BALLS IN BINS PROBLEMS

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This is a collection of problems about balls in bins. Some have exact answers, while others only have asymptotics.

(1) Start with n black, n white balls in a bin. Remove uniformly random balls until 1 color is left. About how many are left? What if n and $2n$?

(2) Start with n black, n white balls. Repeatedly pick a random ball and remove one of the opposite color until a color runs out. About how many are left?

(3) You have n balls all colored differently. You repeatedly pick 2 balls and randomly pick 1 to change to the other color. About how long do you expect this to take?

(4) You have n balls all colored differently. You repeatedly pick 2 differently colored balls and randomly pick 1 to change to the other color. About how long do you expect this to take?

(5) You have n balls colored evenly in 2 colors. You repeatedly pick a random ball, then another random ball which must be colored differently. You change the color of the first to the second. About how long do you expect this to take?

(6) You have n balls all colored differently. You repeatedly pick a random ball, then another random ball which must be colored differently. You change the color of the first to the second. About how long do you expect this to take?

(7) You have n balls colored evenly in 2 colors. You repeatedly pick a random ball, then another random ball which must be colored differently. You change the color of the second to the first. About how long do you expect this to take?

(8) How many balls must be uniformly thrown into n bins until the bins are likely to have all distinct numbers of balls?

(9) Given
$$n$$
i.i.d. uniform points in
$$[0, 1]$$
, on average what is the smallest distance between 2 adjacent points? The largest distance? The median distance?