spawner to emigrant transition models

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Possible models (From Barrowman et al. 2003 Eco. Apps.)

Beverton-Holt

$$R_{s,l,t} = \frac{\alpha_{s,l} S_{s,l,t}}{1/\alpha_{s,l} + S_{s,l,t}/\left(R_{\max,s,l}\right)} e^{\varepsilon_{s,l,t}}$$

Intrinsic productivity $\alpha_{l,s}$ for life history l in stream s
Asymptotic median recruitment $R_{\max,l,s}$
Annual errors $\varepsilon_{s,l,t}$

- Depensatory Beverton-Holt I
  - I think depensation could fit depensation as well as density dependant migration (i.e., fewer early emigrants at low spawner density), which is evident in the data.

$$R_{s,l,t} = \frac{\alpha_{s,l} S_{s,l,t}^{\gamma_{s,l}}}{1/\alpha_{s,l} + S_{s,l,t}^{\gamma_{s,l}}/\left(R_{\max,s,l}\right)} e^{\varepsilon_{s,l,t}}$$

extent of depensation $\gamma_{s,l}$

Smooth hockey stick

$$R_{s,l,t} = \alpha_{s,l}\theta_{s,l}\mu_{s,l}\left(1 + e^{-1/\theta_{s,l}}\right) \times \left\{ \frac{S_{s,l,t}}{\theta_{s,l}\mu_{s,l}} - \log\left[\frac{1 + e^{(S_{s,l,t} - \mu_{s,l})(\theta_{s,l}\mu_{s,l})}}{1 + e^{-1/\theta_{s,l}}}\right] \right\} \times e^{\varepsilon_{s,l,t}}$$

$\mu_{s,l}$ is inflection point of spawner abundance
$\theta_{s,l}$ is a smoothness paramater chosen *a priori*

Alternative depensation

Any model can be multiplied by $\frac{S_{s,l,t}}{S_{s,l,t}+d_{s,l}}$. $d_{s,l}$ controls depensation extent. Interpretable as number of spawners that reduce expected recruits by 50%. Original model obtained when $d = 0$.

Model parameters

Switching from subscript to square bracket notation here.

- Intrinsic productivity $\alpha[s, l]$ in stream s for life history l
- Asymptotic median recruitment $R_{\max}[s, l]$ (Beverton-Holt) or inflection point (Smooth hockey stick) $\mu[s, l]$
- Depensation $\gamma[s, l]$ or $d[s, l]$
- Annual errors $\varepsilon[s, l, t]$ in year t

Possible modeling of intrinsic productivities $\alpha[l, s]$, but asymptotic median recruitment $R_{\max}[l, s]$, inflection points $\mu_[s, l]$, and depensation paramaters $\gamma[l, s]$ or $d[l, s]$ would be modeled in the same way. Annual errors will be modeled differently because of the extra time dimension. See below.

$$log(\alpha[l, s]) = \beta_\alpha + \delta_\alpha[l] + \epsilon_\alpha[s] + \zeta_\alpha[l, s]$$
$$\delta_\alpha[l] \sim N(0, \sigma_{\delta_\alpha})$$
$$\epsilon_\alpha[s] \sim N(0, \sigma_{\epsilon_\alpha})$$
$$\zeta_\alpha[l, s] \sim N(0, \sigma_{\zeta_\alpha})$$

Note: Buhle et al. 2018 modeled covariance among paramaters (i.e., between α, and $R_{\max}$), wheras Barrowman et al. 2003 found that modeling covariance between paramaters had little effect and so modeled them as independant.

Modeling lognormal annual errors $\varepsilon[l, s, y]$.

$$\varepsilon[l, s, y] = \beta_\varepsilon + \delta_\varepsilon[t] + \epsilon_\varepsilon[l, t] + \zeta_\varepsilon[s, t] + \eta_\varepsilon[l, s, t]$$
$$\delta_\varepsilon[t] \sim N(0, \sigma_{\delta_\varepsilon})$$
$$\epsilon_\varepsilon[s, t] \sim N(0, \sigma_{\epsilon_\varepsilon})$$
$$\zeta_\varepsilon[l, t] \sim N(0, \sigma_{\zeta_\varepsilon})$$
$$\eta_\varepsilon[l, s, t] \sim N(0, \sigma_{\eta_\varepsilon})$$

Alternatively, some or all of these could be AR1 if the errors are correlated. I think what Mark was suggesting was an AR1 for $\delta_\varepsilon[t]$ and independant errors like I have written for $\eta_\varepsilon[l, s, t]$.