☐ ISYE 6669 Deterministic Optimization

Homework 4

February 14, 2021

1.1

$f(x) = e^x - x$

$f'(x) = e^x - 1$

$f''(x) = e^x$

Step 1)

$x^0 = -1$

$f\left(x^0\right) = 1.367$

$x^1 = x^0 - f'(x)/f''(x)$

$x^1 = -1 + \frac{0.632}{0.367}$

$x^1 = .722$

$f\left(x^1\right) = 1.33$

Step 2)

$x^2 = x^1 - f'(x)/f''(x)$

$x^2 = .722 - \frac{1.0585}{2.0585}$

$x^2 = 0.722 - 0.514$

$x^2 = 0.208$

$f\left(x^2\right) = 1.023$

Step 3)

$x^3 = x^2 - f'(x)/f''(x)$

$x^3 = 0.208 - \frac{0.2312}{1.2312}$

$x^3 = 0.208 - 0.1878$

$x^3 = 0.020$

$f\left(x^3\right) = 1.0002$

Step 4)

$x^4 = x^3 - f'(x)/f''(x)$

$$x^4 = 0.020 - \frac{0.0202}{1.0204}$$
$$x^4 = 0.020 - 0.1878$$
$$x^4 = 0.020 - 0.0197$$
$$x^4 = 0.0002$$
$$f\left(x^4\right) = 1.00000002$$

Step 5)

$$x^5 = x^4 - f'(x)/f''(x)$$

$$x^5 = 0.0002 - \frac{0.000203}{1.000203}$$

$$x^5 = 0.0002 - \frac{0.000203}{1.000203}$$
$$x^5 = 0.0002 - 0.0002$$
$$x^5 \approx 0$$
$$f\left(x^5\right) \approx 1$$

Step 6)

$$x^6 = x^5 - f'(x)/f''(x)$$

$$x^6 = 0 - \frac{0}{1}$$

Therefore, we can see at the 6th step, $|f'(x)| < 10^{-5}$

1.2

The solution in terms of $y = k(x-a)+b$, when substituting k , a, b and express them in x^k, $f'(x)$, $f''(x)$, we get the following:

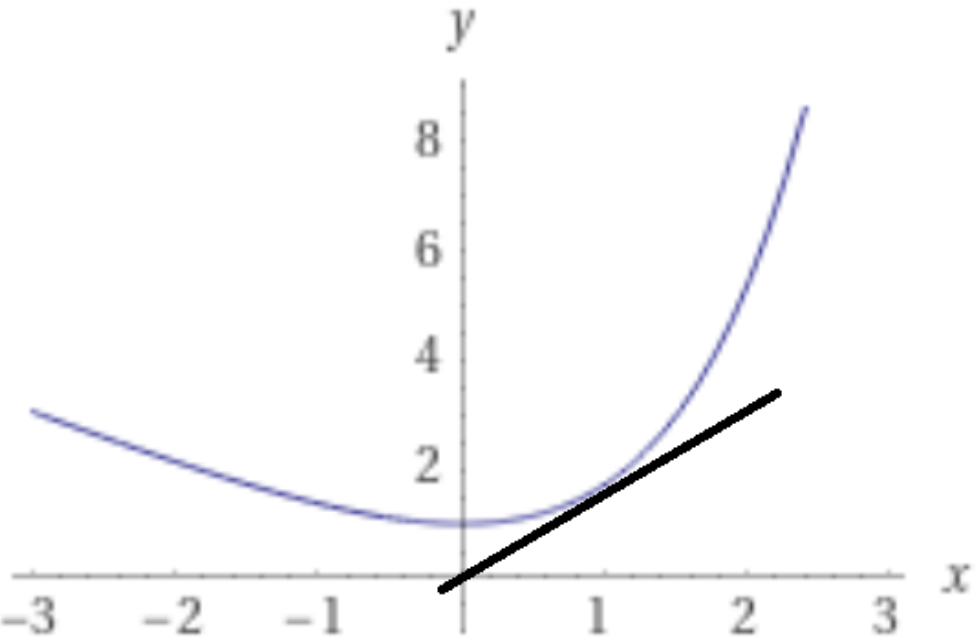
Step 1)

$k(x-a)+b$ = 0

$$-1(x+0.632)+0.368 = 0$$
$$-x-0.632+0.368 = 0$$
$$x = -0.264$$
$$y = f'(x)$$
$$y = f'(-0.264)$$

$y = -0.232026$

You can take this equation to start finding arbitrary points across the x-axis, and you get the following tangent line.



Step 2)

$-0.264(x + 0.232026) + 0.7679735 = 0$

$0.706719 - 0.264\,x = 0$

$x^2 = -\dfrac{0.706719}{0.264}$

$x^2 = -2.676$

$y = f'(x)$

$y = f'(-2.676)$

$y = -0.9311620$

2.

Please see attachment hw4_2.html for jupyter notebook output.