Chapter 1

Known Results on Finite Generation

We know that if $\mathcal{U}$ is the unipotent subgroup of any excpetional rank 2 Chevalley group then $\langle \mathcal{U}_\alpha, \mathcal{U}_\beta \rangle \subsetneq \mathcal{U}$ where α, β are the simple roots of the root system. However, we do have the following results about generation.

Lemma 1. *Let $\alpha, \beta, \beta+\alpha, \beta+2\alpha$ be the positive roots of a root system of type C_2 and $\mathcal{U}$ the unipotent subgroup of $C_2(\mathbb{F}_2)$. Then $\mathcal{U} = \langle \mathcal{U}_\alpha, \mathcal{U}_\beta, \mathcal{U}_{\beta+\alpha} \rangle = \langle \mathcal{U}_\alpha, \mathcal{U}_\beta, \mathcal{U}_{\beta+2\alpha} \rangle$.*

Lemma 2. *Let $\alpha, \beta, \beta+\alpha, \beta+2\alpha, \beta+3\alpha, 2\beta+3\alpha$ be the positive roots of a root system of type G_2 and $\mathcal{U}$ the unipotent subgroup of $G_2(\mathbb{F}_3)$. Then $\mathcal{U} = \langle \mathcal{U}_\alpha, \mathcal{U}_\beta, \mathcal{U}_{\beta+\alpha} \rangle = \langle \mathcal{U}_\alpha, \mathcal{U}_\beta, \mathcal{U}_{\beta+3\alpha} \rangle$.*

Lemma 3. *Let $\alpha, \beta, \beta+\alpha, \beta+2\alpha, \beta+3\alpha, 2\beta+3\alpha$ be the positive roots of a root system of type G_2 and $\mathcal{U}$ the unipotent subgroup of $G_2(\mathbb{F}_2)$. Then $\mathcal{U} = \langle \mathcal{U}_\alpha, \mathcal{U}_\beta, \mathcal{U}_{\beta+\alpha} \rangle$ but $\langle \mathcal{U}_\alpha, \mathcal{U}_\beta, \mathcal{U}_{\beta+3\alpha} \rangle \subsetneq \mathcal{U}$.*