Find the equation of the tangent line to the curve

LEMNISCATE = ∞

$$2(x^2+y^2)^2 = 25(x^2-y^2) \quad \text{point } (-3,-1)$$

$$\frac{dy}{dx}\, 2(x^2+y^2)^2 = \frac{dy}{dx}\, 25(x^2-y^2)$$

Breakdown $\frac{dy}{dx}$ of each side (left side)

$$\frac{dy}{dx}\, 2\cdot(x^2+y^2)^2$$

Lets start with the chain rule for the inner part of the expression $(x^2+y^2)^2$

let $u = x^2+y^2$

let $2u^2 = 2(x^2+y^2)$

Chain Rule $= \frac{dy}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

$$\therefore \frac{dy}{dx}\, 2(x^2+y^2)^2 = 2\cdot 2(x^2+y^2) \cdot \frac{d}{dx}(x^2+y^2)$$

$$= 4(x^2+y^2) \cdot \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$= 4x^2 + 4y^2\,(2x + 2y\cdot y')$$

Right side

$$\frac{dy}{dx}\, 25(x^2-y^2)$$

constant multiple rule

$$\frac{d}{dx}\, c\, y(u) = c \cdot \frac{d}{du}\, y(u$$

$$\frac{d}{dx}\, 25(x^2-y^2) = 25\,\frac{d}{dx}(x^2-y^2) = 25\cdot\ 2x - 2y\,y' = 50x - 50y\,y'$$

$$= 50(x - y\,y')$$

left & Right sides together:

$$4(x^2+y^2)\,(2x + 2y\cdot y') = 50(x - y\cdot y')$$

lets distributes we can move expression around.

$$4(x^2+y^2)\cdot 2\,(x + y\,{}^{dy}/_{dx}) = 50(x - y\,{}^{dy}/_{dx})$$

$$= 8(x^2+y^2)\cdot x + y\,{}^{dy}/_{dx} = 50\,(x - y\,{}^{dy}/_{dx})$$

$$\cancel{= 8(x^2+y^2)(x) + 8(x^2+y^2)(y\,{}^{dy}/_{dx}) =}$$

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$$8(x^2+y^2)\cdot x+y\frac{dy}{dx}=50\left(x-y\frac{dy}{dx}\right)$$

$$8(x^2+y^2)(x)+8(x^2+y^2)\left(y\frac{dy}{dx}\right)=50\left(x-y\frac{dy}{dx}\right)$$

$$8(x^2+y^2)(x)+8(x^2+y^2)\left(y\frac{dy}{dx}\right)=50x-50y\frac{dy}{dx}$$

move all like terms to one side

$$8(x^2+y^2)(x)+8(x^2+y^2)\left(y\frac{dy}{dx}\right)+50y\frac{dy}{dx}=50x$$

$$8(x^2+y^2)\left(y\frac{dy}{dx}\right)+50y\frac{dy}{dx}=50x-8(x^2+y^2)(x)$$

$$\cancel{\frac{\frac{dy}{dx}\left[8(x^2+y^2)(y)+50y\right]}{8(x^2+y^2)(y)+50y}}=\frac{50x-8(x^2+y^2)(x)}{8(x^2+y^2)(y)+50y}$$

$$\frac{dy}{dx}=\frac{50x-8(x^2+y^2)(x)}{8(x^2+y^2)(y)+50y}$$

$$\therefore m=\frac{50x-8(x^2+y^2)(x)}{8(x^2+y^2)(y)+50y}$$

$$=\frac{50x-8(x^2+y^2)(x)}{8y(x^2+y^2)+50y}$$

$(-3,-1)$

$$=\frac{50(-3)-8(9+1)(-3)}{8(9+1)(-1)+50(-1)}=\frac{90}{-130}=-\frac{9}{13}$$

since $m = -\frac{9}{13}$ & we have point $(-3, -1)$

always (x_1, y_1)

$$y_0 - y_1 = -\frac{9}{13}(x_0 - x_1)$$

$$y - -1 = -\frac{9}{13}(x - -3)$$

$$y + 1 = -\frac{9}{13}(x + 3)$$

$$y + 1 = -\frac{9}{13}x - \frac{9}{13}(3)$$

$$-1 \qquad\qquad\qquad -1$$

$$y = -\frac{9}{13}x - \frac{27}{13} - \frac{13}{13}$$

$$\boxed{y = -\frac{9}{13}x - \frac{40}{13}}$$

Can we use slope equation to find tangent line equation?

$$-\frac{9}{13} = \frac{\Delta y}{\Delta x} = \frac{y_0 - y_1}{x_0 - x_1} = \frac{y - -1}{x - -3} = \frac{y + 1}{x + 3}$$

$$-\frac{9}{13} = \frac{y + 1}{x + 3} = \quad -9(x + 3) = 13(y + 1)$$

$$9x - 27 = 13y + 13$$

$$\frac{9x-40}{13}=\frac{13y}{13}$$

$$\frac{9x}{13}-\frac{40}{13}=y$$