1 Section 7.3: Binomial Distribution

A binomial trial is an experiment with 2 possible outcomes. This is the simplest (non-trivial) experiment. We will typically denote the two outcomes as "success" and "failure". Some examples include whether a treatment is effective for a condition, testing a product as defective or not, or winning a game.

When considering binomial trials we will denote the probability of succes as p and the probability of failure as q. Since this is a probability experiment we will have $0 \leq p \leq 1$ and $p + q = 1$. In most practical examples p will not be 0 or 1 because these are uninteresting experiments.

Now consider the following experiment. We take some binomial trial and repeat it n times. We assume each binomial trial is independent so the probabilities do not affect on another. The main problem we have to answer is what is the probability of a certain number of successes.

Now let's be a little more formal. Suppose we have n independent binomial trials, each of which has a probability of success of p. Let X denote the number of sucesses in the n trials. What is the probability distribution of X?

How do we find $P(X = k)$? What does a typical sequences of successes and failures look like. What is the probability of a sequence with exactly k successes? How many sequences have k successes. This gives us the following result

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

Example: Suppose we roll a fair standard die 5 times and we want to count the number of 6's we get. What is the probability we get 3 6's.

Before we do more examples, lets verify this is a probability assignment. What is the sum $P(X = 0) + \cdots P(X = n)$? We get

$$\binom{n}{0} p^0 q^n + \binom{n}{1} p^1 q^{n-1} + \cdots + \binom{n}{n-1} p^{n-1} q^1 + \binom{n}{n} p^n q^0 = (p + q)^n = 1$$

so this does work.

Example: a lightbulb manufacturer sends bulbs in packs of 20 to stores. If production tolerances mean that 1/50 of all bulbs are defective, what is the probability a pack up bulbs has more than 1 defective bulb in it?

What happens when $p = q$? Then both must be 1/2 and $p^k q^{n-k} = (1/2)^n$. This puts us back in the coin flip model where all outcomes are equally likely.

Some other notation:
If X is a random variable with probability distribution which is a binomial distribution, we can write $X \sim \text{Binom}(n, p)$ and say X is distributed binomially with parameters n and p. For example, suppose you play your friend in a game but they win 3/5 of the time. You decide to play 5 times. If X is the number of times you win then we can say $X \sim \text{Binom}(5, 2/5)$.

Concept questions:

- What does a binomial distribution look like if $n = 1$
- When n becomes very large, what happends to $P(X = k)$.
- If $X \sim \text{Binom}(n, p)$ and $Y \sim \text{Binom}m, p$ where X and Y are independent, describe the distribution for $X + Y$.

A tennis match consists of at most 5 sets, where the first player to 3 sets wins. If Nadal and Djokovic play then we estimate there is probability 11/20 that Nadal will win a set. Assume the sets are independent.

1. Find the probability Nadal wins in 5 sets.
2. Find the probability Nadal wins in 4 sets.
3. Find the probability Djokovic wins.
4. Calculate the previous probability in another way (pretend to play all sets, Djokovic must win 3 or more)

Question: What is the "average" outcome of one of these experiments. How do we define this?