Proof that $\sum_{i=1}^{n} i = \frac{1}{2}n(n+1) \quad 1 \leq n < \infty$

1 Introduction

S is a set such that $n \in S$ if $\displaystyle\sum_{i=1}^{n} i = \frac{1}{2}n(n+1)$

By proving that $n \in S$ for $i \leq n < \infty$

it will prove that $\displaystyle\sum_{i=1}^{n} i = \frac{1}{2}n(n+1) \quad 1 \leq n < \infty$

2 The Induction Axiom

The fifth of Peano's axioms, which states: If a set S of numbers contains zero and also the successor of every number in S, then every number is in S. [1]

Zero refers to the first element of the set, which in this case is 1.

3 Proof that $1 \in S$

$$\sum_{i=1}^{1} i = 1$$

$$\frac{1}{2}(1)(1+1) = 1$$

4 Proof that if $n \in S$ then $n+1 \in S$

$$\sum_{i=1}^{n+1} i = \left(\sum_{i=1}^{n} i\right) + n + 1$$

$$f :\rightarrow f(n) = \frac{1}{2}n(n+1)$$

$$f(n+1) = f(n) + n + 1$$

$$\frac{1}{2}(n+1)(n+1+1) = \frac{1}{2}n(n+1) + n + 1$$

$$\frac{1}{2}(n^2 + 3n + 2) = \frac{1}{2}(n^2 + n) + n + 1)$$

$$\frac{n^2}{2} + \frac{3n}{2} + 1 = \frac{n^2}{2} + \frac{n}{2} + n + 1$$

[1] from http://mathworld.wolfram.com/InductionAxiom.html