

1b $\sqrt{x} + \sqrt{y} = 12$ & $y(9) = 81$, find $y'(9)$ by implicit differentiation.

A. We need to differentiate this expression with respect to x .

$$\frac{dy}{dx} [\sqrt{x} + \sqrt{y} = 12], \text{ distribute } \frac{dy}{dx} \text{ to all terms}$$

$$\frac{dy}{dx} \sqrt{x} + \frac{dy}{dx} \sqrt{y} = \frac{dy}{dx} 12, \text{ rewrite \& solve } \frac{dy}{dx}$$

$$\frac{dy}{dx} x^{1/2} + \frac{dy}{dx} y^{1/2} = 0$$

$$\frac{1}{2} x^{-1/2} + \frac{1}{2} y^{-1/2} \frac{dy}{dx} = 0, \text{ isolate } \frac{dy}{dx} \text{ \& like terms}$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \left(\frac{dy}{dx} \right) = 0$$

$$\frac{1}{2\sqrt{y}} \left(\frac{dy}{dx} \right) = -\frac{1}{2\sqrt{x}}, \text{ solve for } \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-2\sqrt{y}}{2\sqrt{x}} = -\frac{\sqrt{y}}{\sqrt{x}}$$

B. We are given $f(9) = 81$ \therefore we have a point $(9, 81)$ that can be used to find $f'(9)$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}, \text{ evaluate @ } (9, 81)$$

$$= -\frac{\sqrt{81}}{\sqrt{9}} = -\frac{9}{3} = -3$$