Measure Theory: A Primer for Dummies

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1 σ-algebras

A σ-**algebra** is a collection of subsets of Ω with nice properties:

- $\emptyset \in \sigma$
- if $A \in \sigma$ then $A^c \in \sigma$
- closed under countable unions

(an **algebra** requires the above but only under finitely many operations)

2 Measurable Space

A **measurable space** is a collection of **events**, β, and a sample space Ω (outcomes).
A sample space makes it possible to talk about complements of an event.
still a bit unclear

3 Measure

A **measure** μ on a set $A \subseteq \Omega$ is a map from $A \to \mathbb{R}^+$.
*idea of a measure is to generalize the notion of volume or length
A **measure space** is a measure space is a measureable space + a measure, written (Ω, β, μ).
A **support** of a measure is all sets with nonzero measure.
a **measure with compact support** means the sets of nonzero measure form a compact set.
(in $\mathbb{R}$ this is closed and bounded, by Heine-Borel)

4 Lebesgue

The **Lebesgue Measure** $\mu_L(A)$ = volume or length of a set A.
e.g., $\mu_L([0,1]) = 1$.
A huge result from measure theory is **Lebesgue's Dominated Convergence Theorem**
For $f_n \to f$,

$$\int f\,dx = \lim \int f_n\,dx$$

which doesn't hold for the Riemann integral.

5 Inequalities

For $a_i, b_i \in \mathbb{C}$,

Cauchy-Schwarz

$$(\sum a_i b_i)^2 \leq \sum a_i^2 \sum b_i^2$$

Holder's Inequality Generalized of Cauchy-Schwarz and used to prove Minkowski's inequality.

$$(\sum |a_i + b_i|^p)^{1/p} \leq (\sum |a_i|^p)^{1/p} (\sum |b_i|^q)^{1/q} \qquad \text{(for } 1/q + 1/p = 1 \text{ and } 1 \leq p.\text{)}$$

Minkowski's Inequality

$$(\sum |a_i + b_i|^p)^{1/p} \leq (\sum |a_i|^p)^{1/p} (\sum |b_i|^q)^{1/q}$$

Resources

Frank Jones, "Lebesgue Integration on Euclidean space"

Analysis Facts

Cauchy in $\mathbb{R} \iff$ convergent. (in general convergent $\implies$ cauchy, but not the other way!)

Complete means every Cauchy sequence converges.

continue http://webbuild.knu.ac.kr/ trj/Analysis/Chandalia.pdf