

Quiz 2

Fundamentals of Calculus I

Name: _____

Explain and justify your thought process.

Write your answers in the space provided. No calculators allowed.

1. Find all solutions to $\frac{1}{e^x} = e^{5(x+2)}$.

2. Solve $\log_5((25)^{100}) = (x - 1)(x - 5) + 195$.

3. What is the minimum value of $x^2 + 8x + 15$?

4. Find all solutions to $\log_2(x^2 - 6x) = 3 + \log_2(1 - x)$.

No justification necessary.

5. Sketch the graph of $x^{100} + \pi$.

6. Provide one application where logarithms are useful.

True or False. No justification necessary.

7. _____ The horizontal asymptote of $\frac{4}{x-5} + 8$ is 5.

8. _____ $\log_a(x + y) = \log_a(x) * \log_a(y)$

9. _____ The domain of $\log_3 x$ is all real number except 0.

Bonus (+1 point): How many digits are in 8^{1000} ? (hint: $\log 2 = .3010$)

Solutions

Explain and justify your thought process.

Write your answers in the space provided. No calculators allowed.

1. Find all solutions to $\frac{1}{e^x} = e^{5(x+2)}$.

We want to find x values such that

$$e^{-x} = e^{5(x+2)}.$$

$$\text{So, } -x = 5x + 10 \implies -5/3.$$

2. Solve $\log_5((25)^{100}) = (x-1)(x-5) + 195$.

Using the definition of log, we have

$$\begin{aligned} 5^{(x-1)(x-5)+195} &= 25^{100} \\ &= 5^{200}. \end{aligned}$$

Therefore, $(x-1)(x-5) + 195 = 200$, meaning $(x-1)(x-5) = 5$.
Now we solve,

$$x^2 - 6x + 5 = 5 \implies x(x-6) = 0$$

So we have $x = 0$ or $x = 6$.

3. What is the minimum value of $x^2 + 8x + 15$?

We can determine the minimum value by relating the function to x^2 :

$$x^2 + 8x + 15 = (x+4)^2 - 1$$

The function is x^2 shifted to the left by 4 and down by -1.
Therefore, the minimum value of the function is -1.

4. Find all solutions to $\log_2(x^2 - 6x) = 3 + \log_2(1 - x)$.

We can rewrite the equation as

$$\log_2(x^2 - 6x) - \log_2(1 - x) = 3.$$

Next we rewrite the logarithms as

$$\log_2(x^2 - 6x) + \log_2(1 - x)^{-1} = \log_2((x^2 - 6x)(1 - x)^{-1}) = 3.$$

By the definition of log we have

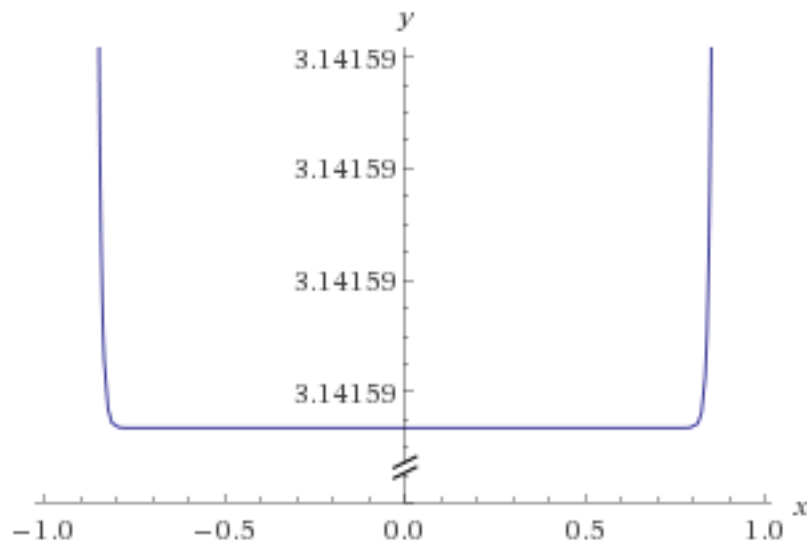
$$2^3 = (x^2 - 6x)(1 - x)^{-1}$$

So,

$$\begin{aligned} 8 - 8x &= x^2 - 6x \implies x^2 + 2x - 8 = 0 \\ \implies (x+4)^2 - 7 &= 0 \implies x = -1 \pm \sqrt{7}. \end{aligned}$$

No justification necessary.

5. Sketch the graph of $x^{100} + \pi$.



The shape is parabolic with a y-intercept of π .

6. Provide one application where logarithms are useful. Answers can range from measuring PH levels to making large numbers (like the hotness of a pepper) human-friendly.

True or False. No justification necessary.

7. False The horizontal asymptote of $\frac{4}{x-5} + 8$ is 5.
8. False $\log_a(x + y) = \log_a(x) * \log_a(y)$
9. False The domain of $\log_3 x$ is all real number except 0.

Bonus (+1 point): How many digits are in 8^{1000} ? (hint: $\log 2 = .3010$)

We know $\log 8^{1000}$ is the power we need to raise ten by to get 8^{1000} .
 This gives us the number of digits in 10s, 100th, 1000th, ... places.
 So 8 has $\log 8^{1000} + 1$ digits (rounded down).
 To compute $\log 8^{1000}$ we have

$$\begin{aligned}\log 8^{1000} &= 1000 \log 8 \\ &= 1000 \log 2^3 = 3000 \log 2 \\ &= 3000 * .3010 = 903\end{aligned}$$

Therefore, 8^{1000} has 904 digits.

Common Mistakes

- Basic algebra such as the zero product property. For example in question $2x(x - 6) = 0$ implies $x = 0$ or $x = 6$, both are solutions.
- Confusing the minimum of a parabola with the y-intercept.
- Attempting to find the minimum of $x^2 + 8x + 15$ by setting the expression equal to zero (or trying to plug in zero for x).
- Claiming the minimum of $x^2 + 8x + 15$ is 15, because it's the constant.
- Incorrectly evaluating logarithms: $\log_5 25 = 2$, not 5.