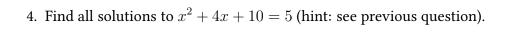# Quiz 1

Fundamentals of Calculus I

Name: _____

Explain and justify your thought process.
Write your answers in the space provided.

1. What's the equation of the line going through $(2, 5)$ and $(3, 10)$?

2. For $f(x) = 1x + 5$ and $g(x) = 3x + 10$, find all solutions to $3x = g(f(x))$.

3. Graph $x^2 + 4x + 10$.

4. Find all solutions to $x^2 + 4x + 10 = 5$ (hint: see previous question).

For questions 5 and 6, note Apple can build an iphone 6 factory for $100,000. Each iphone costs $100 to produce.

5. What's the total cost of producing 800 iphones?

6. If Apple sells each iphone for $500, how many iphones does Apple need to sell to earn $80,000 in profit?

Solutions

1. What's the equation of the line going through $(2, 5)$ and $(3, 10)$?
   First we find the slope. Slope answers the question: how much does y change by when x increases by 1?
   When x inceases by 1, y increases from 5 to 10, implying the slope is 5. Therefore we have $y = 5x + b \implies 5 = 10 + b \implies b = -5$. Thus the equation of the line is $y = 5x - 5$.

2. For $f(x) = 1x + 5$ and $g(x) = 3x + 10$, find all solutions to $3x = g(f(x))$.
   No solution, as the lines are parallel after evaluating the function:
   $$g(f(x)) = 3(x + 5) + 10$$
   $$= 3x + 15 + 10 = 3x + 25.$$

3. Graph $x^2 + 4x + 10$. Complete the square to understand the function:
   $$x^2 + 4x + 10 = (x + 2)^2 + 6$$
   Therefore the function is x^2 shifted to the left by 2 and up by 6.

4. Find all solutions to $x^2 + 4x + 10 = 5$ (hint: see previous question). We determined the function is x^2 shifted to the left by 2 and up by 6. Thus, the function never achieves a value of 5, meaning there are no solutions.

   For questions 5 and 6, note Apple can build an iphone 6 factory for $100,000. Each iphone costs $100 to produce.

5. What's the total cost of producing 800 iphones?
   if we let x be the number of iphones we have: cost = 100 x + 100,000 We evaluate our function at an input of 800: cost = 100*800 + 100,000 = 80,000 + 100,000 = 180,000.

6. If Apple sells each iphone for $500, how many iphones does Apple need to sell to earn $80,000 in profit? If x is the number of iphones sold,
   $$\text{profit} = 500x - \text{cost}$$
   $$= 500x - (100x + 100,000)$$
   $$= 400x - 100,000.$$

   We need to find the input (number of iphones sold) that generates an output (profit) of 80,000:
   $$80,000 = 400x - 100,000 \implies 180,000/400 = x = 450.$$

   Therefore, Apple needs to sell 450 iphones to earn 80,000 in profit.

Common Mistakes

- **not reading the question carefully** Many students provided only the slope, not the equation of the line in question 1.

- **not realizing the implications of a false statement such as 0 = 25.** For example in problem 2, many students arrived at at an impossible result 0 = 25, but couldn't answer the question. We know 0 never equals 25. Therefore, the statements leading up to 0=25 can't be true either.

- **not understanding notation** For example many incorrectly interpreted the notation $f(g(x))$ in question 2. This notation means we takes $g(x)$ as an input of the function $f(x)$.

- **distinguishing a single output from a function.** For example in question 6, many assigned cost or profit a fixed value, instead of realizing each is a function which depends on x (the number of iphones produced/sold).

- **repeating procedures from class without understanding the aim of the question.** For example in question 4, many students completed the square or described the domain, without addressing the question. Understanding what "find all solutions" means is the biggest step to in answering this question.