ISYE 6669 Deterministic Optimization

Homework 7

October 8th, 2020

1.

a)

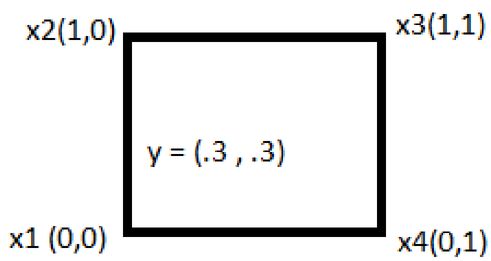


Figure 1: Square with 4 Extreme Points where k ⩾ 3

When taking a look at our example above, we can see that the following point y adds up to (0.3,0.3) from the following formula:

$$\boldsymbol{y} = \frac{1}{2(k-1)}\boldsymbol{x}^1 + \frac{1}{2(k-1)}\boldsymbol{x}^2 + \cdots + \frac{1}{2(k-1)}\boldsymbol{x}^{k-1} + \frac{1}{2}\boldsymbol{x}^k.$$

And therefore,

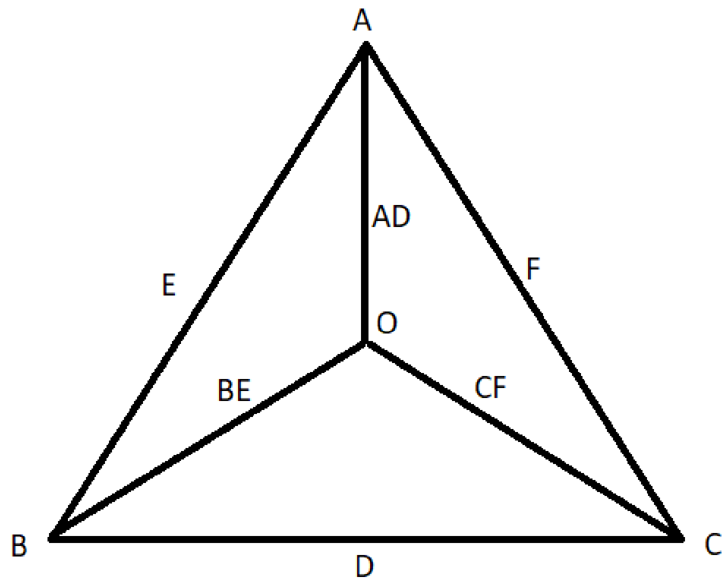
$$= \frac{1}{6}(0,0) \ + \frac{1}{6}(1,0) \ + \ \frac{1}{6}(1,1) + \ \frac{1}{6}(0,1)$$

$$= \frac{1}{6}(0,0) \ + \frac{1}{6}(1,0) \ + \ \frac{1}{6}(1,1) + \ \frac{1}{6}(0,1)$$

$$= \left(\frac{1}{3}, \frac{1}{3}\right)$$

From the result, and the due to the property that we're multiplying each result by a fraction of 1/2(k-1), we know that any coordinate on the new point y will never be > any of the non zero coordinates for our extreme points.

b)



Provided is the convex combination of 0 in graphical representation

2a)
Provided is the feasible region for the linear program in blue:

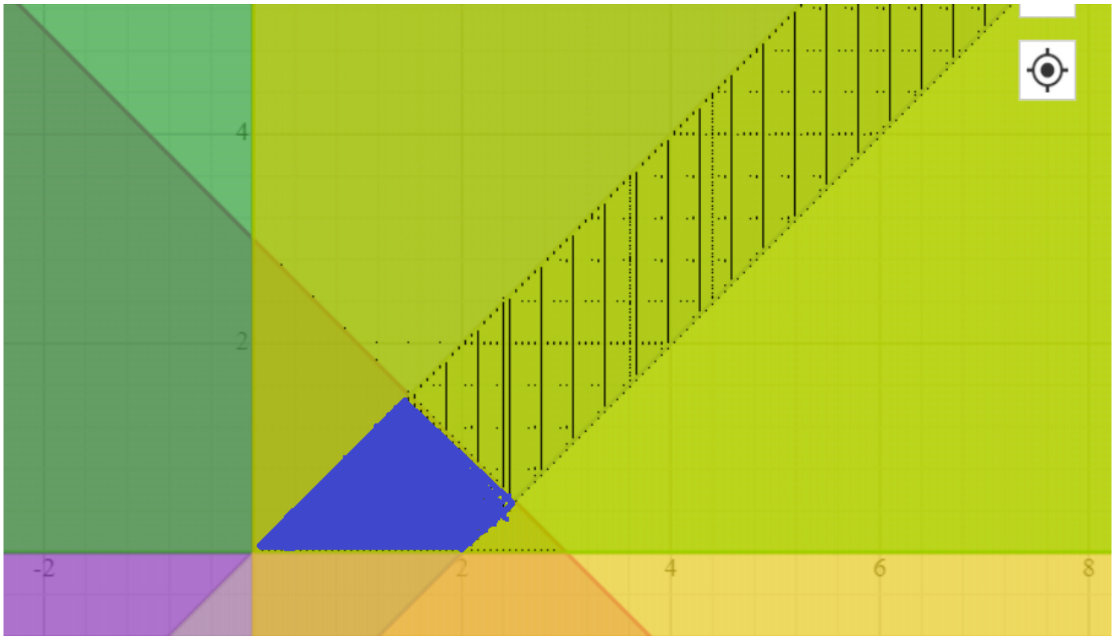


Figure 2: Feasible Region for LP

b)

$$\min \ -2x_1 \tag{1}$$
$$x_1 + x_2 + x_3 = 3 \tag{2}$$
$$-x_1 + x_2 + x_4 = 2 \tag{3}$$
$$x_1 - x_2 + x_5 = 0 \tag{4}$$
$$x_1 \geqslant 0, x_2 \geqslant 0, x_3 \geqslant 0, x_4 \geqslant 0, x_5 \geqslant 0 \tag{5}$$

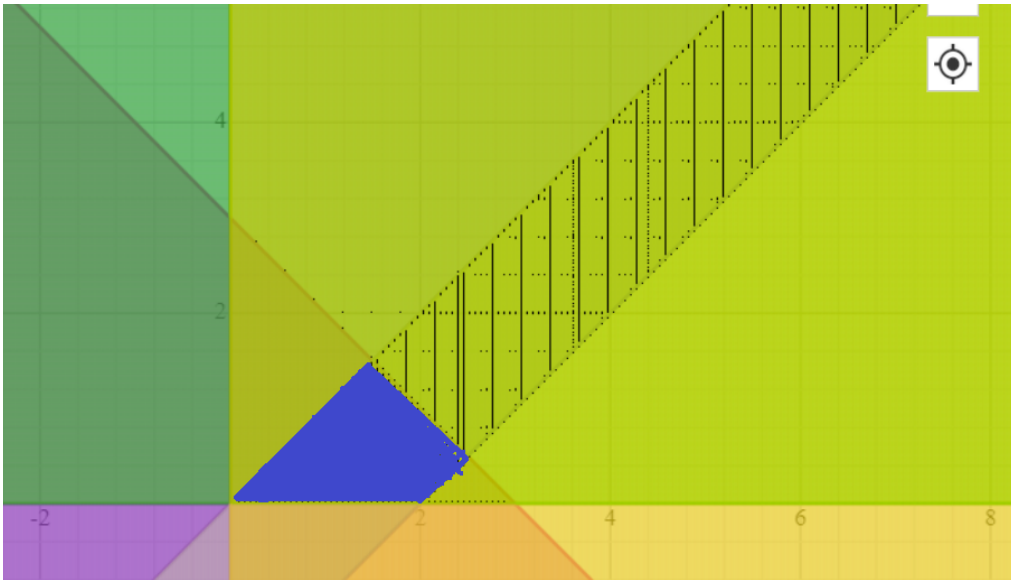
c = $\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, A = $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{bmatrix}$, b = $\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

c)

See python notebook.

d)

Not enough time to graph the output


4
2
-2
2
4
6
8