Foundations for Calculus

1 Properties of Numbers

1.1 Division by Zero

$0 * b = 0$ for all numbers b.
Also, for all numbers $a \neq 0$, $\exists\ a^{-1}$ such that, $a * a^{-1} = 1$.

Therefore, since $0 * b = 0$, we exclude 0^{-1} from the multiplicative inverse above, meaning division by zero can not exist.

note: $(x-1)(x-2) = 0$, implies $x = 1$ or $x = 2$ or BOTH.

1.2 Negative and Positive Numbers

0 is neither positive nor negative,
since positive is defined as all numbers greater than zero (negative less than).

1.3 Absolute Value

For any number a,

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

When solving equations, straightforward appraoch is to use definition.

Geometric Interpretation of $|a|$ is the distance from the origin of a.

To solve $|a| = 4$, we can use the geometric interpretation: numbers that have a distance of 4 from the origin, a = 4 or a = -4.

More generally, for

- $b > 0,$ if $|a| = b$, then $a = b$ or $a = -b$.
- $b = 0, a = 0$
- $b < 0,$ then no a exists, since distance can't be negative!

LOOK MORE INTO ABSOLUTE VALUES.

1.4 Triangle Inequality

Limits

f approaches the limit near a

$$\text{for every } \epsilon > 0, \text{ there exists } \delta > 0 \text{ such that for all } x$$

$$\text{if } 0 < |x - a| < \delta, \text{ then } |f(x) - l| < \epsilon$$

note $x \neq a$!

Observations:

- a function can't approach two different limits [gray]see proof with picture
- $\lim_{x \to a} f(x) + g(x) =$ limit of each sumed (same for products)

redrecall polynomial factor and remainder theorems (long division). see mathisfun.com polynomials remainder factor