# Quiz 2

Fundamentals of Calculus I

Name: ________________________

**Explain and justify your thought process.**
Write your answers in the space provided. No calculators allowed.

1. Find all solutions to $\frac{1}{e^x} = e^{5(x+2)}$.

2. Solve $\log_5((25)^{100}) = (x-1)(x-5) + 195$.

3. What is the minimum value of $x^2 + 8x + 15$ ?

4. Find all solutions to $\log_2(x^2 - 6x) = 3 + \log_2(1 - x)$.

**No justification necessary.**

5. Sketch the graph of $x^{100} + \pi$.

6. Provide one application where logarithms are useful.

**True or False. No justification necessary.**

7. ________ The horizontal asymptote of $\dfrac{4}{x-5} + 8$ is 5.

8. ________ $\log_a(x + y) = \log_a(x) * \log_a(y)$

9. ________ The domain of $\log_3 x$ is all real number except 0.

**Bonus** (+1 point): How many digits are in $8^{1000}$ ? (hint: $\log 2 = .3010$)

## Solutions

**Explain and justify your thought process.**
Write your answers in the space provided. No calculators allowed.

1. Find all solutions to $\dfrac{1}{e^x} = e^{5(x+2)}$.

   We want to find $x$ values such that
   $$e^{-x} = e^{5(x+2)}.$$
   So, $-x = 5x + 10 \implies -5/3$.

2. Solve $\log_5((25)^{100}) = (x-1)(x-5) + 195$.

   Using the definition of log, we have
   $$\begin{aligned} 5^{(x-1)(x-5)+195} &= 25^{100} \\ &= 5^{200}. \end{aligned}$$
   Therefore, $(x-1)(x-5) + 195 = 200$, meaning $(x-1)(x-5) = 5$.
   Now we solve,
   $$x^2 - 6x + 5 = 5 \implies x(x-6) = 0$$
   So we have $x = 0$ or $x = 6$.

3. What is the minimum value of $x^2 + 8x + 15$ ?

   We can determine the minimum value by relating the function to $x^2$:
   $$x^2 + 8x + 15 = (x+4)^2 - 1$$
   The function is $x^2$ shifted to the left by 4 and down by -1.
   Therefore, the minimum value of the function is -1.

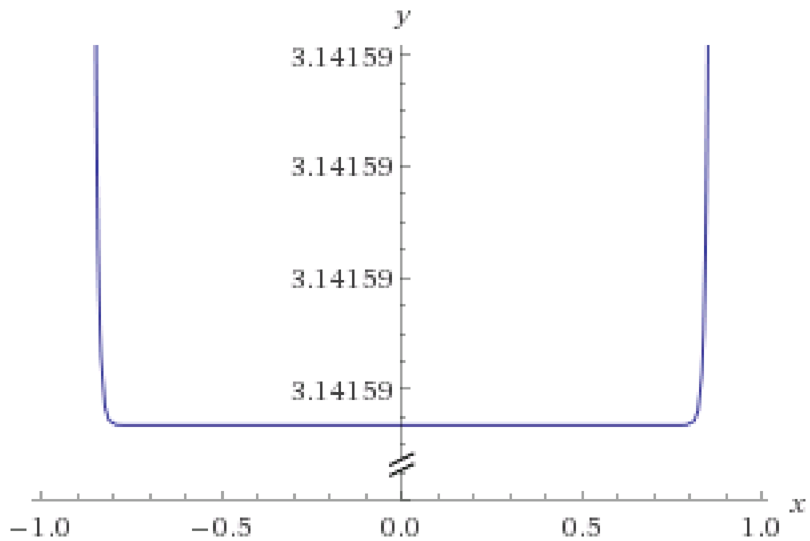
4. Find all solutions to $\log_2(x^2 - 6x) = 3 + \log_2(1-x)$.

   We can rewrite the equation as
   $$\log_2(x^2 - 6x) - \log_2(1-x) = 3.$$
   Next we rewrite the logarithms as
   $$\log_2(x^2 - 6x) + \log_2(1-x)^{-1} = \log_2((x^2 - 6x)(1-x)^{-1}) = 3.$$

   By the definition of log we have
   $$2^3 = (x^2 - 6x)(1-x)^{-1}$$
   So,
   $$\begin{aligned} 8 - 8x = x^2 - 6x &\implies x^2 + 2x - 8 = 0 \\ \implies (x+1)^2 - 7 = 0 &\implies x = -1 \pm \sqrt{7}. \end{aligned}$$

**No justification necessary.**

5. Sketch the graph of $x^{100} + \pi$.




The shape is parabolic with a y-intercept of $\pi$.

6. Provide one application where logarithms are useful. Answers can range from measuring PH levels to making large numbers (like the hotness of a pepper) human-friendly.

**True or False. No justification necessary.**

7. False The horizontal asymptote of $\dfrac{4}{x-5} + 8$ is 5.

8. False $\log_a(x+y) = \log_a(x) * \log_a(y)$

9. False The domain of $\log_3 x$ is all real number except 0.

**Bonus** (+1 point): How many digits are in $8^{1000}$ ? (hint: $\log 2 = .3010$)

We know $\log 8^{1000}$ is the power we need to raise ten by to get $8^{1000}$.
This gives us the number of digits in 10s, 100th, 1000th, . . . places.
So 8 has $\log 8^{1000}$ + 1 digits (rounded down).
To compute $\log 8^{1000}$ we have

$$\begin{aligned}\log 8^{1000} &= 100 \log 8 \\ &= 100 \log 2^3 = 300 \log 2 \\ &= 300 * .3010 = 903\end{aligned}$$

Therefore, $\log 8^{1000}$ has 904 digits.