ISYE 6669 Deterministic Optimization

Homework 7
March 5th, 2021

1.

a)

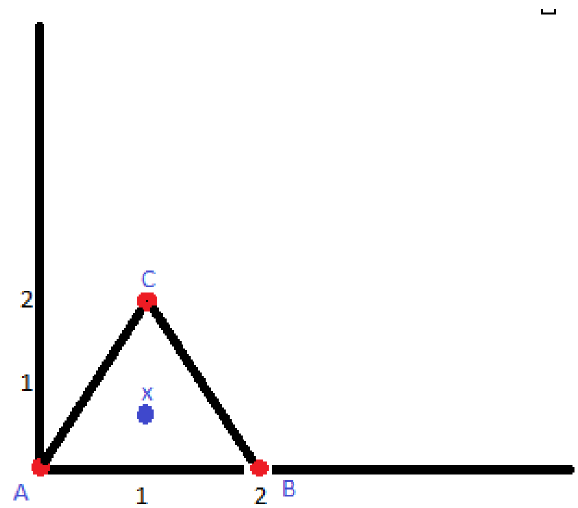

Figure 1: Triangle P with Extreme Points a, b & c

$$\lambda_1, \lambda_2, \lambda_3 = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}^T$$

$$x \; = \; \frac{1}{3}\begin{bmatrix} 0 \\ 0 \end{bmatrix}^T + \frac{1}{3}\begin{bmatrix} 2 \\ 0 \end{bmatrix}^T + \frac{1}{3}\begin{bmatrix} 1 \\ 2 \end{bmatrix}^T$$

$$x \; = \; \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 2/3 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}^T$$

$Therefore,$

$$x \; = \; \begin{bmatrix} 1 \\ 2/3 \end{bmatrix}^T$$

b)

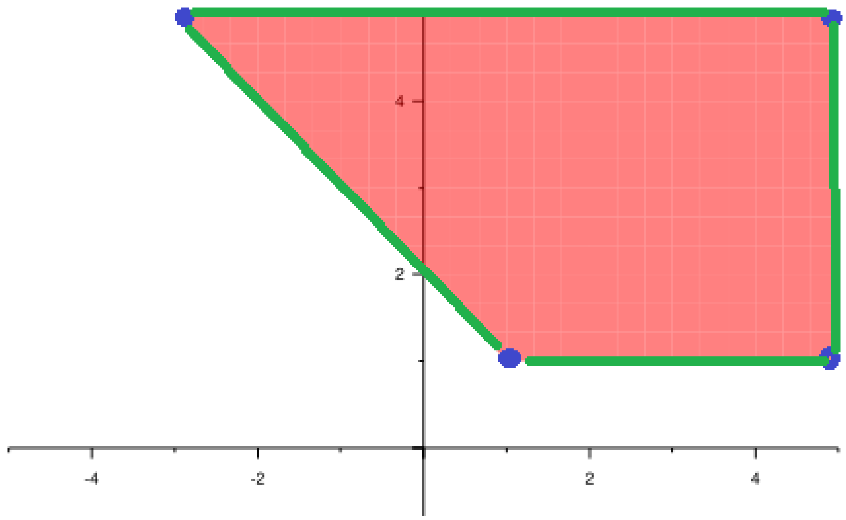

Figure 2: Polyhedron 1b

You can see the polyhedron plotted out with the extreme points shown in blue and the extreme rays shown in green.

c)

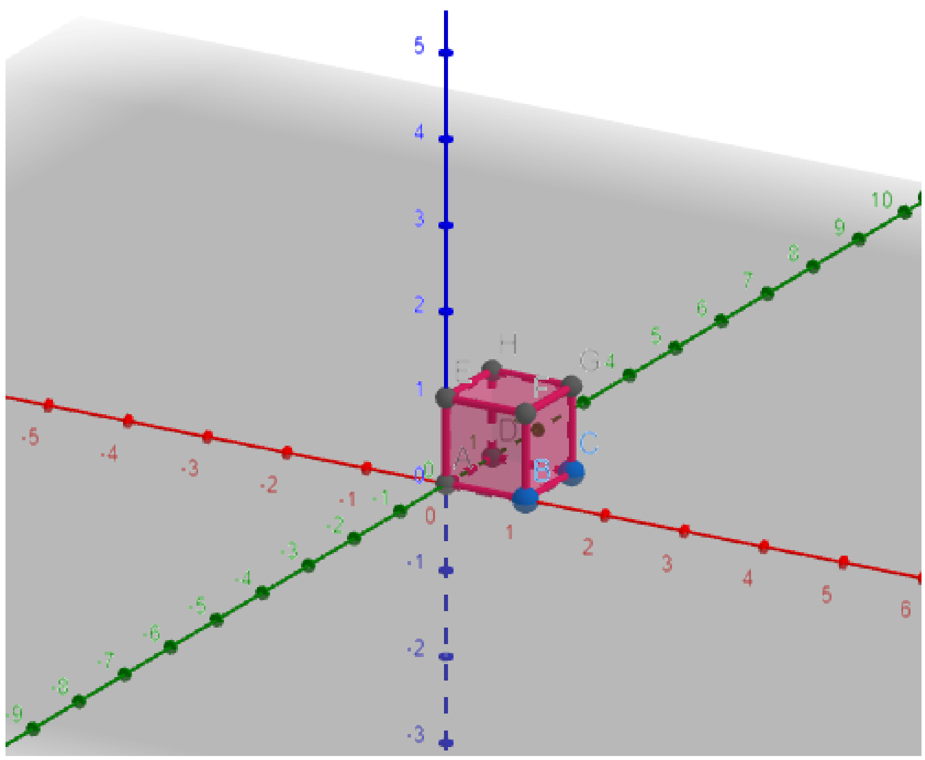

Figure 3: Unit Cube

We can see that there are 8 extreme points on this graph. When cutting this cube with a plane, we can see that when cutting this cube on an angle at any of the extreme points, we can see that the maximum number of extreme points maintained would be 6.

2a)

$$\begin{gathered} min\ \ x\ -y \\ x+y+w=1 \\ 2x+y+z=1 \\ x\leq 0, y\geq 0, w\geq 0,\ z\geq 0 \end{gathered}$$

c = $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, A = $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$, b = $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b)

$$\begin{gathered} min\ \ |x+y|\ +|x-2y| \\ x\leq 10 \\ y\leq 10 \\ x+y\leq 10 \\ x\geq 0, y\geq 0 \end{gathered}$$

3a)

Graph

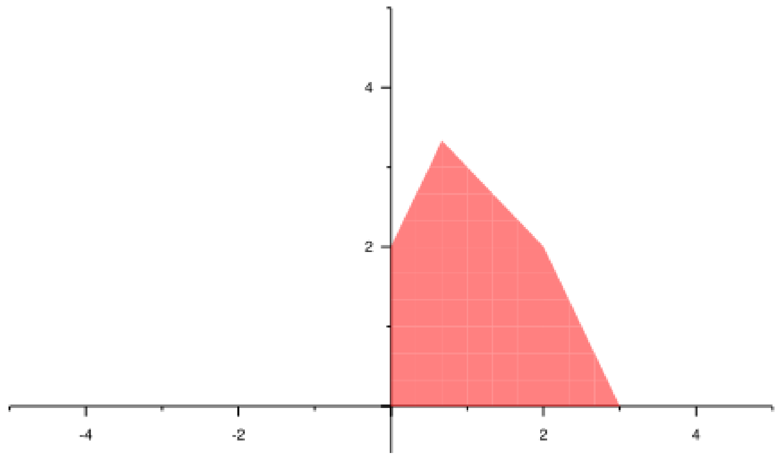

Figure 4: Polyhedron for 3a Feasible Region in Red

b)

$$min \quad -x_1 - 2x_2 \tag{1}$$
$$x_1 \ + x_2 + x_3 = 4 \tag{2}$$
$$-2x_1 \ + x_2 + x_4 = 2 \tag{3}$$
$$2x_1 \ + x_2 + x_5 = 6 \tag{4}$$
$$x_1 \geqslant 0, x_2 \geqslant 0, x_3 \geqslant 0, x_4 \geqslant 0, x_5 \geqslant 0 \tag{5}$$

$$c = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ -2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$$

c)

b1:

$$B_1 = [A_3, A_4, A_5] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}, x_N = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ c_B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \ c_N = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

b2:

$$B_2 = [A_1, A_3, A_5] = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 8 \end{bmatrix}, x_N = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ c_B = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

b3:

$$B_3 = [A_1, A_4, A_5] = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_1 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \\ -2 \end{bmatrix}, x_N = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ c_B = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

b4:

$$B_4 = [A_1, A_3, A_4] = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix}, x_N = \begin{bmatrix} x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ c_B = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

b5:

$$B_5 = [A_2, A_3, A_4] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ -4 \end{bmatrix}, x_N = \begin{bmatrix} x_1 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ c_B = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

b6:

$$B_6 = [A_2, A_4, A_5] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_2 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}, x_N = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ c_B = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

b7:

$$B_7 = [A_2, A_3, A_5] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_2 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}, x_N = \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ c_B = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

b8:

$$B_8 = [A_1, A_2, A_5] = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0.67 \\ 3.33 \\ 1.33 \end{bmatrix}, x_N = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ c_B = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

b9:

$$B_9 = [A_1, A_2, A_4] = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 1 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}, x_N = \begin{bmatrix} x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ c_B = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

b10:

$$B_{10} = [A_1, A_2, A_3] = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 1 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}, x_N = \begin{bmatrix} x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ c_B = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix},\ c_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

d)

Basic Feasible Solutions: b1, b4, b7, b8, b9

Non-basic feasible solutions: b2, b3, b5, b6, b10

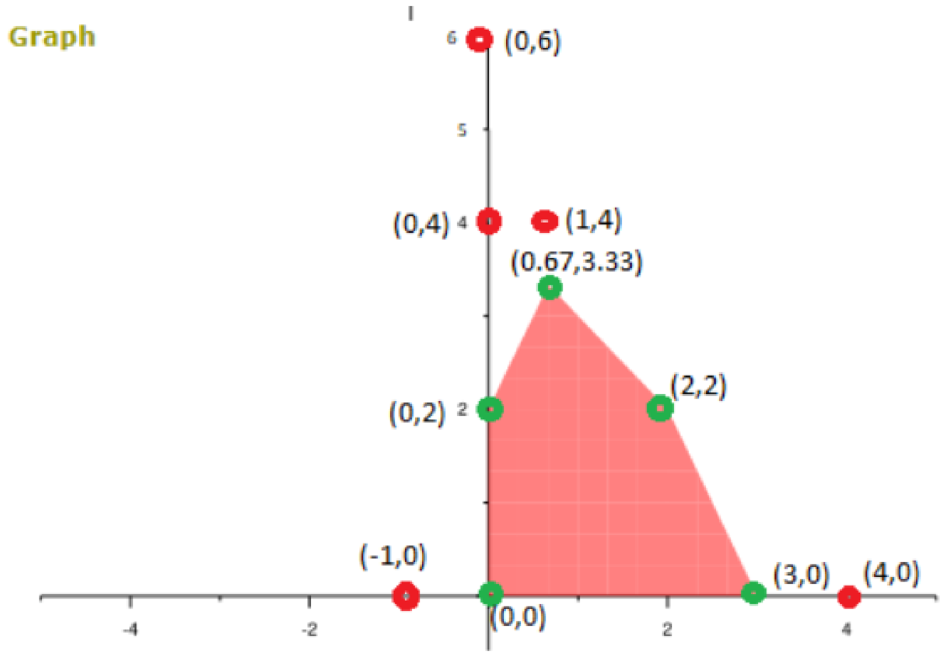

Figure 5: Basic Feasible Solutions