STAT 1500 Assignment 3: Mark Stanley 101311883

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1. Write a function to implement the Linear Congruence Method for pseudo random numbers (PRN). The function should take a length n, the parameters m, a, and c as well as the seed X0 as input and should return a vector X = (X1, X2, . . . , Xn). Test your function by calling it with the parameters m = 8, a = 5, c = 1 and by comparing the output with the results from class.

```
lin_congruence <- function(n, m, a, c, X0) {
  X <- numeric(n)  # store sequence here
  X[1] <- X0  # first value is seed
  for (i in 2:n) {
    X[i] <- (a * X[i - 1] + c) %% m  # find next val in sequence
  }
  return(X)
}

# testing
m <- 8
a <- 5
c <- 1
X0 <- 2
n <- 100
random_sequence <- lin_congruence(n, m, a, c, X0)
print(random_sequence)
```

```
##   [1] 2 3 0 1 6 7 4 5 2 3 0 1 6 7 4 5 2 3 0 1 6 7 4 5 2 3 0 1 6 7 4 5 2 3 0 1 6
##  [38] 7 4 5 2 3 0 1 6 7 4 5 2 3 0 1 6 7 4 5 2 3 0 1 6 7 4 5 2 3 0 1 6 7 4 5 2 3
##  [75] 0 1 6 7 4 5 2 3 0 1 6 7 4 5 2 3 0 1 6 7 4 5 2 3 0 1
```

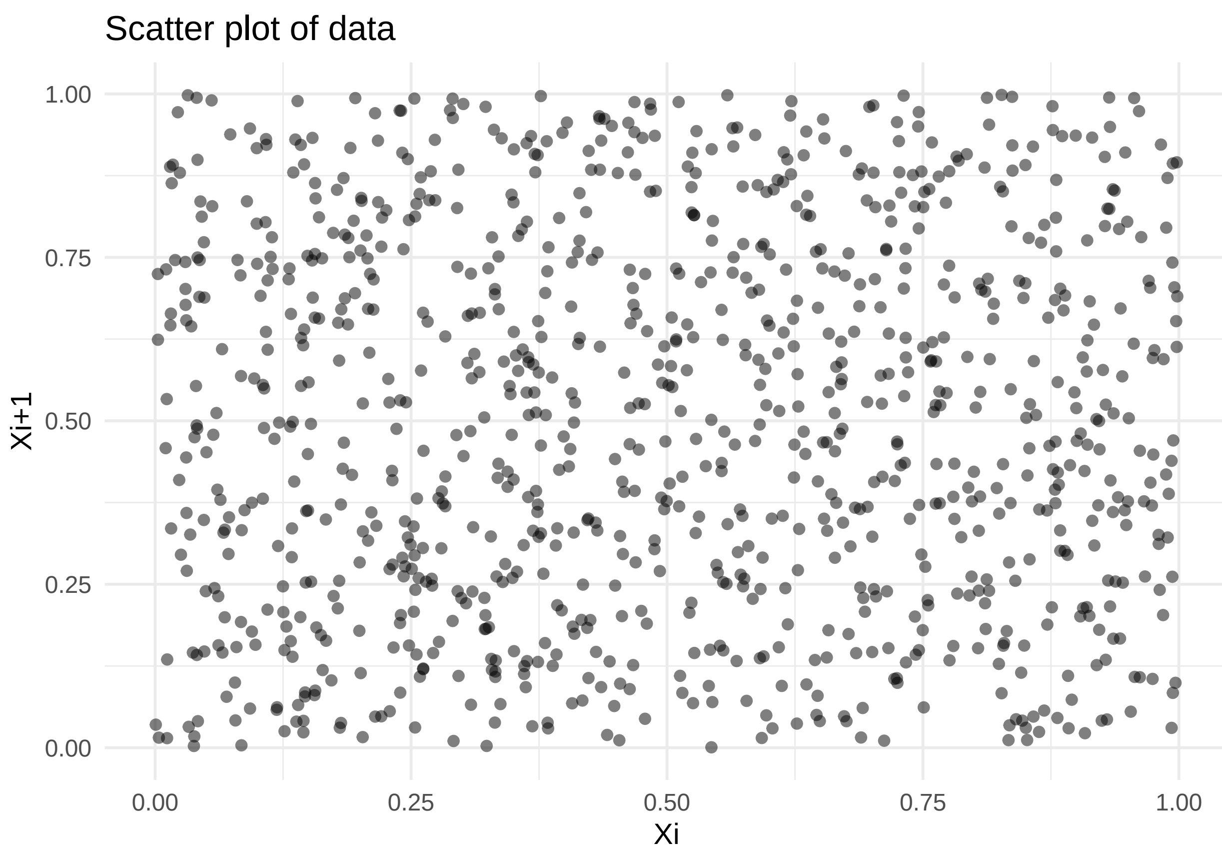
Here the method produces a cycle, as this occurs when c is not very large. This is the reason that most computers use c = 2ˆ31. From class, something similar happened.

2. Given a sequence X1, X2, . . . , Xn of U[0,1]- distributed PRN, we can use a scatter plot of (Xi, Xi+1) for i = 1, . . . , n - 1 in order to try to assess whether the Xi are independent.

(a) Create such a plot using the built in random number generator in R. Can you explain the resulting plot.

```
library(ggplot2)


n <- 1000
random_sequence <- runif(n) # generate distributions

data <- data.frame(Xi = random_sequence[1:(n-1)], Xi1 = random_sequence[2:n])
```

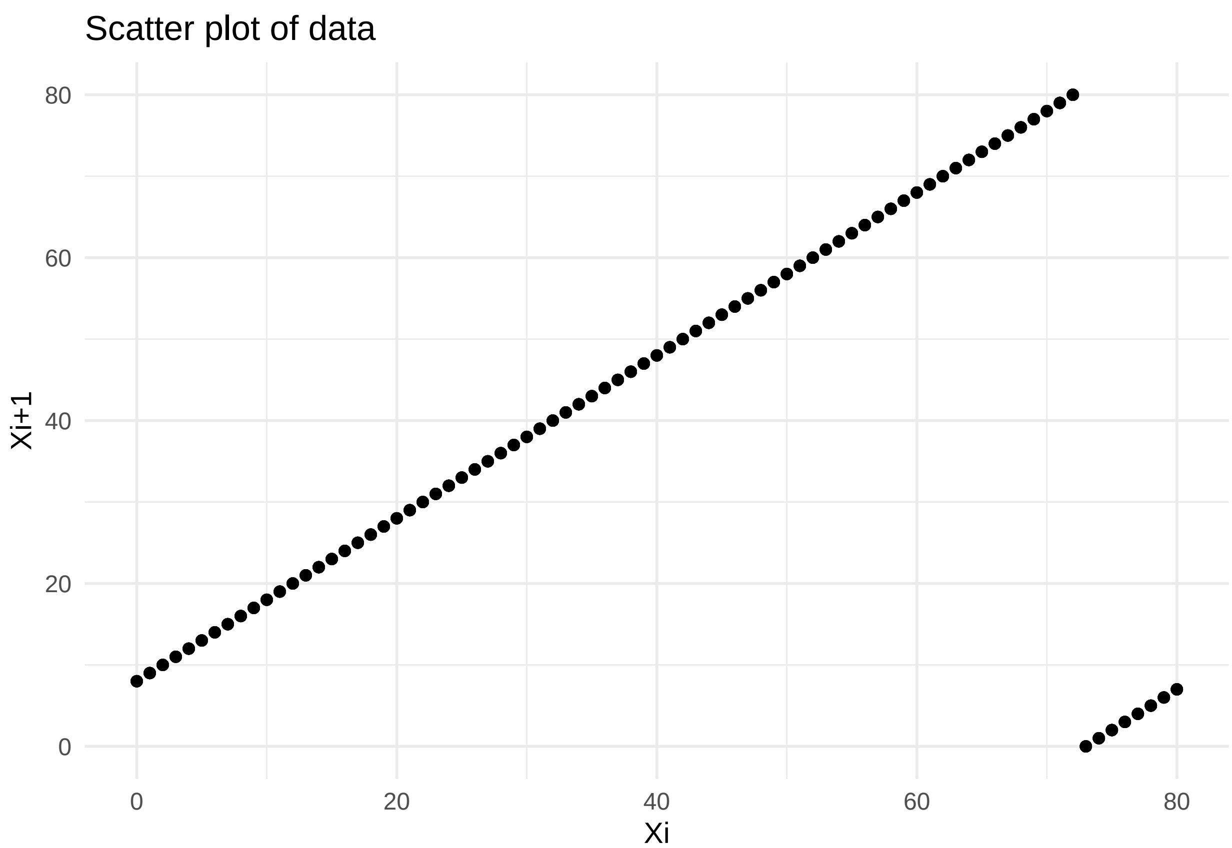
```r
# create scatter plot
ggplot(data, aes(x = Xi, y = Xi1)) +
  geom_point(alpha = 0.5) +
  labs(x = "Xi", y = "Xi+1", title = "Scatter plot of data") +
  theme_minimal()
```



If each value is generated randomly, then pairing up coordinates will be random all the way and create a plot of points that are evenly dispersed all around.

(b) Create a similar plot using your function from exercise 1 with m = 81, a = 1, c = 8 and seed 0. Discuss the resulting plot.

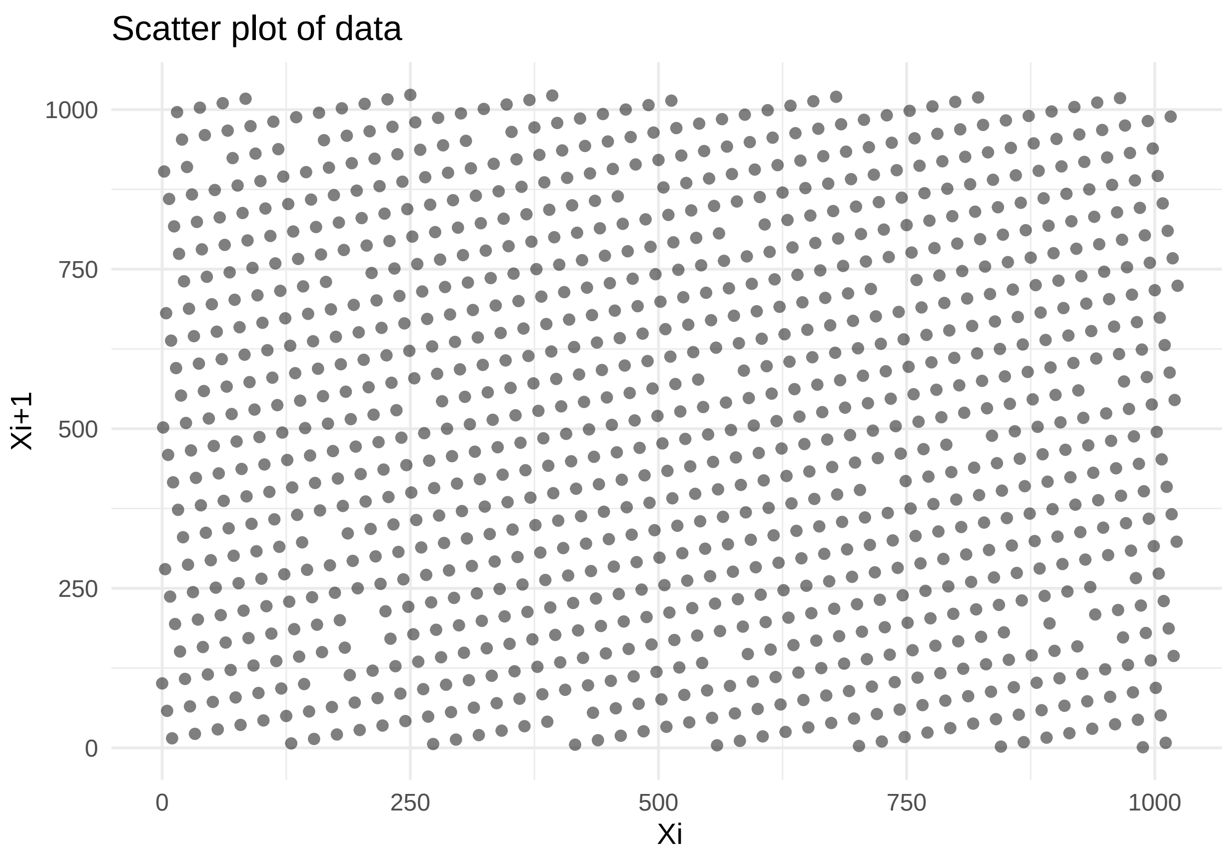
```r
random_sequence <- lin_congruence(1000, 81, 1, 8, 0)

# 1000 random values,

data <- data.frame(Xi = random_sequence[1:(n-1)], Xi1 = random_sequence[2:n])


# make scatter plot
ggplot(data, aes(x = Xi, y = Xi1)) +
  geom_point(alpha = 0.5) +
  labs(x = "Xi", y = "Xi+1", title = "Scatter plot of data") +
  theme_minimal()
```



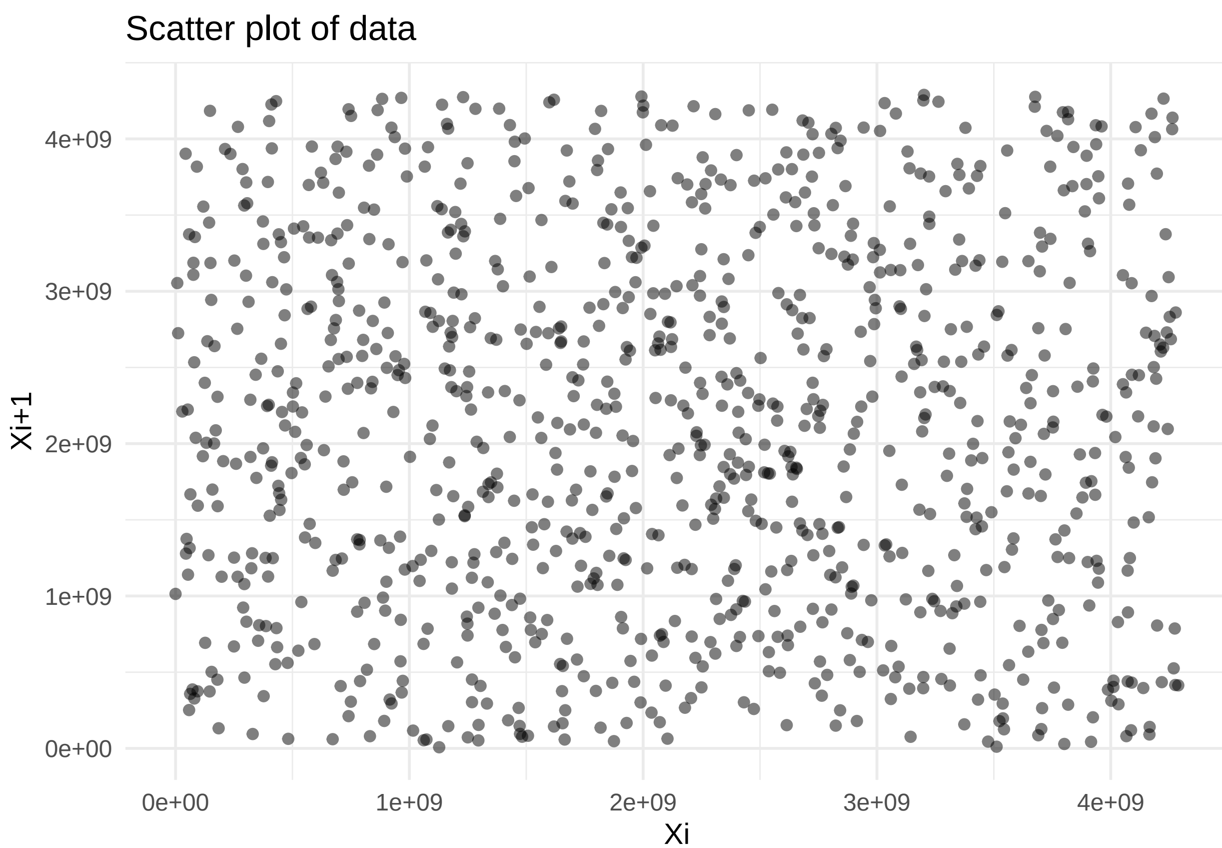
The points are linear here, because the a value is 1 and so we keep adding 8 until the Xi is greater than or equal to 81, which is the modulus and will reset the count.

(c) Repeat the experiment from (b) using the parameters m = 1024, a = 401, c = 101 and m = 232, a = 1664525, c = 1013904223. Discuss the results.

```
random_sequence <- lin_congruence(1000, 1024, 401, 101, 0)


data <- data.frame(Xi = random_sequence[1:(n-1)], Xi1 = random_sequence[2:n])

ggplot(data, aes(x = Xi, y = Xi1)) +
  geom_point(alpha = 0.5) +
  labs(x = "Xi", y = "Xi+1", title = "Scatter plot of data") +
  theme_minimal()
```




```r
random_sequence <- lin_congruence(1000, 2^32, 1664525, 1013904223, 0)

data <- data.frame(Xi = random_sequence[1:(n-1)], Xi1 = random_sequence[2:n])

# make scatter
ggplot(data, aes(x = Xi, y = Xi1)) +
  geom_point(alpha = 0.5) +
  labs(x = "Xi", y = "Xi+1", title = "Scatter plot of data") +
  theme_minimal()
```



In the first test case here, there is a pattern, but the numbers are still distributed pretty evenly. In the second case, the mod is so large that the data is very random, this is why computers use this value to generate values as random as possible. In both these examples, I was not sure if it was required to shrink the data between 0 and 1, but if that were the case, the data would still be evenly distributed.

3. Simulate a Drunkard's walk in two dimensions. The Drunkard's walk is a random process where at each step, the walker randomly chooses one of eight possible directions (North, South, East, West, Northeast, Southeast, Southwest, Northwest) with varying probabilities. Your task is to implement the simulation according to the following specifications:

(a) Define a vector named probabilities representing the probabilities of moving in each direction: North: 10% South: 15% East: 15% West: 20% Northeast: 10% Southeast: 10% Southwest: 10% Northwest: 10%

```r
probabilities <- c(North = 0.1, South = 0.15, East = 0.15, West = 0.2,
                   Northeast = 0.1, Southeast = 0.1, Southwest = 0.1, Northwest = 0.1)

# need probabilities vector
```

Simply create the vector.

(b) Write a function named simulate_step that simulates one step of the drunkard's walk: The function should randomly select one of the eight directions based on the specified probabilities. It should return a vector representing the step in the chosen direction, where the first element indicates the change in the x-coordinate (East/West) and the second element indicates the change in the y-coordinate (North/South).

```r
simulate_step <- function() {
  direction <- sample(names(probabilities), 1, prob = probabilities)

  step <- switch(direction,
                 North = c(0, 1),
                 South = c(0, -1),
                 East = c(1, 0),
                 West = c(-1, 0),
                 Northeast = c(1, 1),
                 Southeast = c(1, -1),
                 Southwest = c(-1, -1),
                 Northwest = c(-1, 1))

  # change in x and y decided by vector
  # used a simple switch statement

  return(step)
}

# test function
step <- simulate_step()
print(step)
```

```
## [1] 1 0
```

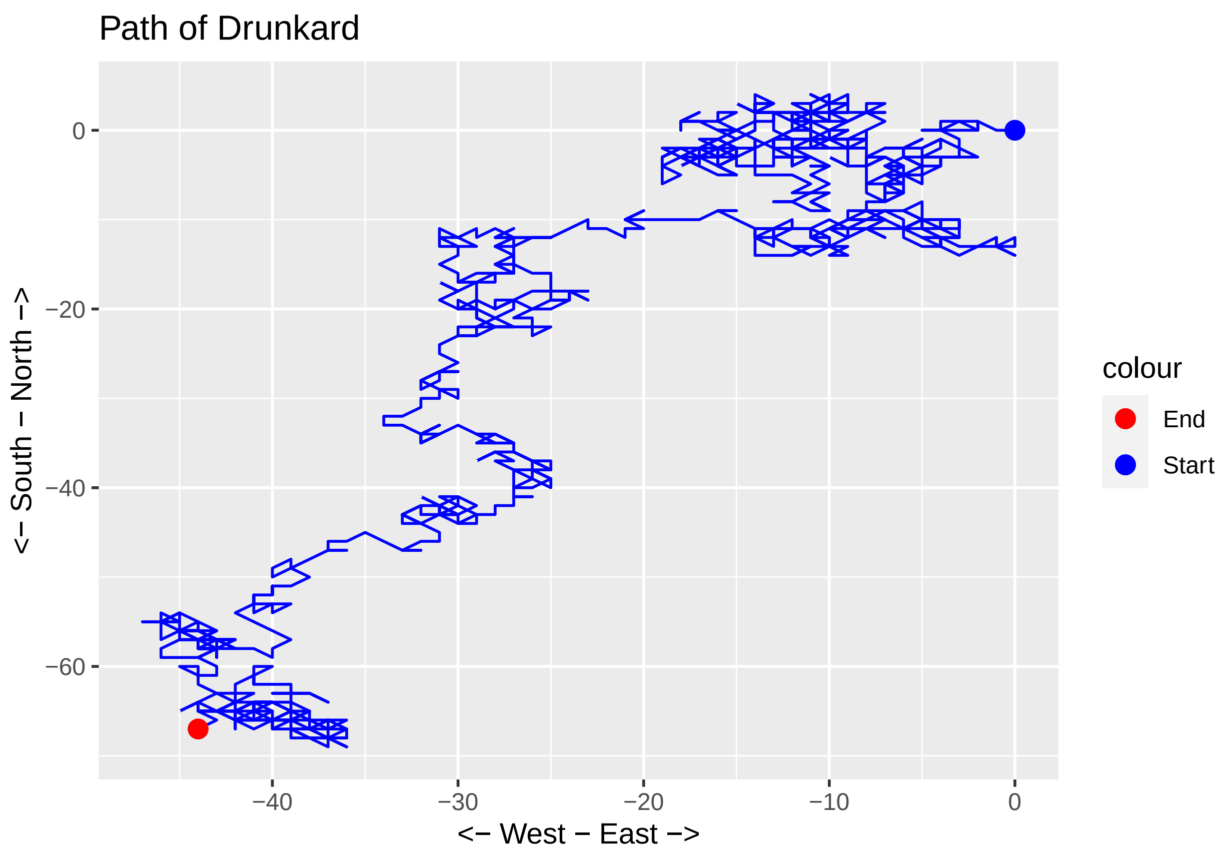
(c) Simulate the drunkard's walk for a total of 1000 steps. Initialize a matrix named positions with dimensions (1001, 2) to store the positions of the drunkard at each step. Use a for loop to iteratively call the simulate_step function and update the drunkard's position accordingly.

```r
num_steps <- 1001

positions <- matrix(0, nrow = num_steps, ncol = 2)


positions[1, ] <- c(0, 0) # start position

for (i in 2:(num_steps)) {
  step <- simulate_step()
  positions[i, ] <- positions[i - 1, ] + step
  # update step positions
}
```

(d) Plot the path of the drunkard's walk in two dimensions. Use the plot function to create a line plot of the positions. Mark the starting point in red and the ending point in blue.

```r
positions_df <- data.frame(Xcoord = positions[, 1], Ycoord = positions[, 2])

# use ggplot

ggplot(positions_df, aes(x = Xcoord, y = Ycoord)) +
  geom_path(color = "blue") +
  geom_point(data = positions_df[1, , drop = FALSE], aes(color = "Start"), size = 3) +
```

```
  geom_point(data = tail(positions_df, n = 1), aes(color = "End"), size = 3) +
  scale_color_manual(values = c("red", "blue")) +
  labs(x = "<- West - East ->", y = "<- South - North ->", title = "Path of Drunkard")
```

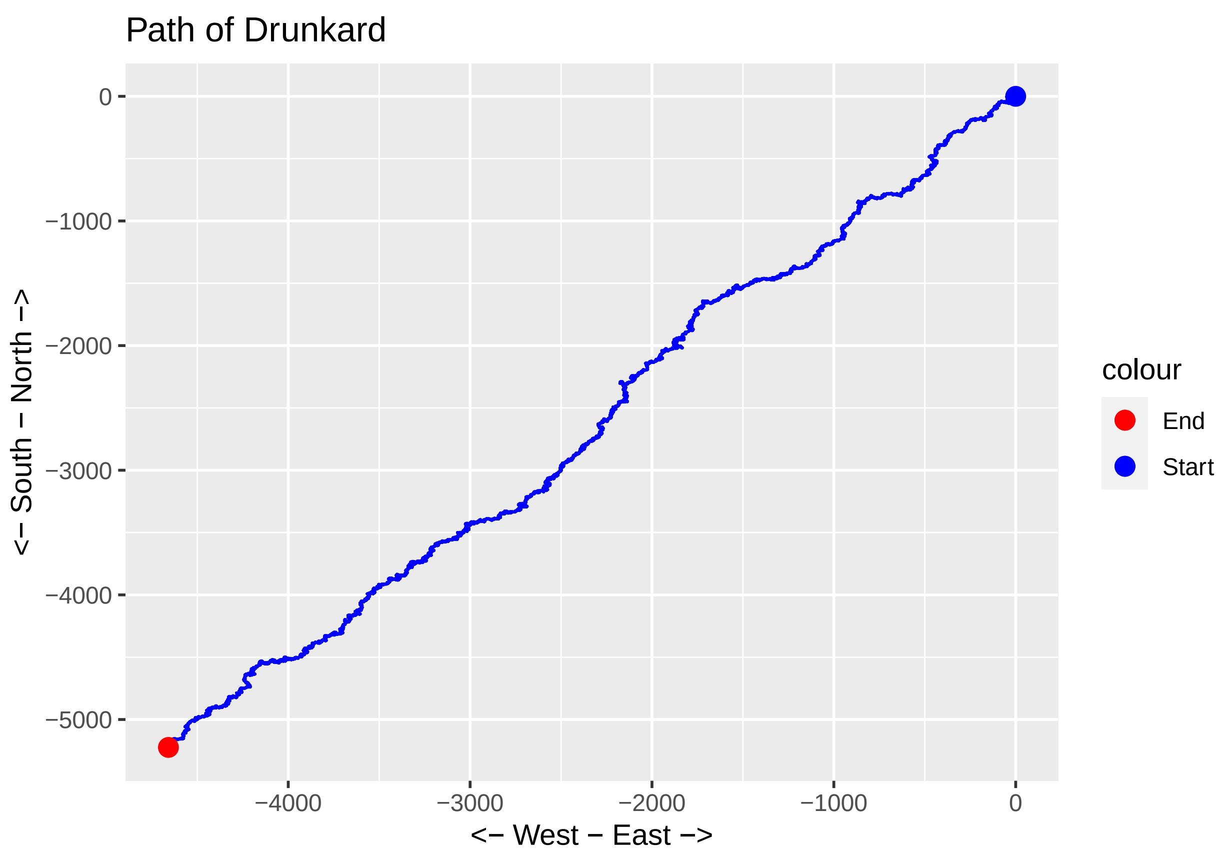


If you add up the probabilities, the walker has a 30% chance of going North, 35% South, 35% East, and 40% West (The probabilities do not add up to 100% as several of the outcomes are possible ex. Northeast will move the drunkard both North AND East). From this, over a longer period of time with more steps, the drunkard should move Southwest, with a slop of approximately 1.

This next portion was not necessary for the assignment, but I thought it would be cool to include. This is the same simulation but for 100000 steps. You will notice the clear Southwest line and the slope of 1.

```
num_steps <- 100001

positions <- matrix(0, nrow = num_steps, ncol = 2)


positions[1, ] <- c(0, 0) # start position

for (i in 2:(num_steps)) {
  step <- simulate_step()
  positions[i, ] <- positions[i - 1, ] + step
  # update step positions
}

positions_df <- data.frame(Xcoord = positions[, 1], Ycoord = positions[, 2])
```

```r
# use ggplot

ggplot(positions_df, aes(x = Xcoord, y = Ycoord)) +
  geom_path(color = "blue") +
  geom_point(data = positions_df[1, , drop = FALSE], aes(color = "Start"), size = 3) +
  geom_point(data = tail(positions_df, n = 1), aes(color = "End"), size = 3) +
  scale_color_manual(values = c("red", "blue")) +
  labs(x = "<- West - East ->", y = "<- South - North ->", title = "Path of Drunkard")
```



4. Assume X ~ Exp(1) and Y ~ N(0, X), that is Y is normally distributed with a random variance. Use Monte Carlo estimation to estimate E(X|Y = 4) and Var(X|Y = 4).

```r
# This question is worded very strangely, so my solution is my attempt
# to translate it

n_samples <- 100000000 # lets do 100 Mil simulations for accuracy.
# (I did 10K then 100K then 1Mil, then 10Mil and 100Mil)
# and the values slowly converged

generate_samples <- function(n) {
  X <- rexp(n,1)
  Y <- rnorm(n,mean = 0, sd = sqrt(X))

  return(list(X=X,Y=Y))
```

```r
}

samples <- generate_samples(n_samples)


X <- samples$X
Y <- samples$Y
valid_X <- X[floor(Y) == 4]


cond_exp <- mean(valid_X) ## expected value

cond_var <- var(valid_X)

cond_exp
```

```
## [1] 3.602084
```

```r
cond_var
```

```
## [1] 2.084849
```

5. Consider a simple stock price model with two states: up and down. The transition probabilities are: If the stock is up today, there's a 60% chance it will be up tomorrow and a 40% chance it will be down. If the stock is down today, there's a 50% chance it will be up tomorrow and a 50% chance it will be down. Simulate this Markov Chain to predict the stock price movement over a certain number of days.

```r
multiply_matrix_n_times <- function(matrix, n) { # create a simple function to multiply a matrix by its
  result <- matrix

  for (i in 2:n) {
    result <- result %*% matrix
  }

  return(result)
}


simulate_stock_price <- function(initial_state, num_days) {

  transition_matrix <- matrix(c(0.6,0.5,0.4,0.5),nrow = 2) # make a 2x2 transition matrix
  simulate_n_times_matrix <- multiply_matrix_n_times(transition_matrix,num_days)

  # now multiply the initial vector

  state_after_n_days <- initial_state %*% simulate_n_times_matrix

  return(state_after_n_days)
}

# testing
```

```r
initial_vector <- c(1,0)
# note here that the first item of the vector is the probability of
#the stock being up, and the second is the probability of the stock being down.

result <- simulate_stock_price(initial_vector,10) # after 10 days probabilities

print("Percent chance the stock will go up")
```

```
## [1] "Percent chance the stock will go up"
```

```r
print(result[1])
```

```
## [1] 0.5555556
```

```r
print("Percent chance the stock will go down")
```

```
## [1] "Percent chance the stock will go down"
```

```r
print(result[2])
```

```
## [1] 0.4444444
```

As expected, the percent chance the stock will go up/down converged.