Assignment 1

Markus Afonso A01333486

COMP 3725

Set: Friday

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1) value of x, for $x > 0$.

a) $\log_3 27x = 5$

$$\frac{27x}{27} = \frac{3^5}{27}$$

$$\boxed{x = 9}$$

b) $\log_3\left(\frac{x}{2}\right) = 3$

$$\frac{x}{2} = 3^3 \times 2$$

$$\boxed{x = 54}$$

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c) $\log_2 4x = \log_4 64x^4 - 6$

$\log_2 4 + \log_2 x = (\log_4 64 + \log_4 x^4) - 6$ → Product Rule

$2 + \log_2 x = 3 + \log_4 x^4 - 6$

$2 - 3 + 6 = \log_4 x^4 - \log_2 x$

$5 = \log_4 x^4 - \log_2 x$

$5 = \frac{\log_2 x^4}{\log_2 (4)_2} - \log_2 x$ → change of bases

$5 = \frac{\log_2 x^4}{2} - \log_2 x$

$5 = \frac{\log_2 x^4 - 2\log_2 x}{2}$

$5 = \frac{\log_2 \left(\frac{x^4}{x^2}\right)}{2}$ → Quotient Rule

$5 = \frac{\log_2 x^2}{2}$ → power Rule

$5 = \log_2 x$

$2^5 = x$

$\boxed{x = 32}$

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d) $\log_b(16-4x) - \log_{b^2} 4x^2 = \log_b 6$

$$\frac{\log(16-4x)}{\log b} - \frac{\log 4x^2}{\log b^2} = \frac{\log 6}{\log b} \rightarrow \text{Change of Bases}$$

$$\frac{\log(16-4x)}{\log b} - \frac{\log(2^2 x^2)^2}{\log b^2} = \frac{\log 6}{\log b} \rightarrow \text{Power Rule}$$

$$\frac{\log(16-4x)}{\log b} - \frac{2\log 2x}{2\log b} = \frac{\log 6}{\log b}$$

$$\frac{\log(16-4x) - \log 2x}{\log b} = \frac{\log 6}{\log b}$$

$$\frac{\log\left(\frac{16-4x}{2x}\right)}{\log b} = \frac{\log 6}{\log b} \rightarrow \text{Quotient Rule}$$

$$\frac{\log\left(\frac{x(8-2x)}{2x}\right)}{\log b} = \frac{\log 6}{\log b}$$

$$\log_b\left(\frac{8-2x}{x}\right) = \log_b 6 \rightarrow \text{Change of Bases}$$

$$\frac{8-2x}{x} = 6x \rightarrow \text{Equal Bases}$$

$$8 = 6x + 2x$$

$$\frac{8}{8} = \frac{8x}{8}$$

$$\boxed{x = 1}$$

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e) $\log_{b^2} \frac{1}{4} + \log_b 8x^2 = \log_b (12x - 9)$

$$\frac{\log \frac{1}{4}}{\log b^2} + \frac{\log 8x^2}{\log b} = \frac{\log (12x - 9)}{\log b}$$

$$\frac{\log \left(\frac{1}{2}\right)^2}{2 \log b} + \frac{\log 8x^2}{\log b} = \frac{\log (12x - 9)}{\log b}$$

$$\log_b \frac{1}{2} + \log_b 8x^2 = \log_b (12x - 9)$$

$$\log_b \left(8x^2 \cdot \frac{1}{2}\right) = \log_b (12x - 9)$$

$$\log_b 4x^2 = \log_b (12x - 9)$$

$$4x^2 = 12x - 9$$

$$4x^2 - 12x + 9 = 0$$

a b c

Quadratic $ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$$

$$x = \frac{12 \pm \sqrt{144 - 144}}{8}$$

$$x = \frac{12}{8}$$

$$\boxed{x = \frac{3}{2} \text{ or } 1.5}$$

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2) $x = \log_b 8$, $y = \log_b 3$

a) $\log_b 648$

$\log_b (8 \cdot 81)$

$\log_b (8 \cdot 3 \cdot 27)$

$\log_b (8 \cdot 3 \cdot 3 \cdot 9)$

$\log_b (8 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$

$\log_b (8 \cdot 3^4)$

$\log_b 8 + \log_b 3^4$ Product Rule

$\log_b 8 + 4 \log_b 3$

given $x = \log_b 8$
and $y = \log_b 3$

$= x + 4y$

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b) $\log_{3b} \frac{9}{64b}$

$\log_{3b} 9 - \log_{3b} 64b$

$\log_{3b} 9 - (\log_{3b} 64 + \log_{3b} b)$

$\frac{\log_b 9}{\log_b 3b} - \left(\frac{\log_b 64}{\log_b 3b} + \frac{\log_b b}{\log_b 3b}\right)$

$\frac{\log_b 9 - (\log_b 64 + \overbrace{\log_b b}^{1})}{\log_b 3b}$

$\frac{\log_b 9 - (\log_b 64 + 1)}{\log_b 3 + \cancel{\log_b b}\ 1}$

$\frac{\log_b 9 - (\log_b 64 + 1)}{\log_b 3 + 1}$

$\frac{\log_b 3^2 - (\log_b 8^2 + 1)}{\log_b 3 + 1}$

$\frac{2\log_b 3 - 2\log_b 8 - 1}{\log_b 3 + 1}$

$= \frac{2y - 2x - 1}{y + 1}$

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3) $s(t) = 4\sin 2t(1-2\sin^2 t) + 3 - 6\sin^2\left(\frac{t}{2} - \frac{\pi}{2}\right)$

$s(t) = 4\sin 2t(1-2\sin^2 t) + 3\left(1-2\sin^2\left(\frac{t}{2} - \frac{\pi}{2}\right)\right)$

Using Double Angle

$1 - 2\sin^2 x = \cos 2x$

$s(t) = 4\sin 2t\left(\cos 2t\right) + 3\left(\cos 2\left(\frac{t}{2} - \frac{\pi}{2}\right)\right)$

$s(t) = 4\sin(2t)\cos(2t) + 3\cos(t - \pi)$

Using Double Angle

$\sin 2x = 2\sin x \cos x$

$s(t) = 2\sin(2(2t)) + 3\cos(t - \pi)$

$s(t) = 2\sin(4t) + 3\cos(t - \pi)$

using sum and differences

$\cos(x - y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$s(t) = 2\sin(4t) + 3(\cos(t)\cancel{\cos(\pi)}^{-1} - \cancel{\sin(t)\sin(\pi)}^{0})$

$s(t) = 2\sin(4t) - 3\cos(t)$

Using terms of complements

$\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$

$s(t) = 2\sin(4t) - 3\sin\left(\frac{\pi}{2} - t\right)$

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b) $S(t) = 2\sin(4t) - 3\sin(\frac{\pi}{2} - t)$

lets say: $a = 2\sin(4t)$ and $b = 3\sin(\frac{\pi}{2} - t)$

a/ peak amplitude: 2V

frequency: $\frac{2}{\pi}$ Hz

$2\pi t = \frac{4}{2\pi}$

$t = \frac{2}{\pi}$ Hz

phase: 0

b/ peak amplitude: 3V

frequency: $\frac{1}{2\pi}$ Hz

$\frac{2\pi}{2\pi} t = \frac{1}{2\pi}$

$t = \frac{1}{2\pi}$

phase: $\pi/2$ or 90°

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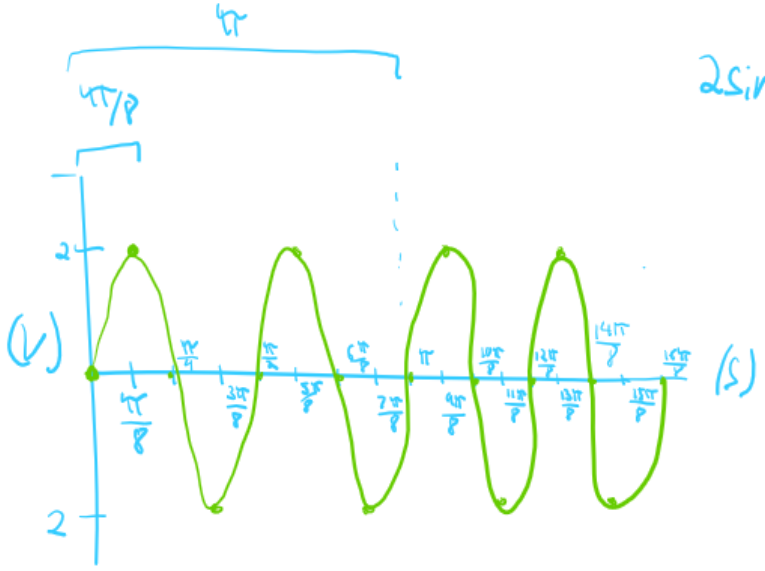
c)

$$a = 2\sin(4t)$$



$2\sin(4t) = 0$

$4t = 0$

$t = 0$

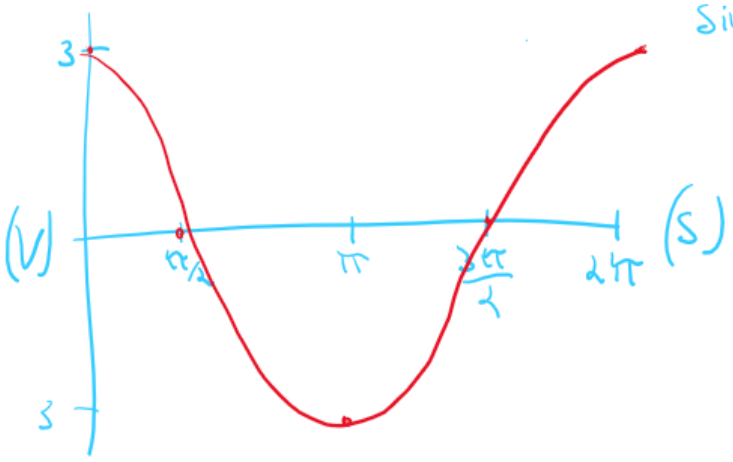
$2\sin(4t) = 2$

$\sin(4t) = 2$

$\frac{4t}{4} = \frac{\pi/2}{4}$

$t = \frac{\pi}{8}$

$$b = 3\sin\left(\frac{\pi}{2} - t\right)$$



$\sin\left(\frac{\pi}{2} - t\right) = 0$

$\frac{\pi}{2} - t = 0$

$\frac{\pi}{2} = 0$

$\sin\left(\frac{\pi}{2} - t\right) = -1$

$\frac{\pi}{2} - t = \frac{3\pi}{2}$

$\frac{\pi}{2} - \frac{3\pi}{2} =$

$\pi = -1$

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d) $s(t) = 2\sin(4t) - 3\sin(\frac{\pi}{2} - t)$

16 points, $a = 2\sin(4t)$, $b = -3\sin(\frac{\pi}{2} - t)$

$0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \pi, \frac{9\pi}{8}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{13\pi}{8}, \frac{7\pi}{4}, 2\pi$

$x = 0$ / $a = 2\sin(4(0))$, $b = -3\sin(\frac{\pi}{2} - t)$

$a = 2(0)$, $b = -3\sin(\frac{\pi}{2})$

$a = 0$, $b = -3$

$y = 0 - 3$

$y = -3$ so $(0, -3)$ at $x = 0$

$x = \frac{\pi}{8}$ / $a = 2\sin(4(\frac{\pi}{8}))$, $b = -3\sin(\frac{4\pi}{8} - \frac{\pi}{8})$

$a = 2\sin(\frac{\pi}{2})$, $b = -3\sin(\frac{3\pi}{8})$

$a = 2$, $b = -2.77$

$y = 2 - 2.77$

$y = -0.77$ so $(\frac{\pi}{8}, -0.77)$

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$x = \frac{\pi}{4}$ $a = 0$, $b = -2.12$

$y = -2.12$ so $(\frac{\pi}{4}, -2.12)$

$x = \frac{3\pi}{8}$ $a = -2$, $b = -1.148$

$y = -3.148$ so $(\frac{3\pi}{8}, -3.148)$

$x = \frac{\pi}{2}$ $a = 0$, $b = 0$

$y = 0$ so $(\frac{\pi}{2}, 0)$

$x = \frac{5\pi}{8}$ $a = 2$, $b = 1.14$

$y = 3.148$ so $(\frac{5\pi}{8}, 3.148)$

$x = \frac{3\pi}{4}$ $a = 0$, $b = 2.1213$

$y = 2.1213$ so $(\frac{3\pi}{4}, 2.1213)$

$x = \frac{7\pi}{8}$ $a = -2$, $b = 2.771$

$y = 0.771$ so $(\frac{7\pi}{8}, 0.771)$

$x = \pi$ $a = 0$, $b = 3$

$y = 3$ so $(\pi, 3)$

$x = \frac{9\pi}{8}$ $a = 2$, $b = 2.77$

$y = 4.77$ so $(\frac{9\pi}{8}, 4.77)$

$x = \frac{5\pi}{4}$ $a = 0$, $b = 2.121$

$y = 2.12$ so $(\frac{5\pi}{4}, 2.12)$

$x = \frac{11\pi}{8}$ $a = -2$, $b = 1.148$

$y = -0.85$ so $(\frac{11\pi}{8}, 0.85)$

$x = \frac{3\pi}{2}$ $a = 0$, $b = 0$

$y = 0$ so $(\frac{3\pi}{2}, 0)$

$x = \frac{13\pi}{8}$ $a = 2$, $b = -1.48$

$y = 0.8519$ so $(\frac{13\pi}{8}, 0.8519)$

$x = \frac{7\pi}{4}$ $a = 0$, $b = -2.121$

$y = -2.12$ so $(\frac{7\pi}{4}, -2.12)$

$x = \frac{15\pi}{8}$ $a = -2$, $b = -2.77$

$y = -4.771$ so $(\frac{15\pi}{8}, -4.77)$

$x = 2\pi$ $a = 0$, $b = -3$

$y = -3$ so $(2\pi, -3)$

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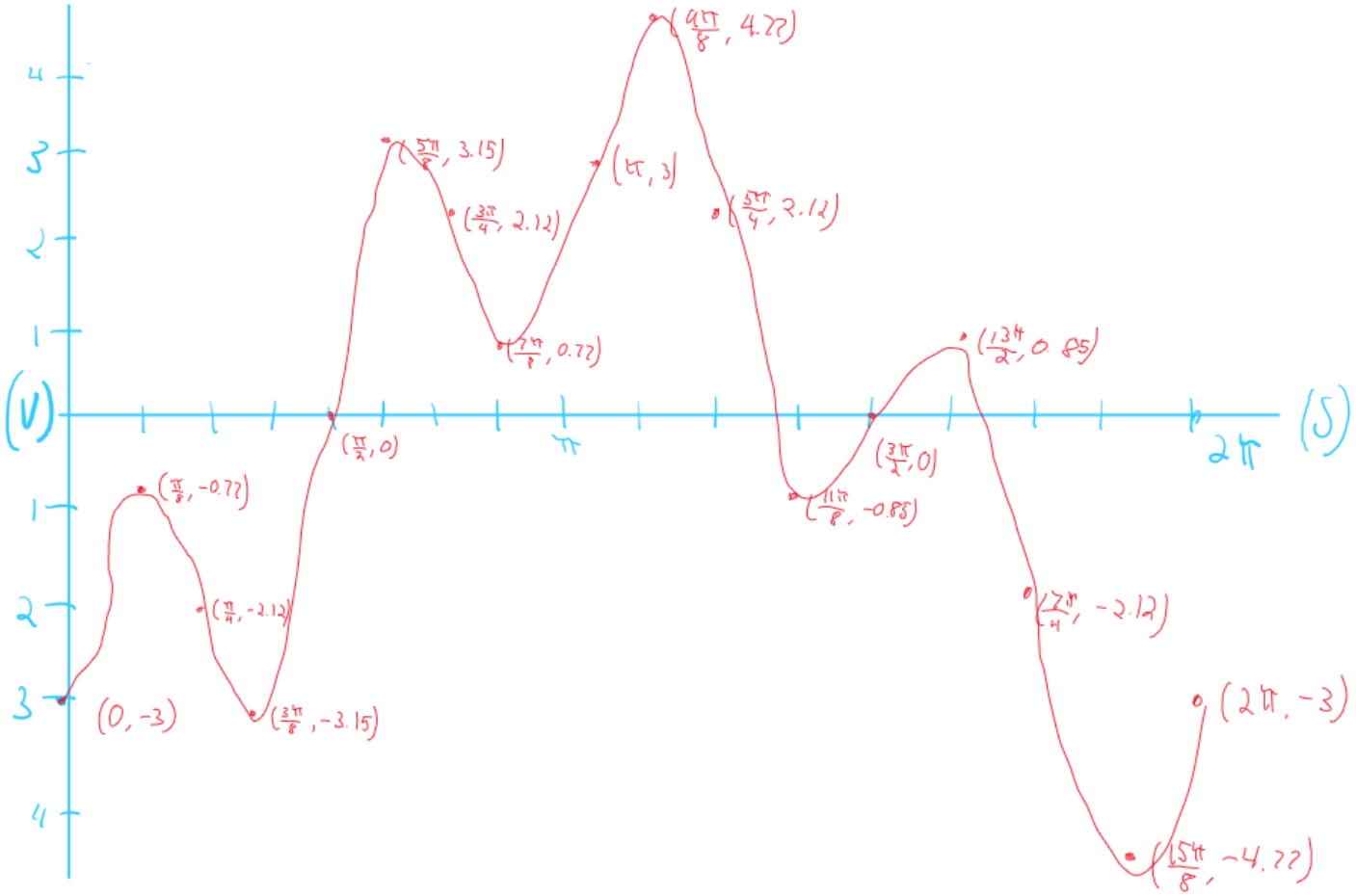

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4) Find file size

bits per frame x frames per second x seconds of runtime

bits per frame = total pixels x bits per pixel

total pixels = 4096 x 2160 bits per pixel = 10 bits for 3 colours

TP = 8847360 BPP = 10 x 3

BPP = 30

bits per frame = TP x BPP

bits per frame = 8847360 x 30

bits per frame = 265,420,800

seconds of runtime = 60 mins x $\frac{60\,s}{1\,min}$ = 3600 s

Bits per frame x frame per second x runtime.

= 265,420,800 x 30 x 3600

= $2.86654464 \times 10^{13}$ bits ÷ 8

= 3.5831808×10^{12} bytes ÷ 10^{12} (tera)

= 3.58 terabytes.

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First find how fast it is by GbE

$$\text{Propagation Speed} = \frac{\text{Distance}}{\text{Prop. Speed}} \qquad \text{Trans. Time} = \frac{\text{message size}}{\text{Trans Rate}}$$

Latency = Prop Speed + Trans. Time (queuing and processing time = 0)

$$\text{Prop Speed} = \frac{15\text{km}}{2\times10^{8}\text{m/s}} = \frac{15000\text{m}}{2\times10^{8}\text{m/s}} = 0.000075\text{s}$$

$$\text{Trans. Time} = \frac{3.58\text{TB}}{125\text{MBs}} \longrightarrow \text{Gigabit Ethernet}$$

$$\text{Trans Time} = \frac{3{,}580{,}000\text{MB}}{125\text{MBS}}$$

Trans Time = 28640s

Latency = 0.000075 + 28640
= 28640.000075s

So drone < 28640.000025s

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Find file load speed

The file needs to be loaded onto the drone's HDD.

$\frac{3.58TB}{3GBS}$ $\frac{2000}{3GBS}$ $\frac{1580}{3GBS}$

$\frac{3580}{3}$ $= 666.66s$ $= 526.66s$

1 ~~[illegible]~~ hard drive is only 2TB

so drone needs to make 2 trips

= file load + trip there + trip back + file load 2 + trip there 2

total trips = 15 + 15 + 15
= 45 km

So then

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$$\text{latency} = \text{file load 1} + \text{file load 2} + \text{total trips}$$

$$28640.000025 > 666.665 + 526.66 + \frac{45km}{s}$$

$$28640.000025 > 1193.33 + \frac{45km}{s}$$

$$27446.67341 > \frac{45km}{s}$$

$$27446.67341^{*s} > \frac{45000m}{s} {}^{*s}$$

$$27446.67341s > \frac{45000m}{27446.67341}$$

$$s > 1.6395\ m/s$$

drone must fly faster

than ~1.64 m/s or 5.9 km/h