Quiz 1

Fundamentals of Calculus I

Name: _____

Explain and justify your thought process.
Write your answers in the space provided.

1. What's the equation of the line going through $(2, 5)$ and $(3, 10)$?

2. For $f(x) = 1x + 5$ and $g(x) = 3x + 10$, find all solutions to $3x = g(f(x))$.

3. Graph $x^2 + 4x + 10$.

4. Find all solutions to $x^2 + 4x + 10 = 5$ (hint: see previous question).

For questions 5 and 6, note Apple can build an iphone 6 factory for $100,000. Each iphone costs $100 to produce.

5. What's the total cost of producing 800 iphones?

6. If Apple sells each iphone for $500, how many iphones does Apple need to sell to earn $80,000 in profit?

Solutions

1. What's the equation of the line going through $(2, 5)$ and $(3, 10)$?
   First we find the slope. Slope answers the question: how much does y change by when x increases by 1?
   When x inceases by 1, y increases from 5 to 10, implying the slope is 5. Therefore we have $y = 5x + b \implies 5 = 10 + b \implies b = -5$. Thus the equation of the line is $y = 5x - 5$.

2. For $f(x) = 1x + 5$ and $g(x) = 3x + 10$, find all solutions to $3x = g(f(x))$.
   No solution, as the lines are parallel after evaluating the function:

$$\begin{aligned} g(f(x)) &= 3(x + 5) + 10 \\ &= 3x + 15 + 10 = 3x + 25. \end{aligned}$$

3. Graph $x^2 + 4x + 10$. Complete the square to understand the function:

$$x^2 + 4x + 10 = (x + 2)^2 + 6$$

   Therefore the function is x^2 shifted to the left by 2 and up by 6.

4. Find all solutions to $x^2 + 4x + 10 = 5$ (hint: see previous question). We determined the function is x^2 shifted to the left by 2 and up by 6. Thus, the function never achieves a value of 5, meaning there are no solutions.

   For questions 5 and 6, note Apple can build an iphone 6 factory for $100,000. Each iphone costs $100 to produce.

5. What's the total cost of producing 800 iphones?
   if we let x be the number of iphones we have: cost = 100 x + 100,000 We evaluate our function at an input of 800: cost = 100*800 + 100,000 = 80,000 + 100,000 = 180,000.

6. If Apple sells each iphone for $500, how many iphones does Apple need to sell to earn $80,000 in profit? If x is the number of iphones sold,

$$\begin{aligned} \text{profit} &= 500x - \text{cost} \\ &= 500x - (100x + 100,000) \\ &= 400x - 100,000. \end{aligned}$$

   We need to find the input (number of iphones sold) that generates an output (profit) of 80,000:

$$80,000 = 400x - 100,000 \implies 180,000/400 = x = 450.$$

   Therefore, Apple needs to sell 450 iphones to earn 80,000 in profit.