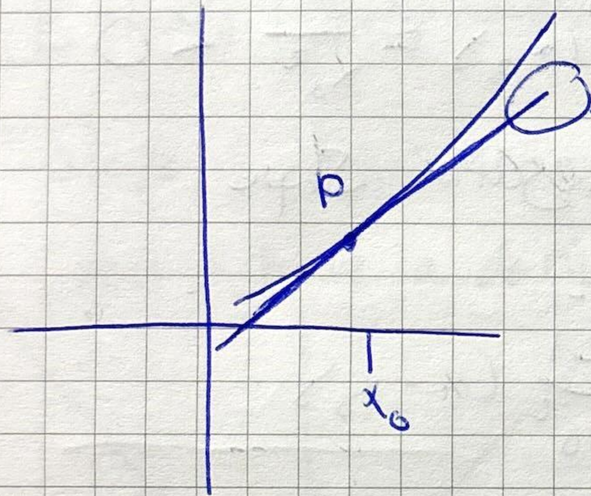
What is a derivative?

A.
- Geometric interpretation
- Physical interpretation
- Importance of derivatives to all measurements

B. How to differentiate Anything

Geometric interpretation:



$\boxed{y - y_0 = m(x - x_0)}$

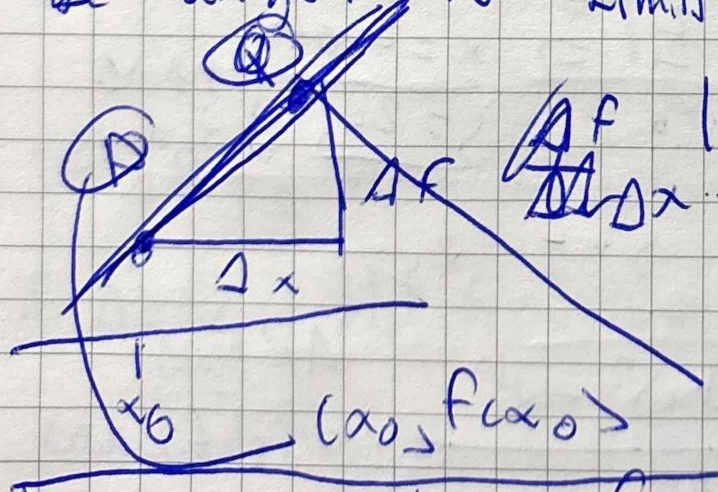
Point $y_0 = f(x_0)$

slope $m = f'(x_0)$

$f'(x_0)$ the derivative of f at the x_0 is the slope of the tangent line to $y = f(x)$ at P.

P fixed

Δ Tangent line = Limits of Secant Lines PQ as $Q \to P$



$$\lim_{\Delta x \to 0} \underbrace{\frac{\Delta f}{\Delta x}}_{\text{slope of secant PQ}} = m$$

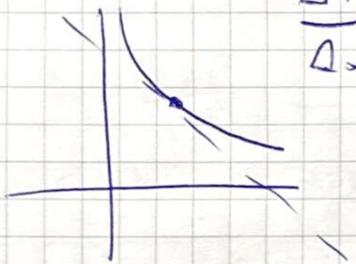
slope of tangent.

$(x_0, f(x_0))$ $(x_0 + \Delta x, f(x_0 + \Delta x))$

$$\boxed{\underbrace{f'(x_0)}_{=m} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}}$$

IMPORTANT

$f(x) = \frac{1}{x}$



$$\frac{\Delta f}{\Delta x} = \frac{\frac{1}{x_0+\Delta x} - \frac{1}{x_0}}{\Delta x} = \frac{1}{\Delta x}\left(\frac{x_0 - (x_0+\Delta x)}{(x_0+\Delta x)x_0}\right)$$

difference quotient

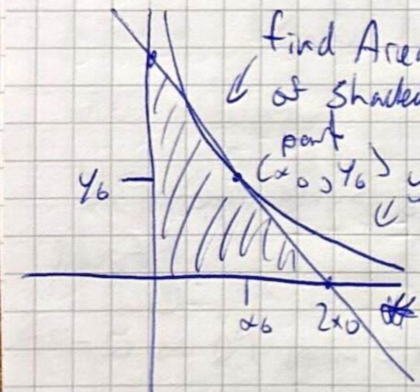
$$\frac{1}{\Delta x}\left(\frac{-\Delta x}{(x_0+\Delta x)x_0}\right)$$

$$= \frac{-1}{(x_0+\Delta x)x_0} \xrightarrow[\Delta x \to 0]{} = \frac{-1}{x_0^2}$$

$$f'(x_0) = \frac{-1}{x_0^2} < 0$$

negative slope

$x \to \infty$ less steep



$$y - y_0 = \frac{-1}{x_0^2}(x - x_0)$$

$x;\ y = 0$

$$0 - y_0 = \frac{-1}{x_0^2}(x - x_0)$$

$$0 = \frac{-1}{x_0^2}(x - x_0) + \frac{1}{x_0}$$

$$\frac{x}{x_0^2} = \frac{2}{x_0} \Rightarrow \left.\begin{array}{l} x = 2x_0 \\ y = 2y_0 \end{array}\right\} \text{using symmetry}$$

$$\frac{1}{2}(2x_0)(2y_0) = 2x_0y_0 = 2$$

$f(x) = x^n$, $n = 1, 2, 3 \ldots$ $y = \frac{1}{x}$ $xy = 1$

$$\frac{d}{dx}x^n = ? \rightarrow \frac{(x+\Delta x)^n - x^n}{\Delta x}$$

Δx moves
x is fixed

$$(x+\Delta x)^n \rightarrow x^n + nx^{n-1}\Delta x + O(\Delta x^2)$$

$$\frac{\cancel{x^n} + nx^{n-1}\Delta x - \cancel{x^n}}{\Delta x} \qquad \frac{\Delta f}{\Delta x} = \frac{1}{\Delta x}\left((x+\Delta x)^2 - x^n\right)$$

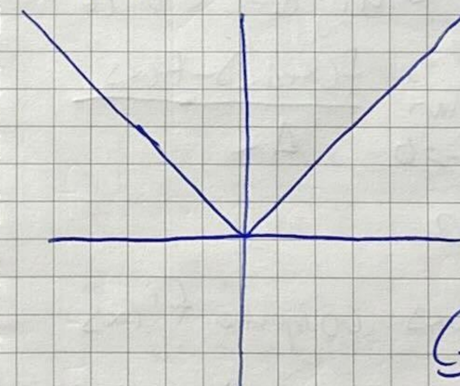
$$\frac{1}{\Delta x}\left(nx^{n-1}\Delta x + O(\Delta x)^2\right) \Rightarrow nx^{n-1} + O(\Delta x)$$

$$\lim_{\Delta x \to 0} nx^{n-1} + O(\Delta x) \rightarrow nx^{n-1} \qquad \frac{d}{dx}x^n = nx^{n-1}$$

Extends to polynomials so $\frac{d}{dx}(x^3 + 5x^{10}) = 3x^2 + 50x^9$

slope of $f(x) = |x|$ changes when $x = 0$

does it have a derivative



$f(x) = |x|$

$$f'(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

$$f'(0) = \frac{f(0+\Delta x) - f(0)}{\Delta x}$$

$$\lim_{x \to 0} \frac{f(\Delta x)}{\Delta x} \leftarrow \text{slope of secant line}$$

if $\Delta x > 0$ then $f(\Delta x) = \Delta x \rightarrow \frac{f(\Delta x)}{\Delta x} = 1$

if $\Delta x < 0$ then $f(\Delta x) = -1\Delta x \rightarrow \frac{f(\Delta x)}{\Delta x} = -1$

can't differentiate at $x = 0$

$f(x) = |x|$, $f'(0)$ is not defined

1) a) $-0.5 = 0.53$, $-0.25 = 0.16$, $0.25 = -0.41$

$0.5 = -0.59$

b) $m \approx -0.16$

c) $\Delta x = -0.01$, or $+0.01$

2) a) $-0.5 = -0.88$, $-0.25 = -0.97$, $0.25 = -0.97$

$0.5 = -0.88$

b) $m \approx -1.0$

c) $-0.46 \leq \Delta x \leq 0.46$

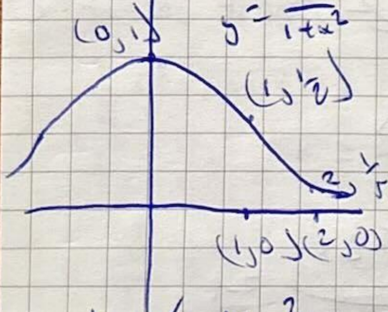
3) a) $-0.5 = -0.59$, $-0.25 = -0.41$, $0.25 = 0.16$

$0.5 = 0.53$

b) $m \approx -0.16$ $\quad f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

c) $-0.1 \leq \Delta x \leq 0.08$ $\quad \Delta x \neq 0$

4) $f(x) = \frac{1}{1+x^2}$ $\quad y = f(x)$ compute $f'(x)$



$$= \lim_{\Delta x \to 0} \frac{\frac{1}{1+(x+\Delta x)^2} - \frac{1}{1+x^2}}{\Delta x}$$

$$\frac{1}{\Delta x}\left(\frac{1}{1+(x+\Delta x)^2} - \frac{1}{1+x^2}\right)$$

$$\frac{1}{\Delta x}\left(\frac{1+x^2}{(1+x^2)(1+(x+\Delta x))^2} - \frac{1+(x+\Delta x)^2}{(1+x^2)(1+(x+\Delta x))^2}\right)$$

$$\frac{1}{\Delta x}\left(\frac{(1+x^2) - (1+(x+\Delta x)^2)}{(1+x^2)(1+(x+\Delta x)^2)}\right)$$

	x	$+\Delta x$
x	x^2	$x\Delta x$
Δx	$x\Delta x$	Δx^2

$\to \cancel{x^2} + 2x\Delta x + \Delta x^2$

$$\frac{\cancel{x^2} - (x+\Delta x)^2}{(1+x^2)(1+(x+\Delta x)^2)} \qquad \frac{-2x\Delta x - \Delta x^2}{(1+x^2)(1+x^2+2x\Delta x+\Delta x^2)}$$

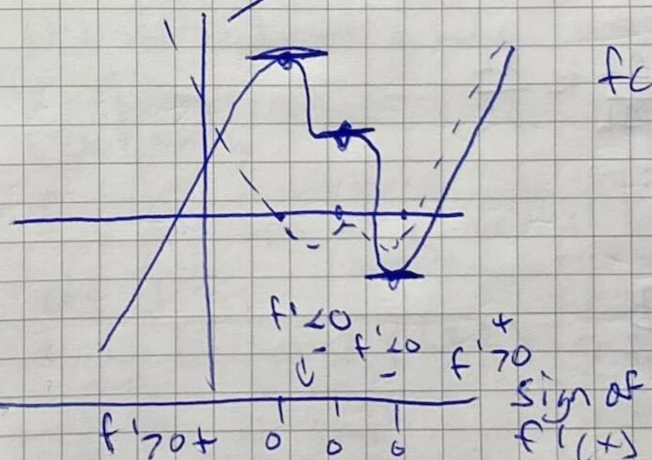
$$\cancel{\frac{-2x\Delta x + \Delta x^2}{1+2x^2+x^4+2x\Delta x+2x^3\Delta x+\Delta x^2+x^2\Delta x^2}}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\cancel{\Delta x}}\left(\frac{-2x\cancel{\Delta x} - \Delta x^{\cancel{2}}}{(1+x^2)(1+(x+\Delta x)^2)}\right)$$

$$= \lim_{\Delta x \to 0} \frac{-2x - \cancel{\Delta x}}{(1+x^2)(1+(x+\cancel{\Delta x})^2)}$$

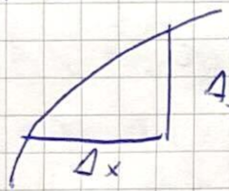
$$= \frac{-2x}{(1+x^2)^2} = \frac{d}{dx}\left(\frac{1}{1+x^2}\right)$$

$f(x) + c = g(x)$



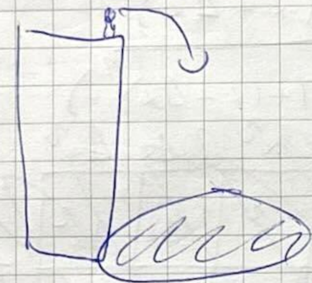
Derivative = Slope of tangent line

Rate of change as interpretation of derivatives



$\frac{\Delta y}{\Delta x} \to \frac{dy}{dx}$ instantaneous change

avg change



q = charge $\frac{dq}{dt}$ = current

s = distance, $\frac{ds}{dt}$ = speed

$h = 80m$ $\quad 80 - 5t^2$ $\quad$ at $t = 4$ $h = 0$

Avg speedchange $\frac{\Delta h}{\Delta t} = \frac{0-80}{4-0} = -20\,ms^{-1}$

instantaneous speed $= \frac{d}{dt} h = 0 - 10t$

$\frac{d}{dt} 80 = 0 \quad \frac{d}{dt} \cancel{10t}\, 5t^2 = 10t$

$t = 4$, $h' = -40\,ms^{-1}$ $\quad \frac{dT}{dx}$ = temperature gradient

Error in h: Δh

ΔL is estimated by $\frac{\Delta L}{\Delta h} \approx \frac{dL}{dh}$

Limits + Continuity:

1. Easy limit

$\lim_{x \to 4} \frac{x+3}{x^2+1} = \frac{4+3}{4^2+1} = \frac{7}{17}$

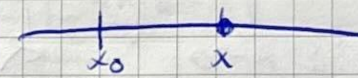
2. Derivatives are always harder:

$\lim_{x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{x - x_0}$ $\quad x = x_0$ gives $\frac{0}{0}$

need cancellation to make sense

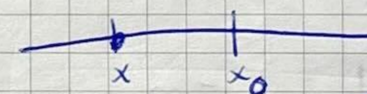
$\lim_{x \to x_0^+} f(x)$ = right-hand limit

$\begin{bmatrix} x \to x_0 \\ x > x_0 \end{bmatrix}$

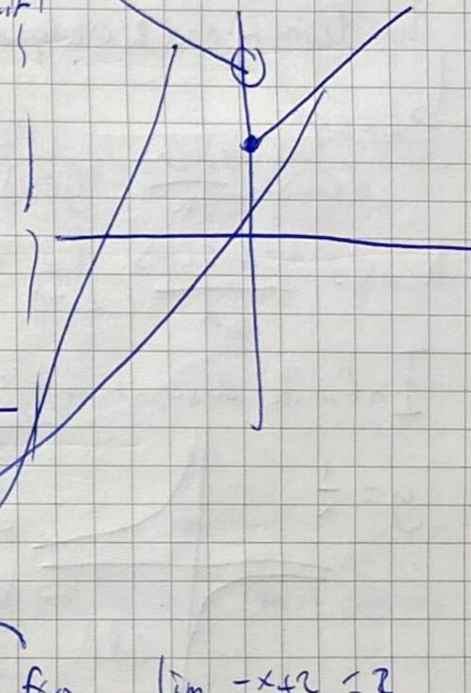


$\lim_{x \to x_0^-} f(x)$ = left-hand limit

$\begin{bmatrix} x \to x_0 \\ x < x_0 \end{bmatrix}$



$f(x) = \begin{cases} x+1, & x > 0 \\ -x+2, & x < 0 \end{cases}$



$\lim_{x \to 0^+} f(x) = \lim_{x \to 0} x+1 = 1$ $\quad \lim_{x \to 0^-} f(x) = \lim_{x \to 0} -x+2 = 2$

Did not need $x < 0$ value

Continuity

f is continuous at x_0 means

Definition

$\boxed{\lim_{x \to x_0} f(x) = f(x_0)}$ $\quad x \neq x_0$

cts at x_0:

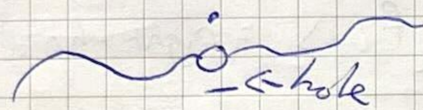
1. $\lim_{x \to x_0} f(x)$ exists $(L = R)$
2. $f(x_0)$ is defined
3. they are equal

Jump discontinuity:

lim from L and right exist but are <u>not</u> equal

Removable discontinuity:

lim from L and R are equal



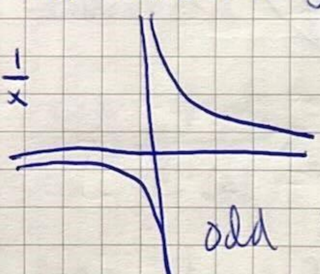
↳ Examples

$g(x) = \frac{\sin(x)}{x}$ $\quad g(0) = ??$ $\quad \lim_{x \to 0} \frac{\sin x}{x} = 1$

$h(x) = \frac{1-\cos(x)}{x}$ $\quad \lim_{x \to 0} \frac{1-\cos(x)}{x} = 0$ ← removable discontinuity at $x=0$

Infinite discontinuity:

$y = \frac{1}{x}$

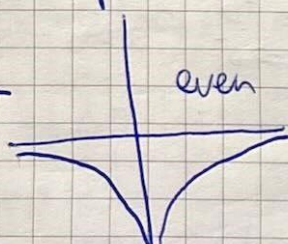


odd

$\lim_{x \to 0^+} \frac{1}{x} = \infty$

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

$y' = \frac{-1}{x^2}$



even

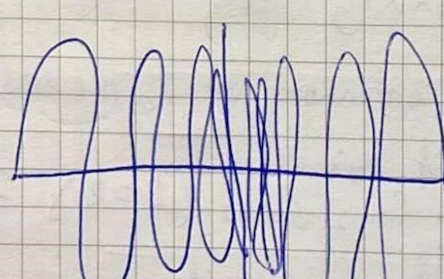
$\lim_{x \to 0} \frac{-1}{x^2} = -\infty$

for both $x \to 0^+$ and $x \to 0^-$

Other discontinuities:

$y = \sin \frac{1}{x}$ as $x \to 0$

no L or R limit →



Theorem (Diff => Cts)

if f is differentiable at x_0 then f is continuous at x_0

Proof:

$\lim_{x \to x_0} f(x) - f(x_0) = 0$

$x \neq x_0$

$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} (x - x_0) = f'(x_0) \cdot 0 = 0$

Problems:

1) $f(t) = -5(t)^2 + 60(t)^2 + 120$ \$

$f'(t) = -10t + 60$

$f'(1) = -10(1) + 60 = 50$ \$

The account balance in February will be larger than in January

2) $f'(10) = -10(10) + 60 = -40$ \$

No the account balance increases up to June and then starts decreasing because June is the 6th month so $f'(6) = 0$

3) The greatest month will be June because that is when the change is 0 due to $f'(6) = 0$, up until this point the balance was increasing each month but goes down from here, s?? of the gradient flips (March = 3) = 255 \$

then $f(3) = 30$ 255 $\quad f'(4) = 20 \pm 305$ $\quad f(5) = ~~315~~ 10$ 315 \$

$$f(x) = \begin{cases} x^2+1 & \text{if } x>1 \\ ax+b & \text{if } x \le 1 \end{cases}$$

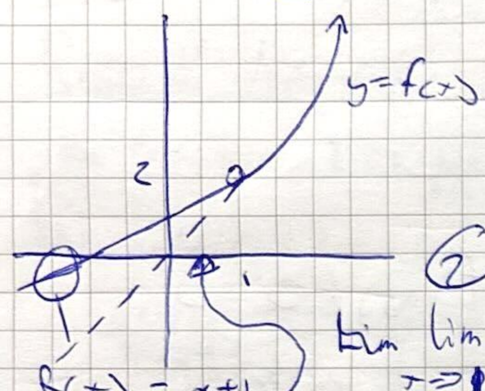
① continuous:

$f(1) = 2$

$(1, 2)$

$2 = a \cdot 1 + b$

$\boxed{2 = a+b}$



$f(x) = x+1$

only time when differentiable is $f(x) = 2x+0$ only

② differentiable:

$\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^+} f'(x)$

$\lim_{x\to 1^-} a = \lim_{x\to 1^+} 2x = 2$

$a=2$

$$f(x) = \begin{cases} ax+b, & x>0 \\ \sin 2x, & x \le 0 \end{cases}$$

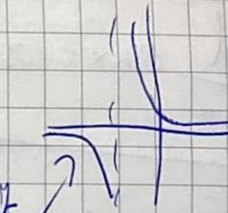
continuous:

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

$\lim_{x\to 0^+} \cancel{ax}+b = \lim b \qquad \lim_{x\to 0^-} \sin 2x = 0$

$b=0$ so $a \neq 0$ $\quad a \le 0$ because we don't want a good tangent at $x=0$ with $y=\sin 2x$ has a positive gradient will make sure it is differentiable.

Limits + Discontinuity Problems:



1. $\lim_{x\to 0} \sqrt{x}$ ← only for positive values

$\lim_{x\to 0^+} \sqrt{x} = 0$ so one sided right hand limit

2. $\lim_{x\to -1} \frac{1}{x+1} \qquad \lim_{x\to -1^-} \frac{1}{x+1} = -\infty \qquad \lim_{x\to -1^+} \frac{1}{x+1} = \infty$

The answer to the limits is different so one sided limit

3. $\lim_{x\to 0} |\sin x| \qquad \lim_{x\to 0^-} |\sin x| = 0 \qquad \lim_{x\to 0^+} |\sin x| = 0$

$\lim_{x\to 0^-} |\sin x| = \lim_{x\to 0^+} |\sin x|$ so double sided limit can be used

4. $\lim_{x\to 1} \frac{1}{(x-1)^4} \qquad \lim_{x\to 1^-} \frac{1}{(x-1)^4} = \infty \qquad \lim_{x\to 1^+} \frac{1}{(x-1)^4} = \infty$ so double can be used

5. $\lim_{x\to 0} \frac{|x|}{x} \qquad \lim_{x\to 0^-} \frac{|x|}{x} = -1 \qquad \lim_{x\to 0^+} \frac{|x|}{x} = 1$ so one sided limit is used.

Derivative Formulas:

Specific: $f'(x) \quad f(x) = x^n, \frac{1}{x}$

General: $(u+v)' = u'+v' \quad (cu)' = cu'$ c constant

need both kinds for polynomials

$$\frac{d}{dx}\sin x \quad \frac{\sin(x+\Delta x) - \sin(x)}{\Delta x} = \frac{\overbrace{\sin x \cos \Delta x + \cos x \sin \Delta x}^{=\cos x} - \sin x}{\Delta x}$$

$$\frac{d}{dx} \sin x \left(\frac{\cos \Delta x - 1}{\Delta x}\right) + \cos x \left(\frac{\sin \Delta x}{\Delta x}\right)$$

$\Delta x \to 0 \to 0$

$$\boxed{\frac{d}{dx}\sin x = \cos x} \qquad \frac{\cos(x+\Delta x) - \cos(x)}{\Delta x}$$

$$\frac{\cos x\cos\Delta x - \sin x\sin\Delta x - \cos(x)}{\Delta x}$$

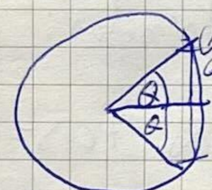
$$\cos x\left(\frac{\cos\Delta x - 1}{\Delta x}\right) + (-\sin x)\left(\frac{\sin\Delta x}{\Delta x}\right)$$

$$\boxed{\frac{d}{dx}\cos x = -\sin x} \qquad \frac{d}{dx}\cos x\Big|_{x=0} = \lim_{\Delta x\to 0}\frac{\cos(\Delta x)-1}{\Delta x} = 0$$

$$\frac{d}{dx}\sin x\Big|_{x=0} = \lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x} = 1$$

Derivatives of sine and cosine at $x=0$ give all values of

$\frac{d}{dx}\sin x$, $\frac{d}{dx}\cos x$



arc length θ

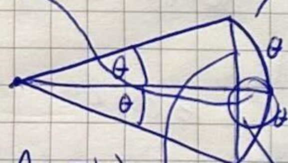
$\sin\theta$

$\frac{2\sin\theta}{2\theta}$ $\frac{\sin\theta}{\theta}$ $\theta \to 0$

Geometric proof : $\Delta x \to 0$

$\frac{1-\cos\theta}{\theta}$

$= -1\left(\frac{\cos\Delta x - 1}{\Delta x}\right) = 0$



short curves are nearly straight

radians

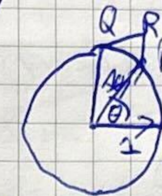
os $\theta \to 0$

$= 1 - \cos\theta \to 0$

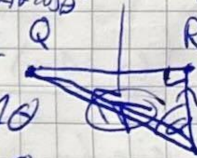
relation is the tending

$$\frac{d}{dx}\sin\theta = \cos\theta \text{ for all values of } \theta$$

$y = \sin\theta$ vertical position of circular motion



$\Delta y = PR \approx \Delta\theta\cos\theta$ practically the straight segment = curved segment



$PQ \approx 0P$ $\quad PQ = \Delta\theta$

PR is vertical $\quad \angle QPR \approx \theta$

$\frac{dy}{d\theta}$ $\quad \frac{\Delta y}{\Delta\theta} = \cos\theta \qquad \lim_{\Delta\theta\to 0}\frac{\Delta y}{\Delta\theta} = \cos\theta$

General rules:

Product rule:

$(uv)' = u'v + uv'$ ← change one at a time

Quotient Rule:

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ $\quad v \neq 0$ $\quad$ Session 6

Constant multiple rule:

$$\frac{d}{dx}(c\cdot f(x)) = c\cdot\frac{d}{dx}(f(x))$$

$$\frac{d}{dx}(3x^2) = 3\cdot\frac{d}{dx}(x^2) = 6x$$

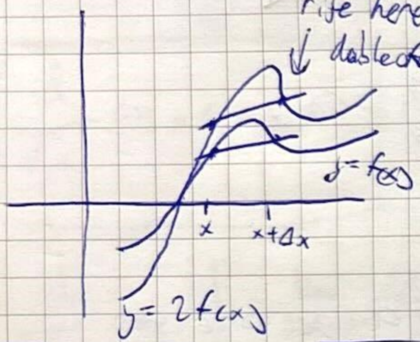
Proof next page ↓

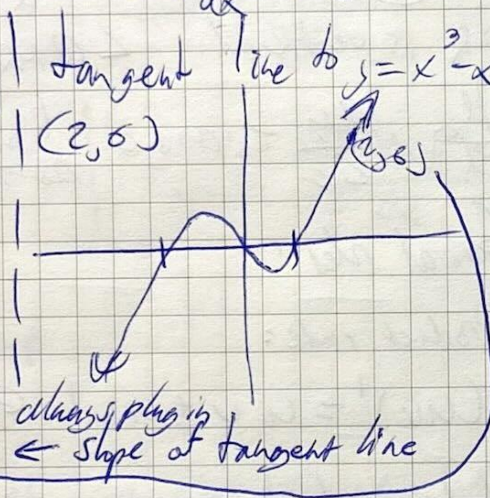
$$\frac{d}{dx}(c\cdot f(x)) = \lim_{\Delta x \to 0} \frac{cf(x+\Delta x) - cf(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} c\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)$$

definition of derivative

$$= c \cdot \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = c \cdot \frac{d}{dx} f(x)$$





$y' = 3x^2 - 1$ $\quad y'(2) = 11$ ← slope of tangent line

$y - 6 = 11(x-2) \to y - 6 = 11x - 22 \quad y = 11x - 16$

$h(x) = \sin x + \sqrt{3}\cos x = R\sin(x+\alpha)$

$1\sin x + \sqrt{3}\cos x = R\sin x\cos\alpha + R\cos x\sin\alpha$

$R\cos\alpha = 1 \qquad R\sin\alpha = \sqrt{3}$

$R = \frac{1}{\cos\alpha} \qquad R = \frac{\sqrt{3}}{\sin\alpha}$

$\frac{1}{\cos\alpha} = \frac{\sqrt{3}}{\sin\alpha} \qquad \sin\alpha = \sqrt{3}\cos\alpha$

$\tan\alpha = \sqrt{3} \qquad \alpha = \arctan(\sqrt{3}) \quad \alpha = \frac{1}{3}\pi$

$R = \frac{1}{\cos(\frac{1}{3}\pi)} \quad R = 2 \quad h(x) = 2\sin(x + \frac{1}{3}\pi)$

$\frac{d}{dx} 2\sin(x + \frac{1}{3}\pi) \to 2\frac{d}{dx}\sin(x+\frac{1}{3}\pi)$

$h'(x) = \cos x - \sqrt{3}\sin x = 0 \quad \cos x = \sqrt{3}\sin x$ Could draw here

$\frac{\cos x}{\sin x} = \sqrt{3} \quad \frac{1}{\tan x} = \sqrt{3} \quad \sqrt{3}\tan x = 1 \quad \tan x = \frac{1}{\sqrt{3}}$

$x = \arctan(\frac{1}{\sqrt{3}}) = \frac{1}{6}\pi, \frac{7\pi}{6}$ ... session 8

Chem Revision

CH_3COCH_3

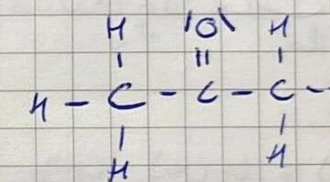
4 3 4 6 4 3

7 10 7

~~11~~ 24

12

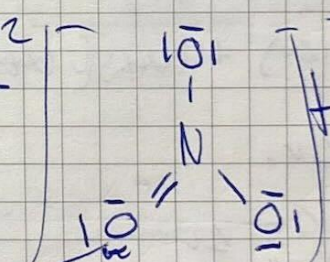
this ~~guy is~~ one is polar so dipole



NO_3^-

5 18

24 12



trigonal planar

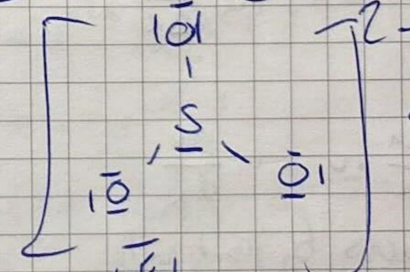
120°

SO_3^{2-}

6 18

24 26

13



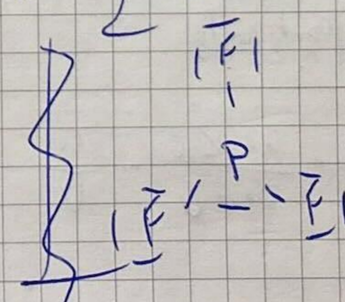
trigonal pyramidal

107°

PF_3

5 21

26



trigonal pyramidal

107°

ClF_2^+

$|\overline{F} - \overline{Cl} - \overline{F}|$

~~20~~ 21

20 10

Session 9:

$\mathrm{Sinc}(x) = \frac{\sin(x)}{x} = \frac{1}{x} \cdot \sin(x)$

$(u+v)' = u' + v'$

$(uv)' = u'v + uv'$ ⇐ Product rule

$\frac{d}{dx}(x^n \sin(x)) = nx^{n-1}\sin(x) + x^n \cos(x)$

$\Delta(uv)$

$= u(x+\Delta x)v(x+\Delta x) - u(x)v(x)$

$= (u(x+\Delta x) - u(x))v(x+\Delta x) + u(x)(v(x+\Delta x) - v(x))$

$= (\Delta u)v(x+\Delta x) + u(x)\Delta v.$

$\frac{\Delta(uv)}{\Delta x} = \frac{\Delta u}{\Delta x}v(x+\Delta x) + u\frac{\Delta v}{\Delta x}$

$\Delta x \to 0$

$\frac{d(uv)}{dx} = \frac{du}{dx}\cdot v + u\frac{dv}{dx}$

$v(x+\Delta x) \to v(x)$ by continuity

Quotient Rule:

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ $\quad \Delta\left(\frac{u}{v}\right) = \frac{u+\Delta u}{v+\Delta v} - \frac{u}{v}$

$= \frac{\cancel{uv} + (\Delta u)v - \cancel{uv} - (\Delta v)u}{v^2 + v(\Delta v)}$

$= \frac{(\Delta u)v - (\Delta v)u}{v^2 + v(\Delta v)}$ $\quad \frac{\Delta\left(\frac{u}{v}\right)}{\Delta x}$

$\hookrightarrow \frac{\frac{\Delta u}{\Delta x}v - \frac{\Delta v}{\Delta x}u}{v^2 + \cancel{v(\Delta v)}}$ $\quad \lim_{\Delta x \to 0}$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}\cdot v - \frac{dv}{dx}u}{v^2}$

Example $u=1$

$\frac{d}{dx}\left(\frac{1}{v}\right) = \frac{-1\cdot v'}{v^2} = -v^{-2}v'$

Sub example:

~~eg~~ $u=1, v=x^n$

$\frac{d}{dx}\left(\frac{1}{x^n}\right) = -x^{-2n}\cdot nx^{n-1} \to \frac{d}{dx}x^{(-n)}$

$= -nx^{-n-1}$

Composition / Chain Rule: Δ

$y = (\sin t)^{10} \quad x = \sin t \quad y = x^{10}$

$\frac{\Delta y}{\Delta t} = \frac{\Delta y}{\Delta x}\cdot\frac{\Delta x}{\Delta t} \to \frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$

$\Delta t \to 0$ Differentiation of a composition is a product

Example:

$$\frac{d}{dt}(\sin(t))^{10} \quad \text{inside } x = \sin(t) \quad \text{outside } y = x^{10}$$

$= 10x^9 \cdot \cos(t)$

$= 10(\sin(t))^9 \cos(t)$

$= 10 \sin^9(t) \cos(t)$

Example 2:

$$\frac{d}{dt}\sin(10t) \quad x = 10t \quad y = \sin(x)$$

$\frac{dy}{dx} = \cos(x) \cdot 10$

$= 10\cos(10t)$

Higher Derivatives:

$u = u(x) \quad u' \quad u'' = (u')' \quad u''' = (u'')'$

$u = \sin x, \; u' = \cos(x), \; u'' = -\sin(x), \; u''' = -\cos(x)$

$u'''' = \sin x$

$(uv)' = u'v + v'u$

$(uvw)' = u' + vw + (vw)'u$

$= u'vw + (v'w + w'v)u$

$= u'vw + v'uw + w'uv$

Example $f(x) = x^2 \sin x \cos x \quad f'(x)$

$(x^2 \sin x \cos x)' = 2x \sin x \cos x + \cos^2 x \, x^2 - \sin^2 x \, x^2$

Worksheet

a)
$$f(x) = \begin{cases} ax^2 + bx + 6, & x \le 0 \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6, & x > 0 \end{cases}$$

b)
$$f(x) = \begin{cases} ax^2 + bx + 6, & x \le 1 \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6, & x > 1 \end{cases}$$

a) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} ax^2 + bx + 6$

$= 6$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 2x^5 + 3x^4 + 4x^2 + 5x + 6$

$= 6$

$$f'(x) = \begin{cases} 2ax + b & x < 0 \\ 10x^4 + 12x^3 + 8x + 5 & x > 0 \end{cases}$$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 2ax + b$

$= b$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} 10x^4 + 12x^3 + 8x + 5$

$= 5$

$b = 5$

$a =$ any real number

b) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} ax^2 + bx + 6$

$= a+b+6$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2x^5+3x^4+4x^2+5x+6$$

$= 2+3+4+5+6$

$= 20$

$a+b+6 = 20$

$a+b = 14$

$$f'(x) = \begin{cases} 2ax+b & x<1 \\ 10x^4+12x^3+8x+5 & x>1 \end{cases}$$

$$\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} 2ax+b$$

$= 2a+b$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 10x^4+12x^3+8x+5$$

$= 35$

$2a+b = 35$

$b = 14-a$ $\quad 2a+(14-a) = 35$

$a+14 = 35$

$a = 21$

$b = -7$

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2} \qquad \frac{d}{dx}(\tan x) \quad \tan x = \frac{\sin x}{\cos x}$$

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) \to \left(\frac{\sin(x)}{(\cos x)}\right)' = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)}$$

$$\frac{d}{dx}(\tan x) = \sec^2(x) \qquad \frac{d}{}\frac{\cos^2(x)}{\cos^2(x)} + \frac{\sin^2(x)}{\cos^2 x} \to 1+\tan^2(x)$$

$\sec^2(x)$

Practice:

a) $\cancel{\frac{}{x+1}} = \frac{d}{dx}\left(\frac{x^2}{x+1}\right) = \frac{2x(x+1)-1(x^2)}{(x+1)^2}$

$$\frac{2x^2+2x-x^2}{x^2+2x+1} \to \frac{x^2+2x}{x^2+x^2+1} = \frac{d}{dx}\left(\frac{x^2}{x+1}\right)$$

b) $\frac{d}{dx}\left(\frac{x^4+1}{x^2}\right) = \frac{4x^3(x^2)-2x(x^4+1)}{x^4} \to \frac{4x^5-2x^5-2x}{x^4}$

$$\frac{2x^5-2x}{x^4} \to \frac{2x^4-2}{x^3} = \frac{d}{dx}\left(\frac{x^4+1}{x^2}\right)$$

c) $\frac{d}{d(x)}\left(\frac{\sin(x)}{x}\right) \to \frac{\cos(x)(x)-1(\sin(x))}{x^2}$

$$\frac{x\cos(x)-\sin(x)}{x^2} = \frac{d}{dx}\left(\frac{\sin(x)}{x}\right)$$

$\frac{d}{d\theta}(\cos^2(\theta^4)) = 0 \to \cos(\theta^4) = x$

$\frac{d}{d\theta}(\cos(\theta^4)) = -\sin(\theta^4)$

$\frac{d}{dx}(x^2) = 2x$

$\cancel{-2\sin(\theta^4) \to -2\sin(4\theta^3)}$

$2x \cdot (-\sin(x)) \cdot 4\theta^3$

$= 2\cos(\theta^4)(-\sin(\theta^4)) \cdot 4\theta^3 \to -8\theta^3\cos(\theta^4)\sin(\theta^4)$

$\theta = 0 \quad \theta^4 = \frac{\pi}{2} \quad \theta = \sqrt[4]{\frac{\pi}{2}}$

Practice:

↓ next page

$$\begin{array}{c|c|c} & x & 1 \\ \hline x & x^2 & x \\ \hline 1 & x & 1 \end{array} = x^2+2x+1$$

a) $\frac{d}{dx}\left(\frac{x^3}{x+1}\right) \to \frac{3x^2(x+1) - 1(x^3)}{(x+1)^2}$

$\to \frac{3x^3+3x^2-x^3}{(x+1)^2} \to \frac{2x^3+3x^2}{(x+1)^2}$

b) $\frac{d}{dx}\left(x^3 + (x+1)^{-1}\right) \to 3x^2 \cdot \frac{1}{x+1}$

$\frac{d}{dx}\left(\frac{1}{x+1}\right) = \frac{0(x+1) + 1(1)}{x+1} = \frac{1}{x+1} \quad \frac{2x^3+3x^2}{(x+1)^2}$

$= 3x^2(x+1)^{-1} + x^3(-(x+1)^{-2})$

$= 3x^2(x+1)^{-1} - x^3(x+1)^{-2}$

$= 3x^2(x+1)^{-2}(x+1) - x^3(x+1)^{-2}$

$= (x+1)^{-2}(3x^2(x+1) - x^3)$

$= (x+1)^{-2}(3x^3+3x^2-x^3)$

$= (x+1)^{-2}(2x^3+3x^2)$

$= \frac{2x^3+3x^2}{(x+1)^2}$

c) $u(x)(v(x))^{-1}$

$= u'(x)(v(x))^{-1} + u(x)(-v(x))^{-2}$

$= u'(x)(v(x))^{-1} - u(x)(v(x))^{-2}v'(x)$

$= u'(x)(v(x))^{-2}v(x) - u(x)(v(x))^{-2}v'(x)$

$= (v(x))^{-2}(u'(x)v(x) - u(x)v'(x))$

$= \frac{u'(x)v(x) - u(x)v'(x)}{(v(x))^2}$

Sesión 12