MST124 18J (June 19 exam) Exam Solutions

Section A

Question	1	2	3	4	5	6	7	8	9	10
	D	E	C	C	B	B	D	B	B	B

Question	11	12	13	14	15	16	17	18	19	20
	B	C	A	D	E	B	A	E	C	B

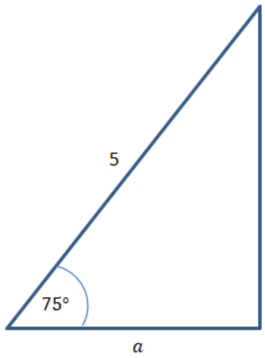
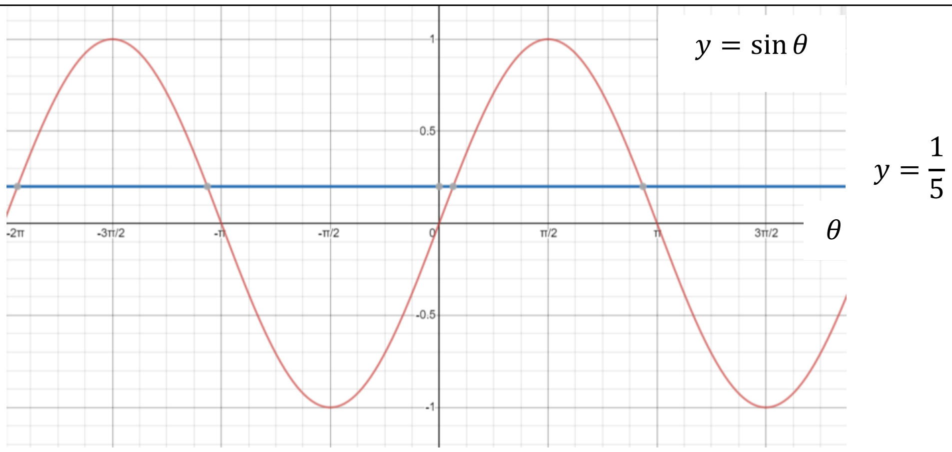
Question	21	22	23	24	25	26	27	28	29	30
	C	E	E	E	C	E	C	D	B	D

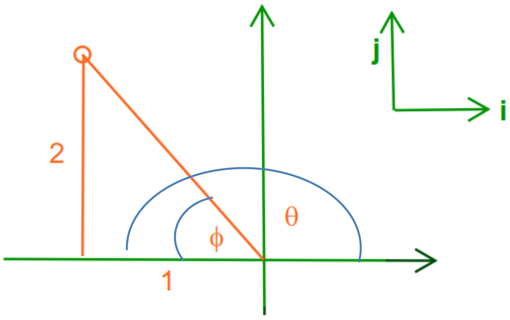
Question	31	32	33	34
	B	E	E	C

Worked Solutions

Q1 **D**	$(x-1)(4x+1)+3x(x-5) = 4x^2+x-4x-1+3x^2-15x$ $= 7x^2-18x-1$
Q2 **E**	$\frac{5}{x+3}-\frac{1}{3x+1} = \frac{5(3x+1)-(x+3)}{(x+3)(3x+1)}$ $= \frac{15x+5-x-3}{(x+3)(3x+1)}$ $= \frac{14x+2}{(x+3)(3x+1)} = \frac{2(7x+1)}{(x+3)(3x+1)}$
Q3 **C** HB, p.18	$b = \sqrt{\frac{3a-3}{a+4}}$ $\Rightarrow \quad b^2 = \frac{3a-3}{a+4} \qquad \Rightarrow \quad b^2(a+4) = 3a-3$ $\Rightarrow \quad b^2a+4b^2 = 3a-3 \qquad \Rightarrow \quad ab^2-3a = -4b^2-3$ $\Rightarrow \quad a(b^2-3) = -4b^2-3 \qquad \Rightarrow \quad a = -\frac{4b^2+3}{b^2-3} = \frac{4b^2+3}{3-b^2}$
Q4 **C** HB p.16	$\frac{5\sqrt{3}}{5\sqrt{3}+4} = \frac{5\sqrt{3}}{5\sqrt{3}+4} \times \frac{5\sqrt{3}-4}{5\sqrt{3}-4} = \frac{5\sqrt{3}\times 5\sqrt{3} - 4\times 5\sqrt{3}}{\left(5\sqrt{3}\right)^2-4^2}$ $= \frac{25\times 3-20\sqrt{3}}{25\times 3-16} = \frac{75-20\sqrt{3}}{59}$

Q5 **B** HB p.19	The coefficient of x is negative, so the graph will slope from top left to bottom right, so we can rule out graphs D and E. The constant is also negative, so the y-intercept is negative. Thus, we can rule out graphs A and C. The correct graph is B.
Q6 **B** HB p.19	The equation is $3y + 4x + 5 = 0$ $\Leftrightarrow\ 3y = -4x - 5 \quad \Leftrightarrow \quad y = \frac{-4x-5}{3} = -\frac{4}{3}x - \frac{5}{3}$ Therefore, the y-intercept is $-\frac{5}{3}$
Q7 **D** HB, p.20	We have $5x - 2y = 3 \ \Leftrightarrow \ 10x - 4y = 6$ (1) (by multiplying through by 2) And $2x - 4y = 14$ (2) Subtracting (2) from (1) $8x = -8$ Divide both side by 8 $x = \frac{-8}{8} = -1$
Q8 **B** HB, p.21	$3x^2 + 12x + 3 = 3[x^2 + 4x] + 3$ $= 3[(x+2)^2 - 4] + 3$ $= 3(x+3)^2 - 12 + 3$ $= 3(x+3)^2 - 9$ So, $s = -9$
Q9 **B** HB, p.25	$f(u) = (u-2)^2 + 4 \qquad -2 \leq u \leq 4$ Since $(u-2)^2$ can never be negative, the least value of $f(u)$ is where $(u-2)^2 = 0$. That is, where $u = 2$. This value is $f(2) = (2-2)^2 + 4 = 4$ Checking the end points of the domain $f(-2) = (-2-2)^2 + 4 = 16 + 4 = 20$ $f(4) = (4-2)^2 + 4 = 4 + 4 = 8$ Thus, the least value of $f(u)$ is 4 and the greatest value of $f(u)$ is 20, both are included. So, the image set is $[4,20]$
Q10 **B** HB, p.26	The domain of the function appears to be the whole of the real number line, so A is not correct. The function is decreasing only on negative x values, so C is not correct. The function is increasing only on postive x values, so E is not correct. There is more than one x value which gives the same value of $f(x)$, meaning the function is not one-to-one. Thus, D is not correct. The image set does not go below 0, so B is correct.
Q11 **B** HB, p.26	Let $y = f(x) = 3x - 5$ Therefore, $y + 5 = 3x \quad \Rightarrow \quad x = \frac{y+5}{3}$ And so, the inverse is $f^{-1}(x) = \frac{x+5}{3}$
Q12 **C** HB, p.6	$\frac{1}{2}\ln(9x^2) + \ln 3x = \ln(9x^2)^{\frac{1}{2}} + \ln 3x$ $= \ln(3x) + \ln(3x)$ $= \ln(3x)^2$ $= \ln(9x^2)$

Q13 **A** HB p.32	$\frac{4}{2x+3} \leq 3x$ $\Leftrightarrow \quad \frac{4}{2x+3} - 3x \leq 0$ $\Leftrightarrow \quad \frac{4}{2x+3} - \frac{3x(2x+3)}{2x+3} \leq 0$ $\Leftrightarrow \quad \frac{4-6x^2-9x}{2x+3} \leq 0$ $\Leftrightarrow \quad \frac{6x^2+9x-4}{2x+3} \geq 0$
Q14 **D** HB, p.4 and p.33	 $\cos 75° = \frac{\text{adj}}{\text{hyp}} = \frac{a}{5}$ So $a = 5 \times \cos 75° = 1.2940 \ldots = 1.3$ (to 1 d.p.)
Q15 **E** HB, p.37	We have three sides and want one angle, so we need the Cosine Rule $7^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos\theta$ $\cos\theta = \frac{36+64-49}{96} = \frac{17}{32}$ $\theta = \cos^{-1}\left(\frac{17}{32}\right) = 57.9100 \ldots° = 58°$ (to the nearest degree)
Q16 **B** HB, p.37	We need the Sine Rule $\frac{a}{\sin 51°} = \frac{9}{\sin 55°} \Rightarrow a = \frac{9 \times \sin 51°}{\sin 55°} = 8.5384 \ldots = 8.5$ (to 1 d.p.)
Q17 **A** HB, p.35 - 36	 The diagram shows that there are 3 values of θ which give $\sin\theta = \frac{1}{5}$ in the range $-\frac{3\pi}{2} \leq \theta \leq \frac{3\pi}{2}$
Q18 **E** HB, p.38	The distance is $\sqrt{(y_2-y_1)^2+(x_2-x_1)^2} = \sqrt{(-4-4)^2+(3-1)^2}$ $= \sqrt{64+4} = \sqrt{68} = 2\sqrt{17}$

Q19 C HB, p.43	 $\tan\phi = \frac{2}{1} = 2$, so $\phi = tan^{-1}(2) = 63.4349\ldots°$ Therefore, $\theta = 180° - 63.4349\ldots° = 116.5650\ldots°$ $= 117°$ (to nearest degree)
Q20 B HB, p.7, p.44	$\frac{d}{dx}(15x^4 - 4x^3 + 2) = 15 \times 4x^3 - 4 \times 3x^2 = 60x^3 - 12x^2$
Q21 C HB, p.7, p.44	$f'(x) = \frac{d}{dx}(3x^3 - 27x^2 + 72x - 13) = 9x^2 - 54x + 72$ The stationary points are where $f'(x) = 0$. That is, where $9x^2 - 54x + 72 = 0$ Factorising gives $9(x-2)(x-4) = 0$ The stationary points are at $x = 2$ and $x = 4$
Q22 E HB, p.48, p.7	Using the product rule $$\frac{d}{dx}(e^x \tan x) = e^x \tan x + e^x \sec^2 x$$
Q23 E HB, p.7, p.48	The first derivative is $\frac{d}{dx}(x^4 \sin x) = 4x^3 \sin x + x^4 \cos x$ The second derivative is $$\frac{d}{dx}(4x^3 \sin x + x^4 \cos x) = 12x^2 \sin x + 4x^3 \cos x + 4x^3 \cos x - x^4 \sin x$$ $$= 12x^2 \sin x + 8x^3 \cos x - x^4 \sin x$$
Q24 E HB, p.52, p.7	$$\int \left(3\sin x + x^{\frac{5}{3}}\right) dx$$ $$= -3\cos x + \frac{3}{8}x^{\frac{8}{3}} + c$$
Q25 C HB, p.7, p.51	The area is given by $$\int_{-5}^{3} f(x)dx = \int_{-5}^{-4} f(x)dx + \int_{-4}^{-1} f(x)dx + \int_{-1}^{3} f(x)dx$$ $$= -0.1 + 1.2 - 6.6 = -5.5$$
Q26 E HB, p.7,p.51 - 52	$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\cos(4x)dx = \left[\frac{1}{2}\sin(4x)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $$= \frac{1}{2}\left[\sin\left(\frac{4\pi}{3}\right) - \sin\left(\frac{2\pi}{3}\right)\right] = \frac{1}{2}\left[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right] = -\frac{\sqrt{3}}{2}$$

Q27 C HB, p.7, p.51	$$\int_1^2 (2x^4+4x)dx = \left[\frac{2}{5}x^5+2x^2\right]_1^2 = \left(\frac{2}{5}\times 2^5 + 2\times 2^2\right) - \left(\frac{2}{5}+2\right)$$ $$= \frac{104}{5} - \frac{12}{5} = \frac{92}{5}$$
Q28 D HB, p.54-55	$\mathbf{P} = \begin{pmatrix} -5 & -2 \\ 6 & -1 \end{pmatrix}$ so $2\mathbf{P} = \begin{pmatrix} -10 & -4 \\ 12 & -2 \end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix} 3 & -3 \\ -8 & -8 \end{pmatrix}$ so $3\mathbf{Q} = \begin{pmatrix} 9 & -9 \\ -24 & -24 \end{pmatrix}$ Therefore $2\mathbf{P} - 3\mathbf{Q} = \begin{pmatrix} -10 & -4 \\ 12 & -2 \end{pmatrix} - \begin{pmatrix} 9 & -9 \\ -24 & -24 \end{pmatrix} = \begin{pmatrix} -19 & 5 \\ 36 & 22 \end{pmatrix}$
Q29 B HB, p.55	$\mathbf{T} = \begin{pmatrix} -5 & -6 \\ 2 & -2 \end{pmatrix}$ so $\mathbf{T}^2 = \begin{pmatrix} -5 & -6 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} -5 & -6 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 25-12 & 30+12 \\ -10-4 & -12+4 \end{pmatrix} = \begin{pmatrix} 13 & 42 \\ -14 & -8 \end{pmatrix}$
Q30 D HB, p.56	$$\det\begin{pmatrix} -9 & 9 \\ -1 & 8 \end{pmatrix} = -9\times 8 - 9\times(-1) = -72+9 = -63$$
Q31 B HB, p.8	First term, $a = 18$, common difference $d = 3$ and last term is $l = 900$ Sum $= \frac{1}{2}n(2a+(n-1)d) = \frac{1}{2}n(a+l)$ Which gives $2a+(n-1)d = a+l$ and so $a-l = -(n-1)d$ This gives $n = \frac{l-a}{d}+1 = \frac{900-18}{3}+1 = 295$ So, Sum $= \frac{1}{2}n(a+l) = \frac{1}{2}\times 295(18+900) = 135405$
Q32 E HB, p.60	We have $r = -\frac{7}{4}$ Since $r < -1$, the sequence alternates in sign and is unbounded. Multiplying by -3 doesn't change its behaviour.
Q33 E HB, p.65 and p.68	By the quadratic formula, with $a = 1$, $b = -2$ and $c = 10$ $$z = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{2\pm\sqrt{(-2)^2-4\times 1\times 10}}{2\times 1}$$ $$= \frac{2\pm\sqrt{-36}}{2} = \frac{2\pm 6i}{2} = 1\pm 3i$$
Q34 C HB, p.67	$z = 6\left(\cos\left(\frac{5\pi}{9}\right) + i\sin\left(\frac{5\pi}{9}\right)\right)$ and $w = 4\left(\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right)$ $$\frac{z}{w} = \frac{6}{4}\left(\cos\left(\frac{5\pi-2\pi}{9}\right) + i\sin\left(\frac{5\pi-2\pi}{9}\right)\right)$$ $$= \frac{3}{2}\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$$

Section B

Question	35	36	37	38	39	40	41	42
	A	B	A	C	A	A	E	D

Worked Solutions

Q35
A
HB, p.41

From A to B is $-3\mathbf{i} - 2\mathbf{j}$ and from B to C is $\mathbf{i} + \mathbf{j}$ so from A to C is

$-3\mathbf{i} - 2\mathbf{j} + \mathbf{i} + \mathbf{j} = -2\mathbf{i} - \mathbf{j}$

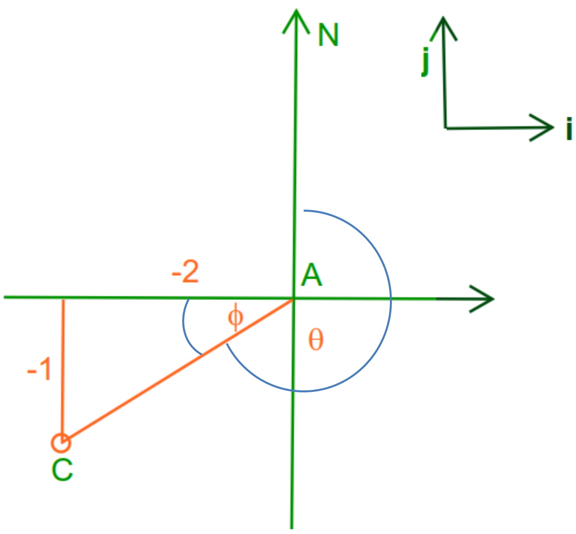
Drawing this resultant vector on the diagram, we have

$\tan\phi = \frac{1}{2}$ and so $\phi = \tan^{-1}\left(\frac{1}{2}\right) = 26.56505\ldots°$

The bearing is measured from the North clockwise,

So, bearing is $270° - 26.5650\ldots° = 243.4349\ldots°$

$= 243°$ (to nearest degree)



Q36
B
HB, p.47, p.7

The velocity is given by $v = \frac{ds}{dt} = -\frac{5\pi}{4}\sin\left(\frac{\pi t}{4}\right) - \pi\cos\left(\frac{\pi t}{4}\right)$

When $t = 2$, $v = -\frac{5\pi}{4}\sin\left(\frac{\pi}{2}\right) - \pi\cos\left(\frac{\pi}{2}\right) = -\frac{5\pi}{4}$

Q37
A
HB, p.48

The quotient rule says $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg\prime}{g^2}$

So, $\frac{d}{dx}\left(\frac{9+\cos x}{e^x+4}\right) = \frac{(e^x+4)(-\sin x) - (9+\cos x)e^x}{(e^x+4)^2}$

$$= -\frac{e^x(9+\cos x) + (e^x+4)\sin x}{(e^x+4)^2}$$

Q38
C
HB, p.48, p.7

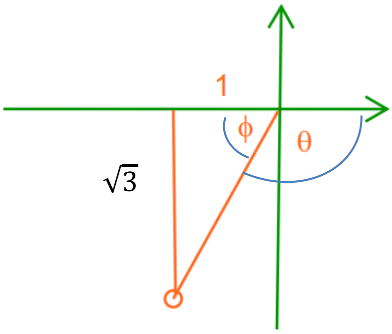
Use the chain (composite) rule

$$\frac{d}{dx}(\cos((3+4x)^2)) = -\sin((3+4x)^2) \times \frac{d}{dx}((3+4x)^2)$$

$$= -\sin((3+4x)^2) \times 2(3+4x) \times 4$$

$$= -8(3+4x)\sin((3+4x)^2)$$

Q39 A HB, p.52 - 53	$$\int 4xe^{x/2}dx$$ By parts, we have $f(x) = 4x$, so $f'(x) = 4$ and $g'(x) = e^{x/2}$, so $g(x) = 2e^{x/2}$ So $\int 4xe^{x/2}dx = fg - \int f'g$ $= 8xe^{x/2} - \int 4 \times 2e^{x/2}dx = 8xe^{x/2} - 8\int e^{x/2}dx$ $= 8xe^{x/2} - 8 \times 2e^{\frac{x}{2}} + c$ $= -16e^{\frac{x}{2}} + 8xe^{\frac{x}{2}} + c$
Q40 A HB p.52	$$\int_{\pi/8}^{\pi/4} \frac{\cos(2x)}{(\sin(2x))^4}dx$$ We need to use substitution. Let $u = \sin(2x)$, which gives $\frac{du}{dx} = 2\cos(2x)$ or $\frac{1}{2}du = \cos(2x)dx$ Changing limits When $x = \frac{\pi}{8}$, $u = \sin\left(2 \times \frac{\pi}{8}\right) = \frac{1}{\sqrt{2}}$ and when $x = \frac{\pi}{4}$, $u = \sin\left(2 \times \frac{\pi}{4}\right) = 1$ So, $$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cos(2x)}{(\sin(2x))^4}dx = \int_{\frac{1}{\sqrt{2}}}^{1} \frac{1}{u^4} \times \frac{1}{2}du$$ $$= \frac{1}{2}\int_{\frac{1}{\sqrt{2}}}^{1} u^{-4}du$$ $$= \frac{1}{2}\left[-\frac{1}{3}u^{-3}\right]_{\frac{1}{\sqrt{2}}}^{1}$$ $$= \frac{1}{2}\left[-\frac{1}{3} + \frac{1}{3}\left(\sqrt{2}\right)^3\right]$$ $$= \frac{-1 + 2\sqrt{2}}{6} = \frac{2\sqrt{2} - 1}{6}$$

Q41 **E** HB p.61	The expression is $(6-3t^4)^{10}$ and we want the coefficient of the term in t^{20} By the binomial theorem, each term has the form $^{10}C_k \times 6^{10-k} \times (-3t^4)^k$ $= {}^{10}C_k \times 6^{10-k} \times (-3)^k \times t^{4k}$ As we need the term in t^{20}, we let $k = 5$ So, the coefficient is $^{10}C_5 \times 6^5 \times (-3)^5$ $= 252 \times 7776 \times (-243)$ $= -476171136$
Q42 **D** HB p.66, p.67	Let $z = -1-\sqrt{3}i$ The modulus is $\lvert z\rvert = \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{1+3} = 2$  From the diagram $\tan\phi = \frac{\sqrt{3}}{1}$, so $\phi = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ The principal argument is $\theta = -\left(\pi - \frac{\pi}{3}\right) = -\frac{2\pi}{3}$ So, $z = 2\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$ By de Moire's formula $$z^4 = 2^4\left(\cos\left(4\times\left(-\frac{2\pi}{3}\right)\right) + i\sin\left(4\times\left(-\frac{2\pi}{3}\right)\right)\right)$$ $$= 16\left(\cos\left(-\frac{8\pi}{3}\right) + i\sin\left(-\frac{8\pi}{3}\right)\right)$$ $$= 16\left(-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$$ $$= -8\left(1+\sqrt{3}i\right) = -8-8\sqrt{3}i$$