ISYE 6669 Deterministic Optimization

Homework 13

April 18th, 2021

1.a)

Lets use variables to represent both conditions:

$A:\ x_1\ \geq 100$
$B:\ x_2\ \leq 100$

$We\ can\ represent\ this\ with\ the\ boolean\ condition\ that\ A\ implies\ B$

$Meaning\ that\ (A\ or\ \neg B)\ =\ false$
$Therefore\ if\ the\ above\ is\ false\ then\ it\ holds\ that:$

$\neg(A\ or\ \neg B)\ =\ true$
$Which\ translates\ to:$

$x_1\ <\ 100\ or\ x_2 \leq\ 100$
$We\ now\ need\ to\ introduce\ an\ epsilon\ variable\ that\ so\ that\ the\ first\ constraint\ can\ contain\ the\ equality\ component:$
$x_1 \leq\ 100\ +\ \epsilon\ or\ x_2 \leq 100$

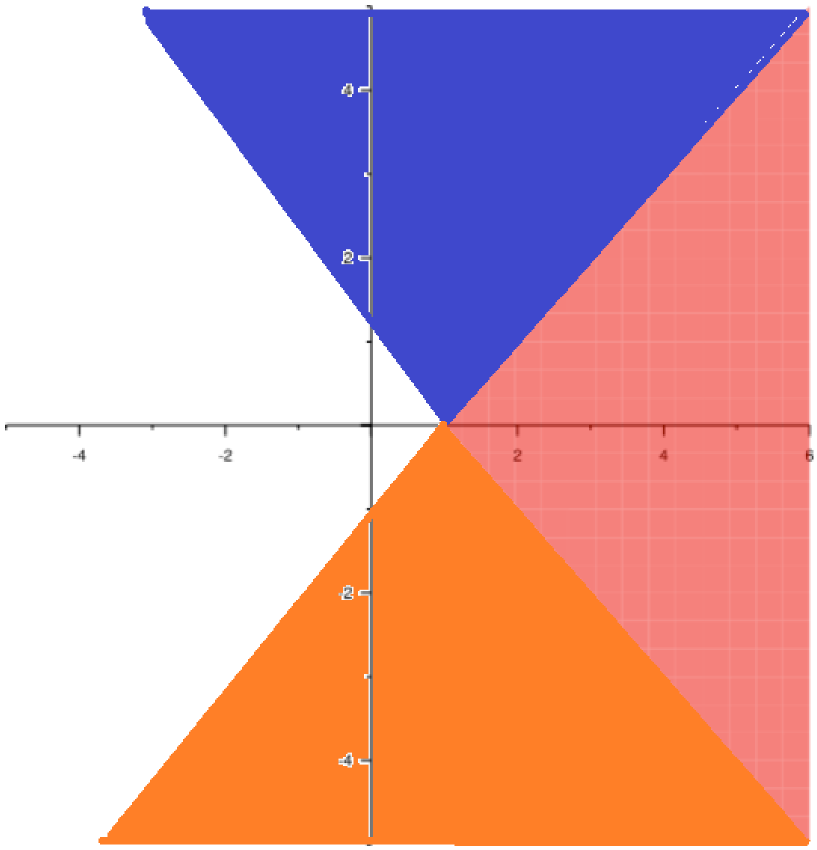
$Now\ introducing\ our\ boolean\ variable\ z\ \in\ \{0,1\},\ M\ upper\ bound\ for\ both\ constraints\ we\ get\ the\ following\ formulation:$

$x_1 \leq\ 100 +\ \epsilon + Mz\ \ or\ x_2 \leq 100 + M(1-z)$
$z\ \in\ \{0,1\}$

b)

When using the same conventions for this question, we get the following reformulation and convert the first constraint to ⩾:
$x_1 + x_2 \geq 1\ +\ \epsilon - Lz$
$x_1 - x_2 \geq 1 - L(1-z)$
$z\ \in\ \{0,1\}$

Red region shows the intersection of both constraints where both conditions are true. The blue region shows the addition to the set if only the first constraint is true, and the orange region shows region where only the 2nd constraint is true.



We can conclude that we have a valid convex set in the context of this question.

2.

$max\ \ p^T x$

$p:\ total\ profit$

$x: binary\ array\ representing\ the\ chosen\ investment$

$b: total\ budget$

$r: array\ eventual\ expected\ profit\ for\ each\ investment$

$c:\ array\ cost\ per\ investment$

You must choose at least one of the investments.

$$\sum_{i=1}^{7} x_i \geq 1 \qquad \text{(a)}$$

Investment 1 cannot be chosen if investment 3 is chosen.

$$x_1 + x_3 \leq 1 \qquad \text{(b)}$$

Investment 4 can be chosen only if investment 2 is also chosen

$$x_4 \leq x_2 \tag{c}$$

You must choose either both investments 1 and 5 or neither

$$x_1 - x_5 = 0 \tag{d}$$

You must choose either at least one of the investments 1, 2, 3 or at least two investment from 2, 4,5, 6

$$\frac{x_1 + x_2 + x_3}{3} + \frac{x_2 + x_4 + x_5 + x_6}{4} \geq 1$$

$$\mathit{Then\ using\ common\ denominator\ logic:}$$

$$\frac{4(x_1 + x_2 + x_3) + \ 3(x_2 + x_4 + x_5 + x_6)}{12} \geq 1$$

$$\frac{4x_1 + 7x_2 + 4x_3 + 3x_4 + 3x_5 + 3x_6}{12} \geq 1$$

Other constraints:

$$\sum_{i=1}^{7} c_i^T x_i \leq b \tag{1}$$

$$p = r - c \tag{2}$$

3.

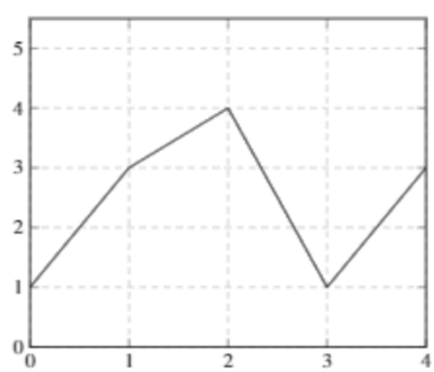


Figure 1: Piecewise linear function $f(x)$.

$We\ have\ a\ piecewise\ linear\ function\ denoted\ the\ following:$

$$f(x) = \left\{ \begin{array}{c} 2x\ for\ 0 \le x \le 1 \\ x\ for\ 1 \le x \le 2 \\ -3x\ for\ 2 \le x \le 3 \\ 2x\ for\ 3 \le x \le 4 \end{array} \right\}$$

$f\ is\ a\ continuous,\ univariate,\ piecewise-linear,\ non-convex\ function\ as\ follows:$
$f:[\ell_1, \ell_5] \epsilon R$

$\ell:\ represents\ breakpoint\ or\ the\ value\ of\ the\ x\ axis\ for\ the\ interval$
$y\ :\ binary\ variable\ to\ select\ which\ interval\ to\ select\ for\ each\ adjacent\ variable$
$(\lambda_k, \lambda_{k+1})$
$\lambda\ :\ used\ to\ enforce\ the\ adjency\ criteria$
$k\ :\ specifying\ our\ interval\ values\ = [1,\ 2,\ 3,\ 4]$

$we\ can\ describe\ the\ function\ as\ follows:$

$$x\ =\ \sum_{k=1}^{4} \lambda_k \ell_k$$

$s.t:$
$y\ \in\ \{0,1\}^4$
$\lambda_k \ge 0\ k\ = 1,...4$
$\lambda_1\ \le y_1$
$\lambda_2\ \le y_1 + y_2$
$\lambda_3\ \le y_2 + y_3$
$\lambda_4\ \le y_3 + y_4$
$\lambda_5\ \le y_4$

$y_1 + y_2 + y_3 + y_4\ = 1$
$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 1$