Quiz 2

College Algebra
Solutions

Only Tools Needed

- basics:
  - commutativity of $+, \times$: a + b = b + a
  - associativity of $+, \times$: $(a + b) + c = a + (b + c)$
  - distribution: $c(a + b) = ca + cb$
- definition of exponents: $a^3 = a * a * a$ and $a^{-3} = \frac{1}{a*a*a}$
- words: y-intercept, function, slope, domain

1. F
2. F
3. T

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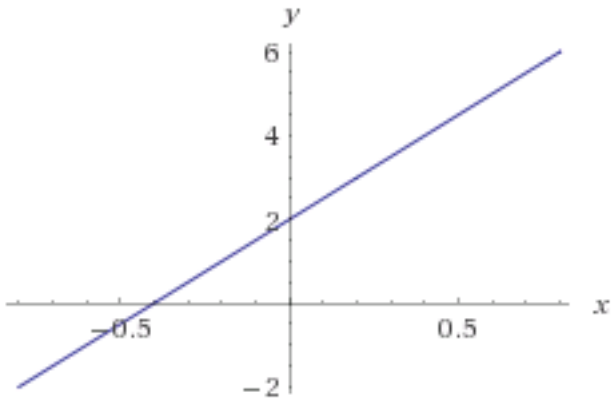
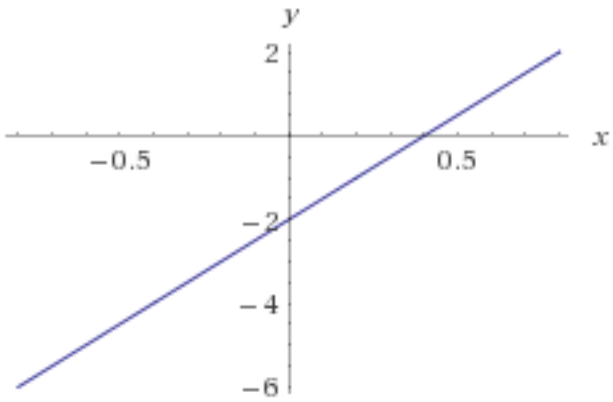
4. E
5. Note two versions:
   The first has *** (three) with $f(x) = 5x + 2$.
   The second has **** (four) with $f(x) = 5x - 2$.
   Otherwise, quizes are identical.



(x from −0.75 to 0.75)

$f(x) = 5x + 2$
OR



(x from −0.75 to 0.75)

$f(x) = 5x - 2$

Two points are needed to plot the line.
Good justifications include:
two points: inputs and outputs
y-intercept, x-intercept, or slope

6. We want to find numbers a making the equation true.
(Left equals Right)

Left side:

$$\begin{aligned} 3 * (a+x) * (a+x) + 4 & \textbf{ by definition of exponents} \\ 3 * ((a+x) * (a+x)) + 4 & \textbf{ by associativity of} \times \\ 3(a^2 + 2ax + x^2) + 4 & \\ = 3a^2 + 6ax + 3x^2 + 4. & \textbf{ by distribution} \end{aligned}$$

Right side:

$$3x^2 + 6ax + 3a^2 - 6. \textbf{ by distribution}$$

Add $(-3x^2 + -6ax + -a^2)$ to both sides.
Use **associativity and commutativity of** $+$ to find:

$$0 + 4 = -6 + 0 \text{ uh oh!}$$

The number 4 never equals -6.
So, no value of a can make the equation true.

Common Errors:
Thinking $3(a+x)^2$ is...

- $\neq (3a + 3x)^2$
  Is $3 * (2+1)^2$ equal to $(6+3)^2$? No: $27 \neq 81$.
- $\neq 3(a^2 + x^2)$
  Is $3 * (2+1)^2$ equal to $3 * (9+1)$? No: $48 \neq 30$.

7. Any real number.
   The domain is all possible inputs of my function.
   $f(x)$ is a line into which I can plug in any value, negative, zero, or positive.

8. Two versions: $f(x) = 5x + 2$ and $f(x) = 5x - 2$

   We know

$$f(x) = g(x)$$

So, for $f(x) = 5x + 2$,

$$\begin{aligned} 5x + 2 &= 3x + 2 \\ \textbf{by definition of } f(x) \textbf{ and } g(x) & \\ +(-2 + (-3x)) + 5x + 2 &= +(-2 + (-3x)) + 3x + 2 \\ \textbf{add (-2 + (-3x)) to both sides} & \\ 2x &= 0 \\ \textbf{associativity of } + & \\ x &= 0 \\ \textbf{multiplied by } \frac{1}{2} & \end{aligned}$$

Errors

- $5x = 3x$, means $5x - 3x = 0$, so $x(5 - 3) = 0$ by distribution. Then $2x = 0$. NOT: $x = \frac{5}{3}$.
- $2x = 0$ does not mean there are no solutions. I can plug in 0 for x and the equation is true.

For $f(x) = 5x - 2$ (same tools),

$$x = 4$$

9. Halving the input means taking half of what goes into my function. So I plug in $\frac{x}{2}$ into $f(x)$:

$$f(\frac{x}{2}) = (\frac{x}{2})^2 = \frac{x}{2}\frac{x}{2} = \frac{x^2}{4}$$

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10. Y-intercept is where the line hits the y-axis, meaning $x = 0$. So, $y = 0 + 612554 = 612554$. No computation is needed.

Quiz 2

College Algebra

$f(x) = 5x + 2$ OR $f(x) = 5x - 2$
(see note above)
$g(x) = 3x + 2$
$h(x) = x^2$

True or False (no work necessary)

1. $\dfrac{x^2a^{-3}y}{x^{-2}y^2} = \dfrac{x^4y^{-1}}{a^{-3}}$

2. $f(x)$ intersects with $y = 5x + \pi + 1000293.2938$

3. $300x + 200$ generates an output 100 times that of $g(x)$.

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4. Select the correct response. What's the range of possible outputs of $h(x)$?
   a. positive $\mathbb{Z}$
   b. $\mathbb{R}$
   c. positive $\mathbb{Q}$ and zero
   d. $\mathbb{N}$
   e. positive $\mathbb{R}$ and zero

State **thought process** and **justification**.

5. Graph $f(x)$

6. Find all values of a satisfying $3(a + x)^2 + 4 = 3x^2 + 6ax + 3(a^2 - 2)$

7. What's the domain of $f(x)$?

8. Find all solutions to $f(x) = g(x)$

9. What's the function generated by halving every input of $h(x)$?

10. What's the y-intercept of $y = (9482 * (203 + 2\pi))x + 612554$?