Università degli Studi di Torino

Corso di Laurea Magistrale in Astrofisica e Fisica Teorica

Titolo

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Abstract

Contents

1 QCD on the Lattice **4**
1.1 The QCD Continuum Action 5
1.1.1 Spinor Fields 5
1.1.2 Quantum Electrodynamics 5

2 Computer Simulation of Gauge Theories **7**

3 Gauge Theories Simulation on non-hypercubic lattice F4 **8**

4 Simulation Results **9**

5 Conclusions **10**

QCD on the Lattice

1.1 The QCD Continuum Action

In order to write the action of QCD on the lattice, I must first recall how the theory is formulated in the continuum.

1.1.1 Spinor Fields

Let us take into consideration a (free) quantum field theory describing a fermion, such as a quark or a lepton, in a 4-dimensional spacetime with metric $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$. Its action (in natural units, where $c = \hbar = 1$) can be written as:

$$S_F[\psi(x),\overline{\psi}(x)] = \int \mathrm{d}^4x \left(\mathrm{i}\overline{\psi}\partial\!\!\!/\psi - m\overline{\psi}\psi\right) \tag{1.1.1}$$

from which, upon the application of the variational principle, the Dirac equation follows:

$$\left(\mathrm{i}\partial\!\!\!/ - m\right)\psi(x) = 0 \tag{1.1.2}$$

It can now be easily checked by direct computation that this action is invariant under a rigid phase transformation:

$$\begin{aligned} \psi &\to \psi' = e^{-\mathrm{i}\alpha}\psi \\ \overline{\psi} &\to \overline{\psi}' = \overline{\psi}e^{\mathrm{i}\alpha} \end{aligned} \tag{1.1.3}$$

where α is a constant that does not depend on the spacetime coordinate x, while if $\alpha = \alpha(x)$ the action (1.1.1) would not be invariant because of the kinetic term.

1.1.2 Quantum Electrodynamics

As the free field theory itself is non interacting, it does not provide any real-world prediction, so it is useful to write an interacting action where the spinor field is coupled, for instance, to a vector field A_μ, i.e. the photon. The action for the vector field is written in terms of its field-strength, namely: [1]

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1.1.4}$$

where two different fields A_μ and A'_μ describe the same physics if one can be obtained from another throug a gauge transformation:

$$\begin{aligned} A'_\mu(x) &= A_\mu(x) - \frac{1}{g}\partial_\mu\Lambda(x) \\ F'_{\mu\nu} &= F_{\mu\nu} - \frac{1}{g}\left(\partial_\mu\partial_\nu - \partial_\nu\partial_\mu\right)\Lambda = F_{\mu\nu} \end{aligned} \tag{1.1.5}$$

with $\Lambda(x)$ being any (at least C^2) scalar function.
Thus, the free action for the vector field is:

$$S_{EM} = -\frac{1}{4}\int \mathrm{d}^4x F_{\mu\nu}F^{\mu\nu} \tag{1.1.6}$$

[1] Actually, this is not a definition, as $F_{\mu\nu}$ is defined as the curvature tensor, obtained through the commutator of the covariant derivative: $F_{\mu\nu} \equiv -\mathrm{i}g\left[D_\mu, D_\nu\right]$

That is also gauge invariant, i.e. invariant under (1.1.5), as $F_{\mu\nu}$ is gauge invariant.
In order to write a fully covariant, gauge-invariant interacting action, the covariant derivative on the spinor has to be defined as follows:

$$D_\mu \psi \equiv (\partial_\mu + \mathrm{i} g A_\mu)\, \psi \tag{1.1.7}$$

Computer Simulation of Gauge Theories

Gauge Theories Simulation on non-hypercubic lattice F4

Simulation Results

Conclusions