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We prove (11). We have

$$
\begin{aligned}
p(x, z \mid \theta) &= \prod_{n=1}^{N} p(x_n \mid z_n) p(z_1) \prod_{n=1}^{N-1} p(z_{n+1} \mid z_n) \\
&= \prod_{n=1}^{N} \exp\{(x_n - Cz_n)^T R^{-1} (x_n - Cz_n)\} (2\pi)^{-\prod J_m/2} |R|^{-1/2} p(z_1) \prod_{n=1}^{N-1} p(z_{n+1} \mid z_n) \\
\implies \log p(x, z|\theta) &= \sum_{n=1}^{N} (x_n - Cz_n)^T R^{-1} (x_n - Cz_n) + \cdots \\
&= \sum_{n=1}^{N} \operatorname{tr}\{(x_n - Cz_n)^T R^{-1} (x_n - Cz_n)\} + \cdots \\
&= \sum_{n=1}^{N} \operatorname{tr}\{R^{-1} (x_n - Cz_n)(x_n - Cz_n)^T\} + \cdots \\
&= \sum_{n=1}^{N} \operatorname{tr}\{R^{-1} (x_n x_n^T - 2 z_n x_n^T C + C z_n z_n^T C^T)\} + \cdots \\
&= \sum_{n=1}^{N} \operatorname{tr}\{R^{-1} (C z_n z_n^T C^T - 2 z_n x_n^T C)\} + \cdots \\
&= \sum_{n=1}^{N} \operatorname{tr}\{R^{-1} (z_n z_n^T C^T C - 2 z_n x_n^T C)\} + \cdots \\
&= \operatorname{tr}\left\{ R^{-1} C \sum_n z_n z_n^T C^T - 2R^{-1} \sum_n z_n x_n^T C \right\} + \cdots \\
&= \operatorname{tr}\left\{ \Omega C \Psi C^T - 2\Omega\Phi C \right\} + \cdots .
\end{aligned}
$$