

Quiz 1

Fundamentals of Calculus I

Name: _____

Explain and justify your thought process.

Write your answers in the space provided.

1. What's the equation of the line going through $(2, 5)$ and $(3, 10)$?

2. For $f(x) = 1x + 5$ and $g(x) = 3x + 10$, find all solutions to $3x = g(f(x))$.

3. Graph $x^2 + 4x + 10$.

4. Find all solutions to $x^2 + 4x + 10 = 5$ (hint: see previous question).

For questions 5 and 6, note Apple can build an iPhone 6 factory for \$100,000. Each iPhone costs \$100 to produce.

5. What's the total cost of producing 800 iPhones?
6. If Apple sells each iPhone for \$500, how many iPhones does Apple need to sell to earn \$80,000 in profit?

Solutions

1. What's the equation of the line going through (2, 5) and (3, 10)?

First we find the slope. Slope answers the question: how much does y change by when x increases by 1?

When x increases by 1, y increases from 5 to 10, implying the slope is 5. Therefore we have $y = 5x + b \implies 5 = 10 + b \implies b = -5$. Thus the equation of the line is $y = 5x - 5$.

2. For $f(x) = 1x + 5$ and $g(x) = 3x + 10$, find all solutions to $3x = g(f(x))$.

No solution, as the lines are parallel after evaluating the function:

$$\begin{aligned} g(f(x)) &= 3(x + 5) + 10 \\ &= 3x + 15 + 10 = 3x + 25. \end{aligned}$$

3. Graph $x^2 + 4x + 10$. Complete the square to understand the function:

$$x^2 + 4x + 10 = (x + 2)^2 + 6$$

Therefore the function is x^2 shifted to the left by 2 and up by 6.

4. Find all solutions to $x^2 + 4x + 10 = 5$ (hint: see previous question). We determined the function is x^2 shifted to the left by 2 and up by 6. Thus, the function never achieves a value of 5, meaning there are no solutions.

For questions 5 and 6, note Apple can build an iphone 6 factory for \$100,000. Each iphone costs \$100 to produce.

5. What's the total cost of producing 800 iphones?

if we let x be the number of iphones we have: $\text{cost} = 100x + 100,000$ We evaluate our function at an input of 800: $\text{cost} = 100 \cdot 800 + 100,000 = 80,000 + 100,000 = 180,000$.

6. If Apple sells each iphone for \$500, how many iphones does Apple need to sell to earn \$80,000 in profit? If x is the number of iphones sold,

$$\begin{aligned} \text{profit} &= 500x - \text{cost} \\ &= 500x - (100x + 100,000) \\ &= 400x - 100,000. \end{aligned}$$

We need to find the input (number of iphones sold) that generates an output (profit) of 80,000:

$$80,000 = 400x - 100,000 \implies 180,000/400 = x = 450.$$

Therefore, Apple needs to sell 450 iphones to earn 80,000 in profit.

Common Mistakes

- question 1: misreading the question. Many students provided only the slope, not the equation of the line.
- question 2: arriving at an impossible result $0 = 25$, but not concluding the appropriate answer. We know 0 never equals 25. Therefore, the statements leading up to $0=25$ can't be true either.
- misunderstanding what $f(g(x))$ notation means... (see question 2)