ISYE 6669 Deterministic Optimization

Homework 2

September 3rd, 2020

1.

a)

If you look at the following function:

We know this know this functions has an optimal solution at, and it's compact as it's both bounded and closed.

$f(x) \ = \ \begin{Bmatrix} 1, & 0 < x \leq 1 \\ 0, & x = 0 \end{Bmatrix}$ and X = [0]

An example that would not yield an optimal solution is the following:

$f(x) \ = \ \begin{Bmatrix} 1, & x = 0 \\ x, & 0 < x \leq 1 \end{Bmatrix}$

In this case x is discontinous and is in a compact set.

b)

When we look at the following example:

X = R (Real numbers)

$f(x) \ = \ 0$

This is an example yielding no optimal solution, where the set is not bounded.

X = R

$f(x) \ = \ x$

c)

An example for a convex function with a compact set is the following:

$X \subseteq [0, 1] \ s.t. \ f(x) \ = x$

An example that would not yield an optimal solution is the following:

$X \subseteq [0, 1] \ s.t. \ f(x) \ = \ \begin{Bmatrix} x, & 0 < x \leq 1 \\ 1, & x = 0 \end{Bmatrix}$

d)

X = R

$f(x) \ = \ -x^2$

This is an example of an unbounded concave function because this will stretch into negative infinity and no box or hypersphere and can be drawn around this function to contain it

The origin in this case the origin would be our optimal solution if this is framed as minimization problem.

When taking a look at question b, we see that $f(x) \ = \ x$ where X = R, this is an unbounded problem for a convex function that yielded no optimal solution. Therefore, since concave means that -$f(x)$ is convex, if we negative the function, then this would yield the following:
X = R
$f(x) \ = \ -x$

e)

An example for a convex function with an open set is the following:

$X \ \subseteq [0, 1] \, s.t. \ f(x) \ = x$

An example that would not yield an optimal solution is the following:

$X \ \subseteq [0, 1] \, s.t. \ f(x) \ = \ \begin{Bmatrix} x, & 0 < x < 1 \\ 1, & x = 0 \end{Bmatrix}$

2
a)
True
Take the example where f(x) = $-x^2$ and g(x) = x

Then when applying our constraint, the optimal solution is still at our origin. When the constraint is changed to v=1, the optimal solution is still the origin, and is therefore is still less than or equal to v.

b)
False
When looking at the example where X = [0, $+\infty$) and f(x) = $\sqrt{x}$. The only local minimum is x = 0 which is also a global minimum. However, f is not a convex function because if you connect any two point on the graph, they will fall below the objective function value.

c)
True

d)
True. Notice for all x ∈ X we have $[f(x)]^2$ ≥ 0. Since the optimal solution $[f(x)]^2$ = 0, by the definition of optimal solution, we have an optimal solution in this case.

e)
True