Example.

Find an open interval about x_0 on which the inequality $|f(x)-L|<\varepsilon$ holds. ~~Then~~ give the largest value for $\delta>0$ such that for all x satisfying $0<|x-x_0|<\delta$ the equality $|f(x)-L|<\varepsilon$ holds.

Given: $f(x)=3x+5$

$L = 23$

$x_0 = 6$

$\varepsilon = 0.27$

Step 1

$$|f(x)-L| < \varepsilon$$

$$|3x+5-(L)| < 0.27$$

$$-0.27 < 3x-18 < 0.27$$

$$+18 \quad +18 \quad +18$$

$$\frac{17.73}{3} < \frac{3x}{3} < \frac{18.27}{3}$$

$$5.91 < x < 6.09$$

what does this range mean?

The range simply provides: a group of numbers around 6 whereby the function's answer stays very close to 23.

as long as we stay between 5.91 & 6.09 (inclusive) the function will provide an answer close to L by a factor of 0.27

Step 2

Answer the question → How far can we go away from 6 the function to still be close to L.

$$|x - x_0| < \delta \longrightarrow \text{variation in } y$$

$$-\delta < |x - 6| < \delta$$

$$\underline{\ +6 \qquad\quad +6 \qquad\quad +6\ }$$

$$-\delta + 6 < |x| \ \delta + 6$$

use our two end points

$$-\delta + 6 = 5.91 \quad \& \quad \delta + 6 = 6.09$$

$$\underline{\ -6 \quad -6 \qquad\qquad -6 \quad -6\ }$$

$$-\delta = -0.09 \qquad\qquad \delta = +0.09$$

So the furthest we can go from $x_0 = 6$ for our definition to hold is 0.09 in either direction.