ISYE 6669 Deterministic Optimization

Homework 8

October 10th, 2020

1.

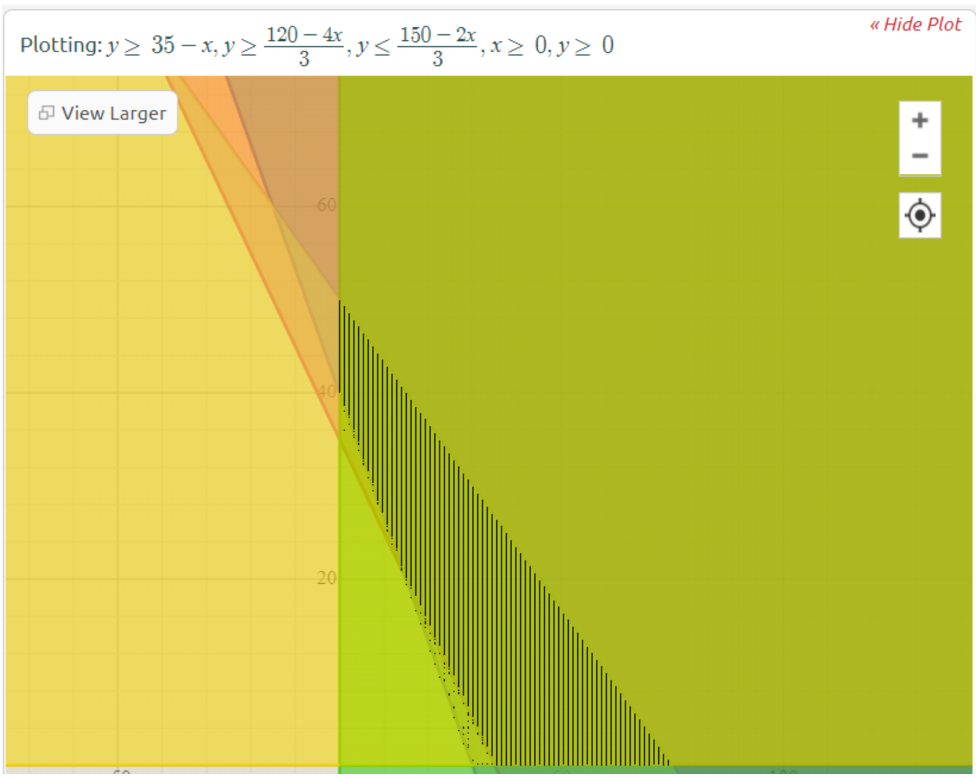

Figure 1: Feasible Region for Solution

2. Converting to standard form yields the following result:

$$min \quad -2x_1 - 3x_2 \tag{1}$$
$$x_1 + x_2 + x_3 = 35 \tag{2}$$
$$4x_1 + 3x_2 + x_4 = 120 \tag{3}$$
$$2x_1 + 3x_2 + x_5 = 150 \tag{4}$$
$$x_1 \geqslant 0, x_2 \geqslant 0, x_3 \geqslant 0, x_4 \geqslant 0, x_5 \geqslant 0 \tag{5}$$

$$c = \begin{bmatrix} -2 \\ -3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 4 & 3 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 35 \\ 120 \\ 150 \end{bmatrix}$$

3.

<u>Iteration 1</u>

a) Choose a starting BFS:

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

<u>Basic solution</u>

$$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} = B^{-1}b = \begin{bmatrix} 35 \\ 120 \\ 150 \end{bmatrix}, \quad X_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and the cost coefficients associated with basic and nonbasic variables:

$$C_B = \begin{bmatrix} C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_N = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

b) Compute reduced costs for non basic vars:

$\bar{c}_1 = -2$

$\bar{c}_2 = -3$

We will use x_2 to enter the basis instead of x_1 in this case.

c)

Compute feasible region direction $d = \begin{bmatrix} d_B \\ d_N \end{bmatrix}$:

$$d_N = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$d_B = \begin{bmatrix} -1 \\ -3 \\ -3 \end{bmatrix}$$

Since some of the components of d_B are negative, we don't have an unbounded optimal soltuion, and we need to decide how are to go along this directions while stil remaining feasible.

d)

$$x + \theta d = \begin{bmatrix} 35 + \theta.(-1) \\ 120 + \theta.(-3) \\ 150 + \theta.(-3) \\ 0 \\ \theta \end{bmatrix} = \begin{bmatrix} 35 - \theta \\ 120 - 3\theta \\ 150 - 3\theta \\ 0 \\ \theta \end{bmatrix}$$

$\theta^*= min\{\frac{35}{1}, \frac{120}{3}, \frac{150}{3}\}$

= 35

So $x_{B(1)} = x_3$ exits the basis.

e) New basis:

$$x_{\overline{B}} = \begin{bmatrix} 35 \\ 15 \\ 45 \end{bmatrix} \quad x_{\overline{N}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

<u>Iteration 2:</u>

a)

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

Cost coefficients for basic and non-basic variables:

$c^B = \begin{bmatrix} c^2 \\ c^4 \\ c^5 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$

$c^N = \begin{bmatrix} c^1 \\ c^3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

b)

$\bar{c}_1 = -2 - \begin{bmatrix} -3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

$= -2 - \begin{bmatrix} -3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

= -5

$\bar{c}_2 = 0 - \begin{bmatrix} -3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

= 3

c)

Feasible direction:

$-B^{-1}A_1 = -\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix}$

Some components of d_B are negative, the optimal solution isn't unbounded.

d)

$x + \theta d = \begin{bmatrix} 35 \\ 15 \\ 45 \end{bmatrix} + \theta \begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} 35 - \theta \\ 15 - 3\theta \\ 45 + 4\theta \end{bmatrix}$

$$\theta^* = min\{\frac{35}{1}, \frac{15}{1}, -\frac{45}{4}\}$$
$$= -11.25$$

e) New basis:

$$x_{\overline{B}} = \begin{bmatrix} 35 \\ 15 \\ 45 \end{bmatrix} \quad x_{\overline{N}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 46.25 \\ 26.25 \\ 0 \end{bmatrix}$$

<u>Iteration 3</u>

a)

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

Cost coefficients for basic and non-basic variables:

$$c^B = \begin{bmatrix} c^2 \\ c^4 \\ c^5 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$$

$$c^N = \begin{bmatrix} c^1 \\ c^3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

b)

$$\overline{c}_1 = -2 - \begin{bmatrix} -3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$= -2 - \begin{bmatrix} -3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

= -5

$$\bar{c}_2 = 0 - \begin{bmatrix} -3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

= 3

c)
Feasible direction:

$$-B^{-1}A_1 = -\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix}$$

Some components of d_B are negative, the optimal solution isn't unbounded.

d)

$$x + \theta d = \begin{bmatrix} 35 \\ 15 \\ 45 \end{bmatrix} + \theta \begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 35 - \theta \\ 15 - 3\theta \\ 45 + 4\theta \end{bmatrix}$$

$$\theta^* = min\{\frac{35}{1}, \frac{15}{1}, -\frac{45}{4}\}$$
$$= -11.25$$

e) New basis:

$$x_{\overline{B}} = \begin{bmatrix} 35 \\ 15 \\ 45 \end{bmatrix} \quad x_{\overline{N}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 46.25 \\ 26.25 \\ 0 \end{bmatrix}$$