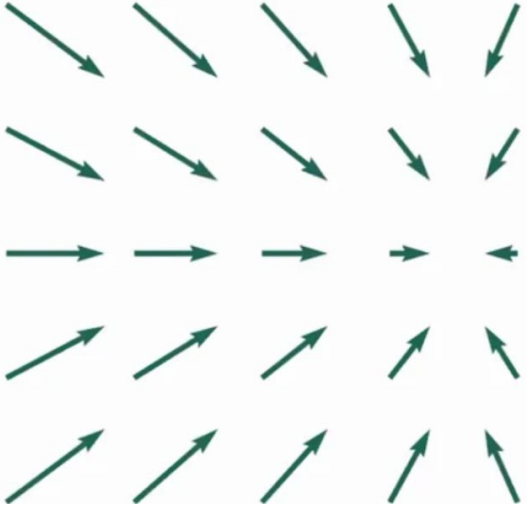
Score Model: $s_\theta(\mathbf{x}) : \mathbb{R}^d \to \mathbb{R}^d \approx \nabla_{\mathbf{x}} \log p_{\text{data}}(\mathbf{x})$

Objective: How to compare two vector fields of scores?

$\nabla_{\mathbf{x}} \log p_{\text{data}}(\mathbf{x})$



$s_\theta(\mathbf{x})$



Estimating the score function of data

Given: $\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N\} \overset{\text{i.i.d.}}{\sim} p_{\text{data}}(\mathbf{x})$

Goal: $\nabla_{\mathbf{x}} \log p_{\text{data}}(\mathbf{x})$

Score Model: $\boldsymbol{s}_{\boldsymbol{\theta}}(\mathbf{x}) : \mathbb{R}^d \to \mathbb{R}^d \approx \nabla_{\mathbf{x}} \log p_{\text{data}}(\mathbf{x})$

Objective: How to compare two vector fields of scores?

$$\frac{1}{2}\mathbb{E}_{p_{\text{data}}(\mathbf{x})}[\|\nabla_{\mathbf{x}} \log p_{\text{data}}(\mathbf{x}) - \boldsymbol{s}_{\boldsymbol{\theta}}(\mathbf{x})\|_2^2]$$

(Fisher divergence)

Estimating the score function of data

$$
\begin{aligned}
&\frac{1}{2}\mathbb{E}_{p_{\text{data}}}[(\nabla_x \log p_{\text{data}}(x) - \nabla_x \log p_\theta(x))^2] \\
=&\frac{1}{2}\int p_{\text{data}}(x)(\nabla_x \log p_{\text{data}}(x) - \nabla_x \log p_\theta(x))^2 \mathrm{d}x \\
=&\underbrace{\frac{1}{2}\int p_{\text{data}}(x)(\nabla_x \log p_{\text{data}}(x))^2 \mathrm{d}x}_{\text{const}} + \frac{1}{2}\int p_{\text{data}}(x)(\nabla_x \log p_\theta(x))^2 \mathrm{d}x \\
&- \int p_{\text{data}}(x)\nabla_x \log p_\theta(x)\nabla_x \log p_{\text{data}}(x)\mathrm{d}x.
\end{aligned}
$$

Estimating the score function of data

$$
\begin{aligned}
&-\int p_{\text{data}}(x)\nabla_x \log p_\theta(x)\nabla_x \log p_{\text{data}}(x)\mathrm{d}x \\
=&-\int \nabla_x \log p_\theta(x)\nabla_x p_{\text{data}}(x)\mathrm{d}x \\
=&-p_{\text{data}}(x)\nabla_x \log p_\theta(x)\Big|_{-\infty}^{\infty}+\int p_{\text{data}}(x)\nabla_x^2 \log p_\theta(x)\mathrm{d}x \\
\overset{(i)}{=}&\mathbb{E}_{p_{\text{data}}}[\nabla_x^2 \log p_\theta(x)],
\end{aligned}
$$

Estimating the score function of data

Given: $\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N\} \overset{\text{i.i.d.}}{\sim} p_{\text{data}}(\mathbf{x})$

Goal: $\nabla_{\mathbf{x}} \log p_{\text{data}}(\mathbf{x})$

Score Model: $s_\theta(\mathbf{x}) : \mathbb{R}^d \to \mathbb{R}^d \approx \nabla_{\mathbf{x}} \log p_{\text{data}}(\mathbf{x})$

Objective: How to compare two vector fields of scores?

$$\frac{1}{2}\mathbb{E}_{p_{\text{data}}(\mathbf{x})}[\| \nabla_{\mathbf{x}} \log p_{\text{data}}(\mathbf{x}) - s_\theta(\mathbf{x}) \|_2^2]$$

(Fisher divergence)

Integration by parts (Gauss's theorem)

Score Matching [Hyvärinen 2005]:

$$\mathbb{E}_{p_{\text{data}}(\mathbf{x})}\left[\frac{1}{2}\|s_\theta(\mathbf{x})\|_2^2 + \text{trace}(\underbrace{\nabla_{\mathbf{x}} s_\theta(\mathbf{x})}_{\text{Jacobian of } s_\theta(\mathbf{x})})\right]$$

Denoising score matching

$\mathbf{x}$

$p_{\text{data}}(\mathbf{x})$

$q_\sigma(\tilde{\mathbf{x}} \mid \mathbf{x})$

$\tilde{\mathbf{x}}$

$q_\sigma(\tilde{\mathbf{x}})$

- Matching the score of a noise-perturbed distribution

$$\frac{1}{2}\mathbb{E}_{q_\sigma(\tilde{\mathbf{x}})}[\|\nabla_{\tilde{\mathbf{x}}} \log q_\sigma(\tilde{\mathbf{x}}) - \mathbf{s}_\theta(\tilde{\mathbf{x}})\|_2^2]$$

- Denoising score matching (Vincent 2011)

$$\frac{1}{2}\mathbb{E}_{p_{\text{data}}(\mathbf{x})}\mathbb{E}_{q_\sigma(\tilde{\mathbf{x}}|\mathbf{x})}[\|\nabla_{\tilde{\mathbf{x}}} \log q_\sigma(\tilde{\mathbf{x}} \mid \mathbf{x}) - \mathbf{s}_\theta(\tilde{\mathbf{x}})\|_2^2]$$

Scalable!

Denoising score matching

$$
\begin{aligned}
\nabla_{\tilde{\mathbf{x}}} \log q_\sigma(\tilde{\mathbf{x}}|\mathbf{x}) &= \nabla_{\tilde{\mathbf{x}}} \log \mathcal{N}(\mathbf{x}, \sigma^2 \cdot \mathbf{I}) \\
&= \nabla_{\tilde{\mathbf{x}}} \log \frac{\exp(-\frac{1}{2}(\tilde{\mathbf{x}} - \mathbf{x})^T \cdot (\sigma^2 \cdot \mathbf{I})^{-1} \cdot (\tilde{\mathbf{x}} - \mathbf{x}))}{\sqrt{(2\pi)^d |\sigma^2 \cdot \mathbf{I}|}} \\
&= \nabla_{\tilde{\mathbf{x}}} \log \exp(-\frac{1}{2}(\tilde{\mathbf{x}} - \mathbf{x})^T \cdot (\sigma^2 \cdot \mathbf{I})^{-1} \cdot (\tilde{\mathbf{x}} - \mathbf{x})) \\
&\quad - \underbrace{\nabla_{\tilde{\mathbf{x}}} \log \sqrt{(2\pi)^d |\sigma^2 \cdot \mathbf{I}|}}_{=0}] \\
&= -\frac{1}{2\sigma^2} \nabla_{\tilde{\mathbf{x}}} (\tilde{\mathbf{x}} - \mathbf{x})^T \cdot \mathbf{I} \cdot (\tilde{\mathbf{x}} - \mathbf{x}) \\
&= -\frac{1}{2\sigma^2} \nabla_{\tilde{\mathbf{x}}} (\tilde{\mathbf{x}} - \mathbf{x})^2 \\
&= -\frac{(\tilde{\mathbf{x}} - \mathbf{x})}{\sigma^2} = \frac{(\mathbf{x} - \tilde{\mathbf{x}})}{\sigma^2}.
\end{aligned}
$$

Denoising score matching

- Sample a minibatch of datapoints $\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n\} \sim p_{\text{data}}(\mathbf{x})$
- Sample a minibatch of perturbed datapoints $\{\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \cdots, \tilde{\mathbf{x}}_n\} \sim q_\sigma(\tilde{\mathbf{x}})$

$$\tilde{\mathbf{x}}_i \sim q_\sigma(\tilde{\mathbf{x}}_i \mid \mathbf{x}_i)$$

- Estimate the denoising score matching loss with empirical means

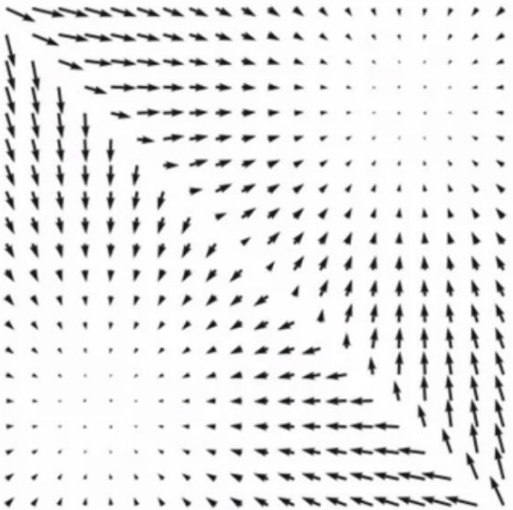
$$\frac{1}{2n}\sum_{i=1}^{n}[\|\boldsymbol{s}_\theta(\tilde{\mathbf{x}}_i) - \nabla_{\tilde{\mathbf{x}}} \log q_\sigma(\tilde{\mathbf{x}}_i \mid \mathbf{x}_i)\|_2^2]$$

- If Gaussian perturbation

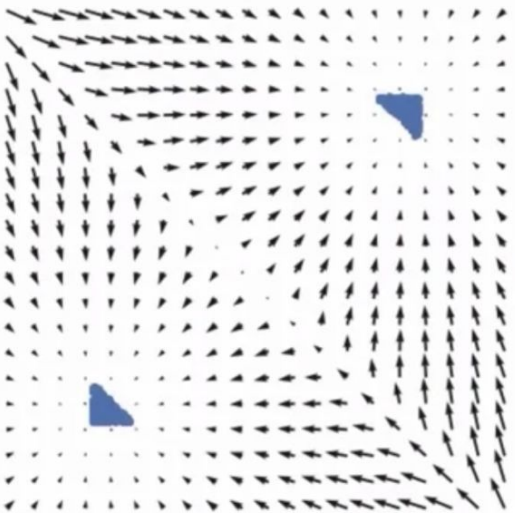
$$\frac{1}{2n}\sum_{i=1}^{n}\left[\left\|\boldsymbol{s}_\theta(\tilde{\mathbf{x}}_i) + \frac{\tilde{\mathbf{x}}_i - \mathbf{x}_i}{\sigma^2}\right\|_2^2\right]$$

- Stochastic gradient descent
- Need to choose a very small σ!

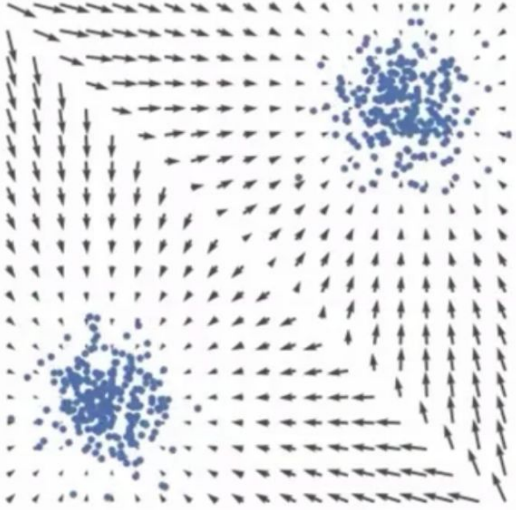
Sampling from score function



Score function



Follow the scores



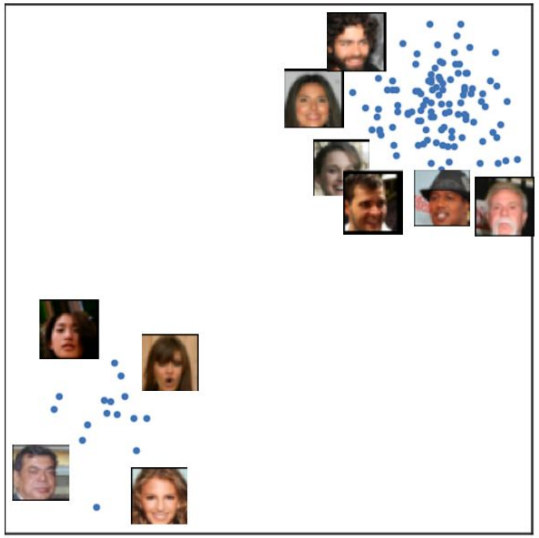
Follow the noisy scores

$$\mathbf{x}_{i+1} \leftarrow \mathbf{x}_i + \epsilon \nabla_{\mathbf{x}} \log p(\mathbf{x})$$

$$\mathbf{x}_{i+1} \leftarrow \mathbf{x}_i + \epsilon \nabla_{\mathbf{x}} \log p(\mathbf{x}) + \sqrt{2\epsilon}\, \mathbf{z}_i$$

Langevin dynamics

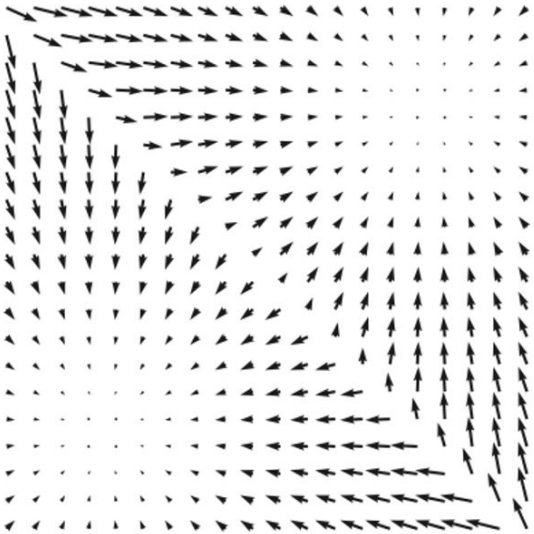
Sampling from score function



Data samples

$$\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N\} \overset{\text{i.i.d.}}{\sim} p(\mathbf{x})$$

score matching



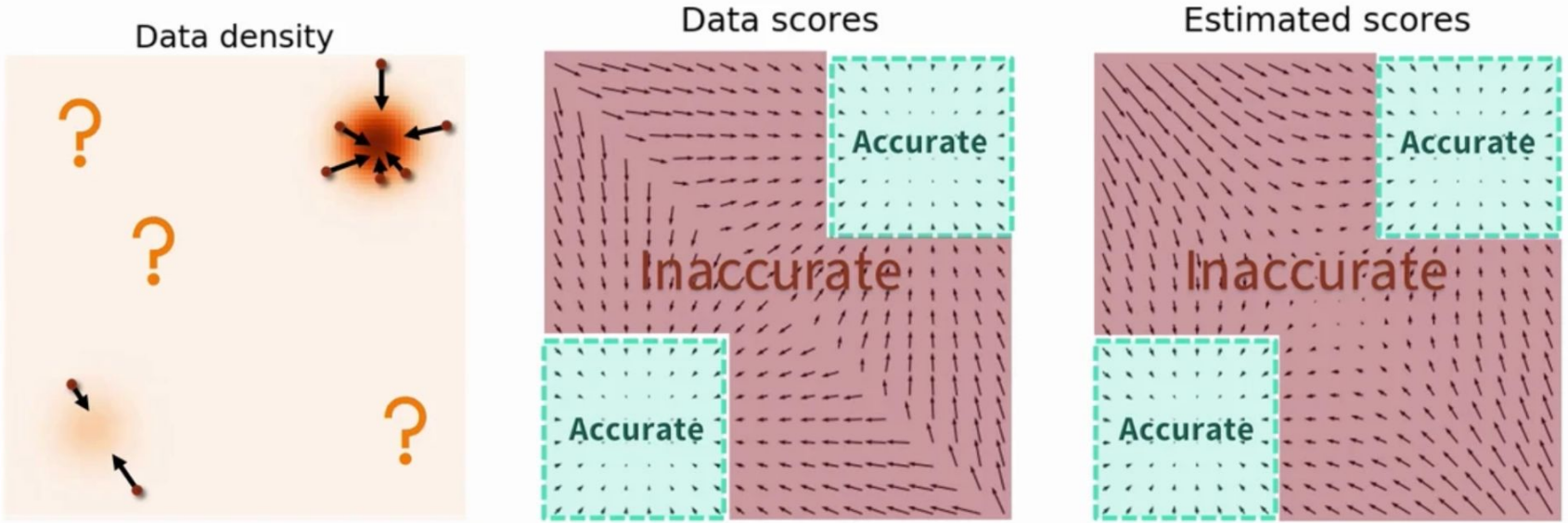
Scores

$$\mathbf{s}_\theta(\mathbf{x}) \approx \nabla_{\mathbf{x}} \log p(\mathbf{x})$$

Langevin dynamics

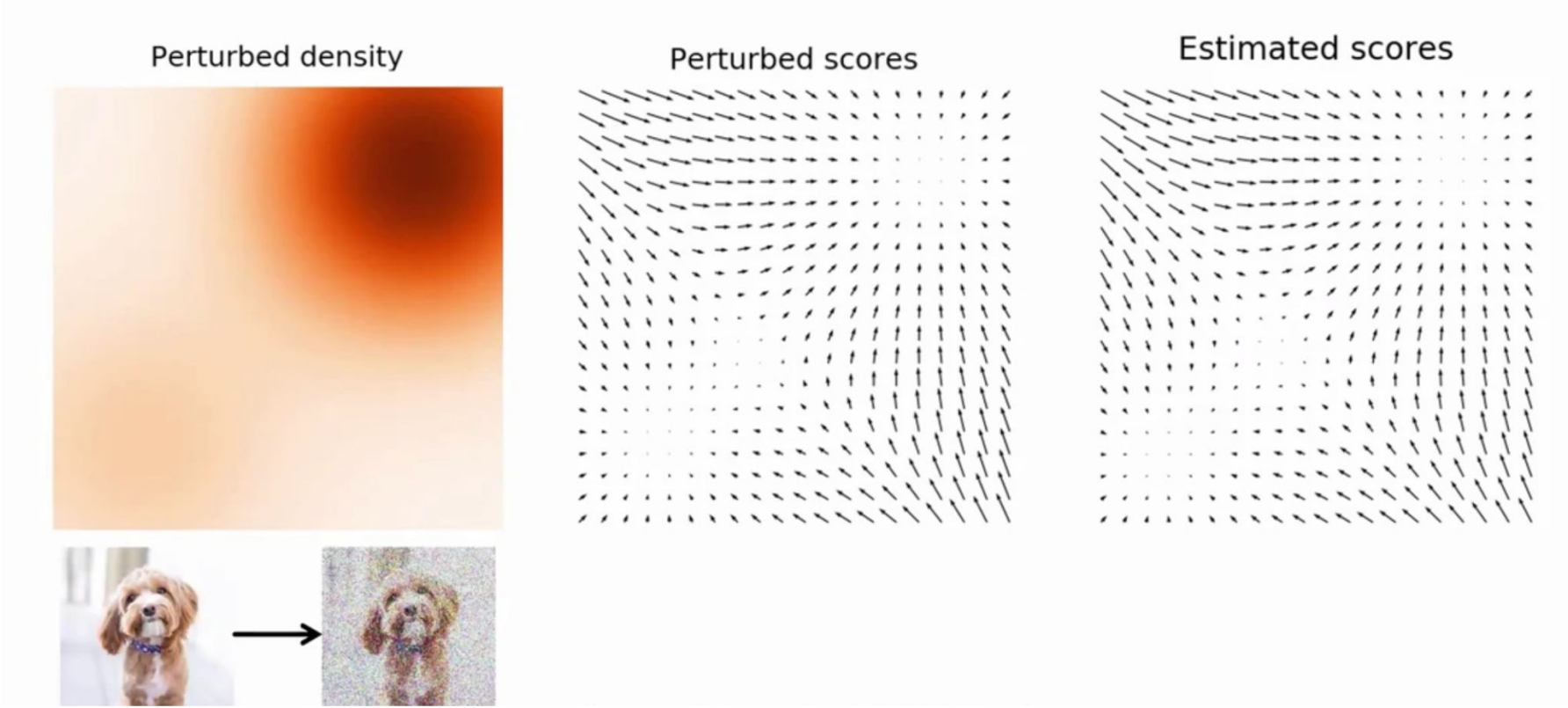
New samples

Challenge in low data regions

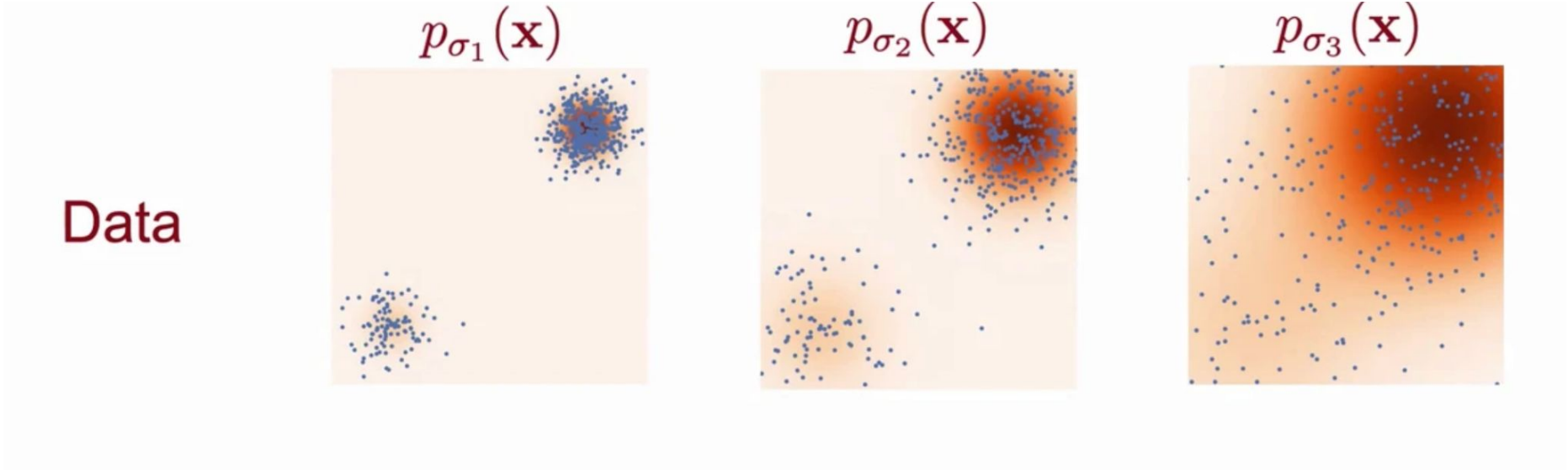


$$\frac{1}{2}\mathbb{E}_{p_{\text{data}}(\mathbf{x})}[\|\nabla_{\mathbf{x}} \log p_{\text{data}}(\mathbf{x}) - \boldsymbol{s}_{\boldsymbol{\theta}}(\mathbf{x})\|_2^2]$$

Challenge in low data regions



Using multiple noise levels

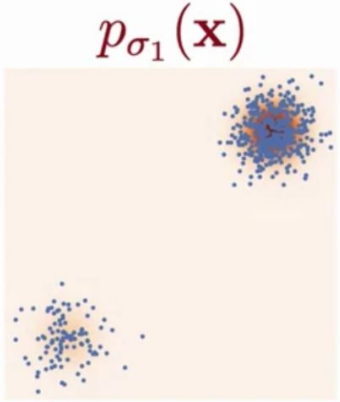


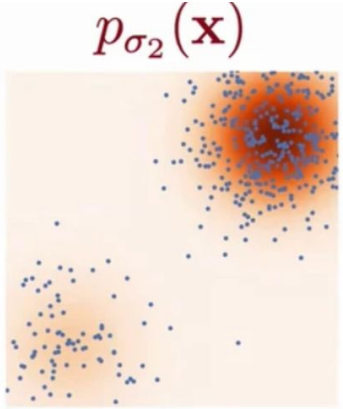
Using multiple noise levels

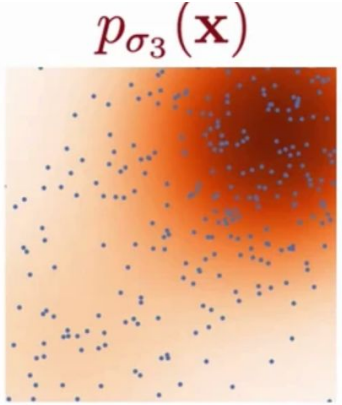
Data

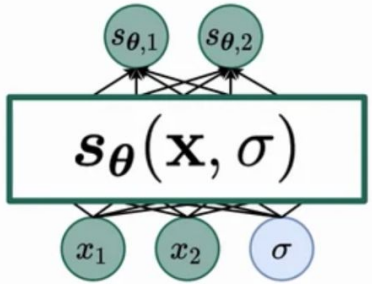
Using multiple noise levels

Data







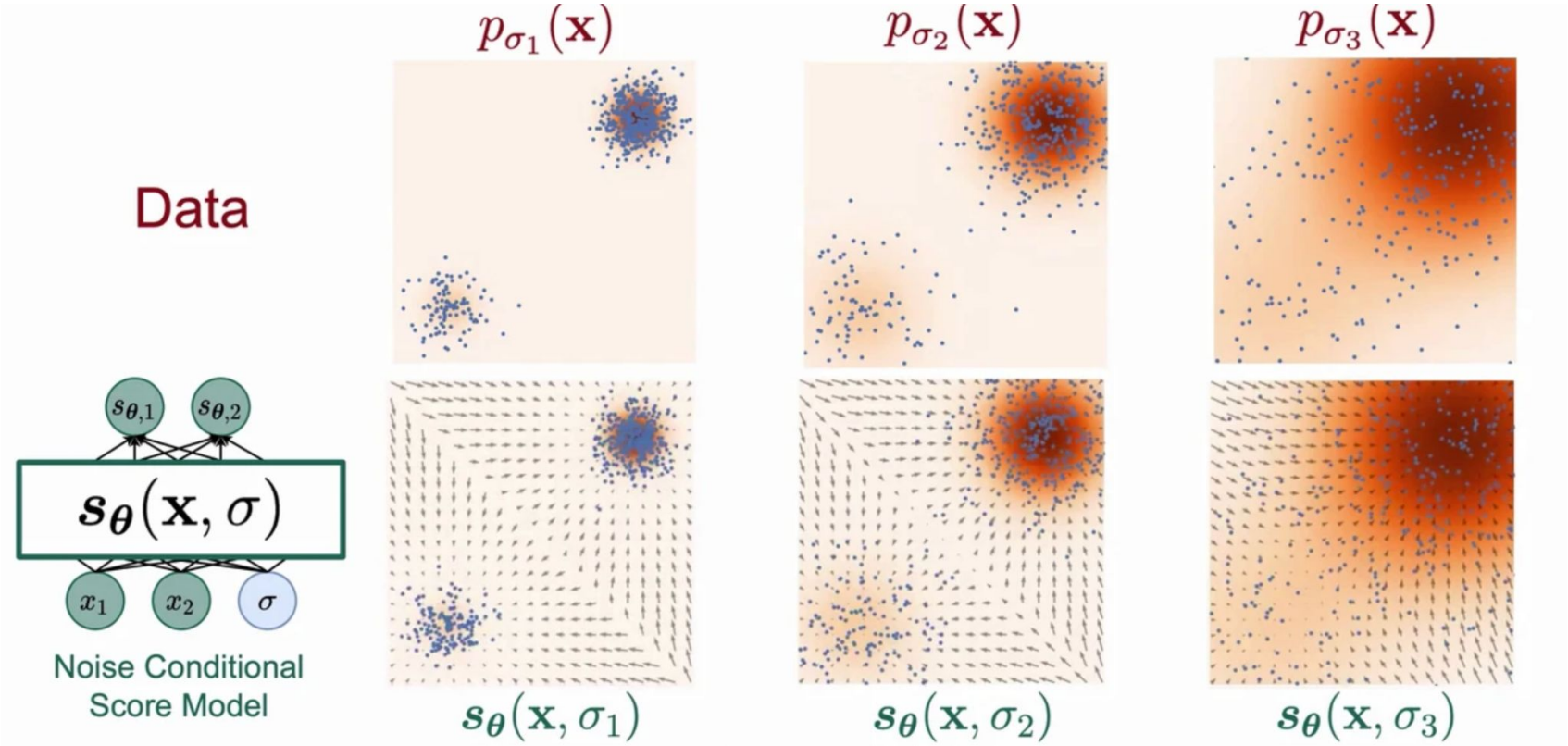


Noise Conditional Score Model

Positive weighting function

$$\frac{1}{N}\sum_{i=1}^{N}\overbrace{\lambda(\sigma_i)}\underbrace{\mathbb{E}_{p_{\sigma_i}(\mathbf{x})}[\|\nabla_{\mathbf{x}}\log p_{\sigma_i}(\mathbf{x}) - s_\theta(\mathbf{x},\sigma_i)\|_2^2]}_{\text{Score matching loss}}$$

Using multiple noise levels



Learning NCSNs via score matching

$$\ell(\boldsymbol{\theta}; \sigma) \triangleq \frac{1}{2}\mathbb{E}_{p_{\text{data}}(\mathbf{x})}\mathbb{E}_{\tilde{\mathbf{x}}\sim\mathcal{N}(\mathbf{x},\sigma^2 I)}\left[\left\|\mathbf{s}_{\boldsymbol{\theta}}(\tilde{\mathbf{x}}, \sigma) + \frac{\tilde{\mathbf{x}} - \mathbf{x}}{\sigma^2}\right\|_2^2\right]$$

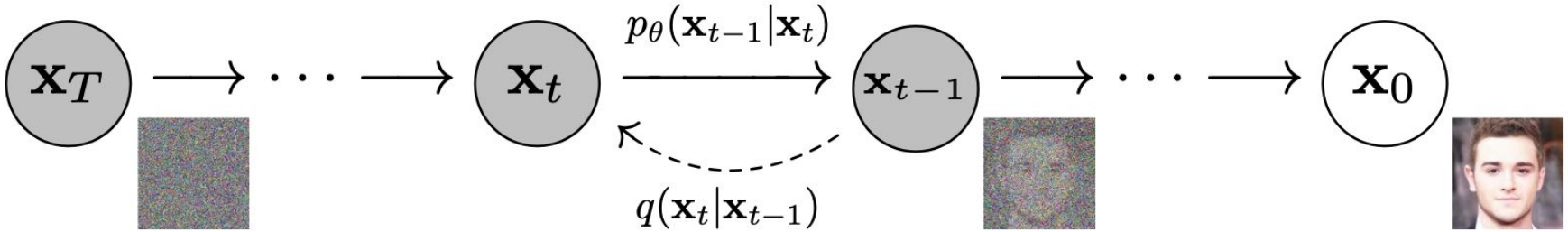
$$\mathcal{L}(\boldsymbol{\theta}; \{\sigma_i\}_{i=1}^L) \triangleq \frac{1}{L}\sum_{i=1}^{L}\lambda(\sigma_i)\ell(\boldsymbol{\theta}; \sigma_i)$$

Algorithm 1 Annealed Langevin dynamics.

Require: $\{\sigma_i\}_{i=1}^L, \epsilon, T$.

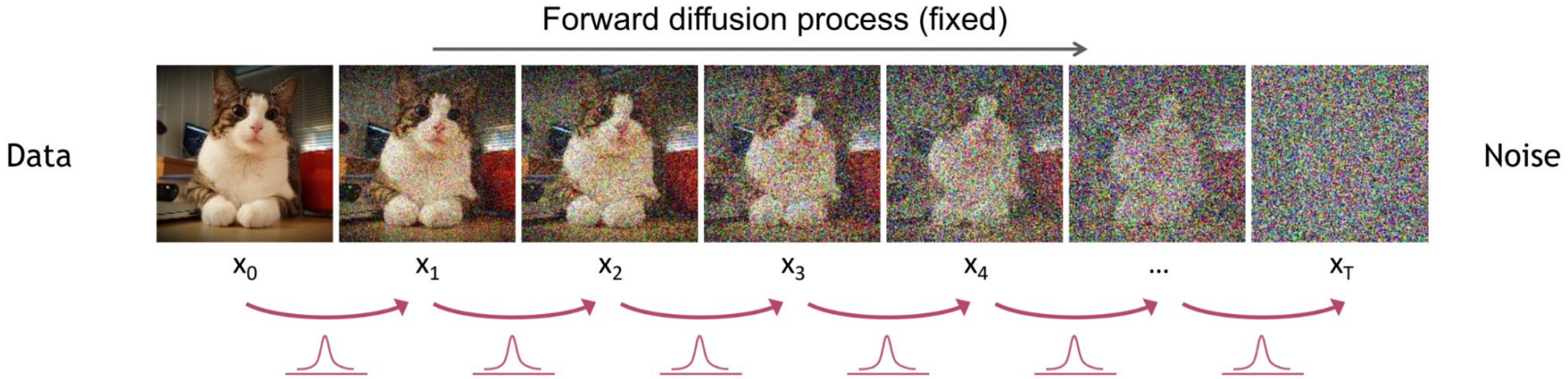
Initialize $\tilde{\mathbf{x}}_0$
for $i \leftarrow 1$ to L **do**
  $\alpha_i \leftarrow \epsilon \cdot \sigma_i^2/\sigma_L^2$ ▷ α_i is the step size.
  for $t \leftarrow 1$ to T **do**
    Draw $\mathbf{z}_t \sim \mathcal{N}(0, I)$
    $\tilde{\mathbf{x}}_t \leftarrow \tilde{\mathbf{x}}_{t-1} + \frac{\alpha_i}{2}\mathbf{s}_{\boldsymbol{\theta}}(\tilde{\mathbf{x}}_{t-1}, \sigma_i) + \sqrt{\alpha_i}\,\mathbf{z}_t$
  end for
  $\tilde{\mathbf{x}}_0 \leftarrow \tilde{\mathbf{x}}_T$
end for
return $\tilde{\mathbf{x}}_T$

Denoising Diffusion Probabilistic Models

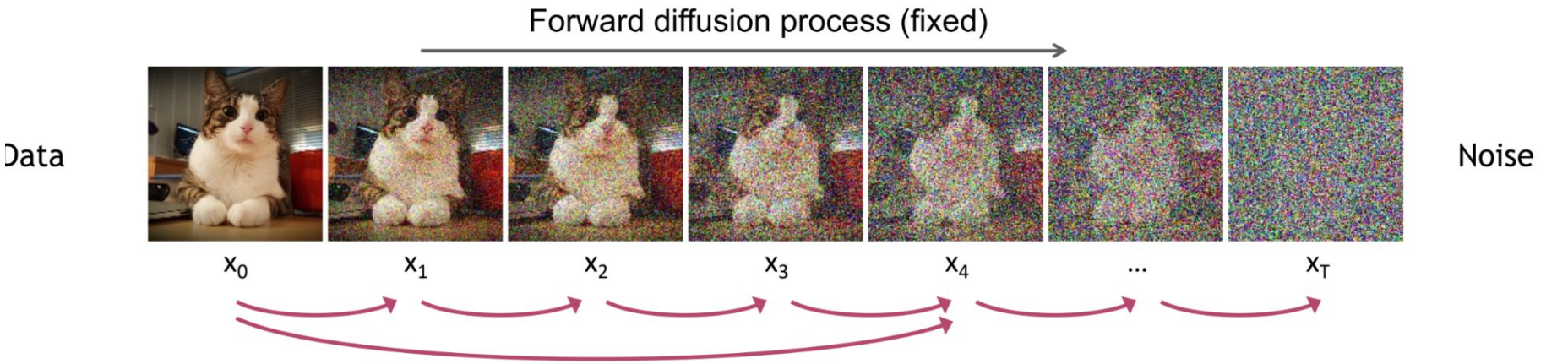


Denoising Diffusion Probabilistic Models

$$q(\mathbf{x}_{1:T}|\mathbf{x}_0) := \prod_{t=1}^{T} q(\mathbf{x}_t|\mathbf{x}_{t-1}), \qquad q(\mathbf{x}_t|\mathbf{x}_{t-1}) := \mathcal{N}(\mathbf{x}_t; \sqrt{1-\beta_t}\mathbf{x}_{t-1}, \beta_t\mathbf{I})$$



Denoising Diffusion Probabilistic Models

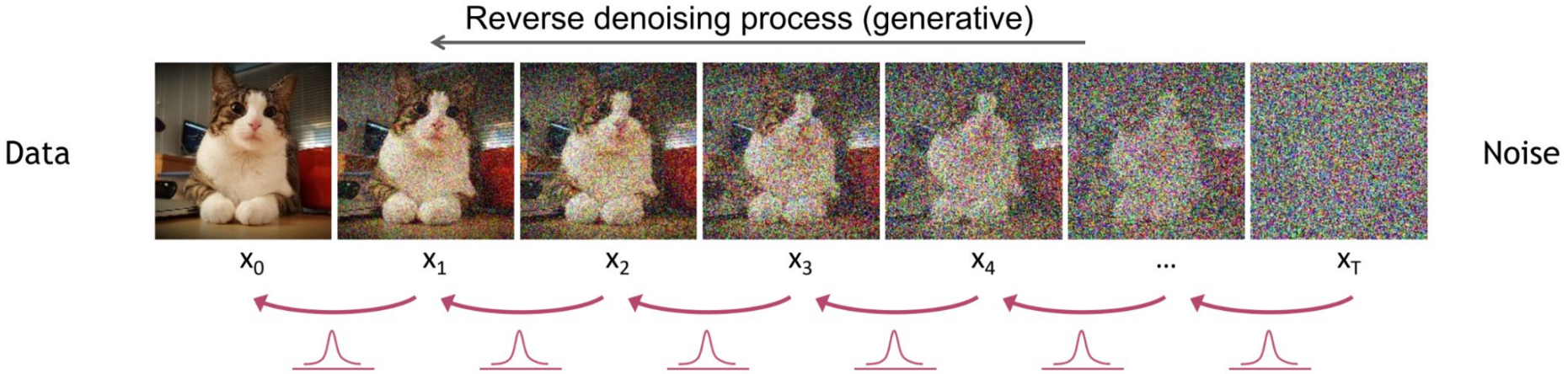


Define $\bar{\alpha}_t = \prod_{s=1}^{t}(1-\beta_s)$ ➡ $q(\mathbf{x}_t|\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_t; \sqrt{\bar{\alpha}_t}\mathbf{x}_0, (1-\bar{\alpha}_t)\mathbf{I}))$ (Diffusion Kernel)

For sampling: $\mathbf{x}_t = \sqrt{\bar{\alpha}_t}\,\mathbf{x}_0 + \sqrt{(1-\bar{\alpha}_t)}\,\epsilon$ where $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

Denoising Diffusion Probabilistic Models

$$p_\theta(\mathbf{x}_{0:T}) := p(\mathbf{x}_T) \prod_{t=1}^{T} p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t), \qquad p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) := \mathcal{N}(\mathbf{x}_{t-1}; \boldsymbol{\mu}_\theta(\mathbf{x}_t, t), \boldsymbol{\Sigma}_\theta(\mathbf{x}_t, t))$$



Denoising Diffusion Probabilistic Models

$$\mathbb{E}\left[-\log p_\theta(\mathbf{x}_0)\right] \leq \mathbb{E}_q\left[-\log \frac{p_\theta(\mathbf{x}_{0:T})}{q(\mathbf{x}_{1:T}|\mathbf{x}_0)}\right] = \mathbb{E}_q\left[-\log p(\mathbf{x}_T) - \sum_{t\geq 1} \log \frac{p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t)}{q(\mathbf{x}_t|\mathbf{x}_{t-1})}\right] =: L$$

$$\mathbb{E}_q\Bigg[\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_T|\mathbf{x}_0) \parallel p(\mathbf{x}_T))}_{L_T} + \sum_{t>1} \underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0) \parallel p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t))}_{L_{t-1}} \underbrace{-\log p_\theta(\mathbf{x}_0|\mathbf{x}_1)}_{L_0}\Bigg]$$

Denoising Diffusion Probabilistic Models

$$q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_{t-1}; \tilde{\boldsymbol{\mu}}_t(\mathbf{x}_t, \mathbf{x}_0), \tilde{\beta}_t \mathbf{I}),$$

$$\text{where} \quad \tilde{\boldsymbol{\mu}}_t(\mathbf{x}_t, \mathbf{x}_0) := \frac{\sqrt{\bar{\alpha}_{t-1}}\beta_t}{1-\bar{\alpha}_t}\mathbf{x}_0 + \frac{\sqrt{\alpha_t}(1-\bar{\alpha}_{t-1})}{1-\bar{\alpha}_t}\mathbf{x}_t \quad \text{and} \quad \tilde{\beta}_t := \frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t}\beta_t$$

$$p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \boldsymbol{\mu}_\theta(\mathbf{x}_t, t), \boldsymbol{\Sigma}_\theta(\mathbf{x}_t, t))$$

$$L_{t-1} = \mathbb{E}_q \left[\frac{1}{2\sigma_t^2} \| \tilde{\boldsymbol{\mu}}_t(\mathbf{x}_t, \mathbf{x}_0) - \boldsymbol{\mu}_\theta(\mathbf{x}_t, t) \|^2 \right] + C$$

Denoising Diffusion Probabilistic Models

$$L_{t-1} = \mathbb{E}_q \left[\frac{1}{2\sigma_t^2} \| \tilde{\boldsymbol{\mu}}_t(\mathbf{x}_t, \mathbf{x}_0) - \boldsymbol{\mu}_\theta(\mathbf{x}_t, t) \|^2 \right] + C$$

$$\boldsymbol{\mu}_\theta(\mathbf{x}_t, t) = \tilde{\boldsymbol{\mu}}_t \left(\mathbf{x}_t, \frac{1}{\sqrt{\bar{\alpha}_t}} (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \boldsymbol{\epsilon}_\theta(\mathbf{x}_t)) \right)$$

$$\mathbb{E}_{\mathbf{x}_0, \boldsymbol{\epsilon}} \left[\frac{\beta_t^2}{2\sigma_t^2 \alpha_t (1 - \bar{\alpha}_t)} \left\| \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \boldsymbol{\epsilon}, t) \right\|^2 \right]$$

Denoising Diffusion Probabilistic Models

$$L_{\text{simple}}(\theta) := \mathbb{E}_{t,\mathbf{x}_0,\boldsymbol{\epsilon}}\left[\left\|\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}, t)\right\|^2\right]$$

Algorithm 1 Training

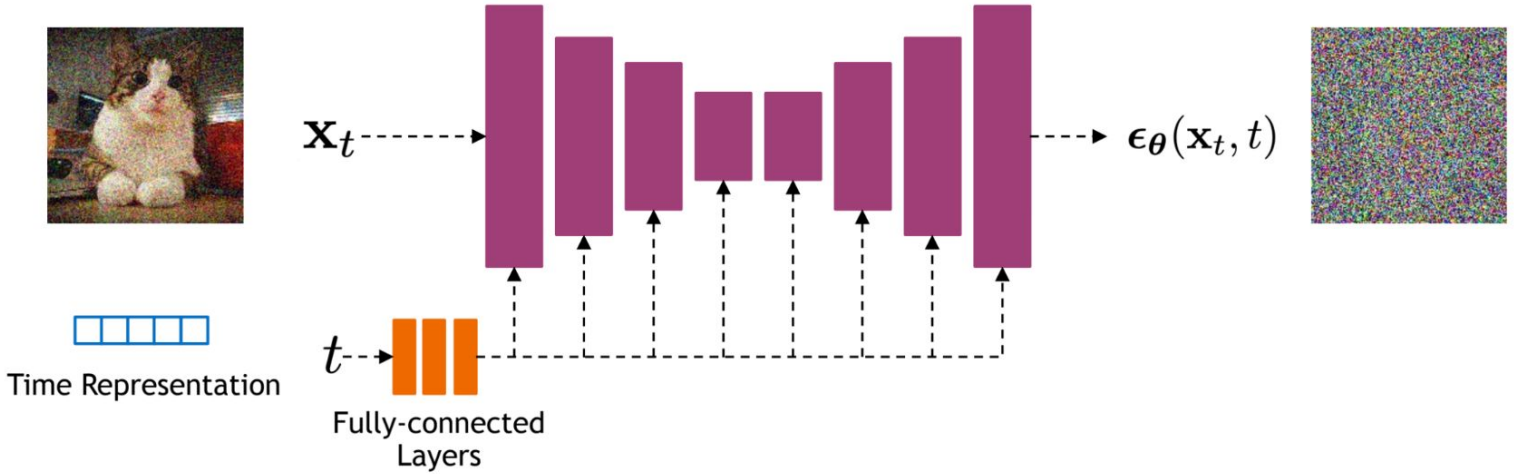
1: **repeat**
2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
3: $t \sim \text{Uniform}(\{1, \dots, T\})$
4: $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
5: Take gradient descent step on
$$\nabla_\theta \left\|\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}, t)\right\|^2$$
6: **until** converged

Algorithm 2 Sampling

1: $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
2: **for** $t = T, \dots, 1$ **do**
3: $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ if $t > 1$, else $\mathbf{z} = \mathbf{0}$
4: $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}}\left(\mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}}\boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t)\right) + \sigma_t \mathbf{z}$
5: **end for**
6: **return** $\mathbf{x}_0$

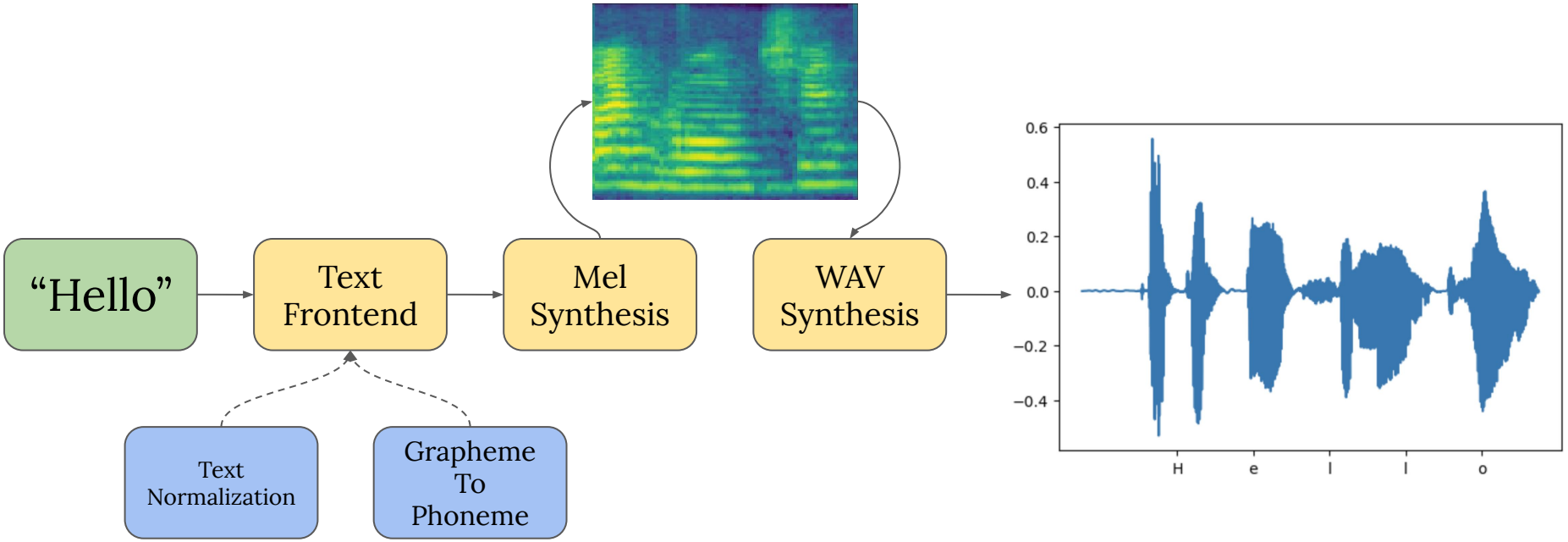
Network architectures

Diffusion models often use U-Net architectures with ResNet blocks and self-attention layers to represent $\epsilon_\theta(\mathbf{x}_t, t)$



Time representation: sinusoidal positional embeddings or random Fourier features.

TTS recap


“Hello”
Text Frontend
Mel Synthesis
WAV Synthesis
Text Normalization
Grapheme To Phoneme
0.6
0.4
0.2
0.0
−0.2
−0.4
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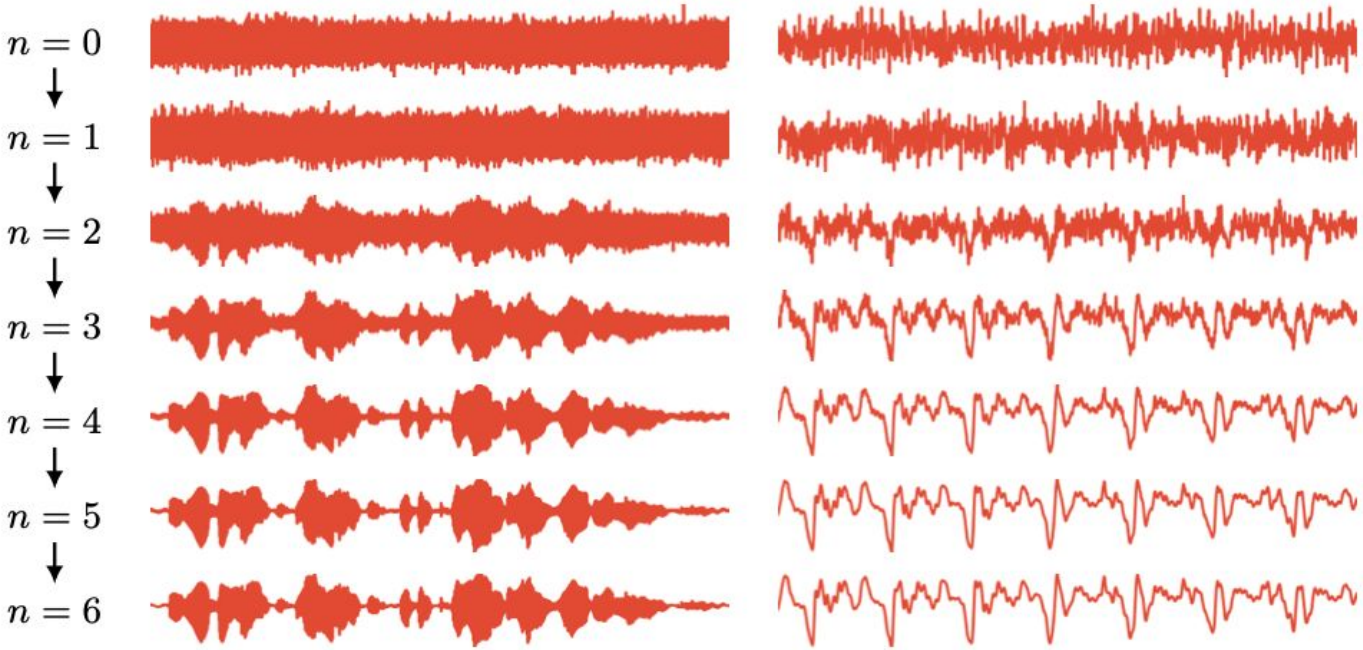
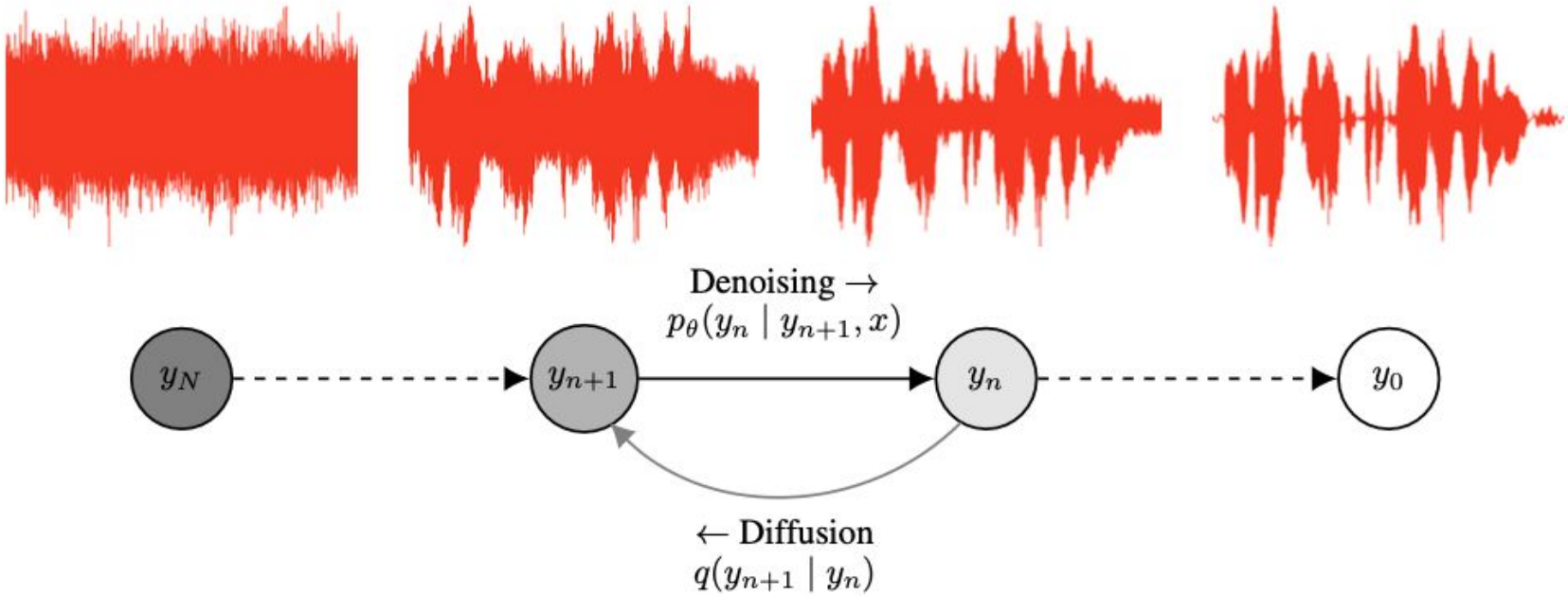
WaveGrad - Vocoder



WaveGrad - Vocoder



WaveGrad - Vocoder

$$q(y_{1:N} \mid y_0) \coloneqq \prod_{n=1}^{N} q(y_n \mid y_{n-1}), \qquad q(y_n \mid y_{n-1}) \coloneqq \mathcal{N}\left(y_n; \sqrt{(1-\beta_n)}\, y_{n-1}, \beta_n I\right),$$

$$y_n = \sqrt{\bar{\alpha}_n}\, y_0 + \sqrt{(1-\bar{\alpha}_n)}\, \epsilon$$

$$p_\theta(y_0 \mid x) \coloneqq \int p_\theta(y_{0:N} \mid x)\, \mathrm{d}y_{1:N}$$

$$\mathbb{E}_{n,\epsilon}\left[C_n \left\| \epsilon_\theta\left(\sqrt{\bar{\alpha}_n}\, y_0 + \sqrt{1-\bar{\alpha}_n}\, \epsilon, x, n\right) - \epsilon \right\|_2^2\right]$$

WaveGrad - Vocoder

$$\mathbb{E}_{\bar{\alpha},\epsilon}\left[\left\|\epsilon_\theta\left(\sqrt{\bar{\alpha}}\,y_0+\sqrt{1-\bar{\alpha}}\,\epsilon,x,\sqrt{\bar{\alpha}}\right)-\epsilon\right\|_1\right],$$

$$l_0=1,\qquad l_s=\sqrt{\textstyle\prod_{i=1}^{s}(1-\beta_i)}.$$

We first sample a segment $s\sim U(\{1,\dots,S\})$, which provides a segment (l_{s-1},l_s), and then sample from this segment uniformly to give $\sqrt{\bar{\alpha}}$.

WaveGrad - Vocoder

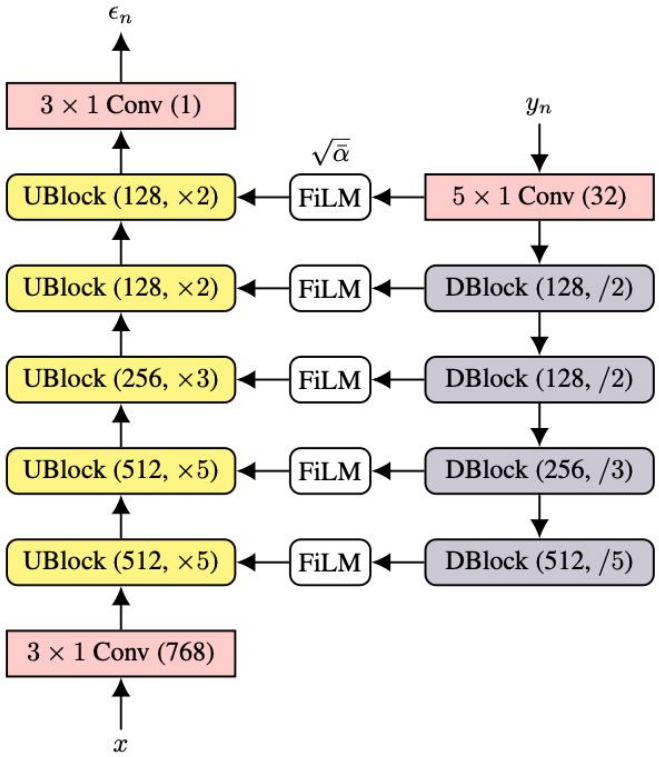
Algorithm 1 Training. WaveGrad directly conditions on the continuous noise level $\sqrt{\bar{\alpha}}$. l is from a predefined noise schedule.

repeat
  $y_0 \sim q(y_0)$
  $s \sim \text{Uniform}(\{1, \dots, S\})$
  $\sqrt{\bar{\alpha}} \sim \text{Uniform}(l_{s-1}, l_s)$
  $\epsilon \sim \mathcal{N}(0, I)$
  Take gradient descent step on
    $\nabla_\theta \left\| \epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}}\, y_0 + \sqrt{1-\bar{\alpha}}\, \epsilon, x, \sqrt{\bar{\alpha}}) \right\|_1$
until converged

Algorithm 2 Sampling. WaveGrad generates samples following a gradient-based sampler similar to Langevin dynamics.

$y_N \sim \mathcal{N}(0, I)$
for $n = N, \dots, 1$ **do**
  $z \sim \mathcal{N}(0, I)$
  $y_{n-1} = \frac{\left(y_n - \frac{1-\alpha_n}{\sqrt{1-\bar{\alpha}_n}}\, \epsilon_\theta(y_n, x, \sqrt{\bar{\alpha}_n})\right)}{\sqrt{\alpha_n}}$
  if $n > 1$, $y_{n-1} = y_{n-1} + \sigma_n z$
end for
return y_0

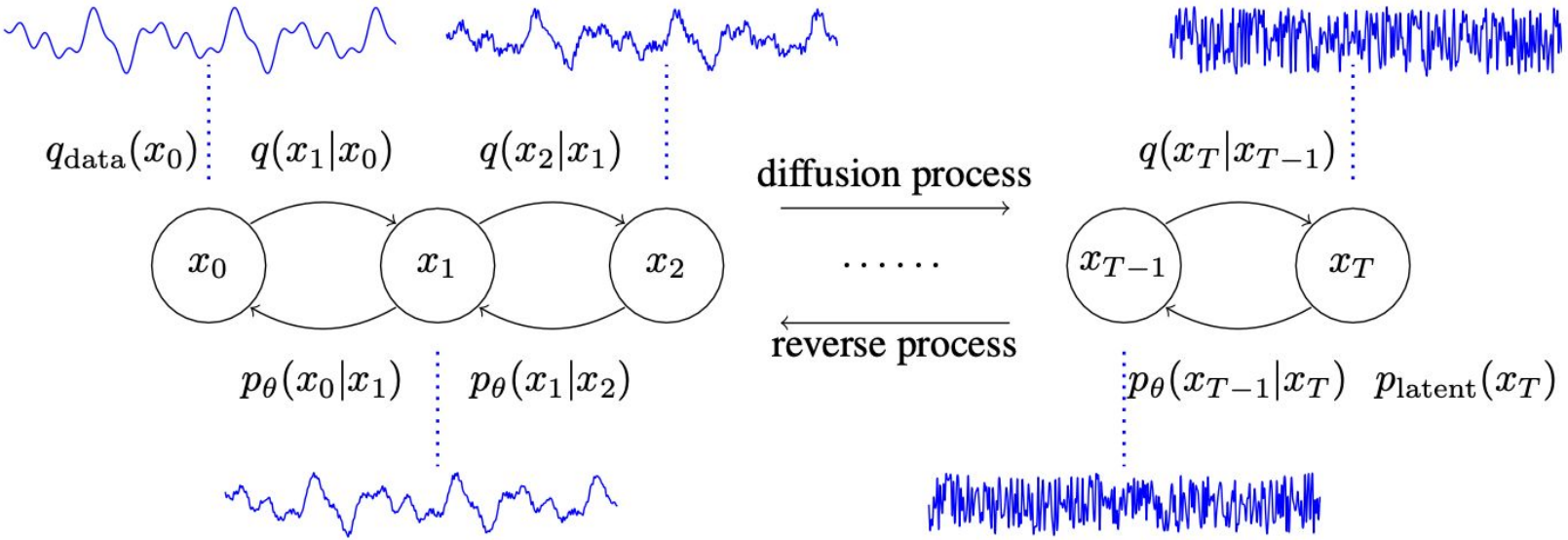
WaveGrad - Vocoder



WaveGrad - Vocoder

Model	**MOS (↑)**
WaveRNN	4.49 ± 0.04
Parallel WaveGAN	3.92 ± 0.05
MelGAN	3.95 ± 0.06
Multi-band MelGAN	4.10 ± 0.05
GAN-TTS	4.34 ± 0.04
WaveGrad	
Base (6 iterations, continuous noise levels)	4.41 ± 0.03
Base (1,000 iterations, discrete indices)	4.47 ± 0.04
Large (1,000 iterations, discrete indices)	4.51 ± 0.04
Ground Truth	4.58 ± 0.05

DiffWave - Vocoder



DiffWave - Vocoder

$$q(x_1, \cdots, x_T|x_0) = \prod_{t=1}^{T} q(x_t|x_{t-1}),$$

$$p_{\text{latent}}(x_T) = \mathcal{N}(0, I), \text{ and } p_\theta(x_0, \cdots, x_{T-1}|x_T) = \prod_{t=1}^{T} p_\theta(x_{t-1}|x_t),$$

$$\min_\theta L_{\text{unweighted}}(\theta) = \mathbb{E}_{x_0,\epsilon,t} \left\|\epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t}x_0 + \sqrt{1-\bar{\alpha}_t}\epsilon,\ t)\right\|_2^2$$

DiffWave - Vocoder

Algorithm 1 Training

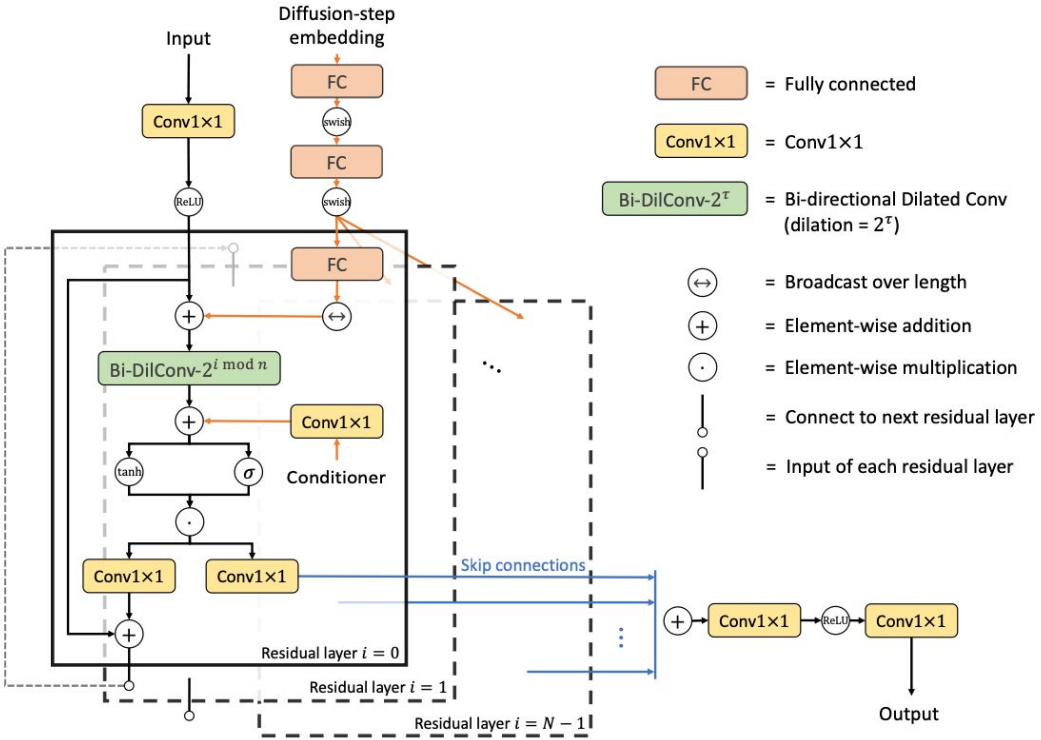
for $i = 1, 2, \cdots, N_{\text{iter}}$ **do**
  Sample $x_0 \sim q_{\text{data}}$, $\epsilon \sim \mathcal{N}(0, I)$, and
    $t \sim \text{Uniform}(\{1, \cdots, T\})$
  Take gradient step on
    $\nabla_\theta \|\epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t} x_0 + \sqrt{1 - \bar{\alpha}_t}\epsilon,\ t)\|_2^2$
    according to Eq. (7)
end for

Algorithm 2 Sampling

Sample $x_T \sim p_{\text{latent}} = \mathcal{N}(0, I)$
for $t = T, T-1, \cdots, 1$ **do**
  Compute $\mu_\theta(x_t, t)$ and $\sigma_\theta(x_t, t)$ using Eq. (5)
  Sample $x_{t-1} \sim p_\theta(x_{t-1}|x_t) =$
    $\mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \sigma_\theta(x_t, t)^2 I)$
end for
return x_0

$$\mu_\theta(x_t, t) = \frac{1}{\sqrt{\alpha_t}} \left(x_t - \frac{\beta_t}{\sqrt{1 - \bar{\alpha}_t}} \epsilon_\theta(x_t, t) \right), \quad \text{and} \quad \sigma_\theta(x_t, t) = \tilde{\beta}_t^{\frac{1}{2}}, \tag{5}$$

DiffWave - Vocoder



DiffWave - Vocoder

Model	T	T_{infer}	**layers**	**res. channels**	**#param(↓)**	**MOS(↑)**
WaveNet	—	—	30	128	4.57M	**4.43** $\pm$ 0.10
ClariNet	—	—	60	64	2.17M	4.27 $\pm$ 0.09
WaveGlow	—	—	96	256	87.88M	4.33 $\pm$ 0.12
WaveFlow	—	—	64	64	5.91M	4.30 $\pm$ 0.11
WaveFlow	—	—	64	128	22.25M	4.40 $\pm$ 0.07
DiffWave $_{\text{BASE}}$	20	20	30	64	2.64M	4.31 $\pm$ 0.09
DiffWave $_{\text{BASE}}$	40	40	30	64	2.64M	4.35 $\pm$ 0.10
DiffWave $_{\text{BASE}}$	50	50	30	64	2.64M	**4.38** $\pm$ 0.08
DiffWave $_{\text{LARGE}}$	200	200	30	128	6.91M	**4.44** $\pm$ 0.07
DiffWave $_{\text{BASE}}$ (Fast)	50	6	30	64	2.64M	*4.37* $\pm$ 0.07
DiffWave $_{\text{LARGE}}$ (Fast)	200	6	30	128	6.91M	*4.42* $\pm$ 0.09
Ground-truth	—	—	—	—	—	4.52 $\pm$ 0.06

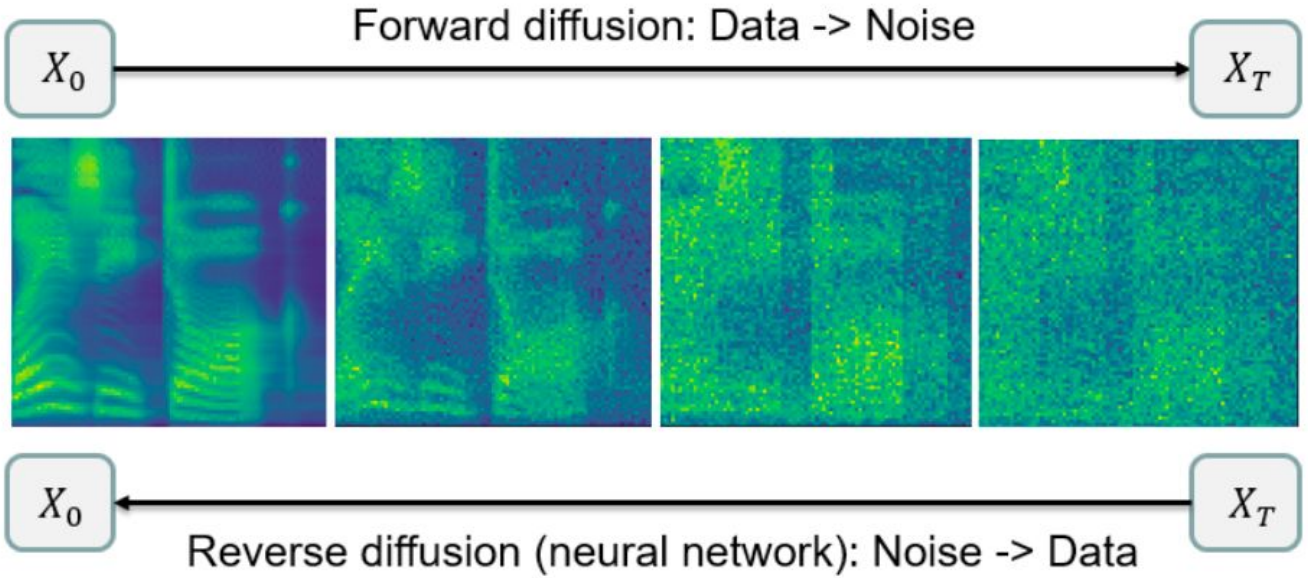
DiffWave - Vocoder

Model	**FID**(↓)	**IS**(↑)	**mIS**(↑)	**AM**(↓)	**NDB**/K(↓)	**MOS**(↑)
WaveNet-128	3.279	2.54	7.6	1.368	0.86	1.34 ± 0.29
WaveNet-256	2.947	2.84	10.0	1.260	0.86	1.43 ± 0.30
WaveGAN	1.349	4.53	36.6	0.796	0.78	2.03 ± 0.33
DiffWave	**1.287**	**5.30**	**59.4**	**0.636**	**0.74**	$\mathbf{3.39} \pm 0.32$
Trainset	0.000	8.48	281.4	0.164	0.00	—
Testset	0.011	8.47	275.2	0.166	0.10	3.72 ± 0.28

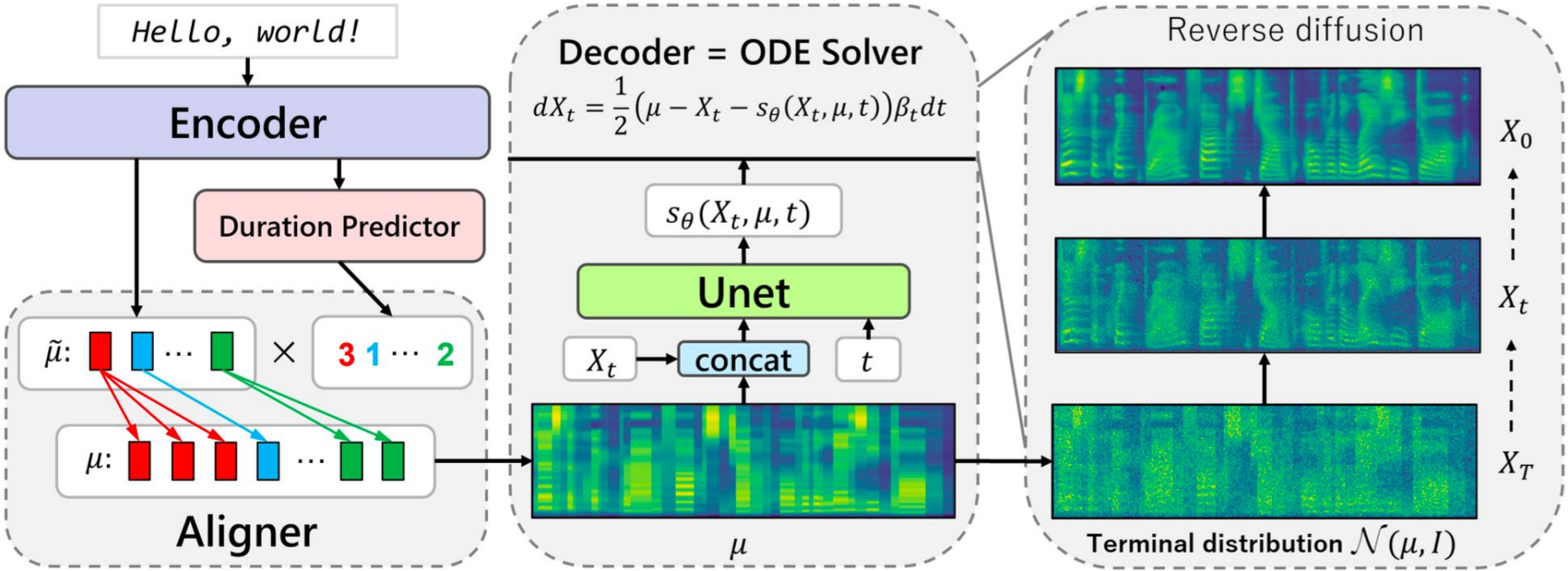
DiffWave - Vocoder

Model	**Accuracy**(↑)	**FID-class**(↓)	**IS**(↑)	**mIS**(↑)	**MOS**(↑)
WaveNet-128	56.20%	7.876±2.469	3.29	15.8	1.46 ± 0.30
WaveNet-256	60.70%	6.954±2.114	3.46	18.9	1.58 ± 0.36
DiffWave	91.20%	1.113±0.569	6.63	117.4	$\mathbf{3.50} \pm 0.31$
DiffWave (deep & thin)	**94.00%**	**0.932**±0.450	**6.92**	**133.8**	3.44 ± 0.36
Trainset	99.06%	0.000±0.000	8.48	281.4	—
Testset	98.76%	0.044±0.016	8.47	275.2	3.72 ± 0.28

GradTTS - Mel synthesis



GradTTS - Mel synthesis



GradTTS - Mel synthesis

Table 2. Model comparison.

Model	Enc params[1]	Dec params	RTF	Log-likelihood	MOS
Grad-TTS-1000	$7.2m$	$7.6m$	3.663	$\mathbf{0.174 \pm 0.001}$	$\mathbf{4.44 \pm 0.05}$
Grad-TTS-100			0.363		4.38 ± 0.06
Grad-TTS-10			0.033		4.38 ± 0.06
Grad-TTS-4			0.012		3.96 ± 0.07
Glow-TTS	$7.2m$	$21.4m$	0.008	0.082	4.11 ± 0.07
FastSpeech	$24.5m$		**0.004**	–	3.68 ± 0.09
Tacotron2	$28.2m$		0.075	–	4.32 ± 0.07
Ground Truth	–		–	–	4.53 ± 0.06

Important papers

- Generative Modeling by Estimating Gradients of the Data Distribution
- Denoising Diffusion Probabilistic Models
- WaveGrad: Estimating Gradients for Waveform Generation
- DiffWave: A Versatile Diffusion Model for Audio Synthesis
- Grad-TTS: A Diffusion Probabilistic Model for Text-to-Speech