

ISYE 6740 Homework 4

Summer 2020

Total 100 points.

1. SVM. (30 points)

- (a) (10 points) Explain why can we set the margin $c = 1$ to derive the SVM formulation?
- (b) (10 points) Using Lagrangian dual formulation, show that the weight vector can be represented as

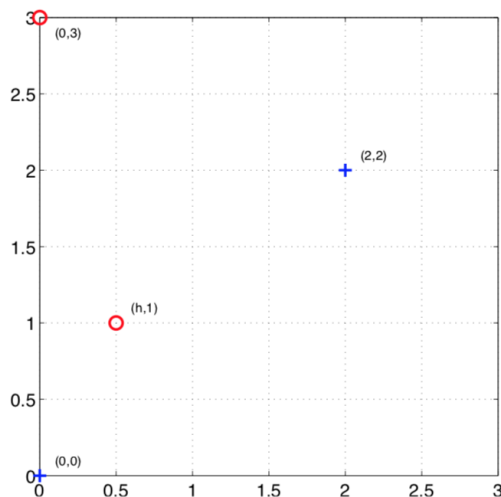
$$w = \sum_{i=1}^n \alpha_i y_i x_i.$$

where $\alpha_i \geq 0$ are the dual variables. What does this imply in terms of how to relate data to w ?

- (c) (10 points) Explain why only the data points on the “margin” will contribute to the sum above, i.e., playing a role in defining w . Hint: use the Lagrangian multiplier derivation and KKT condition we discussed in class.

2. Simple SVM by hand. (20 points)

Suppose we only have four training examples in two dimensions as shown in Fig. The positive samples at $x_1 = (0, 0)$, $x_2 = (2, 2)$ and negative samples at $x_3 = (h, 1)$ and $x_4 = (0, 3)$.



- (a) (10 points) For what range of parameter $h > 0$, the training points are still linearly separable?
- (b) (10 points) Does the orientation of the maximum margin decision boundary change as h changes, when the points are separable?

3. Neural networks. (20 points)

- (a) (10 points) Consider a neural networks for a binary classification using sigmoid function for each unit. If the network has no hidden layer, explain why the model is equivalent to logistic regression.
- (b) (10 points) Consider a simple two-layer network in the lecture slides. Given m training data (x^i, y^i) , $i = 1, \dots, m$, the cost function used to training the neural networks

$$\ell(w, \alpha, \beta) = \sum_{i=1}^m (y^i - \sigma(w^T z^i))^2$$

where $\sigma(x) = 1/(1 + e^{-x})$ is the sigmoid function, z^i is a two-dimensional vector such that $z_1^i = \sigma(\alpha^T x^i)$, and $z_2^i = \sigma(\beta^T x^i)$. Show that the gradient is given by

$$\frac{\partial \ell(w, \alpha, \beta)}{\partial w} = - \sum_{i=1}^m 2(y^i - \sigma(u^i))\sigma(u^i)(1 - \sigma(u^i))z^i,$$

where $u^i = w^T z^i$. Also find the gradient of $\ell(w, \alpha, \beta)$ with respect to α and β and write down their expression.

4. Comparing SVM and simple neural networks. (30 points)

This question is to implement and compare **SVM and simple neural networks** for the same datasets we tried for the last homework (so in the end, we have compared 5 classification algorithms on two datasets). We suggest to use **Scikit-learn**, which is a commonly-used and powerful **Python** library with various machine learning tools. But you can also use other similar libraries in other programming languages of your choice to perform the tasks.

You may use a neural networks function `sklearn.neural_network` with `hidden_layer_sizes=(5, 2)`. Tune the step size so you have reasonable results. You may use `svc` and tune the penalty term C to get reasonable results.

Part One (Divorce classification/prediction). (20 points)

We will compare using the same dataset as the last homework, which is about participants who completed the personal information form and a divorce predictors scale.

The data is a modified version of the publicly available at <https://archive.ics.uci.edu/ml/datasets/Divorce+Predictors+data+set> (by injecting noise so you will not replicate the results on uci website). There are 170 participants and 54 attributes (or predictor variables) that are all real-valued. The dataset was the same as the last homework. The last column of the CSV file is label y (1 means “divorce”, 0 means “no divorce”). Each column is for one feature (predictor variable), and each row is a sample (participant). A detailed explanation for each feature (predictor variable) can be found at the website link above. Our goal is to build a classifier using training data, such that given a test sample, we can classify (or essentially predict) whether its label is 0 (“no divorce”) or 1 (“divorce”).

Build two classifiers using SVM and a simple neural networks. First random shuffle the data set. Then use the first 80% data for training and the remaining 20% for testing. If you use **scikit-learn** you can use `train_test_split` to split the dataset.

- (a) (10 points) Report testing accuracy for each of the two classifiers. Comment on their performance: which performs better and make a guess why it performs better in this setting.
- (b) (10 points) Use the first two features to train two new classifiers. Plot the data points and decision boundary of each classifier. Comment on the difference between the decision boundary for the two classifiers. Please clearly represent the data points with different labels using different colors.

Part Two (Handwritten digits classification). (10 points) Repeat the above part (a) using the **MNIST Data** in our **Homework 3**. Here, give “digit” 6 label $y = 1$, and give “digit” 2 label $y = 0$. All the pixels in each image will be the feature (predictor variables) for that sample (i.e., image). Our goal is to build classifiers such that given a new testing sample, we can tell it is a 2 or a 6. Using the first 80% of the samples for training and remaining 20% for testing. Report the classification accuracy on testing data, for each of the two classifiers. Comment on their performance: which performs better and make a guess why they perform better in this setting.