ISYE 6669 Deterministic Optimization

Homework 1

August 27th, 2020

1.

Consider the following optimization problem since we're converting from maximization to minimization this will yield the following min-{-(x-1)^2+(y-2)^2 : $x + y \leq\ 4,\ x - y \geq\ -2$ 0, x ≥ 0, y ≥ 0}. Plot the feasible region of this problem with the feasible area shaded. Draw (in dashed lines) the contours of the objective function. Based on your drawing, guess an optimal solution and the optimal objective value of this problem.

Provided is the output showing the objective function and constraints as part of the question. We've plotted the feasible region to show what the expected values we should expect to see as part of the minimization of the objective function.

The optimal solution for this objection function can be found at the point (1,2) since this the minimum point on the contour for the objective function. Other solutions can be found all on all points within the shaded red region since it's a non-linear function.

x = 0, y = 1

x = 0, y = 2

x = 1, y = 0

x = 1, y = 1

x = 1, y = 2 (optimal solution)

x = 0, y = 1

x = 0, y = 2

x = 1, y = 0

x = 1, y = 1

x = 1, y = 2

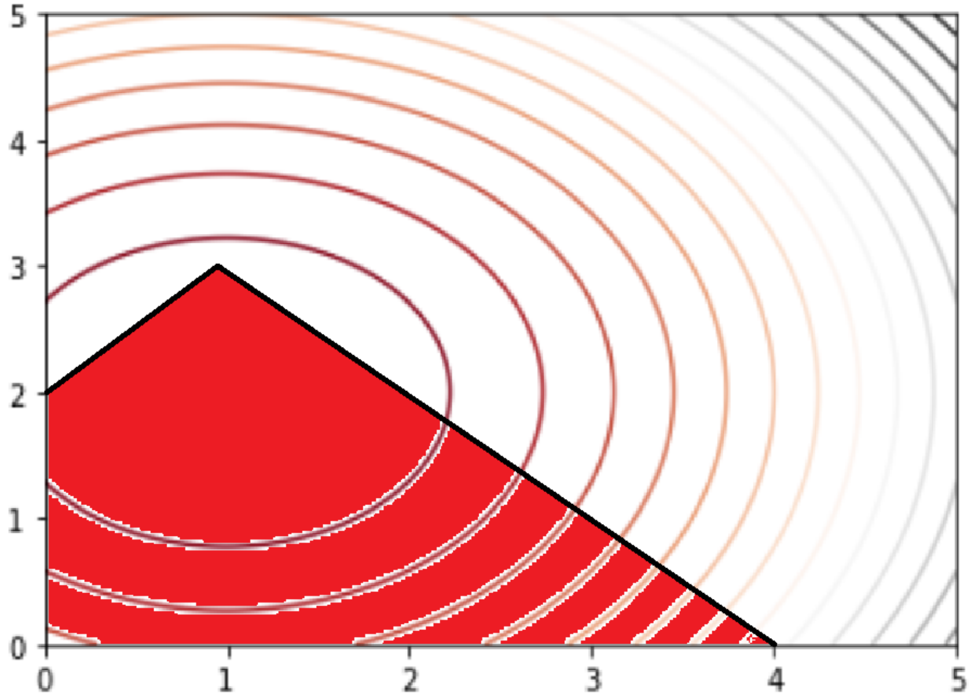

Figure 1: Feasible Region for Objection Function based on Constraints

2. Recall the maximum volume box from Module 1, Lesson 1. Solve the following problem using basic calculus:max{x(1−2x)^2: 0≤x≤1/2}. What is the optimal solution and the optimal objective value?

When we take the derivative of our objective function we get the following result:

$$f'(x) = 12x^2 - 8x + 1 \tag{1}$$

When we take the second order derivative we get the following:

$$f''(x) = 24x - 8 \tag{2}$$

Based on setting the first derivative = 0, and then taking the roots for the equations we get the following:

$x_1^* = \frac{1}{6}, x_2^* = \frac{1}{2}$. The solution to $f'\left(x_2^*\right) = 0$, therefore this solution is not a maximizer. Therefore $\frac{1}{6}$ is out optimal solution, which yields and objective function value of $f'(x)$= $\frac{2}{27}$.

3.

Recall the portfolio optimization problem solved in Module 2, Lesson 3.Collect the prices of

MSFT, V, and WMT from the last 24 months. Usethe code used in the lesson (this will be provided) to solve the exact sameportfolio problem using the new data. Compare and contrast your solutionto the one in the lesson.

```
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MSFT: Exp ret = 0.024611, Risk = 0.058040
V: Exp ret = 0.018237, Risk = 0.042807
WMT: Exp ret = 0.009066, Risk = 0.044461
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Optimal portfolio
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x[MSFT] = 0.582818
x[V] = 0.204324
x[WMT] = 0.212858
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Exp ret = 0.020000
risk    = 0.038256
----------------------
```

```
c:\users\mjpearl\desktop\omsa\isye-6669-oan\env_isye6669\lib\site-packages\cvxpy\
This use of ``*`` has resulted in matrix multiplication.
Using ``*`` for matrix multiplication has been deprecated since CVXPY 1.1.
    Use ``*`` for matrix-scalar and vector-scalar multiplication.
    Use ``@`` for matrix-matrix and matrix-vector multiplication.
    Use ``multiply`` for elementwise multiplication.

  warnings.warn(__STAR_MATMUL_WARNING__, UserWarning)
```

Figure 2: CVXPY Output

We can see very similar results to the output from the class exercise, and the optimal values for the stocks MSFT, V and WMT as part of the portfolio is 0.58, 0.204 and 0.212 respectively.