ISYE 6420 Bayesian Statistics

Homework 1

September 12th, 2021

1.

The following problem can be modelled by depticting the various sub-systems in the circuits and constructing two seperate hypotheses for measuring if the node E5 will be functioning.

When doing so the following sub-systems are formed:

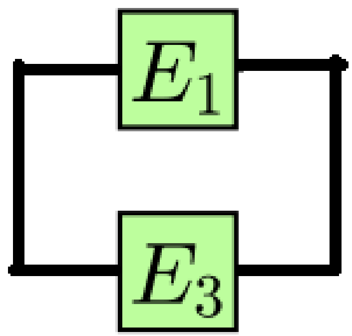

Figure 1: H1 E5 Works

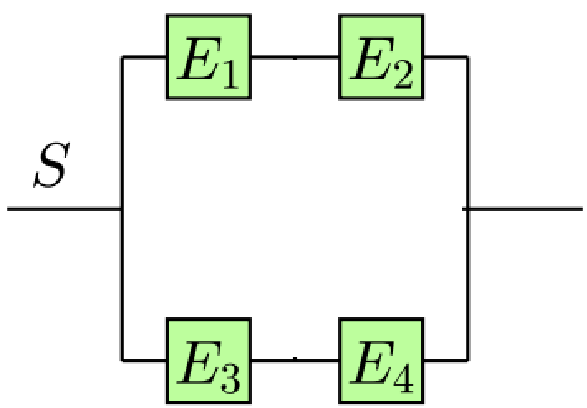

Figure 2: H2 E5 Fails

When plugging in the values of lambda for each subsystem, we derive the following formula:

$$E5 = e^{-3t} + e^{-2t} + e^{-3/2t} - e^{-5/2t} - e^{-4t} + e^{-5/6t} - e^{-23/6t} - e^{-11/6t} + e^{-29/6t}$$

$$E5(1/2) = 0.843$$

b)

Therefore we can use this to determine the probability that component E5 was operational at time t = 1/2, if the system was operational at that time.

$P(E_5|System\ up)\ = P(System\ up\ |\ E5)\ * P(E5)\ /\ P(System\ up)$
= $\left(e^{-\lambda 1} + e^{-\lambda 3t} + e^{-(\lambda 1+\lambda 3)t}\right).\ - e^{-5/2t} / 0.843$
= 0.65687

2.

$Lets\ write\ down\ the\ known\ probabilities:$

$P(NC|A) =\ 0$
$P(NC|B) =\ \ 0.2$
$P(B) = P(A) =\ 0.5$

$P(C|A)\ \ = 1$
$P(C|B)\ =\ 0.8$

$From\ this\ we\ can\ determine\ that\ the\ probability\ of\ conforming\ is\ the\ following:$

$P(Conf) = P(Conf|A) * P(A)\ +\ P(Conf|B) * P(B)$
$=\ 1(0.5)\ +\ (0.8)(05)$
$= 0.9$

$P(B|Conf) =\ P(C|B)* P(B)\ /\ P(Conf)$
$=\ (0.8)\ (0.5)\ /\ 0.9$
$=\ 4/9$

3.

$P\ (\ Actual\ 1)\ =\ 0.01$
$P\ (\ Actual\ 0)\ =\ 0.99$

$P\ (\ Classifer\ 1\ ) = P\ (\ Classifer\ 1\ |\ Actual\ 1) * P(Actual\ 1)\ +\ P\ (\ Classifer\ 1\ |Actual\ 0\ ) * P(Actual\ 0)$

$P\ (\ Actual\ 1\ |\ Classifier\ 1)\ =\ P\ (\ Classifer\ 1\ |\ Actual\ 1)*P(Actual\ 1)\ /\ P\ (\ Classifer\ 1\ )$

$$= \frac{(52/66) * (1/100)}{(52/66) * (1/100) + (18/55) * (99/100)}$$

= 2/83