1. $y = \dfrac{e^x}{2+2x}$

Find y'

Quotient rule $= \left(\dfrac{a}{b}\right)' = \dfrac{a'b - ab'}{b^2}$

$\therefore\ y' = \dfrac{e^x(2+2x) - e^x(2)}{(2+2x)^2}$

$= \dfrac{e^x\left((\cancel{2}+2x) - \cancel{2}\right)}{2\cancel{(1+x)^2}\ (2+2x^2)} = \dfrac{e^x(2x)}{(2+2x^2)}$

$\dfrac{2xe^x}{(2(1+x))^2} = \dfrac{{}^{1}\cancel{2}xe^x}{2\cancel{4}((1+x))^2} = \dfrac{xe^x}{2(1+x)^2}$

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2. $\displaystyle\int 240e^{0.03x}\,dx$

Factor out constant

$\displaystyle 240\int e^{0.03x}\,dx$

Let $u = 0.03x$

$du = 0.03dx$

$dx = \frac{1}{0.03} du$

sub.

$$= 240 \int e^u \cdot \frac{1}{0.03} du = \frac{240}{0.03} \int e^u du$$

$$= 8000 \cdot e^u + \text{some constant.}$$

sub

$$= 8000 \cdot e^{0.03x}$$

$$= 8000 e^{0.03x}$$

$$\lim_{x \to 0} \frac{\sin 7x}{\sin 13x}$$

~~S~~ Sine relationship $\lim_{x \to 0} \frac{\sin(bx)}{bx} = 1$

$$\therefore \lim_{x \to 0} \frac{\sin 7x}{\sin 13x} = \left(\frac{\sin(7x)}{7x} \right) \cdot \frac{13x}{\sin(13x)} \cdot \frac{7x}{13x}$$

$$\lim_{x \to 0} \left(\frac{\sin 7x}{7x} \cdot \frac{13}{\sin(13x)} \cdot \frac{7x}{13x} \right)$$

$$1 \cdot 1 \cdot \frac{7x}{13x} = \frac{7}{13}$$

$$\int \frac{4\,dx}{x \ln(6x)}$$

substitution:

$u = \ln(6x)$

chain rule:

$u = \ln(6x) = \ln 6 + \ln x \longrightarrow \frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\,dx$

$\therefore \frac{dx}{x \ln(6x)} = \frac{1}{\ln(6x)} \cdot \frac{dx}{x} = \frac{du}{u}$

$\therefore \int \frac{4\,dx}{x \ln(6x)} = 4 \int \frac{du}{u} = 4 \ln|u|$

subs u back in

$= 4\ln\left|\ln(\cancel{6x})\right| + \text{constant}$

$= 4\ln\left|\ln(6x)\right| + C$

Facts:

$P_1 = 2k$

P ↑ 2x every 30 minutes

Find P after 20 minutes

exponential growth formula:

$P(t) = P_0 \cdot 2^{t/T}$

$P(20) = 2000 \cdot 2^{20/30} = 2000 \cdot 2^{2/3}$

$2^{2/3} = 2^{0.6667} = 1.5874$

$2000 \cdot 1.5874 = 3174.802 \approx 3175$

Find after 300 minutes

$P(300) = 2000 \cdot 2^{300/30} = 2000 \cdot 2^{10} \rightarrow 1024$

$= \boxed{2000 \times 1024}$

$= 2048000$

$2000 \times 1024 = 2048000$

$\int 210 e^{0.1x} dx$

$210 \int e^{0.10} dx$

$u = 0.10x \longrightarrow du = 0.1 dx \longrightarrow dx = \frac{1}{0.10} du = 10 du$

$210 \int e^u \cdot 10 du$

$= 2100 \int e^u du = 2100 e^u + C$

$2100 e^{0.1x} + C$

$y = \log_5 [x(3x+1)^5]$

given relationship $\log_a A = \frac{\ln A}{\ln 5}$

$\therefore\ y = \frac{\ln[x(3x+1)^9]}{\ln 5}$ or $\frac{1}{\ln 5} \cdot \ln[x(3x+1)^9]$

Chain Rule

$\ln[x(3x+1)^5] = \ln x + \ln(3x+1)^9 = \ln x + 9\ln(3x+1)$

$\therefore\ y = \frac{1}{\ln 5}\left(\ln x + 9\ln(3x+1)\right)$

By section

$\cancel{\frac{dy}{dx}}\ y' = \frac{1}{\ln 5}\left(\ln x' + 9 \cdot [\ln(3x+1)]'\right)$

$= \frac{1}{\ln 5}\left(\frac{1}{x} + 9 \cdot \frac{1}{3x+1} \cdot 3\right)$

$= \frac{1}{\ln 5}\left(\frac{1}{x} + \frac{27}{3x+1}\right)$

antiderivative 6

$\int \cosh(6x+2)$

substitution

let $u = 6x+2$

$\frac{du}{dx} = 6$

$dx = \frac{1}{6} du$

~~regroupin~~ rearrangement

$$\int \cosh(6x+2) = \int \cosh(u) \cdot \frac{1}{6} du$$

$$= \frac{1}{6} \int \cosh(u)$$

$$= \frac{1}{6} \sinh(u)$$

$$= \frac{1}{6} \sinh(6x+2) + C$$

Inverse

by relationships

$y = \sin^{-1}(x)$

$y = \arcsin(x) = \dfrac{1}{\sqrt{1-x^2}}$

Eval. $\displaystyle\int \frac{8+x}{3+x^2}$ rewrite $\displaystyle\int \left(\frac{8}{3+x^2} + \frac{x}{3+x^2}\right) dx$

$\displaystyle\int \frac{8}{\cancel{3+x^2}}$ = what identity?

$y_0 = 180\,mg$

post 27 hours $y = 90$

after 42 hours $y = ?$

exponential decay function maybe.

$y = y_0 e^{-kt}$ $\longrightarrow$ t = known
t = 27
Find k

$$\frac{90}{180} = \frac{180 \cdot e^{-27 \cdot k}}{180}$$

use ln to terminate e.

$$\ln\left[\frac{1}{2} = e^{-27 \cdot k}\right.$$

$\ln(1/2) = -27k$

$\ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2)$
$0 - \ln(2)$
$= -\ln 2$

$\therefore$

$$\frac{-\ln 2}{-27} = \frac{-27k}{-27}$$

$$k = \frac{\ln 2}{27} = \frac{0.693}{27} = 0.02567$$

after 42 hours:

$$y(42) = 180e^{-0.2567 \times 42} \rightarrow 0.34102$$

$$= 180e^{(-1.078)}$$

$$= 180 \times 0.3402 = \cancel{61.2}\ 61.2$$

y

$$y = \ln(x^2 - 10x + 26)$$

kunci

$y' \ln(x) = \frac{1}{x} \frac{dy}{dx}$

sol.

$u = x^2 - 10x + 26$

$\frac{du}{dx} = 2x - 10$

$y' = \frac{1}{x^2 - 10x + 26} \cdot 2x - 10$

$= \frac{2x + 10}{x^2 \cancel{+} 10x + 26}$

$= \frac{2x + 10}{x^2 - 10x + 26}$ $\frac{2x - 10}{x^2 - 10x + 26}$

$$\frac{d}{dx}\left(4^{5x+3}\right)$$

note. for exponential function

$$\frac{d}{dx}a^{u(x)} = a^{u(x)} \cdot \ln(a) \cdot \frac{du}{dx}$$

let:

$a = 4$

$u(x) = 5x+3$

$\frac{du}{dx} = 5$

Rule $\Rightarrow$ $\frac{d}{dx}\left(4^{5x+3}\right) = 4^{5x+3} \cdot \ln(4) \cdot 5$

$\therefore$ $5\ln(4) \cdot 4^{5x+3}$

$y = 7\ln(c5x)$

established $\implies$ $\dfrac{d}{dx}\ln(u) = \dfrac{1}{u}\cdot\dfrac{du}{dx}$

$\therefore$ let $u = \csc x$

$\dfrac{d}{dx}(\csc x) = -\csc x \cot x$

chain Rule

$y' = 7\cdot\dfrac{1}{\csc x}(-\csc x\cot x)$

$y' = 7\cdot(-\cot x) = -7\cot(x)$