Anonymous and Copy-Robust Delegations for Liquid Democracy

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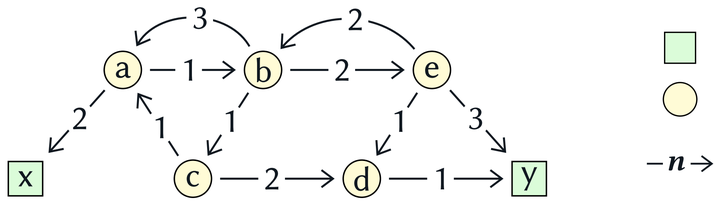
Liquid Democracy with Ranked Delegations

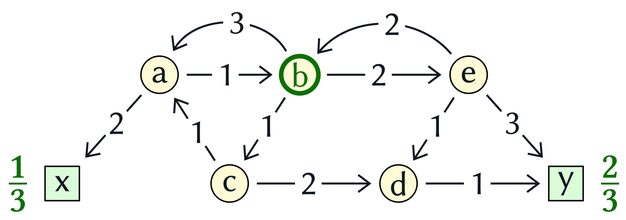
Ranked Delegation Graph

Each voter either casts their vote or delegates their vote by indicating a weak ranking over other voters.



Fractional Delegation Rule

Input: delegation graph and voter v
Output: probability distribution f_v over casting voters



$$f_b(x) = \tfrac{1}{3} \qquad f_b(y) = \tfrac{2}{3}$$

Voting Weight

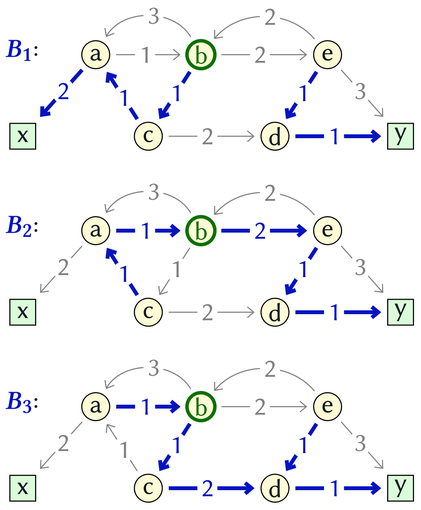
Total voting weight of a casting voter received from all delegating voters:

$$\pi(c) = \sum_v f_v(c)$$

Rules

Mixed Borda

Borda Branching [1]: Minimum cost acyclic subgraph such that every delegating voter has a path to a casting voter.



Mixed Borda Rule:

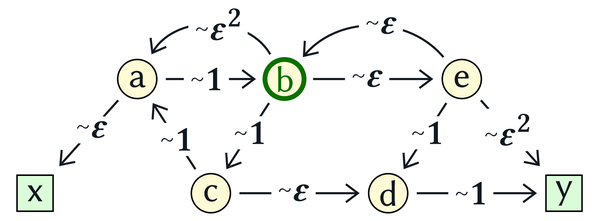
1. Sample a Borda Branching B uniformly at random
2. Define $f_v(w)$ as the probability that delegating voter v reaches casting voter w in B

$$f_b(x) = \frac{1}{3} \qquad f_b(y) = \frac{2}{3}$$

Random Walk Rule

1. Assign each edge with rank r a probability proportional to $\varepsilon^{(r-1)}$
2. For each delegating voter v compute probability $\mathbb{P}_\varepsilon(v \to w)$ of ending in each casting voter w when starting a random walk in v
3. Compute the limit for $\varepsilon \to 0$

$$f_b(x) = \lim_{\varepsilon \to 0} \mathbb{P}_\varepsilon(b \to x)$$

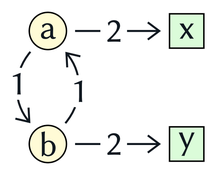


~: weights need to be scaled down proportionally

Axioms

Anonymity

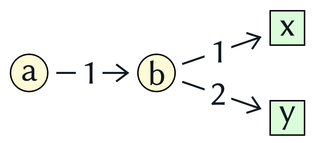
The names of voters do not matter.



$$f_a(x) \overset{!}{=} f_b(y)$$

Confluence

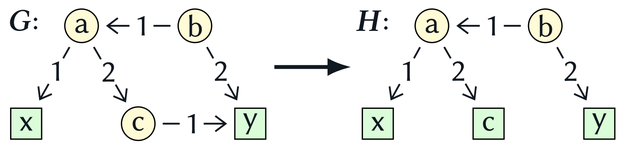
The voting weight that reaches some voter should be passed along in the same way as the own vote of this voter.



$$f_a \overset{!}{=} f_b$$

Copy-robustness

When a delegating voter v decides to cast their vote themselves, the joint voting power of v and its representatives should not change.



$$\pi_G(y) \overset{!}{=} \pi_H(c) + \pi_H(y)$$

Our Results

Equivalence

Mixed Borda and the Random Walk Rule return **the same** probability distribution. (We apply the Markov Chain Tree Theorem.)

Algorithm

We provide a **polynomial time** algorithm for computing the outcome of Mixed Borda (and hence the Random Walk Rule). This algorithm is of independent interest, e.g., in the context of semisupervised learning [2].

Axiomatic Analysis

We show that Mixed Borda (and hence the Random Walk Rule) satisfies **all three axioms**. For the non-fractional case, we prove an **impossibility theorem**, stating that no such rule exists.

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References: [1] M. Brill, T. Delemazure, A.-M. George, M. Lackner, and U. Schmidt-Kraepelin. Liquid Democracy with Ranked Delegations. (AAAI 2022)

[2] E. Fita Sanmartín, S. Damrich, F. A. Hamprecht. Directed Probabilistic Watershed. (NeurIPS 2021)