Chapter 1

Buildings and BN-Pairs

In Chapter **??** we saw that for a Coxeter system (W, S), we can define a simplical complex Σ which will encapsulate the group theoretic structure of W in its geometry. This allows us to understand, W very well, but is somewhat limited as Coxeter groups are very specific. In this chapter we will see how we can generalize some of these notions to other simplical complexes, and then use geometry to study groups which act on them.

Definition 1. A *building* is a simplical complex Δ which can be expressed as a union of subcomplexes Σ, called Apartments, such that

(B0) Every apartment Σ is a Coxeter complex

(B1) For any two simplices $A, B \in \Delta$, there is an apartment containing A and B.

(B2) For any two apartments Σ, Σ', there is an isomorphism from Σ to Σ' which fixes $\Sigma \cap \Sigma'$ pointwise.