ISyE6669 Homework Week 5

Fall 2020

1. Consider the following linear program. Answer the following questions.

$$
\begin{aligned}
\max \quad & 4x_1 + 3x_2 \\
\text{s.t.} \quad & 2x_1 + 2x_2 \leq 5 \\
& 4x_1 + 2x_2 \leq 9 \\
& 2x_1 + 4x_2 \leq 9 \\
& x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

   (a) Draw the feasible region of this LP in (x_1, x_2).

   (b) Find the optimal solution using the picture of the feasible region. Hint: The optimal solution should be one of the corner points of the feasible region.

   (c) Write a CVXPY code to find the optimal solution.

2. Figure 1 below illustrates the water network of Newvillage. The lines are water piplelines numbered from 1 through 13. The arrows on the lines are possible direction(s) of flow of water in these pipelines. The circles are water sources numbered A, B, C. The rectangles are houses D, E, F, G, H. The maximum possible capacity of the water sources are (the sources can operate at less than the maximum capacity): A: 100 Units, B: 100 Units, C: 80 Units Demands of water in the houses are: D: 50 Units, E: 60 Units, F: 40 Units, G: 30 Units, H: 70 Units Since the houses are at different elevation and the pipes are of different diameter, the cost of transporting water is different in the different pipes. These costs per unit of water are: Pipe 1: \$2, Pipe 2: \$3, Pipe 3: \$4, Pipe 4: \$2, Pipe 5: \$3, Pipe 6: \$2, Pipe 7: \$4, Pipe 8: \$1, Pipe 9: \$2, Pipe 10: \$4, Pipe 11: \$5, Pipe 12: \$1, Pipe 13: \$2. Formulate an LP to minimize the total cost of transporting water so as to meet the water demands of each house.
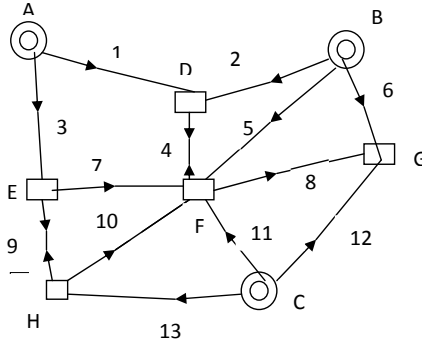
Figure 1: Water Network for Question 2

3. Consider the following electric power network shown in Figure 2. This network is taken from a real-world electric power system. Electricity generators are located at nodes 1, 3, and 5 and producing p_1, p_2, p_3 amounts of electricity, respectively. Electricity loads are located at nodes 2, 4, and 6 and are consuming d_1, d_2, d_3 amounts of electricity, respectively.

The demand is fixed and given as $d_1 = 120$, $d_2 = 95$, $d_3 = 105$.

Each generator i's production must be within an upper and a lower bound as $p_i^{min} \le p_i \le p_i^{max}$. The bounds are given as $p_1^{min} = 20$, $p_1^{max} = 270$, $p_2^{min} = 20$, $p_2^{max} = 250$, $p_3^{min} = 10$, $p_3^{max} = 300$.

The flow limits over lines are given as $f_{12}^{max} = 100, f_{23}^{max} = 120, f_{34}^{max} = 50, f_{45}^{max} = 90, f_{56}^{max} = 60, f_{61}^{max} = 50$.

The line parameters are given as $B_{12} = 11.6$, $B_{23} = 5.9$, $B_{34} = 13.7$, $B_{45} = 9.8$, $B_{56} = 5.6$, $B_{61} = 10.5$. The unit generation costs are given as $c_1 = 10, c_2 = 5, c_3 = 8$.

(a) Formulate the power system scheduling problem using the model discussed in Lecture 2.

(b) Implement and solve the model using CVXPY. Write down the optimal solution.

(c) Find the electricity prices for demand at nodes 2, 4, 6. To do this, use the command constraints[0].dual_value to find the dual variable of constraints[0]. Hint: Recall the electricity price at node i is the dual variable for the flow conservation constraint at node i.
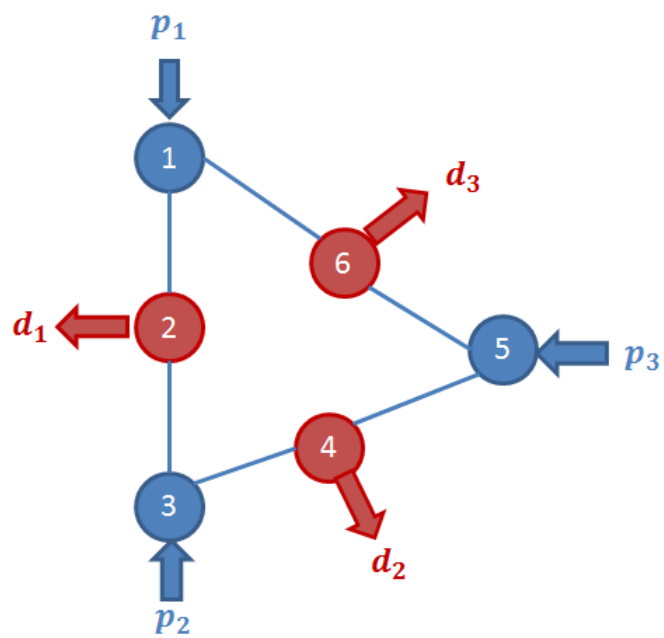
Figure 2: Electric network for Question 3.