<u>Research Question:</u> *To determine how resistance affects the rate at which a capacitor charges*

I. General Overview

Background Information

A capacitor is an electrical component that stores electrical potential energy. Capacitors have many important real world applications such as storing energy for temporary electric power failure, power supply smoothing, and timing circuits. Capacitors play an essential role in the operation of many electronic devices and systems. (Britannica, 2022)

A capacitor is composed of two parallel metal plates separated by an insulator called a dielectric. The medium of the dielectric must have a permittivity such that it prevents a spark containing charge to jump between the plates, for example, a vacuum, air, or a non-conducting material such as plastic.

In *figure 1*, two parallel plates that compose a capacitor are connected to a cell. Initially, the plates are uncharged and the switch is open. When the switch is closed, electrons begin to flow; they move towards the plate connected to the negative terminal. Consequently, a negative charge accumulates at this plate and an equal positive charge collects at the opposite plate. Electrons cannot travel between the plates due to the insulator, and thus there is no current between them. The charge is separated and stored in the system. The energy required to move the electrons is supplied by the cell and stored within the electrons at the negative plate. As the first electron naturally drifts towards the negative plate that is initially uncharged, not much energy is required. However, as the subsequent electrons follow the same path they experience repulsion from the electrons in front of them. A greater amount of energy is required to move each electron to the negative plate in order to overcome the repulsive intermolecular forces. This process ends once there is insufficient potential energy to move any more electrons. Therefore, when measured, the potential difference across the plates of the capacitor must be equal to the emf of the cell. However, once a resistance is added to the circuit, some energy is dissipated and the maximum charge stored in the capacitor decreases. (Homer, D., Bowen-Jones, M., 2014)

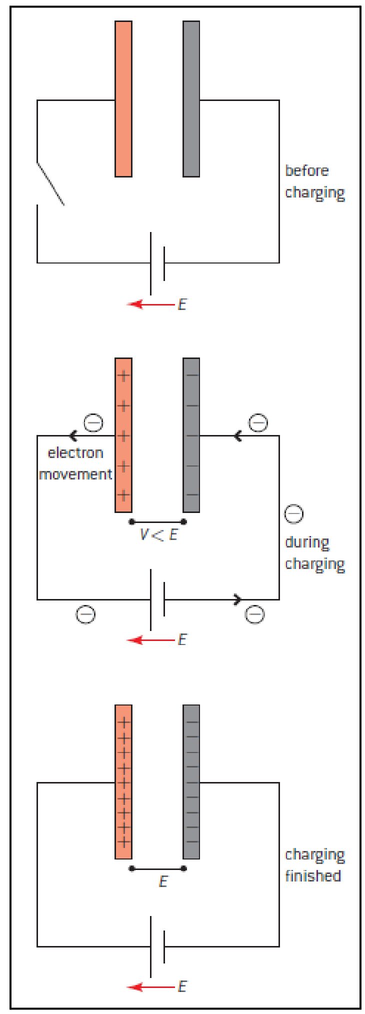


figure 1: Charging a capacitor (Homer, D., Bowen-Jones, M., 2014)

The capacitance of the capacitor is defined as the ratio of the charge stored in the capacitor to the potential difference between the plates of the capacitor. This can be written as $C = \frac{Q}{V_C}$, where C is capacitance of the capacitor measured in farads, Q is the charge on the capacitor, and V_C is the potential difference across the capacitor.

The goal of this investigation is to determine how resistance affects the rate at which a capacitor charges. To theoretically model this relationship, a formula for the potential difference across a capacitor must be derived. As represented in *figure 2*, ε is the emf of the cell, I is the total current in the cell which changes over time but is constant throughout the circuit, R is the total resistance of the resistors connected in series, C is the capacitance of the capacitor, and V_C is the potential difference across the capacitor. When the switch is closed, current begins to flow, and charge builds up on the capacitor thus potential difference across the capacitor increases. By applying Kirchoff's second law that states the sum of the emf in a closed loop is equal to the sum of all potential differences around that loop, as the potential difference across the capacitor increases, the potential difference across the resistors must decrease. This can be represented as $\varepsilon = V_R + V_C$. Ohm's law can be applied to the resistor such that $V_R = IR$, the equation now becomes $\varepsilon = IR + V_C$. The total current is constant throughout the circuit, therefore it can be represented as the current through the capacitor, $I_C = C\frac{dV_C}{dt}$ and by substituting this into the equation it becomes $\varepsilon = RC\frac{dV_C}{dt} + V_C$. Rearranging this equation yields the first order differential equation $\frac{dV_C}{dt} = \frac{\varepsilon - V_C}{RC}$ which can be solved by integrating both sides $\int_0^{V_C} \frac{dV_C}{dt} = \int_0^{t} \frac{\varepsilon - V_C}{RC}$ to give $V_C = \varepsilon(1 - e^{\frac{-t}{RC}})$. (Williams, D., 2021)

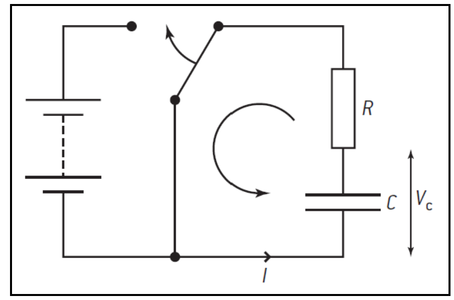


figure 2: RC circuit diagram
(Homer, D., Bowen-Jones, M., 2014)

This equation can be simplified by substituting $\tau = RC$ as the time constant to give $V_C = \varepsilon(1 - e^{\frac{-t}{\tau}})$. The time constant represents the amount of time it takes for the potential difference across the capacitor to reach 63.2 % of its maximum value. The larger the time constant, the slower the rate of charging and the longer it takes for the capacitor to reach its maximum charge. (Tsokos, K., A., 2014)

By substituting these variables with values, a theoretical graph to show the relationship between the mean potential difference of a capacitor and time could be modelled. This relationship of proportionality of $V_C \propto e^{\frac{-t}{RC}}$ indicates that as resistance increases, the time constant increases, and following the inversely proportional relationship, the rate of increasing the potential difference across the capacitor decreases. This means that as resistance is increased, the rate of charging a capacitor decreases.

Hypothesis

The aim of this investigation is to determine how resistance affects the rate at which a capacitor charges. As explained in the *Background Information*, when a capacitor is charged, electrons flow into the plate connected to the negative terminal of the cell. By adding a resistor to the circuit the current decreases. This is because the resistance opposes the flow of current, decreasing the voltage, and thus the current in the circuit. Therefore, the electrons take longer to reach the capacitor and the time to charge the capacitor increases. This theoretical evidence supports the overall hypothesis of this investigation that states that as the resistance of the circuit increases, the rate at which the capacitor charges decreases. Furthermore the hypothesis is supported with the inversely proportional relationship derived in the *Background Information*. Another expected trend is that when a resistance is added to the circuit, the maximum potential difference across the capacitor will decrease.

II. Setup and Methodology

Prior to the investigation preliminary trials were conducted in preparation. In which, the method was refined and improved. During the preliminary trials the capacitor was charging too quickly. This was due to the low capacitance of the capacitor as well as the low resistance of the circuit. Consequently, a capacitor with a greater capacitance and resistors with greater resistance were chosen for this investigation. The findings within the preliminary trials further support the hypothesis of increasing resistance, decreases the rate of charging the capacitor.

Independent Variable: The resistance in the circuit.

The independent variable in this investigation is the resistance in the circuit. This value will be varied by adding or removing resistors to the circuit in series. Each resistor has a resistance of 100 kΩ. When connected in series the total resistance in the circuit is the sum of the resistance of each resistor. Eight different resistances will be used in this investigation: 100, 200, 300, 400, 500, 600, 700, and 800 kΩ. The uncertainty of this measurement is ± 5 % as indicated by the gold tolerance band on the resistors.

Dependent Variable: The time taken to charge the capacitor (the rate of charging the capacitor).

The dependent variable in this investigation is the time taken to fully charge the capacitor. To determine this length of time, the values for potential difference as well as time are required. A video recording of the trial will be conducted. The values for potential difference will be measured when the capacitor is charged using a voltmeter connected in parallel across the capacitor. The uncertainty of this measurement is ± 0.01 volts. The values for time will be measured from the video recording. The uncertainty of this measurement is ± 0.01 seconds. The time taken to fully charge the capacitor is the point in which the potential difference reaches a maximum and no longer increases.

Control Variables:

Control variable	Value of variable	Why and how variable is controlled
Capacitance of the capacitor	100 μF	The capacitance of the capacitor must remain constant throughout the investigation for reliable results. If the capacitance is not kept constant, the amount of charge stored in the capacitor changes, and thus the rate at which the capacitor charges is affected leading to imprecise results.
Resistance in the circuit between repeat trials	100, 200, 300, 400, 500, 600, 700, or 800 ± 0.1 kΩ depending on trial	The resistance in the circuit between repeat trials must be kept constant to yield precise results. Increasing or decreasing the resistance between repeats affects the current through the circuit and thus the rate at which the capacitor is charged. To keep the resistance constant between trials the number of resistors in the circuit must not be changed between the repeat trials.
Type of capacitor and resistor	Electrolytic capacitor, metal film resistor	The type of capacitor and resistor must remain constant throughout the investigation to ensure that the results of the investigation are consistent and reliable. If the type of capacitor or resistor is changed, then any changes in the results could be attributed to this change rather than the change in resistance leading to inconclusive results.
Potential difference supplied to the circuit	4.00 ± 0.01 V	The potential difference supplied to the circuit must be kept constant during the investigation. If the potential difference is varied, the current in the circuit will also change, and thus the rate at which the capacitor charges is affected leading to imprecise results.
Initial potential difference across the capacitor	0.00 ± 0.01 V	The initial potential difference across the capacitor must remain constant throughout the investigation. If the capacitor has stored charge before the investigation begins, imprecise results will be collected. This variable will be kept constant by fully discharging the capacitor between trials.
Temperature of the circuit components (capacitor and resistors)	Room temperature at 22 °C	The temperature of the components must remain constant as temperature affects their electrical properties. A temperature too high or too low can cause a change in the value of the capacitance or resistance, causing the results to be imprecise. Keeping the temperature of the components constant ensures that they remain within their specified electrical parameters and that their performance is not impacted. The capacitor used in this investigation has an optimal temperature range between -40 and 105 °C. The temperature will be kept constant by only powering the circuit when required and by quickly discharging the capacitor after each trial to prevent the components from overheating.

Apparatus

- ☐ 1 100 µF capacitor
- ☐ 8 100 kΩ resistors
- ☐ 1 voltmeter
- ☐ 1 lamp
- ☐ 1 power supply (minimum output of 4.00 ± 0.01 V)
- ☐ 18 crocodile clips
- ☐ 11 wires (minimum length of 50 ± 1 cm)
- ☐ 1 video camera

Risk Assessment

Hazard	Risk	Precautions
Contact with live exposed wiring (crocodile clips), faulty plugs or sockets (power supply), and unsafe work practices	Electric shock	Avoid contact with live wires and unnecessary equipment. Minimise the length of the exposed portion of the wire. Only provide power to the circuit when required. Before starting the investigation verify the electrical equipment is in good condition and undamaged. Use the recommended voltage with the relevant equipment. Do not expose the wires to liquids; avoid drinks near the investigation setup.
	Electric burns	
	Electrocution	
Faulty electrical equipment, wiring, plugs, or sockets	Arc flashes and blasts (high-power sparks)	Before starting the investigation verify the electrical equipment and wiring is in good condition and undamaged. Ensure flammable materials, such as clothing, are a safe distance from equipment and live wires. Ensure a class C fire extinguisher is readily available in the event of electrical fires. Use the recommended voltage with the relevant equipment and only provide power to the circuit when required.
	Fires	
	Explosions	

Method

1. Prepare all the necessary equipment required for this investigation.
2. Complete a thorough assessment of the equipment checking for damage or other visible faults. This is a precaution to ensure all equipment is safe and to limit the amount of possible risks.
3. Construct the charging circuit following the circuit diagram provided in *figure 3*. This will require four wires, four crocodile clips, the capacitor, one resistor, the voltmeter, the lamp, and the power supply.

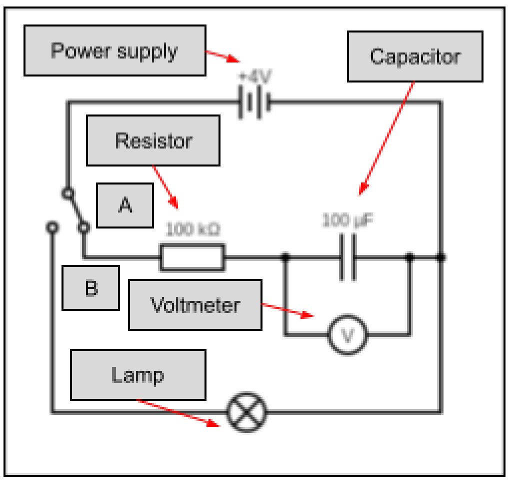


figure 3: Circuit diagram (loop A and B)

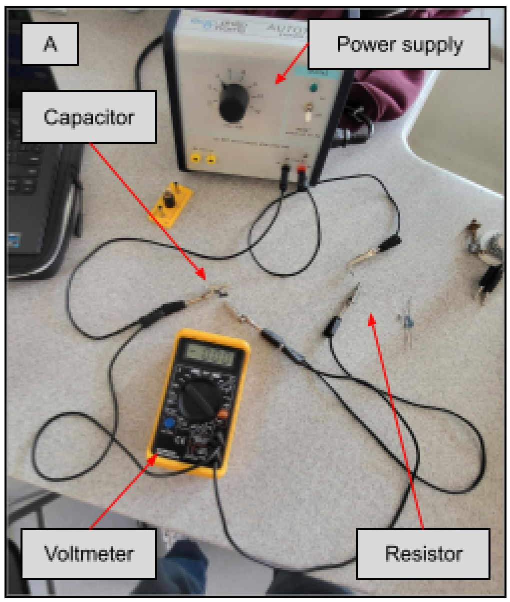


figure 4: Charging circuit (loop A)

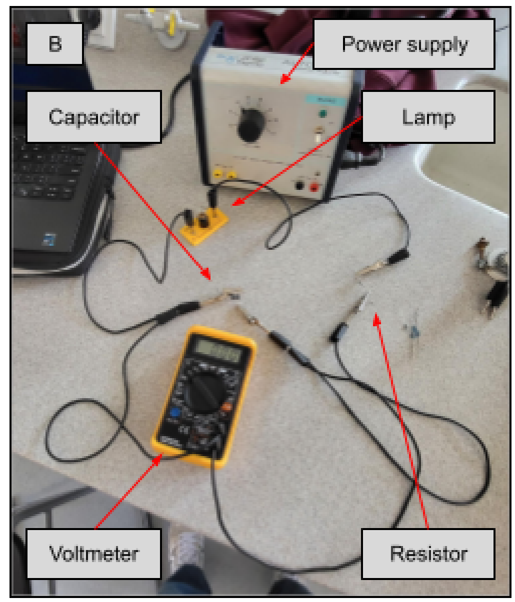


figure 5: Discharging circuit (loop B)

4. Test the circuit is connected correctly. Plug in the power supply, set it to a voltage of 4 volts, and turn on the power supply. The circuit is working successfully if the capacitor begins to charge and the potential difference across the capacitor increases.
5. Discharge the capacitor by connecting the branch containing the capacitor to the branch containing the lamp. Reconnect the circuit back to its charging loop as seen in *figure 3*.
6. The complete investigation setup can be referred to in *figure 4* and *figure 5* respectively.
7. Set up the video camera in a position where the voltmeter reading is clear.
8. Start the video recording.
9. Power on the charging circuit.
10. Once the voltmeter reading reaches a maximum stop the recording and power off the circuit.
11. Discharge the capacitor fully or until the voltmeter reads 0.00 V.
12. Repeat steps 8-11 five times with the same number of resistors connected in series to reduce random errors and to increase the precision of the data.
13. Repeat steps 8-12 each time connecting an additional resistor in series.
14. Record the data (potential difference per second) from the video recordings.

III. Experimental Analysis

During this investigation 2035 total measurements were recorded and for this reason full sets of data for all trials are not included in this paper. Instead, sample data gathered from the first 30 seconds of the trials are included. However, in *figure 6*, the graph represents all of the mean values calculated during the investigation. It should be noted that anomalous results were disregarded when calculating mean values.

*Data Table 1: Sample of mean potential difference across capacitor in 100 k***Ω** *circuit over a time period of 30 seconds*

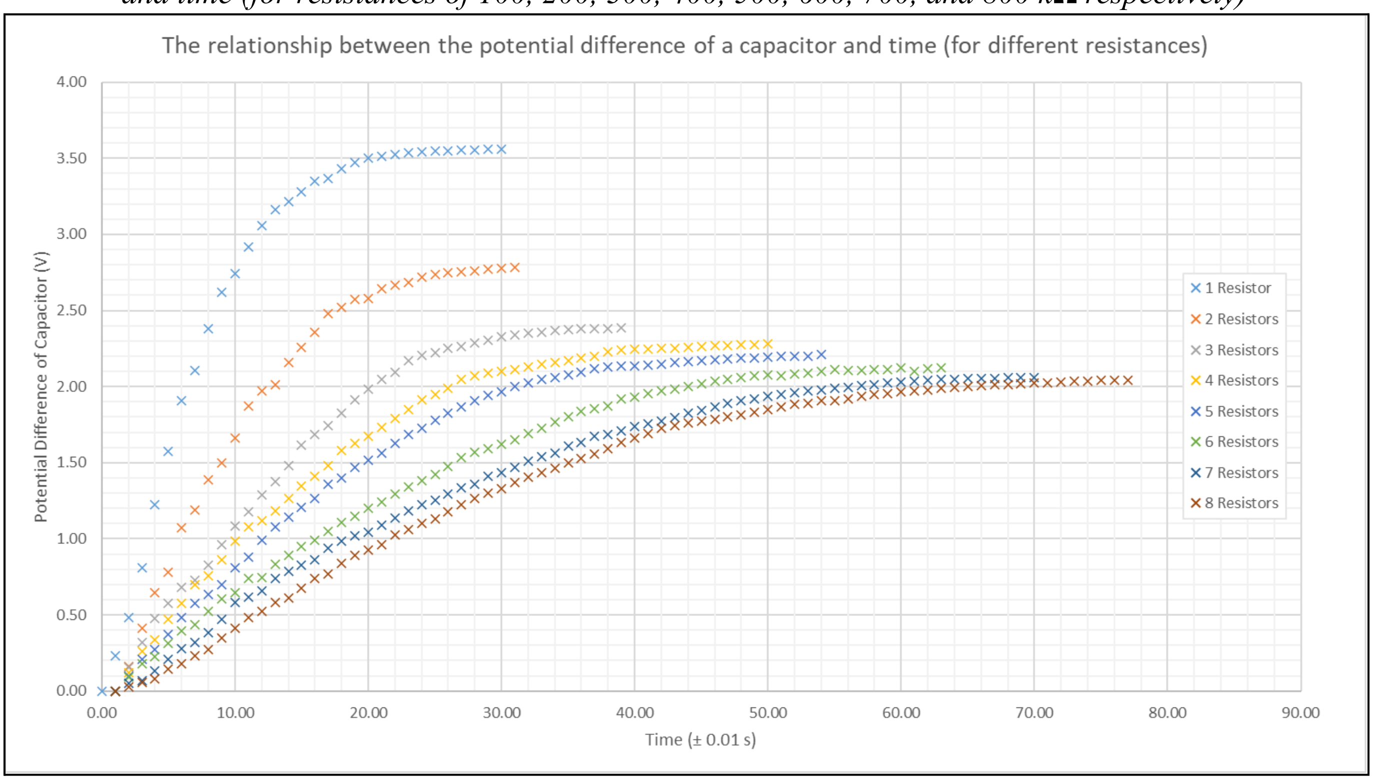
Time, t (± 0.01 s)	**Mean Potential Difference, V (± 0.01 V)**					
	Repeat 1	Repeat 2	Repeat 3	Repeat 4	Repeat 5	Mean
0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.80	0.81	0.80	0.82	0.83	0.81
6.00	1.91	1.89	1.90	1.92	1.92	1.91
9.00	2.61	2.62	2.65	2.60	2.61	2.62
12.00	3.05	3.08	3.06	3.05	3.05	3.06
15.00	3.28	3.29	3.30	3.27	3.26	3.28
18.00	3.43	3.44	3.44	3.43	3.43	3.43
21.00	3.52	3.51	3.52	3.52	3.51	3.52
24.00	3.55	3.55	3.54	3.54	3.54	3.54
27.00	3.56	3.56	3.55	3.56	3.55	3.56
30.00	3.56	3.57	3.56	3.56	3.56	3.56

Data Table 2: Sample of mean potential difference across capacitor for different resistances over a time period of 30 seconds

Time, t (± 0.01 s)	**Mean Potential Difference, V (± 0.01 V)**							
	1 Resistor	2 Resistors	3 Resistors	4 Resistors	5 Resistors	6 Resistors	7 Resistors	8 Resistors
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.81	0.64	0.48	0.34	0.27	0.23	0.13	0.08
6.00	1.91	1.19	0.73	0.70	0.58	0.44	0.32	0.23
9.00	2.62	1.66	1.09	0.99	0.81	0.64	0.58	0.41
12.00	3.06	2.01	1.37	1.19	1.08	0.84	0.74	0.58
15.00	3.28	2.36	1.69	1.41	1.27	0.99	0.86	0.74
18.00	3.43	2.57	1.92	1.63	1.47	1.15	1.02	0.89
21.00	3.52	2.66	2.10	1.79	1.63	1.29	1.14	1.03
24.00	3.54	2.74	2.22	1.95	1.78	1.43	1.25	1.13
27.00	3.56	2.76	2.29	2.07	1.91	1.57	1.36	1.27
30.00	3.56	2.79	2.34	2.11	2.00	1.65	1.47	1.37

** It should be noted that anomalous results were disregarded when calculating mean values*

figure 6: A graph to show the relationship between the mean potential difference of a capacitor and time (for resistances of 100, 200, 300, 400, 500, 600, 700, and 800 kΩ respectively)

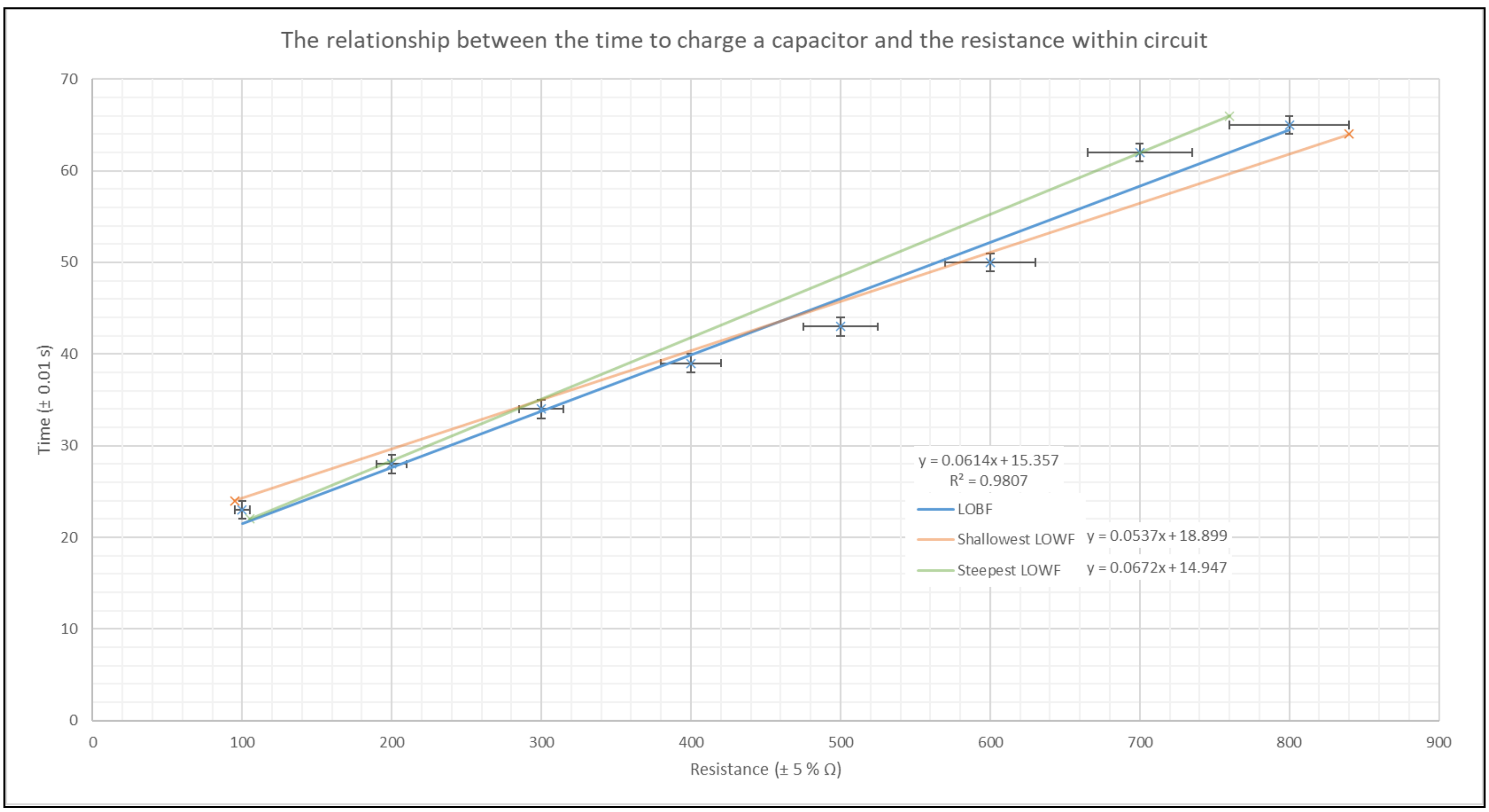


Data Table 3: The time taken to charge a capacitor for varying resistances and its uncertainties

Resistance, R (± 5 % Ω)	Time, t (s)	Time percentage uncertainty (%)	Time absolute uncertainty
100	23	0.0435	1.00
200	28	0.0357	1.00
300	34	0.0294	1.00
400	39	0.0256	1.00
500	43	0.0233	1.00
600	50	0.0200	1.00
700	62	0.0161	1.00
800	65	0.0154	1.00

** Measurements of the time when capacitor is charged is determined by presence of an asymptote*

figure 7: A graph to show the relationship between the time to charge a capacitor and the resistance within the circuit



Analysis

When analysing the graph of the potential difference across the capacitor against time (see *figure 6*) the trend of the gradient of the curve decreasing with time is present. Therefore it can be established that the rate of charging the capacitor decreases with time. This relationship holds for all of the resistances. Additionally, as visualised in *figure 6*, when resistors are added in series and the resistance increases, the maximum potential difference across the capacitor decreases at a decreasing rate from 3.56 to 2.04 ± 0.01 V. Moreover, as resistance increases, the time to reach a maximum potential difference also increases.

The linear data of the time for the capacitor to reach a maximum potential difference against the total resistance in the circuit was plotted onto a second graph (see *figure 7*). The equation of the line of best fit is $y = 0.0614x + 15.357$. The positive gradient of 0.0614 indicates a positive correlation between the variables. In other words, as resistance in the circuit increases, the time for a maximum potential difference across the capacitor to be reached also increases.

Considering the general trend in *figure 7* is linear, it can be determined that time is directly proportional to resistance. In this proportional relationship where $t \propto R$, a constant of proportionality can be calculated so that $t \propto kR$, where k is the constant of proportionality. In linear functions the constant of proportionality is equal to the gradient. Therefore, in this case, the constant of proportionality, k is equal to 0.0614 so that $t \propto 0.0614R$. In a circuit with a constant capacitance, as resistance increases, the time to charge the capacitor also increases. This aligns with the theory of the time constant, $\tau = RC$, which is a measure of the rate of charging.

To determine the reliability of the graphs, the percentage uncertainty of the gradient of the line of best fit must be calculated. For this reason, the shallowest and steepest lines of worst fit with the equations of $y = 0.0537x + 18.899$ and $y = 0.0672x + 14.947$ respectively are plotted in *figure 7*. The percentage uncertainty of the gradient can be calculated as follows:

$$\% \, Unc.(m) = \frac{m(steepest\ LOWF) - m(shallowest\ LOWF)}{2} / m(LOBF) \cdot 100\% = \frac{0.0672 - 0.0537}{2} / 0.0614 \cdot 100\% \approx 11.0\,\%$$

The percentage uncertainty of the line of best fit is 11.0 %.

Another method to determine the reliability of the relationship is using the coefficient of determination (R^2), which is a measure that reflects the ability of a linear regression model to predict an outcome. In *figure 7* the value for the coefficient of determination is equal to 0.9807.

It should be noted that within the complete collection of data there were 22 anomalous results present. This is relatively low as out of the 2035 total measurements recorded only approximately 1.08 % of them were anomalous. Therefore, they did not significantly impact the mean values.

Conclusion

The aim of this investigation is to determine how resistance affects the rate at which a capacitor charges. As calculated in the *Analysis*, the constant of proportionality between time and resistance is equal to 0.0614. This relationship indicates that as the resistance in the circuit increases, the time to reach a maximum potential difference across the capacitor increases, and therefore the time to charge the capacitor also increases. This is because as resistance increases, the flow of electrons, i.e. current decreases causing the capacitor to take a longer time to charge and overall decreasing the rate of charging the capacitor.

The hypothesis for this investigation predicted a proportional relationship between resistance and the time to charge a capacitor. As explained above, this relationship holds and therefore the hypothesis is accepted. In addition, these results are consistent with previous scientific studies which overall confirm the findings in this investigation. The results of these studies are explained in detail in textbooks such as Physics for the IB Diploma (Tsokos, K., A., 2014).

IV. Furtherance

Evaluation

To effectively evaluate the investigation, strengths and weaknesses in the method must be identified, assessed, and improved.

A significant strength within the investigation is the low uncertainty values throughout the measurements of both time and resistance which led to a relatively low overall uncertainty on the line of best fit of 11.0 %. Low uncertainty values indicate precise results and the absence of many systematic errors. They also give direct feedback on the choice of equipment used to measure readings. In this case, low uncertainties verify that the appropriate equipment was chosen. A prime example of this is the measurements for time. The low uncertainty values of time can be visualised in *figure 7* within the magnitude of the vertical error bars. The mode uncertainty of time is approximately 0.02 %, this very low uncertainty can be attributed to the timer in the video recording which had an uncertainty of ± 0.01 seconds. Furthermore, with the use of a video recording the effects of the random error due to human reaction time is also decreased significantly. An improvement to the method to completely eliminate human error and to further decrease uncertainties would be to utilise a datalogger to collect data automatically. Additionally, the use of an accurately calibrated datalogger would decrease the amount of anomalous results present in the data. Approximately 1 % of the measurements recorded were determined to be anomalous which is low considering the number of trials and repeats. For this reason, these anomalous results did not have a significant impact on the investigation, however they still could have impacted the results to a limited extent. The amount of trials and repeats in this investigation is a notable strength within the method.

Another notable strength present in the investigation is the value for the coefficient of determination which is 0.9807. This value is very high and falls within the accepted range where a maximum value of 1 indicates that the line of best fit perfectly fits the data. (Britannica, 2022) Therefore, the line of best fit that represents the data is reliable. This evaluation is strengthened by the relatively low uncertainty of the line of best fit of 11.0 %.

On the other hand, there were a number of weaknesses present in the method of the investigation. For example, a digital multimeter was set to measure voltage which updates between 2 and 4 times a second. This update rate may impact the results negatively as the displayed value could lag behind the actual value. This is a systematic error that causes inaccurate results. To remove the possibility of this systematic error either an analogue voltmeter could be used or a datalogger.

Another weakness in the investigation is the rising temperature of the capacitor even after waiting sufficient time for the surface of the capacitor to cool. The temperature of the capacitor, even if it does not exceed the recommended temperature limit, could affect the results negatively as it increases. For this reason, to improve the method and the accuracy of the results, a capacitor of the same capacitance could be investigated and results can be compared to determine if this control variable had any effect on the outcome.

Overall, the investigation yielded precise and accurate results as verified by the low uncertainty of the line of best fit of 11.0 % as well as by the value of the coefficient of determination of 0.9807. With the implementation of the suggested improvements, weaknesses in the investigation will be avoided. This will reduce the overall impact of systematic and random errors further bolstering the integrity of the results from the investigation.

Further Discussion

To further extend the research conducted in this investigation, it would be interesting to explore the effects resistance has on different types of capacitors, for example, in ceramic, electrolytic, variable, or mica capacitors and determine the optimal capacitor for specific uses. This could be completed by switching the capacitors in the same circuit and by comparing and contrasting the results. Another extension of this research is adding multiple capacitors to the circuit in different configurations such as in series and in parallel to determine the best configuration to charge multiple capacitors the fastest. Additionally, it would be interesting to investigate how long a capacitor is able to store electrical energy and how this links to capacitors real world applications.

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