Mark Rogers

markrogersjr@hkn.eecs.berkeley.edu

We prove (11). Let $x_n = \operatorname{vec}(\mathcal{X}_n)$, $z_n = \operatorname{vec}(\mathcal{Z}_n)$, $C = \operatorname{mat}(\mathcal{C})$, and $R = \operatorname{mat}(\mathcal{R})$. We have

$$\begin{aligned}
p(\mathcal{X}_1, \mathcal{Z}_1, \ldots, \mathcal{X}_N, \mathcal{Z}_N \mid \theta) &= p(x_1, z_1, \ldots, x_N, z_N \mid \operatorname{vec}\theta) \\
&= \prod_{n=1}^{N} p(x_n \mid z_n) p(z_1) \prod_{n=1}^{N-1} p(z_{n+1} \mid z_n) \\
&= \prod_{n=1}^{N} \exp\{(x_n - Cz_n)^{\mathrm{T}} R^{-1} (x_n - Cz_n)\} (2\pi)^{-\prod J_m/2} |R|^{-1/2} p(z_1) \prod_{n=1}^{N-1} p(z_{n+1} \mid z_n) \\
\implies \log p(\mathcal{X}_1, \mathcal{Z}_1, \ldots, \mathcal{X}_N, \mathcal{Z}_N \mid \theta) &= \sum_{n=1}^{N} (x_n - Cz_n)^{\mathrm{T}} R^{-1} (x_n - Cz_n) + \cdots \\
&= \sum_{n=1}^{N} \operatorname{tr}\{(x_n - Cz_n)^{\mathrm{T}} R^{-1} (x_n - Cz_n)\} + \cdots \\
&= \sum_{n=1}^{N} \operatorname{tr}\{R^{-1} (x_n - Cz_n)(x_n - Cz_n)^{\mathrm{T}}\} + \cdots \\
&= \sum_{n=1}^{N} \operatorname{tr}\{R^{-1} (x_n x_n^{\mathrm{T}} - 2Cz_n x_n^{\mathrm{T}} + Cz_n z_n^{\mathrm{T}} C^{\mathrm{T}})\} + \cdots \\
&= \sum_{n=1}^{N} \operatorname{tr}\{R^{-1} (Cz_n z_n^{\mathrm{T}} C^{\mathrm{T}} - 2Cz_n x_n^{\mathrm{T}})\} + \cdots \\
&= \operatorname{tr}\left\{ R^{-1} C \sum_n z_n z_n^{\mathrm{T}} C^{\mathrm{T}} - 2R^{-1} C \sum_n z_n x_n^{\mathrm{T}} \right\} + \cdots \\
\implies \mathrm{E}(\log p(\mathcal{X}_1, \mathcal{Z}_1, \ldots, \mathcal{X}_N, \mathcal{Z}_N \mid \theta)) &= \operatorname{tr}\left\{ R^{-1} C \sum_n \mathrm{E}\left(z_n z_n^{\mathrm{T}}\right) C^{\mathrm{T}} - 2R^{-1} C \sum_n \mathrm{E}\left(z_n x_n^{\mathrm{T}}\right) \right\} + \cdots \\
&= \operatorname{tr}\left\{ \Omega C \left[\Psi C^{\mathrm{T}} - 2\Phi^{\mathrm{T}} \right] \right\} + \cdots .
\end{aligned}$$