



**UNIVERSITÀ
DI TORINO**

UNIVERSITÀ DEGLI STUDI DI TORINO

CORSO DI LAUREA MAGISTRALE IN ASTROFISICA E FISICA TEORICA

Titolo

TESI DI LAUREA MAGISTRALE

Relatore:

Prof.
Panero Marco

Candidato:

Aliberti Marco
Matricola 855766

ANNO ACCADEMICO 2022/2023

Abstract

Contents

1	QCD on the Lattice	1
1.1	The QCD Continuum Action	2
1.1.1	Spinor Fields	2
1.1.2	Quantum Electrodynamics	2
1.1.3	Nonabelian Gauge Theories	3
2	Computer Simulation of Gauge Theories	4
3	Gauge Theories Simulation on non-hypercubic lattice F4	5
4	Simulation Results	6
5	Conclusions	7

QCD on the Lattice

1.1 The QCD Continuum Action

In order to write the action of QCD on the lattice, I must first recall how the theory is formulated in the continuum.

1.1.1 Spinor Fields

Let us take into consideration a (free) quantum field theory describing a fermion, such as a quark or a lepton, in a 4-dimensional spacetime with metric $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. Its action (in natural units, where $c = \hbar = 1$) can be written as:

$$S_F[\psi(x), \bar{\psi}(x)] = \int d^4x (\bar{\psi} \not{\partial} \psi - m \bar{\psi} \psi) \quad (1.1.1)$$

from which, upon the application of the variational principle, the Dirac equation follows:

$$(\not{\partial} - m) \psi(x) = 0 \quad (1.1.2)$$

It can now be easily checked by direct computation that this action is invariant under a rigid (global) phase transformation, also called global $U(1)$ transformation:

$$\begin{aligned} \psi &\rightarrow \psi' = e^{-i\alpha} \psi \\ \bar{\psi} &\rightarrow \bar{\psi}' = \bar{\psi} e^{i\alpha} \end{aligned} \quad (1.1.3)$$

where α is a constant that does not depend on the spacetime coordinate x , while if $\alpha = \alpha(x)$ the action (1.1.1) would not be invariant because of the kinetic term.

1.1.2 Quantum Electrodynamics

As the free field theory itself is non interacting, it does not provide any real-world prediction, so it is useful to write an interacting action where the spinor field is coupled, for instance, to a vector field A_μ , i.e. the photon. One way to implement this interaction is to ask for local, instead of global, invariance of the action (1.1.1) under the phase transformation (1.1.3), where now $\alpha = \alpha(x)$. In order to do so, one has to define the covariant derivative as follows:

$$D_\mu \equiv (\partial_\mu + igA_\mu) \quad (1.1.4)$$

where g is the coupling constant.¹

The vector field's kinetic term is written in terms of its field-strength, namely:

$$\begin{aligned} F_{\mu\nu} &\equiv -\frac{i}{g} [D_\mu, D_\nu] = \\ &= -\frac{i}{g} (D_\mu (\partial_\nu + igA_\nu) - D_\nu (\partial_\mu + igA_\mu)) = \\ &= -\frac{i}{g} (\cancel{\partial_\mu \partial_\nu} + ig\partial_\mu A_\nu - g^2 A_\mu A_\nu - \cancel{\partial_\nu \partial_\mu} - ig\partial_\nu A_\mu + g^2 A_\nu A_\mu) = \\ &= \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] = \\ &= \partial_\mu A_\nu - \partial_\nu A_\mu \end{aligned} \quad (1.1.5)$$

¹Usually g is called e , the electron charge, though I will be using g in analogy to nonabelian gauge theories.

where the term with the commutator is $= 0$ in the abelian theory.

Two different fields A_μ and A'_μ describe the same physics if one can be obtained from another through a gauge transformation:

$$\begin{aligned} A'_\mu(x) &= A_\mu(x) - \frac{1}{g} \partial_\mu \alpha(x) \\ F'_{\mu\nu} &= F_{\mu\nu} - \frac{1}{g} (\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) \alpha(x) = F_{\mu\nu} \end{aligned} \quad (1.1.6)$$

Thus, the free action for the vector field is:

$$S_{EM} = -\frac{1}{4} \int d^4x F_{\mu\nu} F^{\mu\nu} \quad (1.1.7)$$

That is also gauge invariant, i.e. invariant under (1.1.6), as $F_{\mu\nu}$ is gauge invariant.

The term that broke the local phase invariance of the action (1.1.1) can now be “absorbed” by A_μ through a gauge transformation (1.1.6), thus making the full action gauge invariant:

$$S = \int d^4x \left(i\bar{\psi} \not{\partial} \psi - m\bar{\psi} \psi - g\bar{\psi} A \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) \quad (1.1.8)$$

1.1.3 Nonabelian Gauge Theories

Computer Simulation of Gauge Theories

Gauge Theories Simulation on non-hypercubic lattice F4

Simulation Results

Conclusions
