ISYE 6669 Deterministic Optimization

Homework 14

April 25th, 2021

1.

a)

$$If\ x_1 = 1,\ then \sum_{t=2}^{T} x_t \leq \frac{T-1}{2}$$

Which translates to:

$$\sum_{t=2}^{T} x_t \leq M(1-x_1) + \left(\frac{T-1}{2}\right)x_1$$

b)

When you consider the absolute value in this case, we know our condition changes to $2x_1 - x_2 - x_3 \geq 2\ OR\ -(2x_1 - x_2 - x) \geq 2$. *When letting* $z\ \Subset\ \{0,1\}$, *which* $=0\ if$ $2x_1 - x_2 - x_3 \geq\ 0$ *and* 1 *otherwise. We can deduce the following:*

$$2x_1 - x_2 - x_3 \geq 2y - M(1-y)$$
$$2x_1 - x_2 - x_3 \leq\ -2(1-y)\ +\ My$$

$M\ \geq\ 6$ *based on the max of* $|2x_1 - x_2 - x_3|\ =\ (3,0,0)$ *and* $y\ \Subset \{0,1\}$

c) When introducing a binary variable $y\ \Subset\ \{0,1\}$. *If* $x_1 + x_2 \leq 10$, *then* $y = 1$, *and* $y\ = 0$ *when* $2x_1 - x_2 \geq 5$. *Therefore we can deduce the constraints from taking the union of all the different permutations of the binary values:*

$y =\ 1$, *then* $x_1 + x_2 \leq 10y\ +\ M(1-y)$ *and* $2x_1 - x_2 \leq 4y\ +\ M(1-y)$

otherwise, $x_1 + x_2 \geq 11(1-y) - My$ *and* $2x_1 - x_2 \geq 5y\ (1-y) - My$

such that $x_1, x_2\ \Subset\ [0,10]$

2.

d_{ij}: *Dose disposed for cell j per unit intensity at position i*

x_i: *intensity of the radiation at position i* $\forall$ 1,..., 100 *positions*

y_j: *sum of dosages for cell j for all beam positions i*

z: *binary to indicate if cell j gets at least* 75 *units* $\{0,1\}$

•minimize the total dose deposited on all normal cells
•the dose deposited on each cancerous cell should be at least 70 units
•at least 90% of the cancerous cells should each get a dose deposition of at least 75 units
•the dose deposited on each normal cell should not exceed 25 units

$$min \sum_{j=1001}^{5000} \sum_{i=1}^{100} x_i d_{ij}$$

$s.t.$

$$\sum_{i=1}^{100} x_i d_{ij} = y_j \qquad \forall j = 1 \ to \ 1000$$

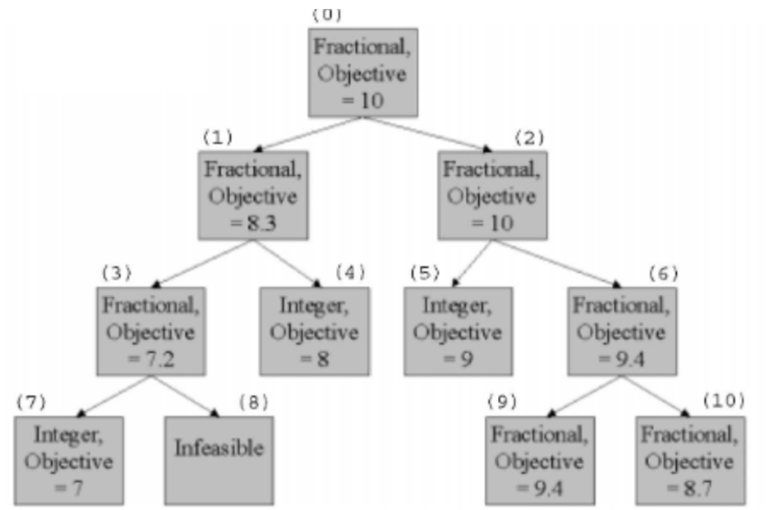
$$y_j \geq 70 + 5z_j \ \ \forall j = 1 \ to \ 1000$$

$$\sum_{j=1}^{1000} z_j \geq 900$$

$$\sum_{j=1001}^{5000} x_i d_{ij} \leq 25 \ \forall j = 1001 \ to \ 5000$$

$$x_i \geq 0 \ \forall \ 1 \ to \ 100, \ y_j \geq 0, \ z_j \Subset \{0, 1\} \ \forall j = 1 \ to \ 1000$$

3.



(a) Is this tree for a minimization or a maximization problem?

(b) Which nodes do you still need to branch from? Why?

(c) Which nodes do you not need to branch from? Why?

(d) What is the gap between the best solution and the best bound found so far?

(e) In what order were the three integer solutions found in the branch-and-bound process?

Figure 1: Branch-Bound Tree

a) maxamization
b) We still need to branch from node 9 because it's the higher of the two fractional values for nodes 9 and 10.
c) We do not need to branch from nodes 4, 5 and 7 because we've reached an integer solution therefore do not need to continue, and a stopping condition also exists at 8 since it's infeasible. In addition to 10, due to the rules of LB.
d) 0.4
e) The traversed order would be 7, 4, then 5.