[1]

# Vectors & Matricies

vector : $v = [v_1, v_2, v_3]$

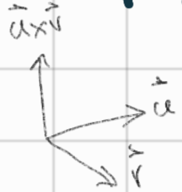
**Magnitude** of v : $|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

**Dot product**: $v \cdot u = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$

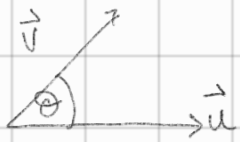
$v_1u_1 + v_2u_2 + v_3u_3$.

**cross product**: $u \times v = |u| \cdot |v| \cdot \sin\theta$

the result is a vector perpendicular to both u & v.




$v \cdot u = |v| \cdot |u| \cos\theta$




if $u \perp v$: dot product = 0.

$\cos(90) = 0$.

* orthogonal & unit vector then: orthonormal.

# Transformations

{A} → orthonormal reference frame , {B} → described wrt {A}.

$^{A}V_{A_0 \to B_0}$ vector from the origin of A to B expressed related to A.

$\left.\begin{matrix} ^{A}x_B \\ ^{A}y_B \\ ^{A}z_B \end{matrix}\right\}$ vectors of B represented in A.

$^{A}r_B$ → reference system, → related system

$^{A}_{B}R$ rotation of coordinate system B wrt A.

$^{A}P_1 = {}^{A}P_{B_0} + {}^{A}_{B}R\,{}^{B}P_1$

$^{A}P_{B_0}$ → location of $B_0$ in A.

$^{A}_{B}R$ → rotate to express in A.

$^{B}P_1$ → some point 1 in frame B

# Homogeneous Transformation

$$\overset{\text{result}}{\begin{bmatrix} {}^{A}P_1 \\ 1 \end{bmatrix}} = \underbrace{\begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{BO} \\ 0_{1\times3} & 1 \end{bmatrix}}_{} \begin{bmatrix} {}^{B}P_1 \\ 1 \end{bmatrix}$$

rotation, translation, point

$T[B \to A]$ ${}^{A}T_{B}$ transforms coordinate system $\{B\}$ to coordinate system $\{A\}$.

$${}^{B}T_{A} = \begin{bmatrix} {}^{A}_{B}R^{T} & -{}^{A}_{B}R^{T} \cdot {}^{A}P_{BO} \\ 0_{1\times3} & 1 \end{bmatrix}$$