Proof that $\binom{n}{m} + \binom{n}{m+1} = \binom{n+1}{m+1}$

Find identities for each term

(1) $\displaystyle \binom{n}{m} = \frac{n!}{m!(n-m)!}$

(2) $\displaystyle \binom{n}{m+1} = \frac{n!}{(m+1)!(n-(m+1))!}$

$(m+1)! = (m+1)m!$

$\displaystyle (n-(m+1))! = (n-m-1)! = \frac{(n-m)!}{n-m}$

(3) $\displaystyle \binom{n}{m+1} = \frac{n!}{(m+1)m!\left[\frac{(n-m)!}{n-m}\right]}$

(4) $\displaystyle \binom{n}{m+1} = \frac{n!}{\left[\frac{(m+1)}{n-m}\right]m!(n-m)!}$

(5) $\displaystyle \binom{n+1}{m+1} = \frac{(n+1)!}{(m+1)!((n+1)-(m+1))!}$

(6) $\displaystyle \binom{n+1}{m+1} = \frac{(n+1)n!}{(m+1)m!(n-m)!}$

Substitute in our new identities

$\displaystyle \binom{n}{m} + \binom{n}{m+1} = \binom{n+1}{m+1}$

(7) $\displaystyle \frac{n!}{m!(n-m)!} + \frac{n!}{\left[\frac{(m+1)}{n-m}\right]m!(n-m)!} = \frac{(n+1)n!}{(m+1)m!(n-m)!}$

Prove the equality

Divide by $\displaystyle \frac{n!}{m!(n-m)!}$

(8) $\displaystyle 1 + \frac{1}{\left[\frac{m+1}{n-m}\right]} = \frac{n+1}{m+1}$

Multiply by $m+1$

(9) $m+1+n-m = n+1$