1 N decay rates

The sterile neutrino has couplings to the Z and $W^{\pm}$ bosons as well as the new Z'. This means that its decay rates are a combination of the standard heavy sterile weak decay rates (pure leptonic and semileptonic rates), combined with novel processes driven by the $Z\prime$.

In our model, the W-vertices are unchanged (they are still only modified by the PMNS matrix), and so at tree-level we can use the usual expressions for the decay rates of any process with only CC diagrams. However, the Z-boson mediated decays should be recomputed. Not only does the Z-boson have modified couplings in our model (and crucially slightly modified axial-vector relationships), but for any process with a Z exchange, we could also exchange the Z'. These diagrams should be allowed to interfere.

We might reasonably expect such interference to be small. Especially when Z' mediation produces a significantly enhanced decay rate. However, when pushing the χ^2 value down, this should be borne in mind.

1.1 $N \to \nu\nu\nu$ (TO DO)

This is normally the dominant decay rate below the π mass threshold. It has a very similar structure to the e^+e^- decay computed below, but with different coupling constants. Specifically, the coupling of Z' to two light neutrinos is doubly suppressed by the small sterile mixing angles. It is expected to be negligibly enhanced by the Z'.

1.2 $N \to \nu\pi^0$

This is purely neutral current decay, and would expect diagrams involving an exchanged Z'. Their effect is expected to be small, because of the predominately vectorial couplings of the boson to fermions. At first order in χ, there is actually no coupling of our boson to pseudoscalar mesons. The leading sensitivity then comes from the modifed couplings to the Z boson. However, the coupling between pion and Z' enters with finite Z' mass at first order in χ, or at third order in χ. As it's quick to do, we recompute this rate with both Z and Z' bosons, ensuring these effects are taken into account.

1.2.1 Recap of basic calculation

We define our interaction vertices as in Appendix A. Because we are dealing with a bound state, we must make use of the pion decay constant, and start from the transition matrix element. We consider the process with only the Z mediator to start:

$$\begin{aligned}
\langle k_1, k_2|\, iT\, |p\rangle &= i\mathcal{M}(2\pi)^4\delta^4(p-k_1-k_2), \\
&= \langle k_1, k_2|\, T\left\{e^{i\int d^4x \mathcal{L}_{\mathrm{Int}}}\right\}|p\rangle\Big|_{\text{conn. amput.}} \\
&= \frac{ic_A^u c^{4i}}{2}\left(\frac{g}{c_W}\right)^2 \frac{\overline{u}(k_1)\gamma^\mu P_{\mathrm{L}} u(p)}{k_2^2 - M_Z^2}\int d^4x e^{i(k_1-p)\cdot x}\langle k_2|\left(\overline{u}\gamma^\mu\gamma^5 u - \overline{d}\gamma^\mu\gamma^5 d\right)|0\rangle
\end{aligned}$$

We cannot compute the final amplitude, but use the pion decay constant in the form

$$\int d^4x e^{i(k_1-p)\cdot x}\langle k_2|\left(\overline{u}\gamma^\mu\gamma^5 u - \overline{d}\gamma^\mu\gamma^5 d\right)|0\rangle = if_\pi k_2^\mu (2\pi)^4\delta^4(p-k_1-k_2).$$

All together we find,

$$i\mathcal{M} = \left(\frac{g}{c_W}\right)^2 \frac{c_A^u c^{4i} f_\pi}{2} k_2^\mu \frac{\overline{u}(k_1)\gamma^\mu P_\mathrm{L} u(p)}{k_2^2 - M_Z^2}$$

1.2.2 Full interference calculation

As the matrix element for a single mediator is so simple, we can also include the decays via Z-boson and their interference. The full matrix element is given by the sum of two terms like the one calculated above

$$\begin{aligned} i\mathcal{M} &= \frac{1}{2}\left(\frac{g}{c_W}\right)^2 \left(\frac{c_A^u c^{4i}}{k_2^2 - M_Z^2} + \frac{d_A^u d^{4i}}{k_2^2 - \mu^2}\right) f_\pi k_2^\mu \overline{u}(k_1)\gamma_\mu P_\mathrm{L} u(p), \\ &\equiv F_i f_\pi k_2^\mu \overline{u}(k_1)\gamma_\mu P_\mathrm{L} u(p). \end{aligned}$$

Where F_i is the product of the two initital factors. Although F_i depends on k_2^2, when we impose energy conservation this is just a function of particle masses as $k_2^2 = m_{\pi^0}^2$, so this term is just an overall scaling of the rate. Completing the calculation leads us to

$$\Gamma_i = \frac{1}{4\pi} |F_i|^2 f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_{\pi^0}^2}{m_s^2}\right)^2.$$

This should be summed over the three final mass states $\Gamma = \sum_{i=1}^3 \Gamma_i$. We can do this by hand, by defining a simplified set of couplings,

$$c^{4i} \equiv U_{s4}^* U_{si} \widetilde{c}^4 \qquad \text{and} \qquad d^{4i} \equiv U_{s4}^* U_{si} \widetilde{d}^4,$$

which allows us to factorize out the dependence on i,

$$\sum_{i=1}^3 |F_i|^2 = |U_{s4}|^2 \left(1 - |U_{s4}|^2\right) \frac{G_\mathrm{F}^2}{2} \rho.$$

where we define a new parameter

$$\rho = 16 M_Z^4 \left| \frac{c_A^u \widetilde{c}^4}{k_2^2 - M_Z^2} + \frac{d_A^u \widetilde{d}^4}{k_2^2 - \mu^2} \right|^2.$$

This gives us the final formula

$$\Gamma = \left(1 - \sum_{\alpha \neq s} |U_{\alpha 4}|^2\right) \sum_{\alpha \neq s} |U_{\alpha 4}|^2 \rho \frac{1}{8\pi} G_F^2 f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_{\pi^0}^2}{m_s^2}\right)^2.$$

1.2.3 Sanity check

As a check we set $\chi = 0$, which means $d^{ij} = d_A^u = 0$ and the c coupings become their νSM values, $c_A^u = \widetilde{c}^4 = \frac{1}{2}$. The ρ parameter can be shown, expanding to first order in in m_π/M_Z, to take the value 1,and therefore

$$\Gamma = \left(1 - \sum_{\alpha \neq s} |U_{\alpha 4}|^2\right) \sum_{\alpha \neq s} |U_{\alpha 4}|^2 \frac{1}{8\pi} G_F^2 f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_{\pi^0}^2}{m_s^2}\right)^2.$$

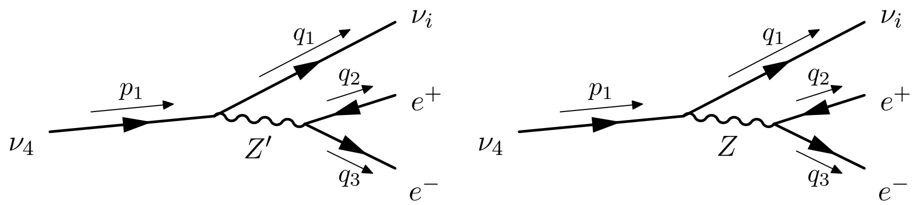

Figure 1. The two NC Feynman diagrams for tree-level N decay into a lepton-antilepton pair of the same flavour.

1.3 $N \to \nu e^+ e^-$

We have three diagrams with different mediators (see Fig. 1 and Fig. 2); however, they turn out to have very similar matrix elements and can be combined before square-summing.

It helps to introduce the following Mandelstam/Dalitz variables,

$$t = (p_1 - q_1)^2 , \qquad u = (p_1 - q_3)^2 \qquad \text{and} \qquad s = (p_1 - q_2)^2 .$$

1.3.1 NC diagrams

The NC diagrams are shown in Fig. 1. We compute the Z' mediated diagram first. Its matrix element reads

$$i\mathcal{M} = -\left(\frac{g}{\sqrt{2}c_W}\right)^2 \frac{d^{4i}}{(p_1 - q_1)^2 - \mu^2} \overline{u}(q_1)\gamma^\mu P_\mathrm{L} u(p_1)\overline{u}(q_3)\gamma_\mu \left(d_V^e - d_A^e\gamma^5\right) v(q_2),$$

where μ is the mass of the Z'. We can write this as

$$\mathcal{M} = \overline{u}(q_1)\gamma^\mu P_\mathrm{L} u(p_1)\overline{u}(q_3)\gamma_\mu \left(V + A\gamma^5\right) v(q_2),$$

where

$$V = -\left(\frac{g}{\sqrt{2}c_W}\right)^2 \frac{d^{4i}d_V^e}{t - \mu^2} \qquad \text{and} \qquad A = +\left(\frac{g}{\sqrt{2}c_W}\right)^2 \frac{d^{4i}d_A^e}{t - \mu^2}.$$

The matrix element for the Z-mediated diagram is identical with the coupling constants and mediator mass swapped,

$$V = -\left(\frac{g}{\sqrt{2}c_W}\right)^2 \frac{c^{4i}c_V^e}{t - M_Z^2} \qquad \text{and} \qquad A = +\left(\frac{g}{\sqrt{2}c_W}\right)^2 \frac{c^{4i}c_A^e}{t - M_Z^2}.$$

1.3.2 CC diagram

Although this diagram looks different, we can actually manhandle it into the same form as the NC diagrams. The matrix element is given by

$$\begin{aligned} i\mathcal{M} &= -\left(\frac{g}{\sqrt{2}}\right)^2 \frac{U_{e4}^* U_{ei}}{(p_1 - q_3)^2 - M_W^2} \overline{u}(q_3)\gamma^\mu P_\mathrm{L} u(p_1)\overline{u}(q_1)\gamma_\mu P_\mathrm{L} v(q_2), \\ &= +\left(\frac{g}{\sqrt{2}}\right)^2 \frac{U_{e4}^* U_{ei}}{(p_1 - q_3)^2 - M_W^2} \overline{u}(q_1)\gamma^\mu P_\mathrm{L} u(p_1)\overline{u}(q_3)\gamma_\mu P_\mathrm{L} v(q_2), \end{aligned}$$

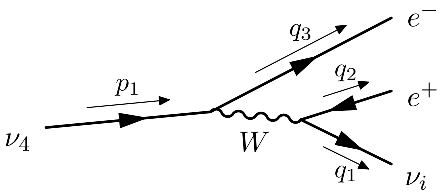

Figure 2. The CC Feynman diagram for tree-level N decay into a lepton-antilepton pair of the same flavour.

where we have used a Fierz identity[1] in the last line. This means we can rewrite it in the form of the other diagrams:

$$\mathcal{M} = \overline{u}(q_3)\gamma^\mu P_\mathrm{L} u(p_1)\overline{u}(q_1)\gamma_\mu \left(V + A\right) v(q_2),$$

where we have defined:

$$V = +\frac{g^2}{4}\frac{U_{e4}^* U_{ei}}{u - M_W^2} \qquad \text{and} \qquad A = -\frac{g^2}{4}\frac{U_{e4}^* U_{ei}}{u - M_W^2}.$$

1.3.3 All the diagrams!

All the matrix elements have identical Dirac structures, and we can group all three together in terms of generalised chiral couplings. These have kinematic dependence because of the propagators, but this will only be a complication when we are trying to integrate over the phase space at the end.

We define the new couplings,

$$\begin{aligned} g_V(t,u) &= -\left(\frac{g}{\sqrt{2}c_W}\right)^2 \frac{d^{4i}d_V^e}{t-\mu^2} - \left(\frac{g}{\sqrt{2}c_W}\right)^2 \frac{c^{4i}c_V^e}{t - M_Z^2} + \frac{g^2}{4}\frac{U_{e4}^* U_{ei}}{u - M_W^2}, \\ g_A(t,u) &= +\left(\frac{g}{\sqrt{2}c_W}\right)^2 \frac{d^{4i}d_A^e}{t-\mu^2} + \left(\frac{g}{\sqrt{2}c_W}\right)^2 \frac{c^{4i}c_A^e}{t - M_Z^2} - \frac{g^2}{4}\frac{U_{e4}^* U_{ei}}{u - M_W^2}. \end{aligned}$$

Using these definitions, the full matrix element is given by,

$$i\mathcal{M} = \overline{u}(q_1)\gamma^\mu P_\mathrm{L} u(p_1)\overline{u}(q_3)\gamma_\mu \left[g_V(t,u) + g_A(t,u)\gamma^5\right] v(q_2),$$

but we'll drop the arguments of the couplings for convenience (we'll need them again on integration). On squared-averaging, we find

$$\overline{|\mathcal{M}|^2} = A^{\mu\nu}B_{\mu\nu},$$

where

$$\begin{aligned} A^{\mu\nu} &\equiv \mathrm{tr}\left[\not{q}_1\gamma^\mu P_\mathrm{L}\left(\not{p}_1 + m_s\right)\gamma^\nu P_\mathrm{L}\right], \\ B^{\mu\nu} &\equiv \mathrm{tr}\left[\left(\not{q}_3 + m_3\right)\gamma^\mu\left(g_V^* + g_A^*\gamma^5\right)\left(\not{q}_2 - m_2\right)\gamma^\nu\left(g_V + g_A\gamma^5\right)\right]. \end{aligned}$$

[1] Nice reference: 0412245

Standard fiddling leaves us with,

$$
\begin{aligned}
A^{\mu\nu} &\equiv 2\left[q_1^\mu p_1^\nu + q_1^\nu p_1^\mu - (p_1\cdot q_1)g^{\mu\nu} + i\varepsilon^{\alpha\mu\beta\nu}(q_1)_\alpha (p_1)_\beta\right],\\
B^{\mu\nu} &\equiv 4\left(|g_V|^2+|g_A|^2\right)\left[q_3^\mu q_2^\nu + q_3^\nu q_2^\mu - (q_2\cdot q_3)g^{\mu\nu}\right]\\
&\qquad + i8\left(\frac{g_V^* g_A + g_V g_A^*}{2}\right)\varepsilon^{\alpha\mu\beta\nu}(q_3)_\alpha (q_2)_\beta - 4m_2 m_3\left(|g_V|^2-|g_A|^2\right)g^{\mu\nu},
\end{aligned}
$$

and therefore,

$$
\begin{aligned}
A^{\mu\nu}B_{\mu\nu} = 16\Big[&|g_V+g_A|^2(q_1\cdot q_2)(p_1\cdot q_3) + |g_V-g_A|^2(q_1\cdot q_3)(p_1\cdot q_2)\\
&+ m_2 m_3\left(|g_V|^2-|g_A|^2\right)(p_1\cdot q_1)\Big].
\end{aligned}
$$

In our Mandelstam-esque variables,

$$
\begin{aligned}
(p_1\cdot q_2) &= \frac{m_s^2+m_2^2-s}{2}, & (p_1\cdot q_3) &= \frac{m_s^2+m_3^2-u}{2}, & (p_1\cdot q_1) &= \frac{m_s^2+m_1^2-t}{2},\\
(q_1\cdot q_3) &= \frac{s-m_1^2-m_3^2}{2}, & (q_1\cdot q_2) &= \frac{u-m_1^2-m_2^2}{2}, & (q_2\cdot q_3) &= \frac{t-m_2^2-m_3^2}{2}.
\end{aligned}
$$

We therefore have,

$$
\begin{aligned}
\overline{|\mathcal{M}|^2} &= A^{\mu\nu}B_{\mu\nu},\\
&= 16\left[|g_V+g_A|^2(q_1\cdot q_2)(p_1\cdot q_3) + |g_V-g_A|^2(q_1\cdot q_3)(p_1\cdot q_2) + m_2m_3\left(|g_V|^2-|g_A|^2\right)(p_1\cdot q_1)\right],\\
&= 4\Big[|g_V+g_A|^2(u-m_1^2-m_2^2)(m_s^2+m_3^2-u)\\
&\qquad + |g_V-g_A|^2(s-m_1^2-m_3^2)(m_s^2+m_2^2-s) + \left(|g_V|^2-|g_A|^2\right)\frac{m_2m_3}{2}\left(m_s^2+m_1^2-t\right)\Big].
\end{aligned}
$$

1.3.4 Dealing with the PMNS

This diagram actually occurs for any outgoing mass state, and we need to sum over these diagrams incoherently (at the matrix element squared level). However, it is easier if we do this summation by hand, as we can then express the answer in terms of the normal $U_{\alpha 4}$ elements (and we minimize the number of times we perform the numerical integrals that will be necessary for the total rate).

Defining some convenient functions,

$$
\begin{aligned}
f(u) &= 4(u-m_1^2-m_2^2)(m_s^2+m_3^2-u),\\
g(s) &= 4(s-m_1^2-m_3^2)(m_s^2+m_2^2-s),\\
h(s,u) &= 2m_2m_3\left(s+u-m_2^2-m_3^2\right),
\end{aligned}
$$

we can rewrite the matrix element as,

$$
\overline{|\mathcal{M}|^2} = |g_V+g_A|^2 f(u) + |g_V-g_A|^2 g(s) + \left(|g_V|^2-|g_A|^2\right)h(s,u).
$$

With all the matrix elements included, we can express the matrix element summed over all outgoing mass states as

$$
\overline{|\mathcal{M}|^2} = \left(1-\sum_{\alpha\neq s}|U_{\alpha 4}|^2\right)\left(\sum_{\alpha\neq s}|U_{\alpha 4}|^2\left[f_1(s,t,u)+\delta_{e\alpha}f_3(s,t,u)\right]\right) + \left(1-|U_{e4}|^2\right)|U_{e4}|^2 f_2(s,t,u).
$$

with the definitions,

$$
\begin{aligned}
f_1(s,t,u) &= \left(\frac{A}{t-\mu^2}+\frac{B}{t-M_Z^2}\right)\left(\frac{Af(u)+Dh(s,u)}{t-\mu^2}+\frac{Bf(u)+Eh(s,u)}{t-M_Z^2}\right), \\
f_2(s,t,u) &= \left(\frac{F}{u-M_W^2}\right)^2 h(s,u), \\
f_3(s,t,u) &= \left(\frac{F}{u-M_W^2}\right)\left(\frac{Ah(s,u)+2Dg(s)}{t-\mu^2}+\frac{Bh(s,u)+2Eg(s)}{t-M_Z^2}\right),
\end{aligned}
$$

and we keep going,

$$
\begin{aligned}
A &= 2\sqrt{2}\left(\frac{M_Z}{m_s}\right)^2 G_F\alpha\frac{d^{4i}}{U_{s4}^*U_{si}}(d_V^e-d_A^e), \\
B &= 2\sqrt{2}\left(\frac{M_Z}{m_s}\right)^2 G_F\beta\frac{c^{4i}}{U_{s4}^*U_{si}}(c_V^e-c_A^e), \\
D &= 2\sqrt{2}\left(\frac{M_Z}{m_s}\right)^2 G_F\alpha\frac{d^{4i}}{U_{s4}^*U_{si}}(d_V^e+d_A^e), \\
E &= 2\sqrt{2}\left(\frac{M_Z}{m_s}\right)^2 G_F\beta\frac{c^{4i}}{U_{s4}^*U_{si}}(c_V^e+c_A^e), \\
F &= -2\sqrt{2}\left(\frac{M_Z}{m_s}\right)^2 G_F\gamma c_W^2.
\end{aligned}
$$

1.3.5 Total rate

The decay rate is given by

$$
d\Gamma = \frac{1}{(2\pi)^5}\frac{1}{64m_s^3}\overline{|\mathcal{M}|^2}dudtd^2\Omega d\phi.
$$

To find the total rate we integrate the matrix element up. The angular integrals just add prefactors, but the Mandelstam integrals require a bit of care (for later use we consider non-zero m_1, but recall that our matrix element assumes $m_1 = 0$ so we have to impose that before doing the integrals),

$$
\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32m_s^3}\int_{(m_2+m_3)^2}^{(m_s-m_1)^2} dt\int_{u_+}^{u_-} du\,\overline{|\mathcal{M}|^2}.
$$

$$
u_\pm = (E_1^*+E_2^*)^2 - (|p_1^*|\pm|p_2^*|)^2\,,
$$

where $(E_i^*)^2-(p_i^*)^2 = m_i^2$ and

$$
|p_1^*|^2 = \frac{\lambda(m_s^2,t,m_1^2)}{4t} \qquad \text{and} \qquad |p_2^*|^2 = \frac{\lambda(t,m_2^2,m_3^2)}{4t}.
$$

These can of course be done numerically, however, doing this repeatedly turned out to be intolerably slow. I'll go a bit further analytically, albeit at the (further) expense of elegance.

The idea is that the functions f_i are made up for fairly simple elementary integrals, all of which can be either precomputed or expressed as special functions. The first set of elementary integrals are

$$
\begin{aligned}
I_{mp} &\equiv \int_{4m_e^2}^{m_s^2} dt \int_{u_+}^{u_-} du \frac{t^m}{(t-\mu^2)^p}, \\
&= \frac{\left(m_s^2 - 4m_e^2\right)^{\frac{5}{2}} \left(4m_e^2\right)^{m-\frac{1}{2}}}{(4m_e^2 - \mu^2)^p} \frac{4}{15} F_1\left(\frac{3}{2}, \frac{1}{2} - m, p, \frac{7}{2}, -\alpha, \beta\right).
\end{aligned}
$$

where $F_1(a, b1, b2, c, x, y)$ is the Appell F1 hypergeometric function. We also need to define another set of integrals,

$$
J_p^{mn} \equiv \int_{4m_e^2}^{m_s^2} dt \int_{u_+}^{u_-} \frac{t^m u^n}{(t-\mu^2)^p}.
$$

I didn't manage to find a general formulae for these integrals, but we will only need $n = 1$ and $n = 2$. These can be expressed as

$$
\begin{aligned}
J_p^{m1} =& \frac{\left(m_s^2 - 4m_e^2\right)^{\frac{7}{2}} \left(4m_e^2\right)^{m-\frac{1}{2}}}{(4m_e^2 - \mu^2)^p} \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(3)}{2\Gamma\left(\frac{9}{2}\right)} F_1\left(\frac{3}{2}, \frac{1}{2} - m, p, \frac{9}{2}, -\alpha, \beta\right) \\
&+ \frac{\left(m_s^2 - 4m_e^2\right)^{\frac{5}{2}} \left(4m_e^2\right)^{m-\frac{1}{2}}}{(4m_e^2 - \mu^2)^p} \frac{m_e^2\Gamma\left(\frac{3}{2}\right)\Gamma(2)}{\Gamma\left(\frac{7}{2}\right)} F_1\left(\frac{3}{2}, \frac{1}{2} - m, p, \frac{7}{2}, -\alpha, \beta\right).
\end{aligned}
$$

Of the $n = 2$ series, we only need J_p^{02} which can be computed

$$
J_p^{02} = \frac{\left(m_s^2 - 4m_e^2\right)^{\frac{5}{2}} \left(4m_e^2\right)^{-\frac{1}{2}}}{(4m_e^2 - \mu^2)^p} \frac{4}{45} \sum_{q=-1}^{2} \left(4m_e^2\right)^q d_q F_1\left(\frac{3}{2}, \frac{1}{2} - q, p, \frac{7}{2}, -\alpha, \beta\right).
$$

where the coefficients d_q are given by

$$
d_{-1} = -m_e^2 m_s^4, \qquad d_0 = 3m_e^4 + 5m_e^2 m_s^2 + m_s^4, \qquad d_1 = -2(2m_e^2 + m_s^2), \qquad d_2 = 1.
$$

I find that the following decomposition holds for the integral of the first function,

$$
\begin{aligned}
\int_{4m_e^2}^{m_s^2} dt \int_{u_+}^{u_-} du\, f_1(s,t,u) =& -4A^2 I_{22} + \left[4A^2(m_s^2 + 2m_e^2) - 2ADm_e^2\right] I_{12} \\
&+ \left[-4A^2 m_e^2(m_s^2 + m_e^2) + 2ADm_e^2 m_s^2\right] I_{02} + \frac{8AB}{M_Z^2} I_{21} \\
&+ \left[-\frac{8AB}{M_Z^2}(m_s^2 + 2m_e^2) + \frac{2m_e^2}{M_Z^2}(AE + BD)\right] I_{11} \\
&+ \left[\frac{8AB}{M_Z^2} m_e^2(m_s^2 + m_e^2) - \frac{2(AE + BD)}{M_Z^2} m_e^2 m_s^2\right] I_{01} - \frac{4B^2}{M_Z^4} I_{20} \\
&+ \left[\frac{4B^2}{M_Z^4}(m_s^2 + 2m_e^2) - \frac{2BEme^2}{M_Z^4}\right] I_{10} \\
&+ \left[-\frac{4B^2}{M_Z^4} m_e^2(m_s^2 + m_e^2) + \frac{2BE}{M_Z^4} m_e^2 m_s^2\right] I_{00}.
\end{aligned}
$$

For the second function,

$$\int_{4m_e^2}^{m_s^2} dt \int_{u_+}^{u_-} du\, f_2(s,t,u) = -\frac{2F^2 m_e^2}{M_W^4} I_{10} + \frac{2F^2 m_e^2 m_s^2}{M_W^4} I_{00}$$

Believe it or not, the third function is even uglier,

$$\begin{aligned}
\int_{4m_e^2}^{m_s^2} dt \int_{u_+}^{u_-} du\, f_3(s,t,u) = & \left[2FB\frac{m_e^2 m_s^2}{M_W^2 M_Z^2} - 8FE\frac{m_e^2(m_s^2+m_e^2)}{M_W^2 M_Z^2}\right] I_{00} \\
& + \left[-2FB\frac{m_e^2}{M_W^2 M_Z^2} + 8FE\frac{m_s^2+2m_e^2}{M_W^2 M_Z^2}\right] I_{10} - \frac{8FE}{M_W^2 M_Z^2} I_{20} \\
& + \left[-2AF\frac{m_e^2 m_s^2}{M_W^2} - 8FD\frac{m_e^2(m_s^2+m_e^2)}{M_W^2}\right] I_{01} \\
& + \left[2AF\frac{m_e^2}{M_W^2} - 8FD\frac{m_s^2+2m_e^2}{M_W^2}\right] I_{11} + \frac{8FD}{M_W^2} I_{21} \\
& + \frac{8FE(2m_e^2+m_s^2)}{M_W^2 M_Z^2} J_0^{01} - \frac{8FE}{M_W^2 M_Z^2} J_0^{02} + \frac{16FE}{M_W^2 M_Z^2} J_0^{11} \\
& - \frac{8FD(2m_e^2+m_s^2)}{M_W^2} J_1^{01} + \frac{8FD}{M_W^2} J_1^{02} + \frac{16FD}{M_W^2} J_1^{11}.
\end{aligned}$$

1.4 $N \to \nu\mu^+\mu^-$ (TO DO)

This would also be enhanced, just as for the electron case. The calculation is identical with masses and mixing elements swapped. Apart from these factors the couplings are universal. I have not coded it up or checked it. In fact, I see no reason for it being suppressed, so it might be a reason to keep the sterile mass under the dimuon threshold.

A Vertices

For reference, here we give our notation for all the vertex factors. Every vertex in our model is identical to the SM apart from the neutrino and neutral current interactions:

$$
\begin{aligned}
-\mathcal{L}_{\rm I} \supset \sum_{f\in\{e,u,d,\dots\}} & \overline{f}\gamma^\mu \left[\frac{g}{2c_W}\left(c_V^f - c_A^f\gamma^5\right) Z_\mu + \frac{g}{2c_W}\left(d_V^f - d_A^f\gamma^5\right) Z'_\mu\right] f, \\
& + \overline{\nu_i}\gamma^\mu \left[\frac{g}{2c_W} c^{ij}\left(1-\gamma^5\right) Z_\mu + \frac{g}{2c_W} d^{ij}\left(1-\gamma^5\right) Z'_\mu\right]\nu_j + \overline{\nu_i}\gamma^\mu f^{i\alpha} W_\mu^+ P_{\rm L} e_\alpha.
\end{aligned}
$$

where the undefined constants are given for the neutrinos by

$$
\begin{aligned}
f^{i\alpha} &= \frac{g}{\sqrt{2}} U^*_{\alpha i}, \\
c^{ij} &= \left(\delta_{ij} - U^*_{si}U_{sj}\right)\frac{c_\beta - s_\beta s_W t_\chi}{2} - U^*_{si}U_{sj}Q_s \frac{g_X}{g}\frac{c_W s_\beta}{c_\chi}, \\
d^{ij} &= \left(\delta_{ij} - U^*_{si}U_{sj}\right)\frac{c_\beta s_W t_\chi + s_\beta}{2} + U^*_{si}U_{sj}Q_s \frac{g_X}{g}\frac{c_W c_\beta}{c_\chi}.
\end{aligned}
$$

And we'll need the following charged fermion couplings,

$$
\begin{aligned}
c_V^e &= c_\beta\left(2s_W^2 - \frac{1}{2}\right) - \frac{3}{2}s_\beta s_W t_\chi, & c_A^e &= -\frac{c_\beta - s_\beta s_W t_\chi}{2}, \\
c_V^u &= c_\beta\left(\frac{1}{2} - \frac{4}{3}s_W^2\right) + \frac{5}{6}s_\beta s_W t_\chi, & c_A^u &= \frac{c_\beta + s_\beta s_W t_\chi}{2}, \\
c_V^d &= c_\beta\left(-\frac{1}{2} + \frac{2}{3}s_W^2\right) - \frac{1}{6}s_\beta s_W t_\chi, & c_A^d &= -\frac{c_\beta + s_\beta s_W t_\chi}{2}, \\
d_V^e &= \frac{3}{2}c_\beta s_W t_\chi + s_\beta\left(-\frac{1}{2} - 2s_W^2\right), & d_A^e &= -\frac{s_\beta + c_\beta s_W t_\chi}{2}, \\
d_V^u &= -\frac{5}{6}c_\beta s_W t_\chi + s_\beta\left(\frac{1}{2} - \frac{4}{3}s_W^2\right), & d_A^u &= \frac{s_\beta + c_\beta s_W t_\chi}{2}, \\
d_V^d &= \frac{1}{6}c_\beta s_W t_\chi + s_\beta\left(-\frac{1}{2} + \frac{2}{3}s_W^2\right), & d_A^d &= -\frac{s_\beta + c_\beta s_W t_\chi}{2}.
\end{aligned}
$$

Throughout the preceding expressions $c_\theta = \cos\theta$, $s_\theta = \sin\theta$ and $t_\theta = s_\theta/c_\theta$. The angle W denotes the Weinberg angle, χ is the kinetic mixing parameter and β is defined by

$$
\tan(2\beta) = \frac{s_W \sin(2\chi)}{\left(\frac{\mu}{v}\right)^2 - \cos(2\chi) - c_W^2 s_\chi^2},
$$

with μ being the (tree-level) mass of the Z' boson and v the Higgs VEV.