Math 1310 – *Technical Math for IT*

Name:________________________

ASSIGNMENT 7

Set:__________________________

Due: Dec 1st, 11:59 PM, online submission, one pdf file

This is how I mark this assignment:

50 percent: I scan all the questions to see if they are solved or not.
50 percent: I select one or two randomly and mark them in details.

Question 1: *(10 marks)*
Solve the linear system:

For x, y and z using
(a) the Gauss-Jordan method. Be sure to show which row operation you are performing at each step.
(b) Cramer's Rule

a)
$$\begin{aligned} 2x - y + 6z &= 10 \\ -3x + 4y - 5z &= 11 \\ 8x - 7y - 9z &= 12 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 6 & 10 \\ -3 & 4 & -5 & 11 \\ 8 & -7 & -9 & 12 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \Rightarrow R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 3 & 5 \\ -3 & 4 & -5 & 11 \\ 8 & -7 & -9 & 12 \end{array}\right]$$

$$\xrightarrow[-8R_1 + R_3 \Rightarrow R_3]{3R_1 + R_2 \Rightarrow R_2} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 3 & 5 \\ 0 & \frac{5}{2} & 4 & 26 \\ 0 & -3 & -33 & -28 \end{array}\right] \xrightarrow{\frac{2}{5}R_2 \Rightarrow R_2} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 3 & 5 \\ 0 & 1 & \frac{8}{5} & \frac{52}{5} \\ 0 & -3 & -33 & -28 \end{array}\right]$$

$$\xrightarrow[3R_2 + R_3 \Rightarrow R_3]{\frac{1}{2}R_2 + R_1 \Rightarrow R_1} \left[\begin{array}{ccc|c} 1 & 0 & \frac{19}{5} & \frac{51}{5} \\ 0 & 1 & \frac{8}{5} & \frac{52}{5} \\ 0 & 0 & -\frac{141}{5} & \frac{16}{5} \end{array}\right] \xrightarrow{-\frac{5}{141}R_3 \Rightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{19}{5} & \frac{51}{5} \\ 0 & 1 & \frac{8}{5} & \frac{52}{5} \\ 0 & 0 & 1 & -\frac{16}{141} \end{array}\right]$$

$$\xrightarrow[-\frac{8}{5}R_3 + R_2 \Rightarrow R_2]{-\frac{38}{10}R_3 + R_1 \Rightarrow R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1499}{141} \\ 0 & 1 & 0 & \frac{1494}{141} \\ 0 & 0 & 1 & -\frac{16}{141} \end{array}\right]$$

b)

$$A = \begin{bmatrix} 2 & -1 & 6 \\ -3 & 4 & -5 \\ 8 & -7 & -9 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

$$x = \frac{A_1}{A} \quad y = \frac{A_2}{A} \quad z = \frac{A_3}{A}$$

$$A = \left|\begin{array}{ccc|cc} 2 & -1 & 6 & 2 & -1 \\ -3 & 4 & -5 & -3 & 4 \\ 8 & -7 & -9 & 8 & -7 \end{array}\right|$$

$-72 + 40 + 126 = 94$

$192 + 70 - 27 = 235$

$94 - 235$

$A = -141$

$$A_1 = \left|\begin{array}{ccc|cc} 10 & -1 & 6 & 10 & -1 \\ 11 & 4 & -5 & 11 & 4 \\ 12 & -7 & -9 & 12 & -7 \end{array}\right|$$

$-360 + 60 - 462 = -762$

$288 + 350 + 99 = 737$

$-762 - 737$

$A_1 = -1499$

$$A_2 = \left|\begin{array}{ccc|cc} 2 & 10 & 6 & 2 & 10 \\ -3 & 11 & -5 & -3 & 11 \\ 8 & 12 & -9 & 8 & 12 \end{array}\right|$$

$-198 - 400 - 216 = -814$

$528 - 120 + 270 = 678$

$-814 - 678$

$= -1492$

$$A_3 = \left|\begin{array}{ccc|cc} 2 & -1 & 10 & 2 & -1 \\ -3 & 4 & 11 & -3 & 4 \\ 8 & -7 & 12 & 8 & -7 \end{array}\right|$$

$96 - 88 + 210 = 218$

$320 - 154 + 36 = 202$

$218 - 202$

$= 16$

$$x = \frac{A_1}{A} \quad y = \frac{A_2}{A} \quad z = \frac{A_3}{A}$$

$$x = \frac{-1499}{-141} \quad y = \frac{-1492}{-141} \quad z = \frac{16}{-141}$$

$$\boxed{x = \frac{1499}{141} \quad y = \frac{1492}{141} \quad z = -\frac{16}{141}}$$

<u>Question 2</u>: *(8 marks)*
Given the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$$

(a) Determine the inverse matrix $\mathbf{A}^{-1}$.
(b) Confirm that your answer to part (a)

2)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$$

a) $[A : I]$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & 5 & 7 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-1R_1 + R_3 \Rightarrow R_3]{-2R_1 + R_2 \Rightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 3 & 4 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-1R_2 \Rightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & 3 & 4 & -1 & 0 & 1 \end{array}\right] \xrightarrow[-3R_2 + R_3 \Rightarrow R_3]{-2R_2 + R_1 \Rightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -3 & 2 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & 0 & -2 & -7 & 3 & 1 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{2}R_3 \Rightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -3 & 2 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & 0 & 1 & \frac{7}{2} & -\frac{3}{2} & -\frac{1}{2} \end{array}\right] \xrightarrow[R_3 + R_1 \Rightarrow R_1]{-2R_3 + R_2 \Rightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -5 & 2 & 1 \\ 0 & 0 & 1 & \frac{7}{2} & -\frac{3}{2} & -\frac{1}{2} \end{array}\right]$$

b)

$$AA^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 3 \\ 1 & 5 & 7 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -5 & 2 & 1 \\ \frac{7}{2} & -\frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} [1\ 2\ 3]\begin{bmatrix} \frac{1}{2} \\ -5 \\ \frac{7}{2} \end{bmatrix} & [1\ 2\ 3]\begin{bmatrix} \frac{1}{2} \\ 2 \\ -\frac{3}{2} \end{bmatrix} & [1\ 2\ 3]\begin{bmatrix} -\frac{1}{2} \\ 1 \\ -\frac{1}{2} \end{bmatrix} \\ [2\ 4\ 3]\begin{bmatrix} \frac{1}{2} \\ -5 \\ \frac{7}{2} \end{bmatrix} & [2\ 4\ 3]\begin{bmatrix} \frac{1}{2} \\ 2 \\ -\frac{3}{2} \end{bmatrix} & [2\ 4\ 3]\begin{bmatrix} -\frac{1}{2} \\ 1 \\ -\frac{1}{2} \end{bmatrix} \\ [1\ 5\ 7]\begin{bmatrix} \frac{1}{2} \\ -5 \\ \frac{7}{2} \end{bmatrix} & [1\ 5\ 7]\begin{bmatrix} \frac{1}{2} \\ 2 \\ -\frac{3}{2} \end{bmatrix} & [1\ 5\ 7]\begin{bmatrix} -\frac{1}{2} \\ 1 \\ -\frac{1}{2} \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 3: *(8 marks)*

Solve the following system of linear equations using matrix inverse method

$2x\,y + \;-\; = 2\;10z$

$y + 10z = -28$

$3\;16y + \;z = -42$

3)
$$\begin{aligned} 2x + y - 2z &= 10 \\ y + 10z &= -28 \\ 3y + 16z &= -42 \end{aligned}$$

$$A = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 1 & 10 \\ 0 & 3 & 16 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ -28 \\ -42 \end{bmatrix}$$

$[A \,\vdots\, I]$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 10 & 0 & 1 & 0 \\ 0 & 3 & 16 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \Rightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 10 & 0 & 1 & 0 \\ 0 & 3 & 16 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[-3R_2 + R_3 \Rightarrow R_3]{-\frac{1}{2}R_2 + R_1 \Rightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -6 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 10 & 0 & 1 & 0 \\ 0 & 0 & -14 & 0 & -3 & 1 \end{array}\right] \xrightarrow{-\frac{1}{14}R_3 \Rightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -6 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 10 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & \frac{3}{14} & -\frac{1}{14} \end{array}\right]$$

$$\xrightarrow[-10R_3 + R_2 \Rightarrow R_2]{6R_3 + R_1 \Rightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{11}{14} & -\frac{3}{7} \\ 0 & 1 & 0 & 0 & -\frac{8}{7} & \frac{5}{7} \\ 0 & 0 & 1 & 0 & \frac{3}{14} & -\frac{1}{14} \end{array}\right]$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{11}{14} & -\frac{3}{7} \\ 0 & -\frac{8}{7} & \frac{5}{7} \\ 0 & \frac{3}{14} & -\frac{1}{14} \end{bmatrix} \begin{bmatrix} 10 \\ -28 \\ -42 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{11}{14} & -\frac{3}{7} \end{bmatrix} \begin{bmatrix} 10 \\ -28 \\ -42 \end{bmatrix} \\ \begin{bmatrix} 0 & -\frac{8}{7} & \frac{5}{7} \end{bmatrix} \begin{bmatrix} 10 \\ -28 \\ -42 \end{bmatrix} \\ \begin{bmatrix} 0 & \frac{3}{14} & -\frac{1}{14} \end{bmatrix} \begin{bmatrix} 10 \\ -28 \\ -42 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} x = 1 \\ y = 2 \\ z = -3 \end{bmatrix}$$

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<u>Question 4</u>: *(9 marks)*

Determine the values of “a “ so that the following system in unknowns x, y and z has

(i) A unique solution

(ii) More than one solutions

Also, can you find a value “a “ that the system has no solution? Why?

$$x\,y\,z+ \;+\; =0$$
$$2\;3x+ \;+\; =y\;az\;0$$
$$x\;ay+ \;+\; =3\;0z$$

$x + y + z = 0$
$2x + 3y + az = 0$
$x + ay + 3z = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & a \\ 1 & a & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$\left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & a & 2 & 3 \\ 1 & a & 3 & 1 & a \end{array}\right|$$

$9 + a + 2a = 9 + 3a$
$3 + a^2 + 6 = a^2 + 9$

$(9 + 3a) - (a^2 + 9) = 0$

$\cancel{9} + 3a - a^2 - \cancel{9}$

$-a^2 + 3a$

$-a(a-3)$

$a = 0, 3$

$$\left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 3 & 2 & 3 \\ 1 & 3 & 3 & 1 & 3 \end{array}\right|$$

$$\left.\begin{array}{l} 3 + 9 + 6 = \\ 9 + 3 + 6 = \end{array}\right\} = 0$$

$$\left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 0 & 2 & 3 \\ 1 & 0 & 3 & 1 & 0 \end{array}\right|$$

when $a = 0, 3$

$$\left.\begin{array}{l} 9 + 0 + 0 = 9 \\ 3 + 0 + 6 = 9 \end{array}\right\} = 0$$

when $a = 0, 3$ the determinate is 0 $\therefore$ when $a \neq 0, 3$ the equation has a unique solution.