Università degli Studi di Torino

Corso di Laurea Magistrale in Astrofisica e Fisica Teorica

Quantum Field Theory on a Highly Symmetric Lattice

Tesi di Laurea Magistrale

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Abstract

The regularization on a Euclidean lattice, first proposed by Kenneth G. Wilson in 1974, remains the only approach to study strongly coupled, non-supersymmetric non-Abelian gauge theories (including, in particular, quantum chromodynamics: the fundamental theory of the strong nuclear interaction in the Standard Model of elementary-particle physics) from first principles.

While normally the theory is discretized on a four-dimensional hypercubic grid, this is not the only possible choice, and the fact that the explicit breaking of Lorentz-Poincaré symmetries due to the discretization has an impact on the approach to the continuum limit is a motivation to consider the regularization also on other, more symmetric, lattices.

The goal of this thesis project consists in studying Yang-Mills theories based on local SU(N) invariance on the lattice of the roots of the exceptional simple Lie group F_4, which is a four-dimensional body-centered cubic lattice, and the most symmetric regular lattice that exists in four dimensions.

Contents

Yang-Mills Theories on the Lattice

1.1 The Yang-Mills Continuum Action

The aim of this chapter is to discretize the Yang-Mills action on a hypercubic lattice in 4 dimensions. In order to do so, the action is obtained firstly in the continuum, beginning from the simplest case, Quantum Electrodynamics.
This first section is based on material that can be found in standard textbooks on quantum field theory [1–5], personal notes and computations.

1.1.1 Scalar Fields

In quantum field theory, a real scalar massive field is described, in a 4-dimensional space-time with metric $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, by the following Lorentz-covariant action (in natural units, where $c = \hbar = 1$):

$$S[\phi] = \int \mathrm{d}^4x \left(-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 + V(\phi) \right) \tag{1.1.1}$$

where $V(\phi)$ is any potential, such as $\frac{g}{6}\phi^3$ or $\frac{g}{4!}\phi^4$. Real scalar fields do not describe any real-world elementary particle, though they are useful to learn basic principles of quantum field theory, as they are the simplest fields that can be written.

1.1.2 Dirac Spinor Fields

Let us now take into consideration a (free) quantum field theory describing a fermion, such as a quark or a lepton. Its action can be written as:

$$S_\psi[\psi(x), \overline{\psi}(x)] = \int \mathrm{d}^4x \left(i\overline{\psi}\slashed{\partial}\psi - m\overline{\psi}\psi \right) \tag{1.1.2}$$

from which, upon the application of the variational principle, the Dirac equation follows:

$$\left(i\slashed{\partial} - m \right)\psi(x) = 0 \tag{1.1.3}$$

It can now be easily checked by direct computation that this action is invariant under a rigid (global) phase transformation, also called a global $U(1)$ transformation:

$$\psi(x) \rightarrow \psi'(x) = e^{-i\alpha}\psi(x) \tag{1.1.4}$$
$$\overline{\psi}(x) \rightarrow \overline{\psi}'(x) = \overline{\psi}(x)e^{i\alpha}$$

where α is a constant that does not depend on the spacetime coordinate x, because if α was a function of x, the kinetic term of the action (1.1.2) would not be invariant under such transformation.

1.1.3 Quantum Electrodynamics

As the free field theory itself is non interacting, it does not provide any real-world prediction, so it is useful to write an interacting action where the spinor field is coupled, for instance, to a vector field A_μ, i.e. the photon. One way to implement this interaction is to ask for local, instead of global, invariance of the action (1.1.2) under the phase transformation (1.1.4), where now $\alpha = \alpha(x)$. In order to do so, the covariant derivative has to be defined as follows:

$$D_\mu \equiv \partial_\mu + igA_\mu \tag{1.1.5}$$

where g is the couplig constant.[1]

The vector field's kinetic term is written in terms of its field-strength, namely:

$$
\begin{aligned}
F_{\mu\nu} &\equiv -\frac{i}{g}[D_\mu, D_\nu] = \\
&= -\frac{i}{g}\left(D_\mu\left(\partial_\nu + igA_\nu\right) - D_\nu\left(\partial_\mu + igA_\mu\right)\right) = \\
&= -\frac{i}{g}\left(\cancel{\partial_\mu\partial_\nu} + ig\partial_\mu A_\nu - g^2 A_\mu A_\nu - \cancel{\partial_\nu\partial_\mu} - ig\partial_\nu A_\mu + g^2 A_\nu A_\mu\right) = \\
&= \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] = \\
&= \partial_\mu A_\nu - \partial_\nu A_\mu
\end{aligned}
\tag{1.1.6}
$$

where $[A_\mu, A_\nu] = A_\mu A_\nu - A_\nu A_\mu = 0$ in the abelian theory.

Two different fields A_μ and A'_μ describe the same physics if one can be obtained from

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[1] Usually, in QED, g is called e, the electron charge, though g will be used in analogy to nonabelian gauge theories.

another through a gauge transformation:

$$A'_\mu(x) = A_\mu(x) + \frac{1}{g}\partial_\mu\alpha(x) \tag{1.1.7}$$

$$F'_{\mu\nu} = F_{\mu\nu} + \frac{1}{g}\left(\partial_\mu\partial_\nu - \partial_\nu\partial_\mu\right)\alpha(x) = F_{\mu\nu}$$

Thus, the free action for the vector field is:

$$S_{EM} = -\frac{1}{4}\int \mathrm{d}^4x\, F_{\mu\nu}F^{\mu\nu} \tag{1.1.8}$$

That is also gauge invariant, i.e. invariant under (1.1.7), as $F_{\mu\nu}$ is gauge invariant.
The term that broke the local phase invariance of the action (1.1.2) can now be "absorbed" by A_μ through a gauge transformation (1.1.7), thus making the full action gauge invariant:

$$S_{QED} = \int \mathrm{d}^4x \left(i\overline{\psi}\slashed{D}\psi - m\overline{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) = \tag{1.1.9}$$

$$= \int \mathrm{d}^4x \left(i\overline{\psi}\slashed{\partial}\psi - m\overline{\psi}\psi - g\overline{\psi}\slashed{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

$$S_{QED} \to S'_{QED} = \int \mathrm{d}^4x \left(i\overline{\psi}\slashed{\partial}\psi + \overline{\psi}\slashed{\partial\alpha}\psi - m\overline{\psi}\psi - g\overline{\psi}\slashed{A}\psi - \overline{\psi}\slashed{\partial\alpha}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) =$$

$$= \int \mathrm{d}^4x \left(i\overline{\psi}\slashed{\partial}\psi - m\overline{\psi}\psi - g\overline{\psi}\slashed{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) = S_{QED}$$

1.1.4 Non-Abelian Gauge Theories

Let us now consider a theory with N fermions, all with the same mass m, described by the spinorial fields $\psi_i(x)$ with $i = 1, \ldots, N$. These N fermions represent the N possible charges of the same particle[2] and are not to be confused with, for instance, the different possible flavors of the quarks, that describe different particles with different masses. Its free action is:

$$S_\psi[\psi_i(x), \overline{\psi}_i(x)] = \sum_{i=1}^{N} \int \mathrm{d}^4x \left(i\overline{\psi}_i\slashed{\partial}\psi_i - m\overline{\psi}_i\psi_i\right) \tag{1.1.10}$$

From now on, the sum over i (and all other repeated latin indexes) will be omitted, unless differently specified. This action is invariant under the global transformation:

$$\psi_i(x) \to \psi'_i(x) = U_{ij}\psi_j(x) \tag{1.1.11}$$

$$\overline{\psi}_i(x) \to \overline{\psi}'_i(x) = \overline{\psi}_j(x)U^\dagger_{ji}$$

if U is any (constant) $N \times N$ matrix such that $UU^\dagger = U^\dagger U = \mathbb{1} \Leftrightarrow U^\dagger = U^{-1}$, or in other words, if $U \in U(N)$. For this reason, this transformation is also called a global $U(N)$ transformation. The phase transformation (1.1.4) is the particular case where $U = e^{-i\alpha} \in U(1)$, that is the only Abelian (commutative) unitary group. As $U(N) = SU(N) \otimes U(1)\ \forall N > 1$, $U \in SU(N)$ instead of $U \in U(N)$ can be imposed,

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[2]For example, if $N = 3$ the 3 possible charges are the color charges of QCD, as will be shown later.

and will be from now on, without loss of generality.

In an analogous way to what has been done in Section 1.1.3 , this invariance can be made local by implementing a proper covariant derivative, similar to (1.1.5). In order to do so, the infinitesimal $SU(N)$ transformation has to be considered:

$$U_{ij}(x) = \delta_{ij} + \mathrm{i}\theta^a(x)\,(T^a)_{ij} + O(\theta^2) \tag{1.1.12}$$

where the indices i and j run from 1 to N (as before) and the index a runs from 1 to $N^2 - 1$ (the dimension of the group $SU(N)$). The matrixes T^a are the $N^2 - 1$ generators of $\mathfrak{su}(N)$ (the Lie algebra of $SU(N)$), thus they are $N \times N$ hermitean and traceless, which obey the commutation relations:

$$\left[T^a, T^b\right] = \mathrm{i}f^{abc}T^c \tag{1.1.13}$$

where f^{abc} are called *structure constants* of $\mathfrak{su}(N)$. The normalization of these matrices can be chosen such that they obey the condition:

$$\mathrm{Tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab} \tag{1.1.14}$$

some examples are:

- $N = 2$, $T^a = \frac{\sigma^a}{2}$, with σ^a the Pauli matrices and $f^{abc} = \varepsilon^{abc}$;

- $N = 3$, $T^a = \frac{\lambda^a}{2}$, with λ^a the Gell-Mann matrices.

where ε^{abc} is the completely antisymmetric Levi-Civita symbol.

The covariant derivative, therefore, is written as:

$$D_\mu \equiv \partial_\mu + \mathrm{i}g\mathbf{A}_\mu(x) \tag{1.1.15}$$

where an $N \times N$ identity matrix $\mathbb{1}$ multiplying ∂_μ has to be understood, and $\mathbf{A}_\mu(x)$ is a gauge field of $SU(N)$, i.e. a traceless, hermitean $N \times N$ matrix, or, in other words, $\mathbf{A}_\mu(x) \in \mathfrak{su}(N)$.

The covariant derivative can be written more explicitly acting on the set of spinors ψ_i:

$$(D_\mu)_{ij}\,\psi_j = \partial_\mu \mathbb{1}_{ij}\psi_j + \mathrm{i}g\,(\mathbf{A}_\mu(x))_{ij}\,\psi_j$$

In order for the action to be gauge invariant, the field $\mathbf{A}_\mu$ must satisfy the gauge transformation property

$$\mathbf{A}_\mu(x) \to \mathbf{A}'_\mu(x) = U(x)\mathbf{A}_\mu(x)U^\dagger(x) - \frac{\mathrm{i}}{g}U(x)\partial_\mu U^\dagger(x) \tag{1.1.16}$$

This expression is a little more complicated than (1.1.7), due to the fact that $\mathbf{A}_\mu$ is now a non-commuting matrix. However if the Abelian case $U(1)$ is taken into consideration, where $U(x) = e^{-\mathrm{i}\alpha(x)}$, (1.1.7) follows directly from (1.1.16).

Now, it can be easily checked that the kinetic term of the Lagrangian

$$\mathcal{L}_K = \mathrm{i}\overline{\psi}_i \slashed{D}\psi_i = \mathrm{i}\overline{\psi}_i \slashed{\partial}\psi_i - g\overline{\psi}_i \slashed{\mathbf{A}}\psi_i$$

is gauge invariant (i.e. invariant under (1.1.11) and (1.1.16)) through direct computation:

$$
\begin{aligned}
\mathcal{L}_K \to \mathcal{L}'_K ={}& \mathrm{i}\overline{\psi}_i U^\dagger \slashed{\partial}\left(U\psi_i\right) - g\overline{\psi}_i \underbrace{U^\dagger U}_{\mathbb{1}}\, \slashed{\mathbf{A}} \underbrace{U^\dagger U}_{\mathbb{1}}\, \psi_i + \mathrm{i}\overline{\psi}_i \underbrace{U^\dagger U}_{\mathbb{1}}\left(\slashed{\partial}U^\dagger\right)U\psi_i = \\
={}& \mathrm{i}\overline{\psi}_i U^\dagger\left(\slashed{\partial}U\right)\psi_i + \underbrace{\mathrm{i}\overline{\psi}_i\slashed{\partial}\psi_i - g\overline{\psi}_i\slashed{\mathbf{A}}\psi_i}_{\mathcal{L}_K} + \mathrm{i}\overline{\psi}_i\left(\slashed{\partial}U^\dagger\right)U\psi_i = \\
={}& \mathcal{L}_K + \mathrm{i}\overline{\psi}_i\gamma^\mu\left(U^\dagger\partial_\mu U + \partial_\mu U^\dagger U\right)\psi_i = \\
={}& \mathcal{L}_K + \mathrm{i}\overline{\psi}_i\gamma^\mu\partial_\mu\left(U^\dagger U\right)\psi_i = \\
={}& \mathcal{L}_K + \mathrm{i}\overline{\psi}_i\gamma^\mu\underbrace{\partial_\mu\left(\mathbb{1}\right)}_{=0}\psi_i = \mathcal{L}_K
\end{aligned}
$$

Because of this fact, it is directly implied that the covariant derivative (1.1.15) must transform, under a gauge transformation, in the adjoint representation:

$$
D_\mu \to D'_\mu = U D_\mu U^\dagger \tag{1.1.17}
$$

The field-strength for the field $\mathbf{A}_\mu$ is obtained, as for the Abelian case, through the commutator of two covariant derivatives. The computation is the same as (1.1.6), but this time the commutator term is non-vanishing:

$$
F_{\mu\nu} \equiv -\frac{\mathrm{i}}{g}[D_\mu, D_\nu] = \partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu + \mathrm{i}g[\mathbf{A}_\mu, \mathbf{A}_\nu] \tag{1.1.18}
$$

This expression can be simplified a little by considering that $\mathbf{A}_\mu$ and $F_{\mu\nu}$ are elements of $\mathfrak{su}(N)$, thus writing them in terms of their components w. r. t. the basis T^a:

$$
\mathbf{A}_\mu(x) = A^a_\mu(x)T^a \tag{1.1.19}
$$

$$
F_{\mu\nu}(x) = F^a_{\mu\nu}(x)T^a \tag{1.1.20}
$$

and by considering the relation (1.1.13):

$$
\begin{aligned}
F^a_{\mu\nu}T^a ={}& \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)T^a + \mathrm{i}g\left[A^b_\mu T^b, A^c_\nu T^c\right] = \\
={}& \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)T^a + \mathrm{i}g A^b_\mu A^c_\nu \underbrace{\left[T^b, T^c\right]}_{\mathrm{i}f^{bca}T^a} = \\
={}& \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc}A^b_\mu A^c_\nu\right)T^a \\
F^a_{\mu\nu} ={}& \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc}A^b_\mu A^c_\nu \tag{1.1.21}
\end{aligned}
$$

In order to write a kinetic action for the field $\mathbf{A}_\mu$, a term proportional to $F_{\mu\nu}F^{\mu\nu}$, like in (1.1.8), is not enough: because of (1.1.17) and the definition (1.1.18), it must transform as $F_{\mu\nu}F^{\mu\nu} \to U F_{\mu\nu}F^{\mu\nu}U^\dagger$, therefore it would not be gauge invariant. In fact a gauge invariant action, called Yang-Mills action, is:

$$
S_{YM} = -\frac{1}{2}\int \mathrm{d}^4x\, \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) \tag{1.1.22}
$$

because of the ciclic property of the trace.[3] This action can be written in components, using (1.1.20) and the trace property (1.1.14):

$$
S_{YM} = -\frac{1}{2}\int \mathrm{d}^4x\, \mathrm{Tr}\left(F^a_{\mu\nu}F^{b\mu\nu}T^aT^b\right) = -\frac{1}{4}\int \mathrm{d}^4x\, F^a_{\mu\nu}F^{a\mu\nu} \tag{1.1.23}
$$

---

[3]Actually, a term proportional to $\det\left(F^a_{\mu\nu}F^{a\mu\nu}\right)$ would be gauge invariant as well, but it would not be a suitable kinetic term as it would involve terms of higher order than 2 in the components $F^a_{\mu\nu}$

Here, there are two remarks that need to be done. The first one is that, if the gauge group is taken to be $U(1)$, the action (1.1.23) reduces to (1.1.8), as $a = 1$ because the group $U(1)$ has only 1 generator. The second one is that, if non-Abelian gauge groups are taken into consideration, this action naturally introduces self-interacting cubic and quartic terms, because the structure constants f^{abc} are non-vanishing. This is, for example, the case for the group $SU(3)$, that is used to describe gluon interaction, i.e. Quantum Chromodynamics (QCD). These self-interactions make the the Yang-Mills action interesting to be studied even alone, without any other fermionic or bosonic interacting field, as it will be shown later.

1.1.5   Wick Rotation

Up to now, actions were written in Minkowskian spacetime, where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In order to have a positive-defined metric $\eta_{\mu\nu} = \delta_{\mu\nu}$ a Wick rotation can be performed by re-defining the time coordinate to be $\tau = \mathrm{i}t = \mathrm{i}x^0$, as is rigorously proven in [6]. The principle behind Wick rotation is that the time coordinate is promoted to be a complex number and the integration (in the action) from $-\infty$ to ∞ is made to be from $-\mathrm{i}\infty$ to $\mathrm{i}\infty$ through a rotation of $\frac{\pi}{2}$ in the complex plane, assuming that correlators and other physical quantities do not present singularities in the first and third quadrant of the complex plane of t. This rotation is not only a matter of convenience, as it is needed in perturbative QFT for the computation of (otherwise oscillating) functional integrals and because a Minkowskian lattice cannot be rigorously defined.

The relation between the Minkowski actions written before and the Euclidean actions is $S^M = \mathrm{i}S^E$, because $\mathrm{d}^4 x^E = \mathrm{i}\mathrm{d}^4 x^M$.

Thus, the Euclidean actions are[4]:

$$S^E[\phi] = \int \mathrm{d}^4 x \left(\frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \frac{1}{2} m^2 \phi^2 + V(\phi) \right) \tag{1.1.24}$$

$$S_F^E = \int \mathrm{d}^4 x \overline{\psi} \left(\gamma^\mu \left(\partial_\mu + \mathrm{i}g\mathbf{A}_\mu \right) + m \right) \psi \tag{1.1.25}$$

$$S_{YM}^E = \frac{1}{4} \int \mathrm{d}^4 x F_{\mu\nu}^a F^{a\mu\nu} \tag{1.1.26}$$

where the superscript E, meaning that the action is written in the Euclidean spacetime, will be omitted from now on.

1.2   Lattice Field Theory

1.2.1   Why Lattice Field Theory

After the Wick rotation, the quantum field theory can be studied perturbatively, through the saddle-point evaluation of the path integral, for instance, where the partition function

$$Z = \int \mathcal{D}\varphi e^{\mathrm{i}S^M[\varphi]} = \int \mathcal{D}\varphi e^{-S^E[\varphi]}$$

---

[4]Note that they are not the same as their Minkowskian counterpart, as some redefinitions of the fields and the γ matrices have been implicitly made, in order to have the respective theories non ill-defined.

has become non-oscillating. This approach, however, works well only if the coupling constants are *small enough* and does not allow to obtain non-perturbative results, i.e. physical quantities that depend on essential singularities in the coupling constant, like glueballs for Yang-Mills theories.

For this reason, a non-perturbative approach has to be taken into consideration and one possibility is Lattice Field Theory: the spacetime is discretized to a lattice, with lattice spacing a, and the fields can assume different values only on the sites of the lattice. More in detail, a scalar field lives on lattice sites, a vector field lives on links between sites and objects with k indices live on k-simplexes. If the continuum limit $(a \to 0)$ is taken, the theory must reproduce results obtained in the continuum, like the ones obtained in perturbation theory in the regime where it can be applied, however the presence of a discrete spacetime provides a natural cutoff for the momenta, allowing ultraviolet divergencies to be kept under control, although they need to be taken into consideration when approaching the continuum limit.

In this section, based on standard quantum field theory textbooks cited before and lattice field theory textbooks [7–9], the regularization of the previous quantum field theories on a Simple Hypercubic (SH) lattice is presented.

1.2.2 Lattice

A lattice Λ in $\mathbb{R}^D$ is defined as the set of all possible linear combinations with integer coefficients of the vectors $\{v_i\}$, which form a basis of $\mathbb{R}^D$. Formally:

$$\Lambda = \left\{ \sum_{i=1}^{D} c_i v_i \ \middle| \ c_i \in \mathbb{Z} \right\} \tag{1.2.1}$$

1.2.2.1 Simple Hypercubic Lattice

If the basis $\{v_i\}$ is taken to be orthonormal, the lattice is called *Simple Hypercubic*[5].
In figure (1.2.1) is represented a portion of a Simple Hypercubic lattice in $D = 3$.
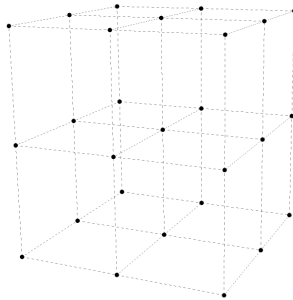

Figure 1.2.1: A cubic lattice.

1.2.3 Scalar Fields on the Simple Hypercubic Lattice

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[5]The word *simple* is used to make a distinction with the Body-Centered Hypercubic lattice, in the following sections.

Computer Simulation of Gauge Theories

Gauge Theories Simulation on non-hypercubic lattice F4

Simulation Results

Conclusions

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