Quantum Field Theory on a Highly Symmetric Lattice

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Why Lattice Quantum Chromodynamics?

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$$\langle f|i\rangle = \int_{\phi_i}^{\phi_f} \mathcal{D}\left[\phi\right] e^{-S[\phi]}$$

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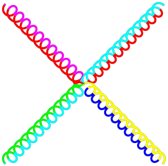

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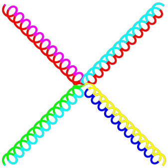
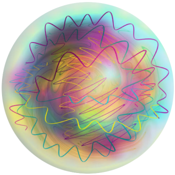


Non-Perturbative



Perturbative vs Non-Perturbative

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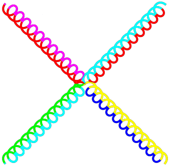

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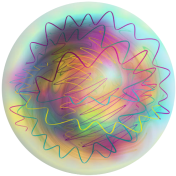

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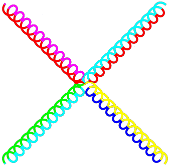

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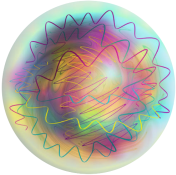

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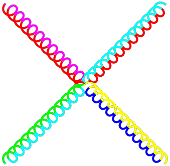

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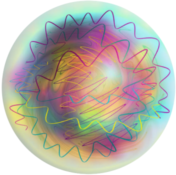

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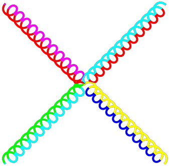

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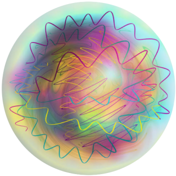

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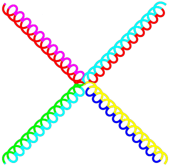

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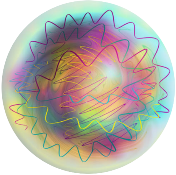

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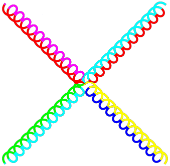

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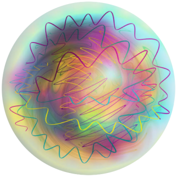

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What is a Lattice?

Definition: Lattice Λ

$\Lambda = \{\sum_{i=1}^{n} a_i e_i \mid a_i \in \mathbb{Z}\}$, with $\{e_i\}$ any basis of $\mathbb{R}^n$

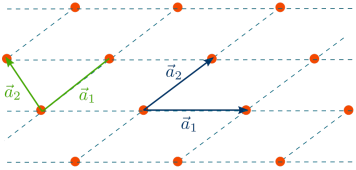

Figure: A bidimensional lattice.

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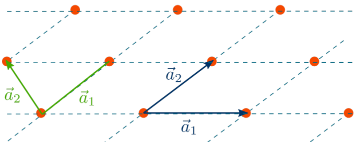

Figure: A bidimensional lattice.

Hypercubic lattice

$\{e_i\}$ is the canonical basis of $\mathbb{R}^n$
a is called *lattice spacing*.

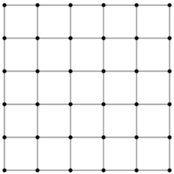

Figure: A square lattice.

Lattice Field Theory

Basic idea

Fields can take values only in given parts of the lattice, $x \to n \in \Lambda$.

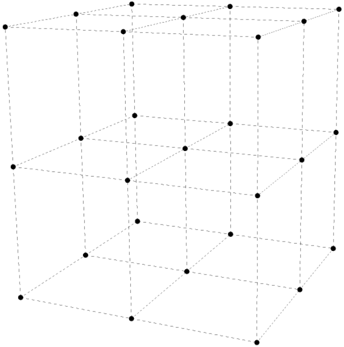

Figure: A (hyper)cubic lattice in $\mathbb{R}^3$.

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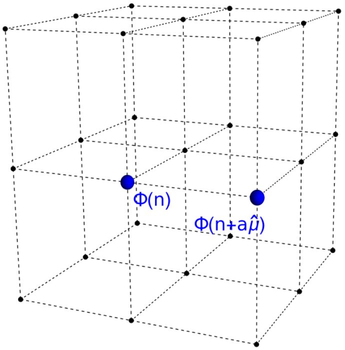

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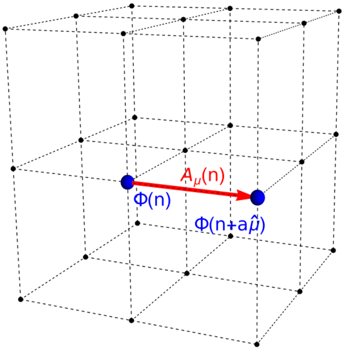

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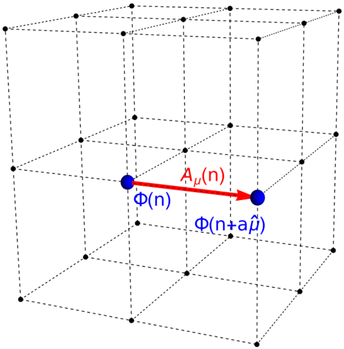

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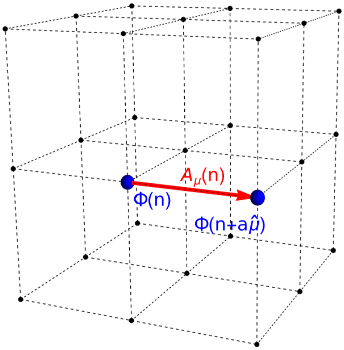

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Attention!

Spinorial fields are trickier to be discretized.