TECHNOLOGICAL UNIVERSITY OF THE PHILIPPINES – CAVITE CAMPUS

BACHELOR OF ENGINEERING TECHNOLOGY

ENGINEERING DATA ANALYSIS

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UNIT 5

CONFIDENCE INTERVALS

FOR PROPORTIONS AND MEANS

OVERVIEW

In Lesson 4, the sampling distributions for the sample statistics assumed we knew the population parameters. In real life, we do not know these parameters, or we would not need statistics). We know what to do when the parameters are known, let's see how we can use that information when they are unknown.

SPECIFIC OBJECTIVES

At the end of this unit, the students are expected to:

1. Describe the role of statistical inference in estimation in terms of the population and sample.

2. Explain the general form of a confidence interval and apply it to different statistics and conditions.

3. Construct a confidence interval to estimate a population mean or proportion.

4. Given a confidence interval, interpret the meaning in terms of the population.

5. Identify when to use the t-distribution as opposed to the normal distribution given the sample size and population distribution.

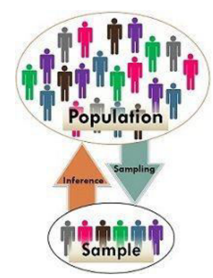
6. Define and interpret the margin of error.

5.1

INTRODUCTION TO CONFIDENCE INTERVALS

5.1 INTRODUCTION TO CONFIDENCE INTERVALS

The real power of statistics comes from applying the concepts of probability to situations where you have data but not necessarily the whole population. The results, called **statistical inference**, give you probability statements about the population of interest based on that set of data.



5.1 INTRODUCTION TO CONFIDENCE INTERVALS

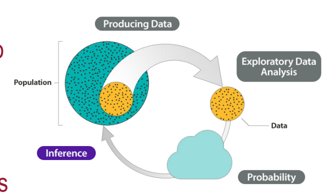
Two Types of Statistical Inference

1. Estimation

Use information from the sample to estimate / predict the parameter of interest.

ex:

result of a poll about the president's current approval rating



5.1 INTRODUCTION TO CONFIDENCE INTERVALS

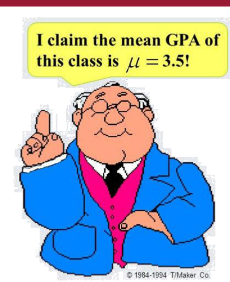
Two Types of Statistical Inference

2. Statistical (Hypothesis) Tests

Use information from the sample to determine whether a certain statement about the parameter of interest is true.

ex:

news station claims of approval rating VS poll data.



5.1 INTRODUCTION TO CONFIDENCE INTERVALS

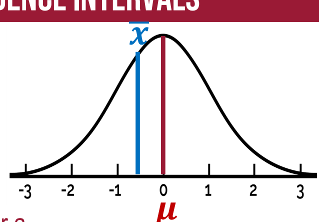
Two Methods of Estimation

1. Point Estimates

An estimate for a parameter that is one numerical value. Ex: $\bar{x}$, $\hat{p}$

2. Interval Estimates

Gives an interval as the estimate for a parameter. Such intervals are built around point estimates.



$\mu = 1.5$
$\bar{x} = 1.3$

5.1 INTRODUCTION TO CONFIDENCE INTERVALS

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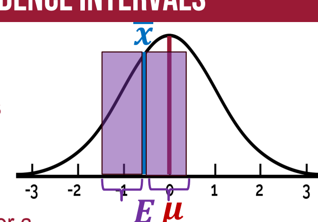

$\mu = 1.5$
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$E = 0.3$

5.1 INTRODUCTION TO CONFIDENCE INTERVALS

Two Methods of Estimation

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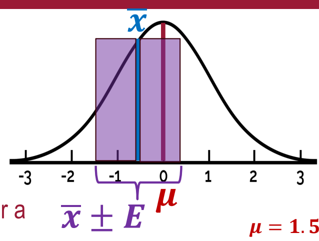


$\mu = 1.5$
$\bar{x} = 1.3$
$E = 0.3$
$\bar{x} \pm E = 1.0 - 1.6$

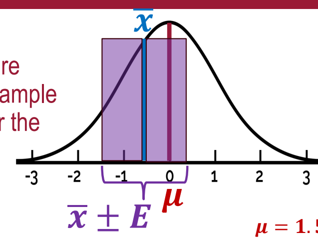
5.1 INTRODUCTION TO CONFIDENCE INTERVALS

Confidence Intervals (CI)

Also called as interval estimates, are interval of values computed from sample data ($\bar{x}$ *or* $\hat{p}$) that is likely to cover the true parameter of interest.

$$CI = \bar{x} \pm E$$

$$CI = \hat{p} \pm E$$



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$\bar{x} = 1.3$
$E = 0.3$
$\bar{x} \pm E = 1.0 - 1.6$

5.1 INTRODUCTION TO CONFIDENCE INTERVALS

Confidence Intervals

Ex: A survey of the current approval rating of the President

"47% of those surveyed approved of the President's reaction. The survey had a 3.5% margin of error, or ± 3.5%."

$$47\% \pm 3.5\%$$

5.1 INTRODUCTION TO CONFIDENCE INTERVALS

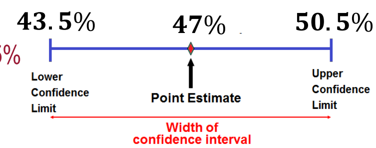
Confidence Intervals

$$47\% \pm 3.5\%$$

$$43.5\% - 50.5\%$$

Ex: A survey of the current approval rating of the President

"47% of those surveyed approved of the President's reaction. The survey had a 3.5% margin of error, or ± 3.5%."

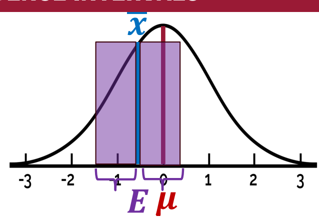


5.1 INTRODUCTION TO CONFIDENCE INTERVALS

Margin of Error (E)

consists of standard error ($\widehat{SE}$) of the sample statistic and some multiplier (M) of this standard error, based on how confident we want to be in our estimate.

$$E = M * \widehat{SE}$$



$\mu = 1.5$
$\bar{x} = 1.3$
$E = 0.3$

REVIEW...

STANDARD ERROR OF MEAN AND PROPORTION

$$SE_{\mu} = \frac{\sigma}{\sqrt{n}} \qquad SE_p = \sqrt{\frac{p(1-p)}{n}}$$

$SE_{\mu} = Standard\ Error\ of\ Mean$
$\sigma = Standard\ Deviation$
$n = Sample\ Size$

$SE_p = Standard\ Error\ of\ Proportion$
$p = Population\ Proportion$
$n = Sample\ Size$

5.1 INTRODUCTION TO CONFIDENCE INTERVALS

ESTIMATED STANDARD ERROR OF MEAN AND PROPORTION

$$\widehat{SE}_{\mu} = \frac{s}{\sqrt{n}} \qquad \widehat{SE}_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

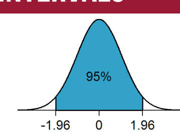
$\widehat{SE}_{\mu} = Estimated\ Standard\ Error$
$s = Sample\ Standard\ Deviation$
$n = Sample\ Size$

$\widehat{SE}_p = Estimated\ Standard\ Error$
$\hat{p} = Sample\ Proportion$
$n = Sample\ Size$

5.1 INTRODUCTION TO CONFIDENCE INTERVALS

Confidence Level (CL)

Most confidence levels use ranges from **90%** confidence to **99%** confidence, with **95%** being the most widely used. In fact, when you read a report that includes a margin of error, **you can usually assume this has a 95% confidence attached to it unless otherwise stated.**



Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
90%	.10	$z_{0.05}$	1.645
95%	.05	$z_{0.025}$	1.960
98%	.02	$z_{0.01}$	2.326
99%	.01	$z_{0.005}$	2.576

*For sample proportion only

5 . 2

CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

General Form of CI of Sample Proportion

To construct a confidence interval, use the following 3 steps:

1. CHECK CONDITIONS

1. $n\hat{p} \geq 5$
2. $n(1-\hat{p}) \geq 5$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

General Form of CI of Sample Proportion

To construct a confidence interval, use the following 3 steps:

2. CONSTRUCT THE GENERAL FORM

$$point\ estimate \pm margin\ of\ error$$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

General Form of CI of Sample Proportion

To construct a confidence interval, use the following 3 steps:

2. CONSTRUCT THE GENERAL FORM

$$point\ estimate \pm \mathbf{M}\ \mathbf{x}\ \widehat{SE}$$

5.1 INTRODUCTION TO CONFIDENCE INTERVALS

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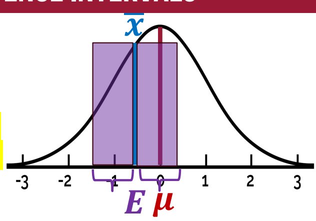

$$E = \mathbf{M} * \widehat{\mathbf{SE}}$$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Multiplier ($z_{\alpha/2}$) from the Co

Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
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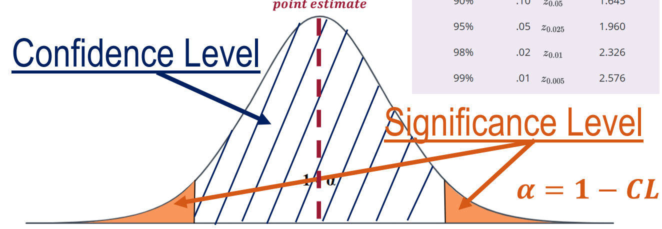

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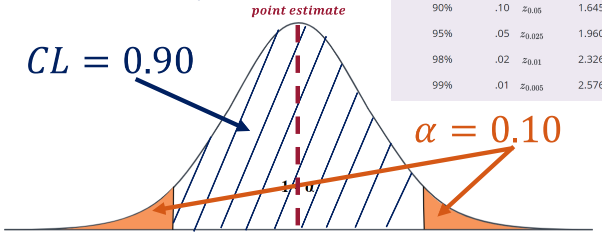

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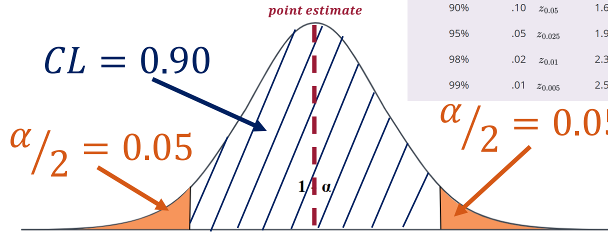

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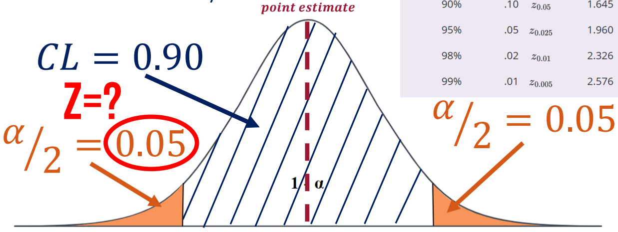
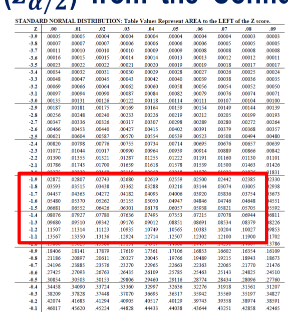


Multiplier ($z_{\alpha/2}$) from the Confidence Level



Multiplier ($z_{\alpha/2}$) from the Confidence Level

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900

Multiplier ($z_{\alpha/2}$) from the Confidence Level

$$\begin{array}{cc} 1.64 & 0.05050 \\ z_{\alpha/2} & 0.05 \\ 1.65 & 0.04947 \end{array}$$

$$\frac{1.64 - z_{\alpha/2}}{1.64 - 1.65} = \frac{0.05050 - 0.05}{0.05050 - 0.04947}$$

$$z_{\alpha/2} = 1.645$$

Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
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5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

General Form of CI of Sample Proportion

To construct a confidence interval, use the following 3 steps:

2. CONSTRUCT THE GENERAL FORM

$$\hat{p} \pm z_{\alpha/2}\widehat{SE}$$

5.1 INTRODUCTION TO CONFIDENCE INTERVALS

ESTIMATED STANDARD ERROR OF MEAN AND PROPORTION

$$\widehat{SE}_\mu = \frac{s}{\sqrt{n}} \qquad \widehat{SE}_p = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$$

$\widehat{SE}_\mu = Estimated\ Standard\ Error$
$s = Sample\ Standard\ Deviation$
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5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

General Form of CI of Sample Proportion

To construct a confidence interval, use the following 3 steps:

2. CONSTRUCT THE GENERAL FORM

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

General Form of CI of Sample Proportion

To construct a confidence interval, use the following 3 steps:

3. INTERPRET THE CONFIDENCE INTERVAL

$$\hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} \qquad \hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$$

Upper Confidence Limit — Lower Confidence Limit

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1:

A random sample of 1500 U.S. adults is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, 660 respond with "approved". Calculate a 95% confidence interval for the overall approval rating of the president.

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1:

A random sample of 1500 U.S. adults is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, 660 respond with "approved". Calculate a 95% confidence interval for the overall approval rating of the president.

1. CHECK CONDITIONS

1. $n\hat{p} \geq 5$ — $660 \geq 5$ ✓
2. $n(1-\hat{p}) \geq 5$ — $(1500-660) \geq 5$ ✓

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1:

A random **sample of 1500 U.S. adults** is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

2. CONSTRUCT THE GENERAL FORM

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1:

A random **sample of 1500 U.S. adults** is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

2. CONSTRUCT THE GENERAL FORM

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} \qquad \hat{p} = \frac{approved}{n} = \frac{660}{1500} = 0.44$$

5.2 CONFIDENCE INTERVALS OF

Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
90%	.10	$z_{0.05}$	1.645
95%	.05	$z_{0.025}$	$\boxed{1.960}$
98%	.02	$z_{0.01}$	2.326
99%	.01	$z_{0.005}$	2.576

Sample Problem 1:

A random **sample of 1500 U.S. ad**... whether they approve or disappr... performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

2. CONSTRUCT THE GENERAL FORM

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} \qquad z_{\alpha/2} = 1.960$$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1: $\hat{p} = 0.44$ $z_{\alpha/2} = 1.960$ $n = 1500$

A random **sample of 1500 U.S. adults** is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

2. CONSTRUCT THE GENERAL FORM

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = 0.44 \pm 1.960\sqrt{\frac{0.44(1-0.44)}{1500}}$$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1: $\hat{p} = 0.44$ $z_{\alpha/2} = 1.960$ $n = 1500$

A random **sample of 1500 U.S. adults** is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

2. CONSTRUCT THE GENERAL FORM

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = 0.44 \pm 1.960\sqrt{\frac{0.44(1-0.44)}{1500}} = 0.44 \pm 0.025$$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1: $\hat{p} = 0.44$ $z_{\alpha/2} = 1.960$ $n = 1500$

A random **sample of 1500 U.S. adults** is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

2. CONSTRUCT THE GENERAL FORM

$44\% \pm 2.5\%$

Point estimate Margin of Error

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1: $\hat{p} = 0.44$ $z_{\alpha/2} = 1.960$ $n = 1500$

A random **sample of 1500 U.S. adults** is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

3. INTERPRET THE CONFIDENCE INTERVAL

$0.44 + 0.025$ $0.44 - 0.025$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1: $\hat{p} = 0.44$ $z_{\alpha/2} = 1.960$ $n = 1500$

A random **sample of 1500 U.S. adults** is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

3. INTERPRET THE CONFIDENCE INTERVAL

$0.44 - 0.025 = 0.415$ $0.44 + 0.025 = 0.465$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1: $\hat{p} = 0.44$ $z_{\alpha/2} = 1.960$ $n = 1500$

A random **sample of 1500 U.S. adults** is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

3. INTERPRET THE CONFIDENCE INTERVAL

41.5% 46.5%

Lower Confidence Limit Upper Confidence Limit

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1: $\hat{p} = 0.44$ $z_{\alpha/2} = 1.960$ $n = 1500$

A random **sample of 1500 U.S. adults** is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

3. INTERPRET THE CONFIDENCE INTERVAL

"The current U.S. approval rating for the president is 44% with a 95% margin of error of 2.5%."

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 1: $\hat{p} = 0.44$ $z_{\alpha/2} = 1.960$ $n = 1500$

A random **sample of 1500 U.S. adults** is taken. They are asked whether they approve or disapprove of the current president's performance so far. Of the 1500 surveyed, **660 respond with "approved"**. Calculate a **95% confidence interval** for the overall approval rating of the president.

3. INTERPRET THE CONFIDENCE INTERVAL

"We are 95% confident that the overall U.S. adult approval rating for the current president is from 41.5% to 46.5%."

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Determining the Required Sample Size

The wider the interval, the poorer the precision. Note that the higher the confidence level, the wider the width of the interval and thus the poorer the precision.

Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
90%	.10	$z_{0.05}$	1.645
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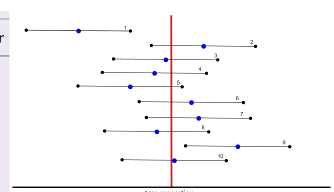

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Determining the Required Sample Size

Since the confidence level reflects the success rate of the method we use to get the confidence interval, we like to have a narrower interval while keeping the confidence level at a reasonably higher level.

Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
90%	.10	$z_{0.05}$	1.645
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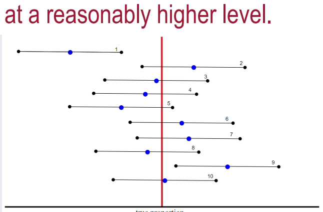


5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Determining the Required Sample Size

If the desired margin of error E is specified and the desired confidence level is specified, the required sample size to meet the requirements can be calculated by

$$n = \frac{{z_{\alpha/2}}^2}{4E^2}$$

$z_{\alpha/2} = CL\ multiplier$
$E = margin\ of\ error$
$n = required\ sample\ size$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 2:

Suppose a television poll states that the "approval rating of the president is 72%." For the next poll of the president's approval rating, we want to get a margin of error of 1% with 95% confidence. How many individuals should we sample?

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 2:

Suppose a television poll states that the "approval rating of the president is 72%." For the next poll of the president's approval rating, we want to get a **margin of error of 1%** with **95% confidence**. How many individuals should we sample?

$$n = \frac{{z_{\alpha/2}}^2}{4E^2}$$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 2: $E = 0.01$

Suppose a television poll states that the "approval rating of the president is 72%." For the next poll of the president's approval rating, we want to get a **margin of error of 1%** with **95% confidence**. How many individuals should we sample?

Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
90%	.10	$z_{0.05}$	1.645
95%	.05	$z_{0.025}$	1.960
98%	.02	$z_{0.01}$	2.326
99%	.01	$z_{0.005}$	2.576

$$n = \frac{{z_{\alpha/2}}^2}{4E^2} = \frac{1.960^2}{4(0.01)^2}$$

5.2 CONFIDENCE INTERVALS OF SAMPLE PROPORTIONS

Sample Problem 2: $E = 0.01$

Suppose a television poll states that the "approval rating of the president is 72%." For the next poll of the president's approval rating, we want to get a **margin of error of 1%** with **95% confidence**. How many individuals should we sample?

Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
90%	.10	$z_{0.05}$	1.645
95%	.05	$z_{0.025}$	1.960
98%	.02	$z_{0.01}$	2.326
99%	.01	$z_{0.005}$	2.576

$$n = \frac{1.960^2}{4(0.01)^2} = 9604$$

A SURVEY CONSISTS OF 120 RESPONDENTS. THE RESPONDENTS ARE ASKED WHETHER THEY TRIED SELLING THINGS ONLINE DURING THE PANDEMIC. OUT OF 120 RESPONDENTS, 20 ANSWERED THAT THEY TRIED TO DO SO.

A). WHAT PERCENTAGE OF THE SAMPLE TRIED SELLING THINGS ONLINE?

B). WHAT WILL BE THE Z-SCORE MULTIPLIER TO BE USED IF THE SURVEY WILL USE 90% CONFIDENCE INTERVAL?

C). WHAT IS THE ESTIMATED STANDARD ERROR (IN PERCENT) OF THE SAMPLE DATA?

D). WHAT IS THE MARGIN OF ERROR (IN PERCENT) OF THE SAMPLE DATA?

E). WHAT IS THE UPPER CONFIDENCE LIMIT (IN PERCENT) OF THE CONFIDENCE INTERVAL?

F). WHAT IS THE LOWER CONFIDENCE LIMIT (IN PERCENT) OF THE CONFIDENCE INTERVAL?

G). WHAT IS THE WIDTH (IN PERCENT) OF THE CONFIDENCE INTERVAL?

H). INSTEAD OF 120 RESPONDENTS, IF A MARGIN OF ERROR OF ONLY 2.5% IS REQUIRED, DETERMINE THE REQUIRED SAMPLE SIZE FOR A 90% CONFIDENCE INTERVAL.

OUT OF 500 STUDENTS, 300 STUDENTS STATED THAT THEY ARE PLAYING ONLINE GAMES.

A). WHAT PERCENTAGE OF THE SAMPLE PLAY ONLINE GAMES?

B). WHAT WILL BE THE Z-SCORE MULTIPLIER TO BE USED FOR THE CONFIDENCE INTERVAL? (USE DEFAULT CONFIDENCE LEVEL)

C). WHAT IS THE ESTIMATED STANDARD ERROR (IN PERCENT) OF THE SAMPLE DATA?

D). WHAT IS THE MARGIN OF ERROR (IN PERCENT) OF THE SAMPLE DATA?

E). WHAT IS THE UPPER CONFIDENCE LIMIT (IN PERCENT) OF THE CONFIDENCE INTERVAL?

F). WHAT IS THE LOWER CONFIDENCE LIMIT (IN PERCENT) OF THE CONFIDENCE INTERVAL?

G). WHAT IS THE WIDTH (IN PERCENT) OF THE CONFIDENCE INTERVAL?

5.3

CONFIDENCE INTERVALS OF SAMPLE MEANS

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

General Form of CI of Sample Mean

1. CHECK CONDITIONS

a. If the sample comes from a Normal distribution

b. If the sample does not come from a normal distribution but the sample size is large $(n \geq 30)$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

General Form of CI of Sample Mean

2. CONSTRUCT THE GENERAL FORM

$$point\ estimate \pm \mathrm{M} \times \widehat{SE}$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

General Form of CI of Sample Mean

2. CONSTRUCT THE GENERAL FORM

$$\bar{x} \pm \mathrm{M} \times \widehat{SE}$$

5.1 INTRODUCTION TO CONFIDENCE INTERVALS

ESTIMATED STANDARD ERROR OF MEAN AND PROPORTION

$$\widehat{SE}_\mu = \frac{s}{\sqrt{n}} \qquad \widehat{SE}_p = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$$

$\widehat{SE}_\mu = Estimated\ Standard\ Error$
$s = Sample\ Standard\ Deviation$
$n = Sample\ Size$

$\widehat{SE}_p = Estimated\ Standard\ Error$
$\hat{p} = Sample\ Proportion$
$n = Sample\ Size$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

General Form of CI of Sample Mean

2. CONSTRUCT THE GENERAL FORM

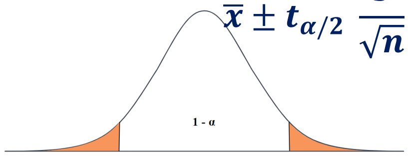
$$\bar{x} \pm \mathrm{M}\,\frac{s}{\sqrt{n}}$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

General Form of CI of Sample Mean

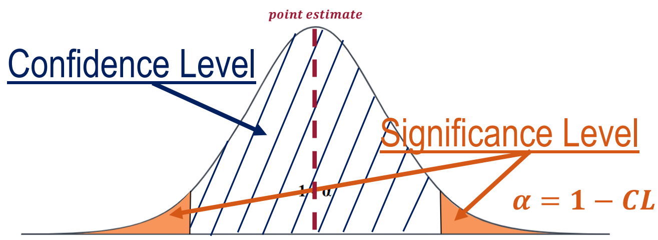
2. CONSTRUCT THE GENERAL FORM

$$\bar{x} \pm t_{\alpha/2}\,\frac{s}{\sqrt{n}}$$



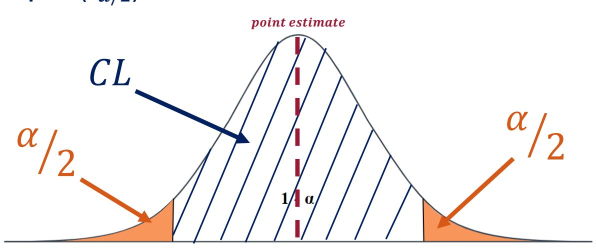
5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Multiplier ($t_{\alpha/2}$) from the Confidence Level



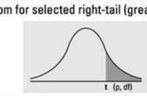
5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Multiplier ($t_{\alpha/2}$) from the Confidence Level



Multiplier ($t_{\alpha/2}$) from the Confidence Level

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



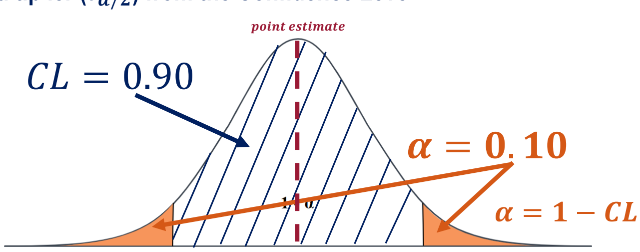
1/2 of Significance level ($\alpha/2$)

Degree of Freedom (dF)
$dF = n - 1$

df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

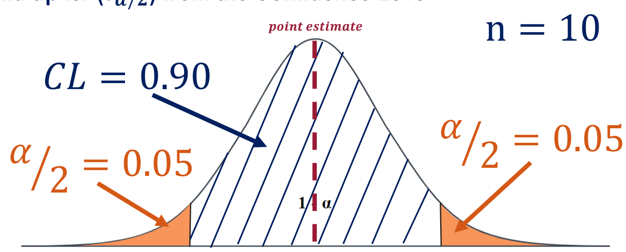
Multiplier ($t_{\alpha/2}$) from the Confidence Level



5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Multiplier ($t_{\alpha/2}$) from the Confidence Level

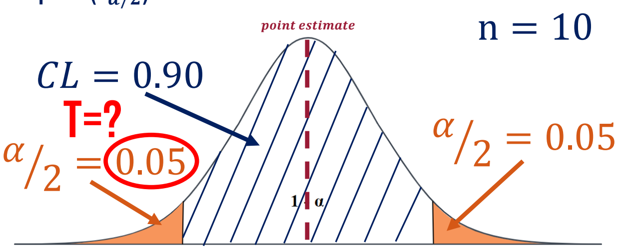
$n = 10$



5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Multiplier ($t_{\alpha/2}$) from the Confidence Level

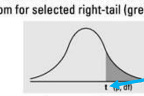
$n = 10$



Multiplier ($t_{\alpha/2}$) from the Confidence Level

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).

$n = 10$
$dF = 9$
$\alpha/2 = 0.05$



1/2 of Significance level ($\alpha/2$)

Degree of Freedom (dF)
$dF = n - 1$

df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
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12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

General Form of CI of Sample Mean

3. INTERPRET THE CONFIDENCE INTERVAL

$\bar{x} + t_{\alpha/2}\dfrac{s}{\sqrt{n}}$ — **Upper Confidence Limit**

$\bar{x} - t_{\alpha/2}\dfrac{s}{\sqrt{n}}$ — **Lower Confidence Limit**

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem:

You are interested in the average emergency room (ER) wait time at your local hospital. You take a random sample of 50 patients who visit the ER over the past week. From this sample, the mean wait time was 30 minutes and the standard deviation was 20 minutes. Find a 95% confidence interval for the average ER wait time for the hospital.

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem:

You are interested in the average emergency room (ER) wait time at your local hospital. You take a random **sample of 50 patients** who visit the ER over the past week. From this sample, the mean wait time was 30 minutes and the standard deviation was 20 minutes. Find a 95% confidence interval for the average ER wait time for the hospital.

1. CHECK CONDITIONS $(50 \geq 30)$ ✓

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem:

You are interested in the average emergency room (ER) wait time at your local hospital. You take a random **sample of 50 patients** who visit the ER over the past week. From this sample, the **mean wait time was 30 minutes** and the **standard deviation was 20 minutes**. Find a **95% confidence** interval for the average ER wait time for the hospital.

2. CONSTRUCT THE GENERAL FORM

$\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}}$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem:

You are interested in the average emergency room (ER) wait time at your local hospital. You take a random **sample of 50 patients** who visit the ER over the past week. From this sample, the **mean wait time was 30 minutes** and the **standard deviation was 20 minutes**. Find a **95% confidence** interval for the average ER wait time for the hospital.

2. CONSTRUCT THE GENERAL FORM

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 30 \pm t_{\alpha/2}\frac{20}{\sqrt{50}}$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem:

You are interested in the average emergency room (ER) wait time at your local hospital. You take a random **sample of 50 patients** who visit the ER over the past week. From this sample, the **mean wait time was 30 minutes** and the **standard deviation was 20 minutes**. Find a **95% confidence** interval for the average ER wait time for the hospital.

2. CONSTRUCT THE GENERAL FORM

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 30 \pm t_{\alpha/2}\frac{20}{\sqrt{50}}$$

$CL = 95\% = 0.95$

$\alpha = 1 - CL = 0.05$

$\alpha/2 = 0.025$

$dF = n - 1 = 49$

$\alpha/2 = 0.025$

$dF = 49$

Probability of exceeding the critical value

ν	0.10	0.05	0.025	0.01	0.005	0.001
40.	1.303	1.684	2.021	2.423	2.704	3.307
41.	1.303	1.683	2.020	2.421	2.701	3.301
42.	1.302	1.682	2.018	2.418	2.698	3.296
43.	1.302	1.681	2.017	2.416	2.695	3.291
44.	1.301	1.680	2.015	2.414	2.692	3.286
45.	1.301	1.679	2.014	2.412	2.690	3.281
46.	1.300	1.679	2.013	2.410	2.687	3.277
47.	1.300	1.678	2.012	2.408	2.685	3.273
48.	1.299	1.677	2.011	2.407	2.682	3.269
49.	1.299	1.677	2.010	2.405	2.680	3.265
50.	1.299	1.676	2.009	2.403	2.678	3.261

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem:

You are interested in the average emergency room (ER) wait time at your local hospital. You take a random **sample of 50 patients** who visit the ER over the past week. From this sample, the **mean wait time was 30 minutes** and the **standard deviation was 20 minutes**. Find a **95% confidence** interval for the average ER wait time for the hospital.

2. CONSTRUCT THE GENERAL FORM

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 30 \pm (2.010)\frac{20}{\sqrt{50}}$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem:

You are interested in the average emergency room (ER) wait time at your local hospital. You take a random **sample of 50 patients** who visit the ER over the past week. From this sample, the **mean wait time was 30 minutes** and the **standard deviation was 20 minutes**. Find a **95% confidence** interval for the average ER wait time for the hospital.

2. CONSTRUCT THE GENERAL FORM

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 30 \pm (2.010)\frac{20}{\sqrt{50}} = 30 \pm 5.69$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem:

You are interested in the average emergency room (ER) wait time at your local hospital. You take a random **sample of 50 patients** who visit the ER over the past week. From this sample, the **mean wait time was 30 minutes** and the **standard deviation was 20 minutes**. Find a **95% confidence** interval for the average ER wait time for the hospital.

3. INTERPRET THE CONFIDENCE INTERVAL

$30 - 5.69 = 24.31$ — **Lower Confidence Limit**

$30 + 5.69 = 35.69$ — **Upper Confidence Limit**

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem:

You are interested in the average emergency room (ER) wait time at your local hospital. You take a random **sample of 50 patients** who visit the ER over the past week. From this sample, the **mean wait time was 30 minutes** and the **standard deviation was 20 minutes**. Find a **95% confidence** interval for the average ER wait time for the hospital.

3. INTERPRET THE CONFIDENCE INTERVAL

The average ER waiting time for the hospital is 30 mins with a 95% margin or error of 5.69 mins.

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Determining the Required Sample Size

To determine the required number for sample means, we use the formula:

$$n = \left(\frac{z_{\alpha/2}\ s}{E}\right)^2$$

Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
90%	.10	$z_{0.05}$	1.645
95%	.05	$z_{0.025}$	1.960
98%	.02	$z_{0.01}$	2.326
99%	.01	$z_{0.005}$	2.576

$z_{\alpha/2} = CL\ multiplier$
$E = margin\ of\ error$
$n = required\ sample\ size$
$s = sample\ SD$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 2:

What is the required number of samples if the mean is 45 cm, and the **standard deviation is 5 cm**, and the **margin of error is 3 cm** using **99% confidence level**?

$$n = \left(\frac{z_{\alpha/2}\ s}{E}\right)^2$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 2:

What is the required number of samples if the mean is 45 cm, and the **standard deviation is 5 cm**, and the **margin of error is 3 cm** using **99% confidence level**?

$$n = \left(\frac{z_{\alpha/2}\ s}{E}\right)^2 = \left(\frac{2.576 * 5}{3}\right)^2$$

Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
90%	.10	$z_{0.05}$	1.645
95%	.05	$z_{0.025}$	1.960
98%	.02	$z_{0.01}$	2.326
99%	.01	$z_{0.005}$	2.576

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 2:

What is the required number of samples if the mean is 45 cm, and the **standard deviation is 5 cm**, and the **margin of error is 3 cm** using **99% confidence level**?

$$n = 18.43 \approx 19\ samples$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem:

You are interested in the average emergency room (ER) wait time at your local hospital. You take a random **sample of 50 patients** who visit the ER over the past week. From this sample, the **mean wait time was 30 minutes** and the **standard deviation was 20 minutes**. Find a **95% confidence** interval for the average ER wait time for the hospital.

3. INTERPRET THE CONFIDENCE INTERVAL

The average ER waiting time for the hospital is 24.31 – 35.69 mins. at 95% confidence level.

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 2:

What is the required number of samples if the mean is 45 cm, and the standard deviation is 5 cm, and the margin of error is 3 cm using 99% confidence level?

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

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What is the required number of samples if the mean is 45 cm, and the **standard deviation is 5 cm**, and the **margin of error is 3 cm** using **99% confidence level**?

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98%	.02	$z_{0.01}$	2.326
99%	.01	$z_{0.005}$	2.576

$$n = \left(\frac{z_{\alpha/2}\ s}{E}\right)^2 = \left(\frac{z_{\alpha/2} * 5}{3}\right)^2$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 2:

What is the required number of samples if the mean is 45 cm, and the **standard deviation is 5 cm**, and the **margin of error is 3 cm** using **99% confidence level**?

$$n = \left(\frac{z_{\alpha/2}\ s}{E}\right)^2 = \left(\frac{2.576 * 5}{3}\right)^2 = 18.43$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 3:

A marketing research firm wants to estimate the average amount a student spends during the Spring break. They want to determine it to within $120 with 90% confidence. One can roughly say that it ranges from $100 to $1700. How many students should they sample?

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 3:

A marketing research firm wants to estimate the average amount a student spends during the Spring break. They want to determine it to within **$120 with 90% confidence**. One can roughly say that it **ranges from $100 to $1700**. How many students should they sample?

$$n = \left(\frac{z_{\alpha/2}\, s}{E}\right)^2$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 3:

A marketing research firm wants to estimate the average amount a student spends during the Spring break. They want to determine it to within **$120 with 90% confidence**. One can roughly say that it **ranges from $100 to $1700**. How many students should they sample?

Confidence Level	α	$z_{\alpha/2}$	$z_{\alpha/2}$ Multiplier
90%	.10	$z_{0.05}$	1.645
95%	.05	$z_{0.025}$	1.960
98%	.02	$z_{0.01}$	2.326
99%	.01	$z_{0.005}$	2.576

$$n = \left(\frac{z_{\alpha/2}\, s}{E}\right)^2$$

$$z_{\alpha/2} = 1.645$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 3:

To estimate SD given range:

$$s \approx \frac{range}{4} = \frac{Max - Min}{4}$$

A marketing research firm wants to estimate the average amount a student spends during the Spring break. They want to determine it to within **$120 with 90% confidence**. One can roughly say that it **ranges from $100 to $1700**. How many students should they sample?

$$z_{\alpha/2} = 1.645$$

$$n = \left(\frac{z_{\alpha/2}\, s}{E}\right)^2$$

$$s \approx \frac{1700 - 100}{4} \approx 400$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 3:

A marketing research firm wants to estimate the average amount a student spends during the Spring break. They want to determine it to within **$120 with 90% confidence**. One can roughly say that it **ranges from $100 to $1700**. How many students should they sample?

$$n = \left(\frac{z_{\alpha/2}\, s}{E}\right)^2$$

$$z_{\alpha/2} = 1.645$$

$$s \approx 400$$

$$E = 120$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 3:

A marketing research firm wants to estimate the average amount a student spends during the Spring break. They want to determine it to within **$120 with 90% confidence**. One can roughly say that it **ranges from $100 to $1700**. How many students should they sample?

$$z_{\alpha/2} = 1.645$$

$$n = \left(\frac{z_{\alpha/2}\, s}{E}\right)^2 = \left(\frac{1.645 * 400}{120}\right)^2$$

$$s \approx 400$$

$$E = 120$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 3:

A marketing research firm wants to estimate the average amount a student spends during the Spring break. They want to determine it to within **$120 with 90% confidence**. One can roughly say that it **ranges from $100 to $1700**. How many students should they sample?

$$z_{\alpha/2} = 1.645$$

$$n = \left(\frac{z_{\alpha/2}\, s}{E}\right)^2 = \left(\frac{1.645 * 400}{120}\right)^2 = 30.07$$

$$s \approx 400$$

$$E = 120$$

5.3 CONFIDENCE INTERVALS OF SAMPLE MEANS

Sample Problem 3:

A marketing research firm wants to estimate the average amount a student spends during the Spring break. They want to determine it to within **$120 with 90% confidence**. One can roughly say that it **ranges from $100 to $1700**. How many students should they sample?

$$n = 30.07 \approx 31 \textit{ students}$$

Other Problems:

1. The average temperature of 30 people entering a mall is 37.5 degrees Celsius with a standard deviation of .5 degrees Celsius. Construct a confidence interval using 90% confidence level.

2. The average score of 65 students taking engineering data analysis for their major exam is 85 points out of 100 with a SD of 3 points. Find the confidence interval of the mean of the scores using 99% confidence level.