Assignment 3.

① 1. Compute all vectors of length 1 perpendicular to the vectors (1, 2, 3) and (2, 3, 4).

· the cross product $x \times y$ is a vector perpendicular to both $x$ and $y$.

The cross product of $x = (1,2,3)$ and $y = (2,3,4)$ is

$$x \times y = \begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = i \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} + (-j) \cdot \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} =$$

$$= \cancel{2i} - i + 2j - k \Rightarrow (-1, +2, -1)$$

length of $x \times y = |x \times y| = \sqrt{(-1)^2 + (+2)^2 + (-1)^2} = \sqrt{1+4+1} = \sqrt{6}$.

· To find the unit vector in the same direction as $x \times y$, divide by the length of $x \times y$.

Hence, a unit vector perpendicular to both (1,2,3) and (2,3,4) is $\left(-\frac{1}{\sqrt{6}}, +\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$. The opposite vector $\left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, +\frac{1}{\sqrt{6}}\right)$ is also a solution.

2. Compute the angle between the vectors $(2,3,-1)$ and $(-1,2,3)$.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|} = \frac{-2+6-3}{\sqrt{14}\cdot\sqrt{14}} = \frac{1}{14}$$

$$\|\vec{u}\| = \sqrt{4+9+1} = \sqrt{14}$$

$$\|\vec{v}\| = \sqrt{1+4+9} = \sqrt{14}$$

$$\theta = \cos^{-1}\left(\frac{1}{14}\right) \approx 85.9^\circ$$

3. Let $\vec{u} = (7,4,-22)$ and $\vec{v} = (-7, \sqrt{3}, 9.9)$. Compute $(\vec{u}+\vec{v})\cdot(\vec{u}\times\vec{v})$.

$$\vec{u}+\vec{v} = (0,\ 4+\sqrt{3},\ -12.1)$$

$$\vec{u}\times\vec{v} = \cancel{\begin{vmatrix} i & j & k \\ -7 & \sqrt{3} & 9.9 \end{vmatrix}} = \begin{vmatrix} i & j & k \\ 7 & 4 & -22 \\ -7 & \sqrt{3} & 9.9 \end{vmatrix} = i\cdot(4\cdot 9.9 + 22\sqrt{3}) - j(7\cdot 9.9 - 22\cdot 7) +$$

$$+ k(7\sqrt{3} + 7\cdot 4) = (39.6 + 22\sqrt{3})i + \overset{84.7}{\cancel{87.4}}\,j + (28 + 7\sqrt{3})k$$

$$(\vec{u}+\vec{v})\cdot(\vec{u}\times\vec{v}) = \left(0,\ (4+\sqrt{3})\cdot 84.7\ ;\ -12.1\cdot 7(4+\sqrt{3})\right) =$$

$$= \left(0,\ 84.7(4+\sqrt{3}),\ -84.7(4+\sqrt{3})\right). =$$

$$= 0 + 84.7(4+\sqrt{3}) - 84.7(4+\sqrt{3}) = 0. \text{ (orthogonal)}$$

4. Consider a plane in 3 dimensions passing through the point $P_0$ $(2,0,-1)$ and having $\overset{(n)}{\text{normal}}$ $(1,1,2)$. Find the implicit equation of the plane.

Let $\vec{p}$ be a variable standing for any vector in the plane.

~~$\vec{p} = (x, y, z)$.~~

~~Then, $n\cdot($~~

Let $P$ be a variable standing for any point in the plane.

$P = (x, y, z)$.

Then, $n \cdot (P - P_0) = 0$ (orthogonal)

With a little abuse of notation, $n \cdot P - n P_0 = 0$

$$0 = (1,1,2) \cdot [(x,y,z) - (2,0,-1)] =$$

$$= (1,1,2) \cdot [(x-2, y, z+1)] =$$

$$= (x-2) + y + 2(z+1) = \underline{x + y + 2z} \cancel{-4}$$

$$x + y + 2z = 0.$$

5. Is the point $(2,5)$ an affine combination of the points $(6,3)$ and $(-9,11)$?

We would like to find $x$ and $y$ such that

$C = xA + yB$ and $x + y = 1$.

$$\begin{cases} 6x - 9y = 2 \\ x + y = 1 \end{cases}$$

$$-9y - 6y = 2 - 6$$

$$-15y = -4$$

$$y = \frac{4}{15} \qquad x = \frac{11}{15} = \frac{11}{15}$$

$$3x + 11y = 5$$

$$\frac{3 \cdot 11}{15} + \frac{11 \cdot 4}{15} = \frac{33}{15} + \frac{44}{15} = \frac{77}{15} \neq 5.$$

Double-check:

$$\begin{cases} 6x - 9y = 2 \\ 3x + 11y = 5 \end{cases} \quad -9 - 22y = 2 - 10; \quad -31y = -8; \quad y = \frac{8}{31}.$$

$$x = \frac{5}{3} - \frac{11y}{3} = \frac{5}{3} - \frac{88}{93} = \frac{67}{93}.$$

$$x + y = \frac{67}{93} + \frac{24}{93} = \frac{91}{93} \neq 1.$$

NOT AN AFFINE COMBINATION.