ISYE 8803 HDDA

Homework 1

May 31st, 2021

1.

$When\ taking\ the\ 2^{nd}\ derivative\ \frac{d^2}{dx}y(x)\ =\ 0\ for\ the\ outside\ interval for\ x\ outside\ (\xi_1, \xi_k)$
$we\ get\ the\ following\ result:$

$Taking\ the\ first\ derivative:$

$$\beta_1 +\ \beta_2 x\ +\ \sum_{k=1}^{K} \theta_k (x - \xi_k)_+^2 = 0$$

$$2\sum_{k=1}^{K} \theta_K (x - \xi_K)\ = 0$$

$Taking\ the\ second\ derivative:$

$$2\sum_{k=1}^{K} \theta_K (x - \xi_K) = 0$$

$$\sum_{k=1}^{K} \theta_K = 0$$

$We\ can\ rearrange\ this\ so\ that:$

$$\theta_K =\ -\sum_{k=1}^{K} \theta_k$$

$We\ know\ that\ \sum_{k=1}^{K} \theta_k (x - \xi_k)_+^2\ =\ \sum_{k=1}^{K-1} \theta_k (x - \xi_k)_+^2 + \theta_K (x - \xi_K)_+^2$

$Then\ when\ I\ plug\ this\ back\ into\ the\ original\ equation\ I\ get\ the\ following:$

$$\beta_1 +\ \beta_2 x\ +\ \sum_{k=1}^{K-1} \theta_k (x - \xi_k)_+^2 - \theta_k (x - \xi_K)_+^2\ = 0$$

$Then\ we\ can\ factor\ out\ the\ summation\ to\ get\ the\ following:$

$$\beta_1 +\ \beta_2 x\ +\ \sum_{k=1}^{K-1} \theta_k \left[(x - \xi_k)_+^2 - (x - \xi_K)_+^2 \right]$$

Therefore, the set of basis functions are:

$$\left\{ 1,\ x,\ \ \left\{(x-\xi_k)_+^2-(x-\xi_K)_+^2\right\}\forall k=1,\ldots,K-1\right\}$$