A company selling widgets has found that the number of items sold x depends upon the price p at which they're sold, according the equation $x = \dfrac{20000}{\sqrt{2p+1}}$.

Due to inflation and increasing health benefit costs, the company has been increasing the price by $2 per month. Find the rate at which revenue is changing when the company is selling widgets at $190 each.

[ ] dollars per month

Known variables

$\frac{dp}{dt} = \$2/\text{month}$

$p_{t_0} = \$190/\text{item}(x)$

Revenue = $P \cdot X$

Revenue $= p \cdot \dfrac{20000}{\sqrt{2p+1}}$

$R = p \cdot \dfrac{20000}{\sqrt{2p+1}}$

$R = \dfrac{20000P}{\sqrt{2p+1}}$

To find $\frac{dR}{dt}$

$\frac{d}{dt}(R) = \frac{d}{dt} \cdot \dfrac{20000P}{(2p+1)^{1/2}}$; apply the quotient rule

$\frac{d}{dx} \dfrac{u(x)}{v(x)} = \dfrac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{(v(x))^2}$

$\frac{dR}{dt} = \dfrac{(2p+1)^{1/2} \cdot 20000 - 20000p \cdot \frac{d}{dt}(2p+1)^{1/2}}{((2p+1)^{1/2})^2}$; apply chain rule to d/dt in numerator & power exponent rule to denominator.

$\frac{d}{dx} g(y(x)) = g'(y(x)) \cdot y'(x)$

&

$((a^m)^n) = a^{mn}$

$\frac{dR}{dt} = \dfrac{(2p+1)^{1/2} \cdot 20000 - 20000p \cdot \frac{1}{2}(2p+1)^{-1/2}(2)}{2p+1}$

$\frac{d}{dt}(2p+1)^{1/2}$

let $u = 2p+1$

$e = u^{1/2}$

$= \frac{d}{dt}\, e'(u(x)) \cdot u'(x)$

$= \frac{1}{2}(2p+1)^{-1/2}(2)$

substitute $p = 190$ $= \dfrac{(2(190)+1)^{1/2} \cdot 20000 - 20000(190) \cdot \frac{1}{2}(2(190+1))^{-1/2}(2)}{2(190)+1}$

$= \dfrac{(381^{1/2}) \cdot 20000 - 20000(190) \cdot (381)^{-1/2}}{381}$

$= 195,704/381 = 513.66$

$513.66 = \cancel{514.44} \times \frac{dp}{dt}$

$= 1027.32$ is the rate @ which revenue is changing when the company is selling widgets @ $190