A company selling widgets has found that the number of items sold x depends upon the price p at which they're sold, according the equation $x = \dfrac{20000}{\sqrt{2p+1}}$.

Due to inflation and increasing health benefit costs, the company has been increasing the price by $\$2$ per month. Find the rate at which revenue is changing when the company is selling widgets at $\$190$ each.

[ ] dollars per month

Known variables

$\frac{dp}{dt} = \$2/\text{month}$

$p_{t_0} = \$190/\text{item}(x)$

$\text{Revenue} = P \cdot x$

$\text{Revenue} = p \cdot \frac{20000}{\sqrt{2p+1}}$

$R = p \cdot \frac{20000}{\sqrt{2p+1}}$

$R = \frac{20000p}{\sqrt{2p+1}}$

To find $\frac{dR}{dt}$

$\frac{d}{dt}(R) = \frac{d}{dt} \cdot \frac{20000p}{(2p+1)^{1/2}}$ apply the quotient rule

$\frac{d}{dx}\frac{u(x)}{v(x)} = \frac{v(x)\cdot u'(x) - u(x)\cdot v'(x)}{(v(x))^2}$

$\frac{dR}{dt} = \frac{(2p+1)^{1/2}\cdot 20000 - 20000p \cdot \frac{d}{dt}(2p+1)^{1/2}}{((2p+1)^{1/2})^2}$ apply chain rule to d/dt in numerator & power exponent rule to denominator -

$\frac{d}{dx} g(y(x)) = g'(y(x)) \cdot y'(x)$

&

$((a^m)^n) = a^{mn}$

$\frac{dR}{dt} = \frac{(2p+1)^{1/2}\cdot 20000 - 20000p \cdot \frac{1}{2}(2p+1)^{-1/2}(2)}{2p+1}$

$\frac{d}{dt}(2p+1)^{1/2}$

let $u = 2p+1$

$e = u^{1/2}$

$= \frac{d}{dt} e'(u(x)) \cdot u'(x)$

$= \frac{1}{2}(2p+1)^{-1/2}(2)$

substitute $p = 190$

$= \frac{(2(190)+1)^{1/2}\cdot 20000 - 20000(190)\cdot \frac{1}{2}(2(190)+1)^{-1/2}(2)}{2(190)+1}$

$= \frac{(381^{1/2})\cdot 20000 - 20000(190)\cdot (381)^{-1/2}}{381}$

$= 196{,}160 / 381 = 514.44$

$= 514.44 \times \frac{dp}{dt}$

$= 1028.88$ is the rate @ which revenue is changing

when the company is selling assets @ #190