Quantum Field Theory on a Highly Symmetric Lattice

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Why Lattice Quantum Chromodynamics?

In quantum field theory scattering amplitudes in the form

$$\langle f|i\rangle = \int_{\phi_i}^{\phi_f} \mathcal{D}\left[\phi\right] e^{-S[\phi]}$$

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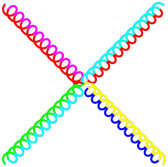

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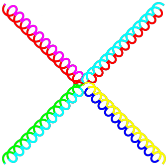
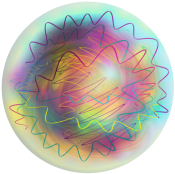


Non-Perturbative



Perturbative vs Non-Perturbative

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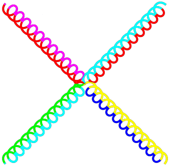

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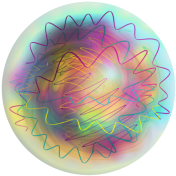

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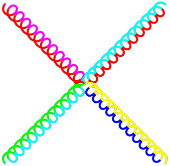

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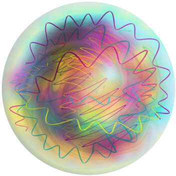

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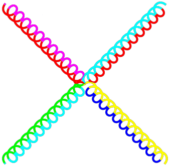

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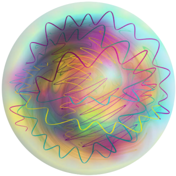

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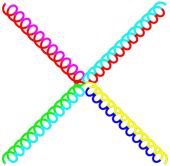

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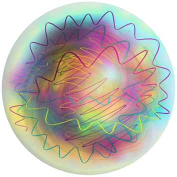

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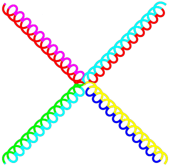

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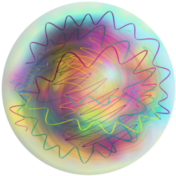

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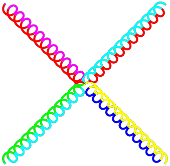

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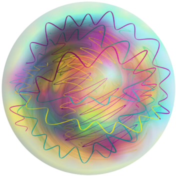

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Lattice Field Theory

Definition: Lattice Λ

$\Lambda = \{\sum_{i=1}^{n} a_i e_i \mid a_i \in \mathbb{Z}\}$, $\{e_i\}$ basis of $\mathbb{R}^n$

Hypercubic lattice: $\{e_i\}$ is the canonical basis of $\mathbb{R}^n$, a is called *lattice spacing*.

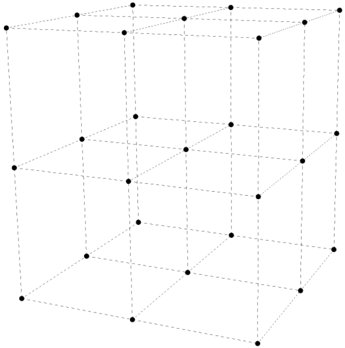

Figure: Example of a cubic lattice in $\mathbb{R}^3$.

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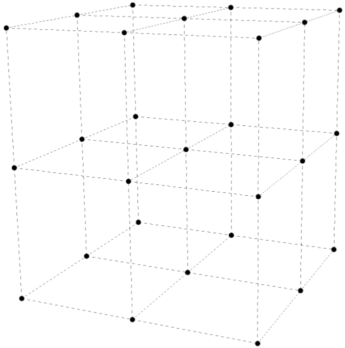
Basic idea: fields can take values only in given parts of the lattice, $x \rightarrow n \in \Lambda$.



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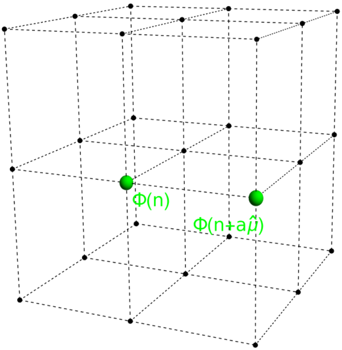

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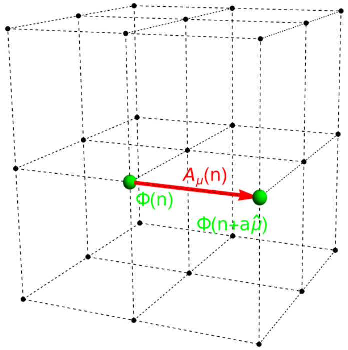

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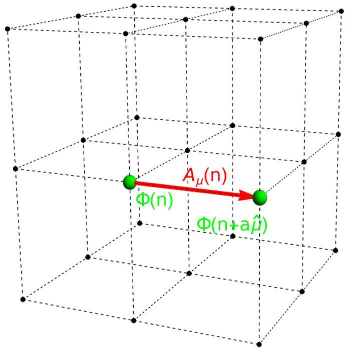

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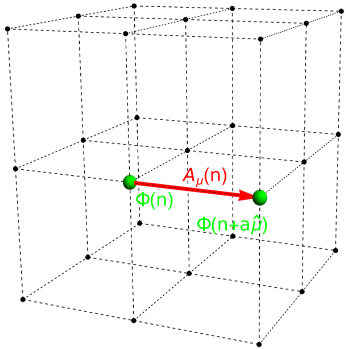
Attention!

Spinorial fields are trickier.



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