Analysis Continued

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Analysis I Primer

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there...

Power Series

def

A **power series** is of the form

$$\sum_{n=0}^{\infty} a_n x^n$$

*can be centered at x_0 we can write it as $\sum_{n=0}^{\infty} a_n (x - x_0)^n$.

uniform convergence

If power series converges at a **single point** say z, then the power series **converges uniformly** on for all $r < |x_0|$.

converge to diff func

The power series converges to a **differentiable** f inside the radius of convergence.

Taylor's Theorem

$f : [a, b] \to \mathbb{R}$, infinitely differentiable, derivatives cont., and $f^{(k)}$ is finite,
then
there exists x_1 for any $c \in [a, b]$ and all $x \neq c$.

$$f(x) = \underbrace{\sum_{k=0}^{n-1} \frac{f^{(k)}(c)}{k!}(x-c)^k}_{\text{Taylor Poly}} + \underbrace{\frac{f^{(n)}(x_1)}{n!}(x-c)^n}_{\text{Taylor Remainder}}$$

*note: x_1 depends on n, x, c

see notes for general form with $f(x)$ and $g(x)$.

take better notes on Power series and Taylor using book

Multivariable Derivatives

Let $f : \mathbb{R}^n \to \mathbb{R}^m$

Two bad attempts: partial derivative, **directional derivative**.

partial derivative denoted $D_k f(c) = \frac{\partial f(c)}{\partial x_k}$)

both are bad because they don't imply continuity as we'd like.
Instead we define the **Total Derivative**, which works as we wish.

directional derivative

The derivative of f in the direction of u is

$$f'(\vec{c}, \vec{u}) = \lim_{h \to 0} \frac{f(\vec{c} + h\vec{u}) - f(\vec{c})}{h}$$

*often other books require $\vec{u}$ to be a unit vector, but not here

linear algebra review in notebook

total derivative

correct def

The function f is **differentiable** at a if there exists a linear transformation T_a such that

$$f(a+v) = f(a) + T_a(v) + ||v||E_c(v)$$

where $E_c(v) \to 0$ as $v \to 0$.

*$||v||E_c(v)$ can be written using "little o" notation as $o(||v||)$.
little o notation: $f = o(g)$ as $x \to c$ if $\lim_{x \to c} \frac{f(x)}{g(x)} = 0$.

cont

If f is differentiable at a, then f is continous at a.

directional deriv

If f is differentiable at a, then $f'(a; u)$ exists and $f'(a; u) = Au$ for any u

Derivatives in Matrices