1 Section 5.1: Sets

A **Set** is any collection of objects. The objects in the set are called **Elements**. To define sets we will use curly braces.

Examples:
The set of all positive prime numbers less than 10: $P = \{2, 3, 5, 7\}$
The set of all NHL teams: $T = \{Penguins, GoldenKnights, \ldots, Capitals\}$ (Order doesn't matter)
Set Builder notation:
The graph of a function $y = f(x)$ is all ordered pairs (x, y) such that $y = f(x)$: $G = \{(x, y)|f(x) = y\}$

We will typically denote sets with capital letters. We also use the symbol $\in$ to denote when an object is an element of a set. For example

$$2 \in P \qquad Redwings \in T \qquad (1, f(1)) \in G$$

Another notion for sets is the sub-set. We say A is a subset of B and write $A \subset B$ or $A \subseteq B$ if every element of A is an element of B. Exampls:
$\{2, 3\} \subset P$
$\{$NHL teams with multiple Stanley Cups$\} \subset T$
$\{$female students in this class$\} \subset \{$all students in this class$\}$

We say two sets are equal if they contain exactly the same elements. On common way to prove $A = B$ is to prove $A \subset B$ and $B \subset A$.

Difference between $\in$ and $\subset$: If $A = \{1, 2, 3\}$ then we could say $1 \in A$ or $\{1\} \subset A$ but we would not say $1 \subset A$.
We can also put a slash through either of these symbols to mean "not".

What if a set has no elements? There is exactly one such set call the **Empty Set** and it is denoted by either $\emptyset$ or $\{\}$. (First one is better) Note that $\emptyset \subset A$ for any set A.

Small group questions: find 2-3 others and discuss the following questions

1. List all of the subsets of $\{2, 3, 5\}$
2. Is $A \subset A$ for any set
3. If $A \subset B$ and $B \subset C$ is it necessarily true that $A \subset C$
4. Is it possible to have $A \not\subset B$ and $B \not\subset A$
5. Is $0 \in \emptyset$

Building New Sets
There are several ways we can get new sets from old sets. The two most common are **Union** and **Intersection**.

Suppose A and B are sets then the Union of A and B, denoted as $A \cup B$ is the set of all elements contained in A <u>or</u> B (or both).

The intersection of A and B, denoted by $A \cap B$ is the set of all elements contained in A <u>and</u> B.

Examples:
Let $A = \{a, b, c, d\}$ and let $B = \{a, c, d, f, g\}$.
To find $A \cup B$ we simply look for any element which is contained in either set. This gives $A \cup B = \{a, b, c, d, f, g\}$.
On the other hand, to find the intersection, look at the elements which lie in both. $A \cap B = \{a, c, d\}$.

Conceptual Questions:
Let A and B be any sets.

1. Is $A \cap B \subset A$ always
2. Is $A \cup B = B \cup A$ always
3. Is $A \cup B \subset A$ always
4. What is $A \cap \emptyset$

5. What is $A \cup \emptyset$

One last way to work with sets. Suppose all of our sets (for now) are subsets of some "Universal Set" which we will call U. Then for any set A we can define the **complement** of A by $A' = \{x \in U | x \notin A\}$. The key words for Union and Intersection were "or" and "and", the key word for complement is "not".

Example:
Let $U = \{1, 2, \ldots, 10\}$. If $A = \{1, 3, 5, 7, 9\}$ then $A' = \{2, 4, 6, 8, 10\}$.

Note: The complement of a set depends heavily on the univerals set U.
Let $A = \{1, 2\}$. Find A' if

$$U = \{1, 2, 3\} \quad U = \{1, 2, 3, 4, 5\} \quad U = \{1, 2, 3, 5, 8, 13\}$$

Questions:

1. What is U'
2. What is $A \cup A'$
3. What is $A \cap A'$

Now consider the following table of weather data for Charlottesville.

Month	Average High Temp. (°F)	Average Precip. (in.)
Jan	45	3.11
Feb	49	3.07
Mar	58	3.86
Apr	69	3.39
May	76	4.65
Jun	84	4.17
Jul	87	5.31
Aug	86	4.06
Sep	79	4.88
Oct	69	3.74
Nov	59	4.09
Dec	48	3.35

Let $A = \{$months with more than 4 in. average rainfall$\}$ and let $B = \{$months with average temperature less than 60°$\}$. Suppose U is the set of all months. For each of the following sets, explicity list the elements, and give a sentence describing their real life meaning.

- A
- B
- $A \cap B$
- $A \cup B$
- A'
- $A' \cup B'$
- $(A \cap B)'$

Leading into next section, do you notice anything about the last two? Is this a coincidence?