ISyE6669 Homework Week 6
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1 Week 6

1. Implement the stochastic inventory control problem in CVXPY with the following data.

   (a) Demand d is forecast at three levels: $d_1 = 10, d_2 = 30, d_3 = 60$, with probability $p_1 = 0.2, p_2 = 0.2, p_3 = 0.6$, respectively.

   (b) Unit cost $c = 10$, retail price $r = 50$, discount price $s = 6$.

   (c) Production capacity $\bar{x} = 80$

   Submit your code and the optimal solution.

2. Reformulate the following nonlinear optimization problem as an equivalent linear program.

$$\begin{aligned} \min \quad & 2|x-y| + 3|2x+3y-3| - \min\{1, -x-y\} & (1)\\ \text{s.t.} \quad & \max\{x+1, x-2y+2, -2x+y-3\} \leq 5, & (2)\\ & x+y \leq 6. & (3) \end{aligned}$$

   Hint: The absolute value function is a convex piecewise linear function, i.e. a max of two linear functions. Similarly, the negative of a minimum of linear functions can be turned into a maximum of linear functions.

3. Given a set of training data $\{\boldsymbol{x}_i, y_i\}_{i=1,\ldots,N}$, where $\boldsymbol{x}_i$ is an n-dimensional feature vector and y_i is a label of value either 0 or 1. Think about each $\boldsymbol{x}_i$ represents a vector of lab test data of a patient i and y_i labels if this person has a certain disease. We want to build a linear classifier, i.e. a linear function $f(\boldsymbol{x}) = \beta_0 + \sum_{j=1}^n \beta_j x_j$, so that for a given feature vector $\boldsymbol{x}$, if $f(\boldsymbol{x}) \geq 0.5$, then $\boldsymbol{x}$ is classified as $y = 1$, otherwise classified as $y = 0$.

   Consider the following robust absolute deviation regression model.

$$\text{(RADR)} \qquad \min_{\beta_0,\ldots,\beta_n} \max_{i=1,2,\ldots,N} \left| y_i - \beta_0 - \sum_{j=1}^n \beta_j x_{ij} \right|,$$

   where x_{ij} is the jth component of vector $\boldsymbol{x}_i$. Notice that the RADR model is a nonlinear optimization problem.

   Answer the following questions.

(a) The objective function $f(\beta_0, \ldots, \beta_n)$ of (RADR) is defined as

$$f(\beta_0, \ldots, \beta_n) = \max_{i=1,2,\ldots,N} \left| y_i - \beta_0 - \sum_{j=1}^{n} \beta_j x_{ij} \right|.$$

Is $f(\beta_0, \ldots, \beta_n)$ a convex function of $\beta_0, \ldots, \beta_n$? Explain why.

Hint: Function f is a maximum of absolute value functions. From the previous exercise, we know each $|y_i - \beta_0 - \sum_{j=1}^{n} \beta_j x_{ij}|$ is a convex piecewise linear function of β. Then, use the operation that preserves convexity to conclude about f.

(b) Write a linear programming reformulation of (RADR).

(c) Code your LP reformulation of (RADR) in CVXPY, using the data file provided.

(d) Write a Python code to plot the data points and the hyperplane obtained from (RADR).