UNIVERSITÀ DEGLI STUDI DI TORINO

CORSO DI LAUREA MAGISTRALE IN ASTROFISICA E FISICA TEORICA

Titolo

TESI DI LAUREA MAGISTRALE

Relatore:
Prof.
Panero Marco

Candidato:
Aliberti Marco
Matricola 855766

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Abstract

Contents

QCD on the Lattice

1.1 The QCD Continuum Action

In order to write the action of QCD on the lattice, I must first recall how the theory is formulated in the continuum.

1.1.1 Spinor Fields

Let us take into consideration a (free) quantum field theory describing a fermion, such as a quark or a lepton, in a 4-dimensional spacetime with metric $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$. Its action (in natural units, where $c = \hbar = 1$) can be written as:

$$S_F[\psi(x),\overline{\psi}(x)] = \int \mathrm{d}^4x \left(\mathrm{i}\overline{\psi}\partial\!\!\!/\psi - m\overline{\psi}\psi\right) \tag{1.1.1}$$

from which, upon the application of the variational principle, the Dirac equation follows:

$$\left(\mathrm{i}\partial\!\!\!/ - m\right)\psi(x) = 0 \tag{1.1.2}$$

It can now be easily checked by direct computation that this action is invariant under a rigid (global) phase transformation, also called global $U(1)$ transformation:

$$\begin{aligned} \psi &\to \psi' = e^{-\mathrm{i}\alpha}\psi \\ \overline{\psi} &\to \overline{\psi}' = \overline{\psi}e^{\mathrm{i}\alpha} \end{aligned} \tag{1.1.3}$$

where α is a constant that does not depend on the spacetime coordinate x, while if $\alpha = \alpha(x)$ the action (1.1.1) would not be invariant because of the kinetic term.

1.1.2 Quantum Electrodynamics

As the free field theory itself is non interacting, it does not provide any real-world prediction, so it is useful to write an interacting action where the spinor field is coupled, for instance, to a vector field A_μ, i.e. the photon. One way to implement this interaction is to ask for local, instead of global, invariance of the action (1.1.1) under the phase transformation (1.1.3), where now $\alpha = \alpha(x)$. In order to do so, one has to define the covariant derivative as follows:

$$D_\mu \equiv (\partial_\mu + \mathrm{i}gA_\mu) \tag{1.1.4}$$

where g is the couplig constant.[1]
The vector field's kinetic term is written in terms of its field-strength, namely:

$$\begin{aligned} F_{\mu\nu} \equiv& -\frac{\mathrm{i}}{g}\left[D_\mu, D_\nu\right] = \\ =& -\frac{\mathrm{i}}{g}\left(D_\mu\left(\partial_\nu + \mathrm{i}gA_\nu\right) - D_\nu\left(\partial_\mu + \mathrm{i}gA_\mu\right)\right) = \\ =& -\frac{\mathrm{i}}{g}\left(\cancel{\partial_\mu\partial_\nu} + \mathrm{i}g\partial_\mu A_\nu - g^2A_\mu A_\nu - \cancel{\partial_\nu\partial_\mu} - \mathrm{i}g\partial_\nu A_\mu + g^2A_\nu A_\mu\right) = \\ =& \partial_\mu A_\nu - \partial_\nu A_\mu + \mathrm{i}g[A_\mu, A_\nu] = \\ =& \partial_\mu A_\nu - \partial_\nu A_\mu \end{aligned} \tag{1.1.5}$$

[1] Usually g is called e, the electron charge, though I will be using g in analogy to nonabelian gauge theories.

where the term with the commutator is $= 0$ in the abelian theory.
Two different fields A_μ and A'_μ describe the same physics if one can be obtained from another throug a gauge transformation:

$$A'_\mu(x) = A_\mu(x) - \frac{1}{g}\partial_\mu \alpha(x) \tag{1.1.6}$$
$$F'_{\mu\nu} = F_{\mu\nu} - \frac{1}{g}\left(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu\right)\alpha(x) = F_{\mu\nu}$$

Thus, the free action for the vector field is:

$$S_{EM} = -\frac{1}{4}\int \mathrm{d}^4 x F_{\mu\nu} F^{\mu\nu} \tag{1.1.7}$$

That is also gauge invariant, i.e. invariant under (1.1.6), as $F_{\mu\nu}$ is gauge invariant.
The term that broke the local phase invariance of the action (1.1.1) can now be "absorbed" by A_μ through a gauge transformation (1.1.6), thus making the full action gauge invariant:

$$S = \int \mathrm{d}^4 x \left(\mathrm{i}\overline{\psi}\partial\!\!\!/\psi - m\overline{\psi}\psi - g\overline{\psi}A\!\!\!/\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) \tag{1.1.8}$$

1.1.3 Nonabelian Gauge Theories

Computer Simulation of Gauge Theories

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