

**MATH 1310 – Technical Math for IT**

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Set: C

**ASSIGNMENT 5**

**Due:** Start of the class, Oct 31 (Set B and Set C), Nov 3 (Set A)

This is how I mark this assignment:

50 percent: I scan all the questions to see if they are solved or not.

50 percent: I select a few parts randomly and mark them in detail.

**24 marks. Complete all work on separate pages. Work must be clear and complete to receive full marks.**

Consider the Boolean expression

$$F = (x + z)' \oplus (xy')' + (x + y)' z$$

- a) [3] Construct the truth table for  $F$ , without any simplification

$x$	$y$	$z$	$x'$	$y'$	$z'$	$x+z$	$(x+z)'$	$xy'$	$(xy')'$	$(x+y)$	$(x+y)'$	$(x+y)'z$	$(x+z)' \oplus (xy')'$	$(x+z)' \oplus (xy')' + (x+y)'z$
0	0	0	1	1	1	0	1	0	1	0	1	0	0	0
0	0	1	1	1	0	1	0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	1	0	1	1	0	0	0	0
0	1	1	1	0	0	1	0	0	1	1	0	0	1	1
1	0	0	0	1	1	1	0	1	0	1	0	0	0	0
1	0	1	0	1	0	1	0	1	0	1	0	0	0	0
1	1	0	0	0	1	1	0	0	1	1	0	0	1	1
1	1	1	0	0	0	1	0	0	1	1	0	0	1	1

b) [2] Find canonical SOP and POS forms using the truth table in part a)

x	y	z	F	
0	0	0	0	$(x + y + z)$
0	0	1	1	$x'y'z$
0	1	0	0	$(x + y' + z)$
0	1	1	1	$x'y z$
1	0	0	0	$(x' + y + z)$
1	0	1	0	$(x' + y + z')$
1	1	0	1	$xy z'$
1	1	1	1	$xyz$

$$\text{SOP: } x'y'z + x'y z + xy z' + xyz$$

$$\text{POS: } (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z')$$

c) [2] Create Karnaugh map corresponding to the truth table in part a)

xy	00	01	11	10
2				
0	0	0	1	0
1	1	1	1	0

d) [4] Find optimal SOP and POS using the karnaugh map

$x \backslash y$	00	01	11	10
0	0	0	1	0
1	1	1	1	0

$$\begin{cases} x = 0 \\ y = \text{any} \\ z = 1 \end{cases} \quad x'z$$

$$\Rightarrow xy + x'z$$

$$\begin{cases} x = 1 \\ y = 1 \\ z = \text{any} \end{cases} \quad xz$$

$x \backslash y$	00	01	11	10
0	0	0	1	0
1	1	1	1	0

$$\begin{cases} x = 0 \\ y = \text{any} \\ z = 0 \end{cases} \quad (x+z)$$

$$\Rightarrow (x+z)(x'+y)$$

$$\begin{cases} x = 1 \\ y = 0 \\ z = \text{any} \end{cases} \quad (x'+y)$$

- e) [6] Starting from original expression, use postulates/theorems of Boolean algebra to reach optimal SOP form you found in section d)

$$(x+z)' \oplus (xy')' + (x+y)' =$$

$$\text{let } (x+z)' = a, (xy')' = b$$

$$a \oplus b$$

$$ab' + a'b + (x+y)' =$$

$$[(x+z)'][(xy')'] + [(x+z)'][(xy')'] + (x+y)' =$$

$$(x'z') (xy') + (x+z)(x' + y'') + (x'y') = \quad \text{T15a, T15b}$$

$$\cancel{x'}^0 \cancel{xy'} + (x+z)(x' + y) + x'y' =$$

$$\cancel{xx'}^0 + xy + zx' + zy + x'y' = \quad \text{L8a}$$

$$xy + zy + zx'(1 + y')$$

$$xy + zy + zx'(1)$$

$$xy + zy(1) + zx' \quad \text{T10a}$$

$$xy + zy(x+x') + zx' \quad \text{T12b}$$

$$xy + zyx + zyx' + zx' \quad \text{L8a}$$

$$xy(\cancel{1+z}) + zx'(\cancel{y+1})' \quad \text{L8a}$$

$$xy(1) + zx'(1)$$

$$xy + zx'$$

- f) [4] Starting from optimal SOP form, use postulates/theorems of Boolean algebra to reach optimal POS from

$$xy + 2x'$$

$$xy(1) + 2x'(1)$$

$$xy(1+2) + 2x'(1+y) \quad L8a$$

$$xy + xy2 + 2x' + 2x'y$$

$$xy + 2x' + xy2 + 2x'y$$

$$xy + 2x' + 2y(\cancel{x+x'})$$

$$xy + 2x' + 2y + 0$$

$$xy + 2x' + 2y + xx'$$

$$\underline{xx'} + \underline{xy} + \underline{2x'} + \underline{2y}$$

$$(x+2)(x'+y)$$

L8a

$$= (x+2)(x'+y)$$

g) [2] Construct truth table for optimal SOP and POS form

$x$	$y$	$z$	$x'$	$xy$	$x'z$	$(x+z)$	$(x'+y)$	$xy+x'z$	$(x+z)(x'+y)$
0	0	0	1	0	0	0	1	0	0
0	0	1	1	0	1	1	1	1	1
0	1	0	1	0	0	0	1	0	0
0	1	1	1	0	1	1	1	1	1
1	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	1	0	0	0
1	1	0	0	1	0	1	1	1	1
1	1	1	0	1	0	1	1	1	1

$\uparrow$        $\uparrow$

h) [1] Do they match with the original truth table in part a)? what is the conclusion?

They are equal and match the original truth table  $\therefore$  proves that POS and SOP forms are equal even though they look different.