FAST SEARCHING / BALANCED TREES

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Algorithms and Data Structures 2
Exercise – 2021W

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BALANCED TREES

Motivation

- Real data is usually not randomly distributed
- Prevent **degeneration** of binary search trees into linear lists!
- Search/Insert in **O(log(n))**

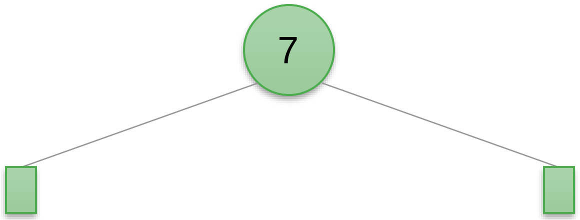
Approach

- Monitor tree structure
- Insert/Remove may require restructuring

Binary search trees that guarantee the execution of search, insert and delete operations in **O(log(n))** even in worst case → height-balanced trees (e.g.: **AVL tree**)

BALANCED TREES :: DEGENERATION

7, 5, 8, 17, 32



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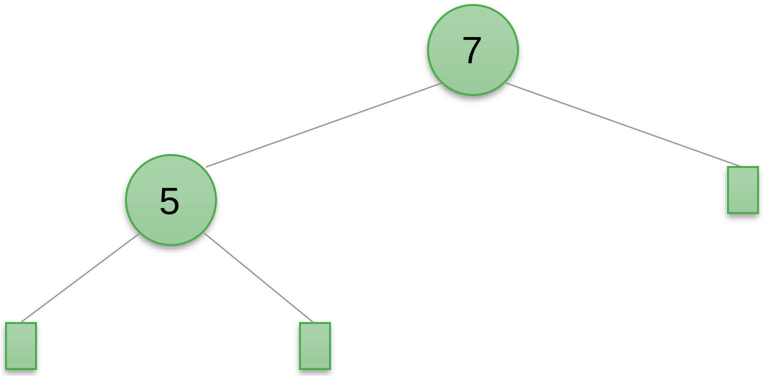

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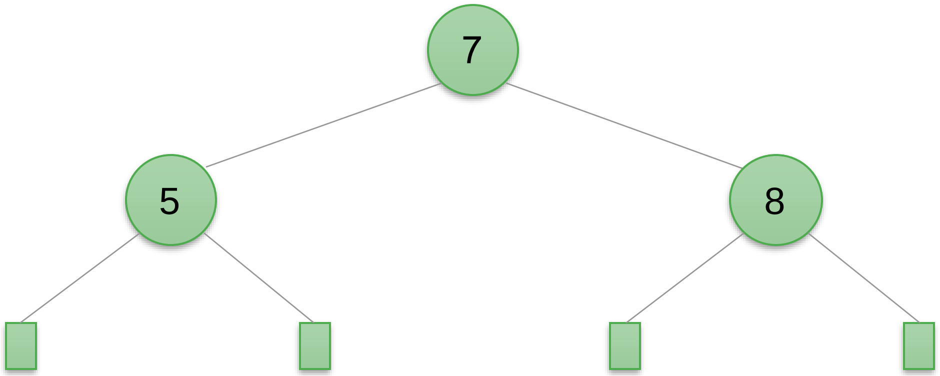

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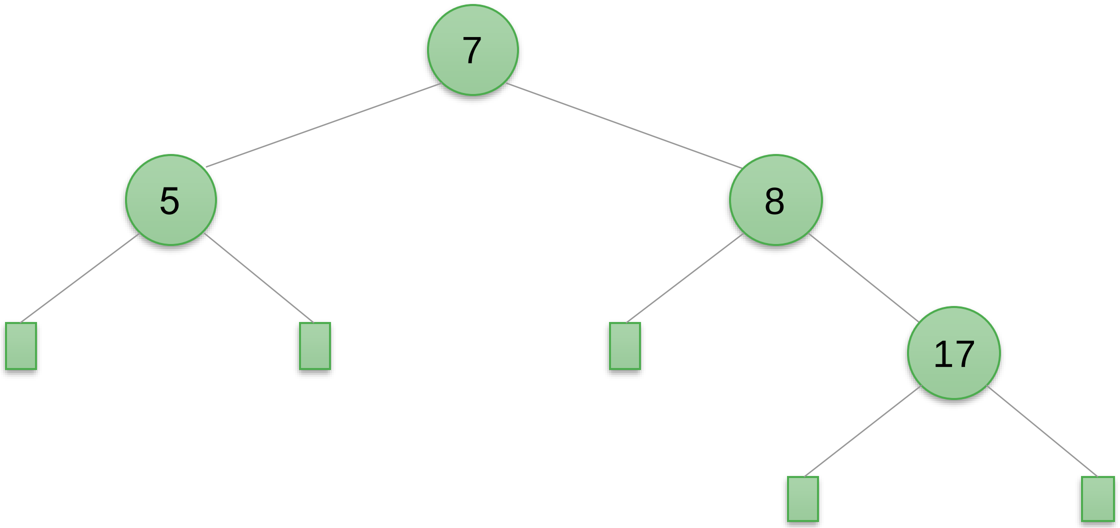

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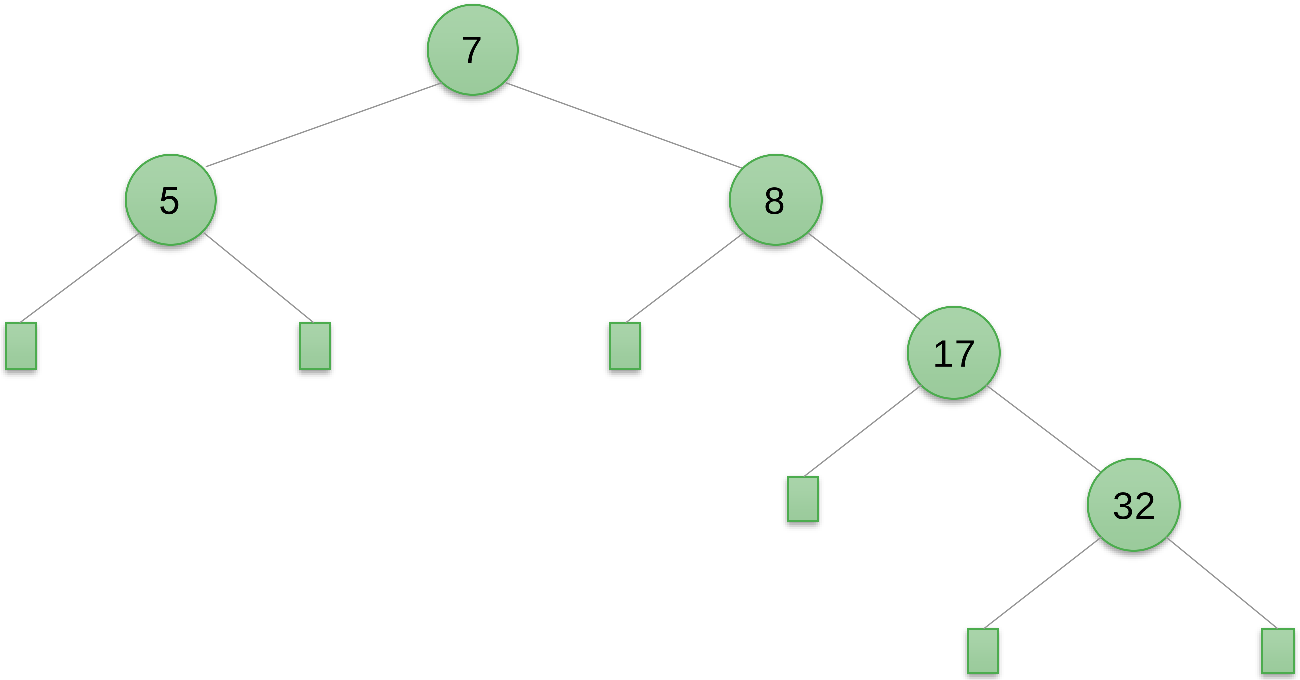

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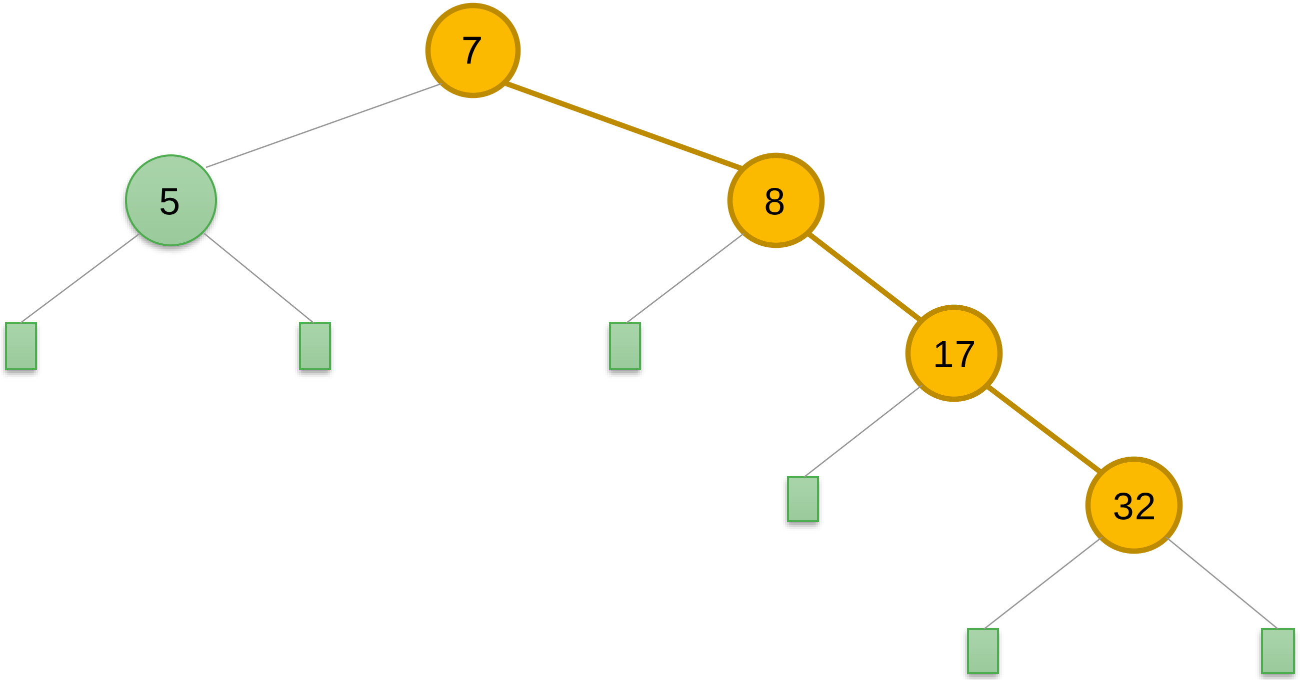


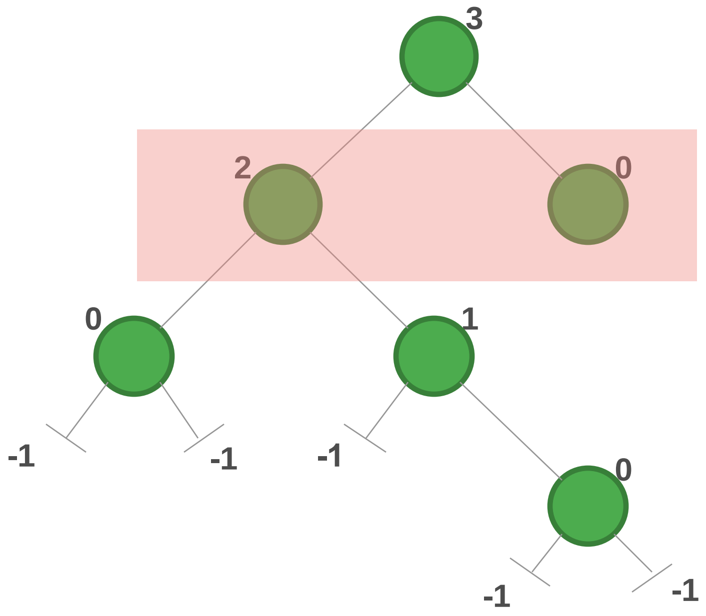
List: Access O(n)

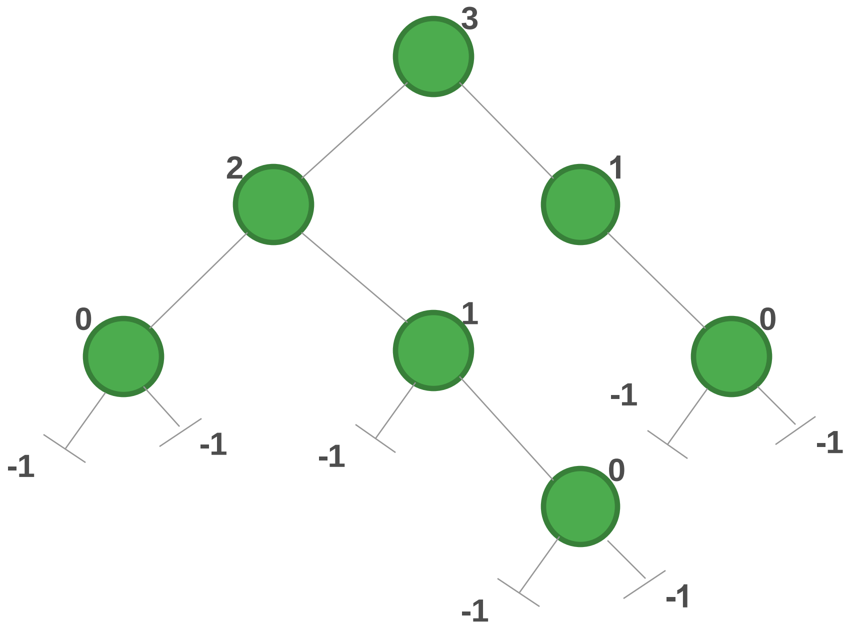
AVL TREE

Properties

- Binary search tree
- for each node, the heights of its two subtrees differ by not more than 1 („balanced“)

Examples





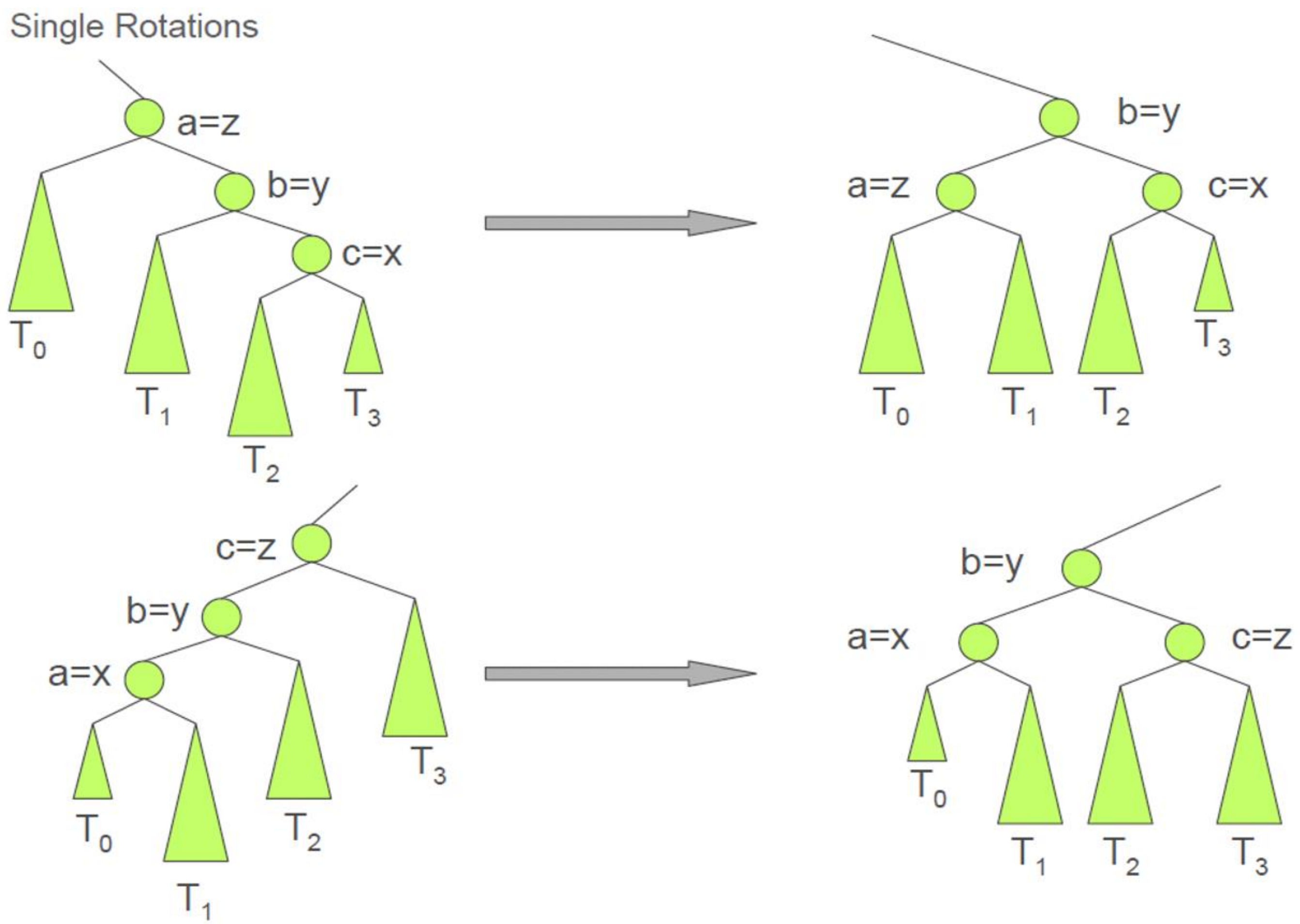
AVL TREE :: INSERT

Insert is in general the same as for the binary search tree but may cause the AVL tree to become unbalanced → restructuring required!

Restructuring

1. Go up from the new node in the tree until the first node **x** is found, whose grandparent **z** is an unbalanced node
2. Define **y** as child of **z** (= the node we passed on the way to z);
   height(y) = height(sibling(y))+2
3. Define **x** as child of **y**
4. Rename **x**,**y**,**z** in **a**,**b**,**c** (according to Inorder traversal!)
5. Replace **z** by **b**
6. Children of **b** are now **a** (left) and **c** (right)
7. Children of **a** and **c** are the subtrees T_0 … T_3, which have been children of **x**, **y** and **z** before → reassign and distinguish **4 cases…**

AVL TREE :: ROTATIONS



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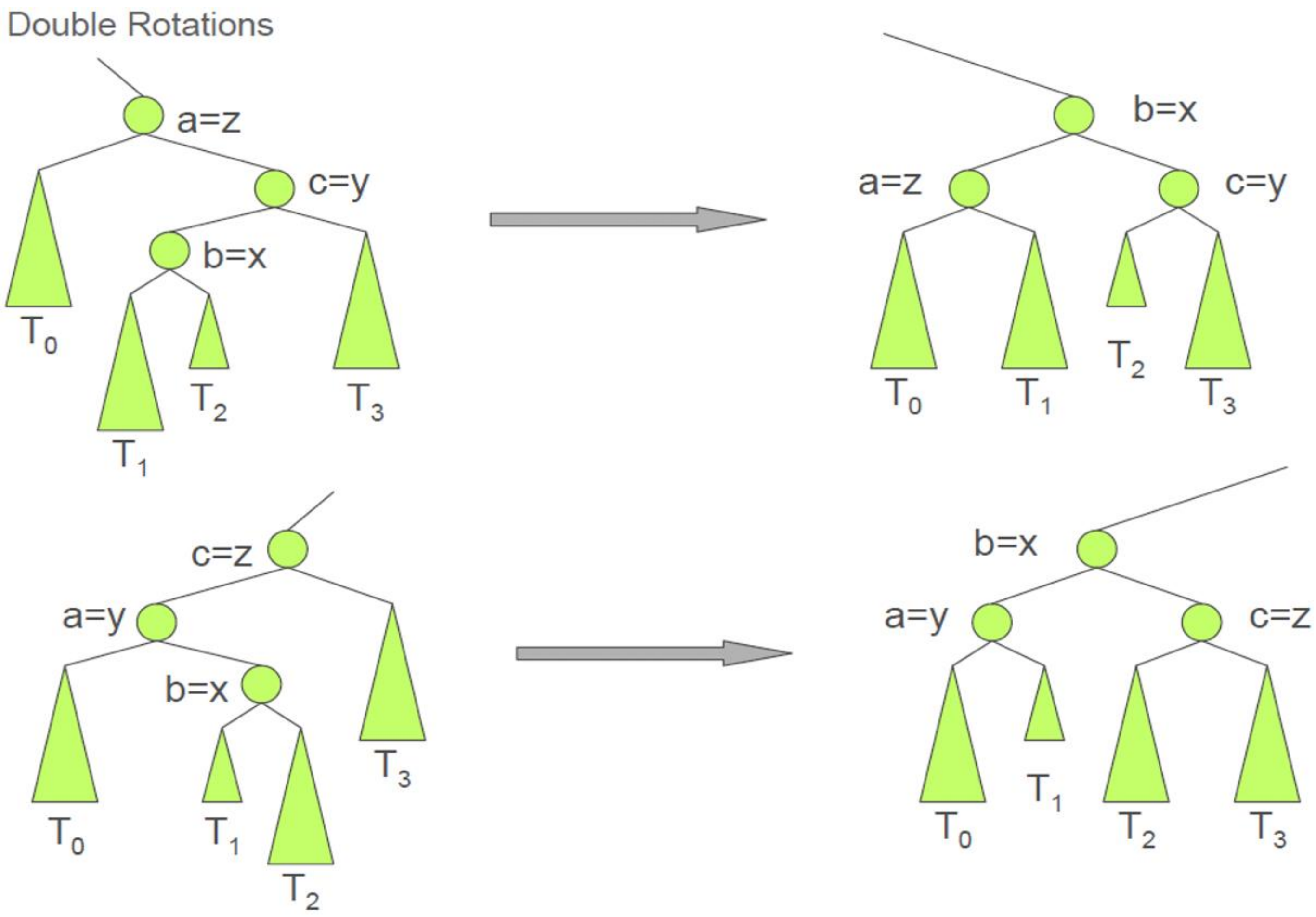

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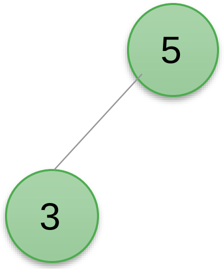

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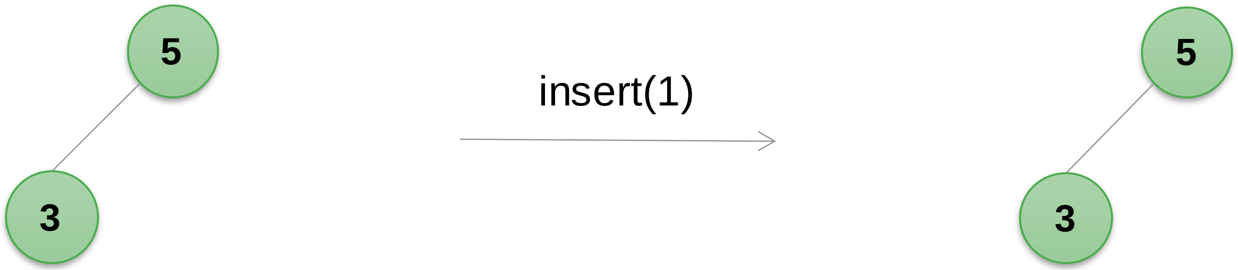

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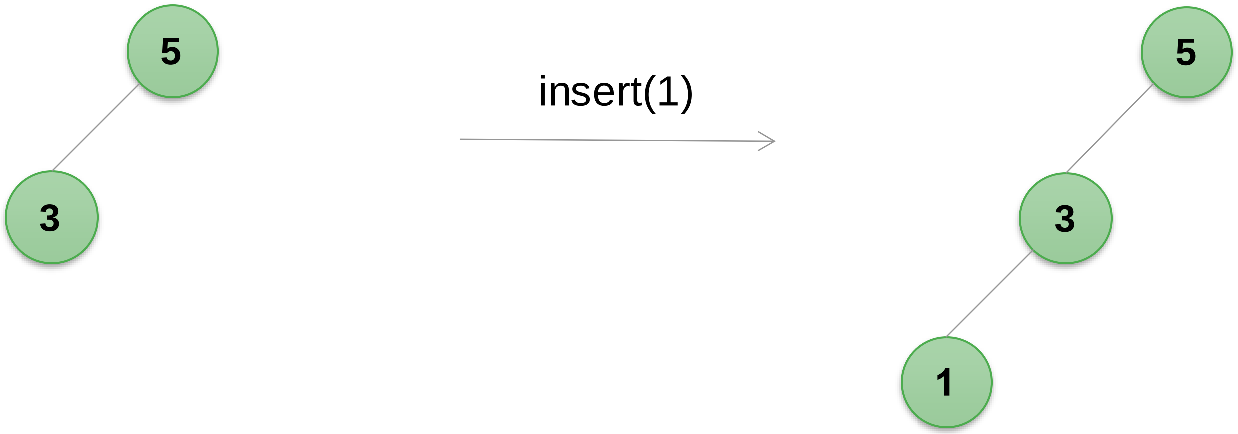

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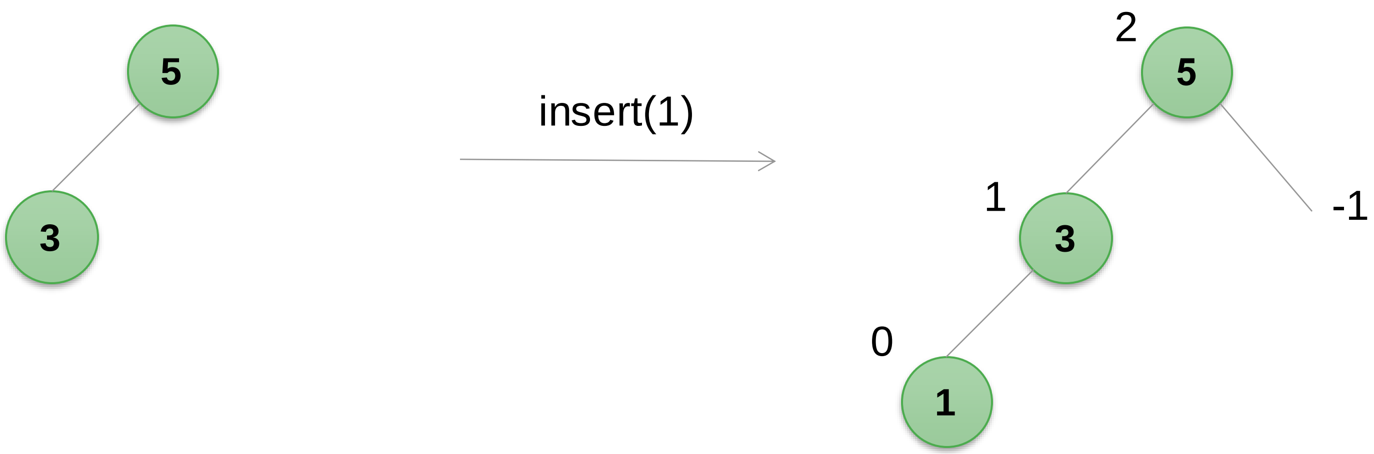

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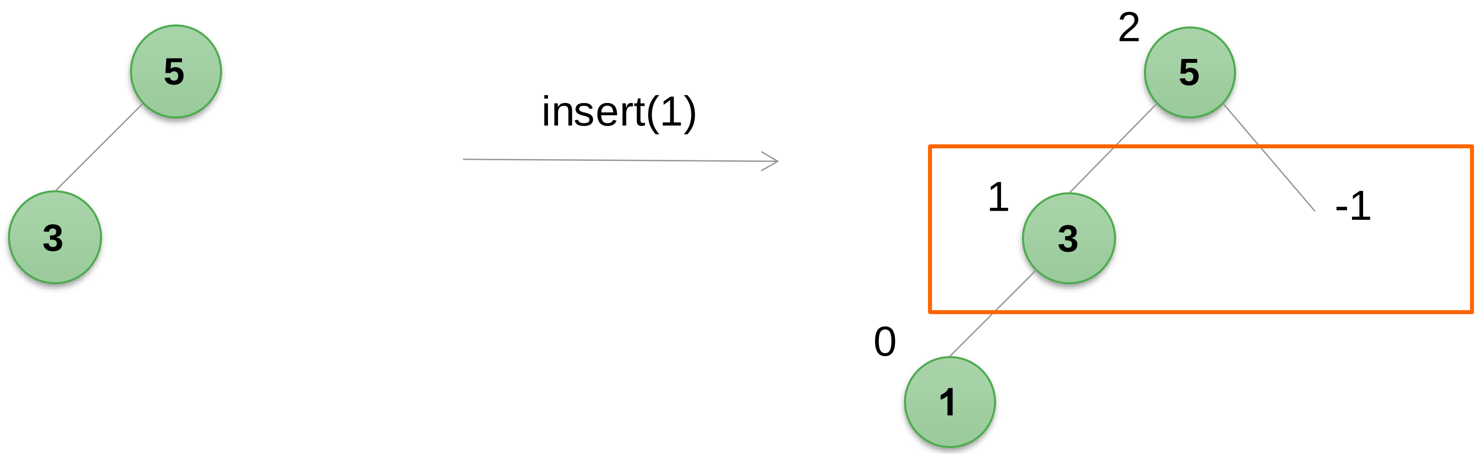

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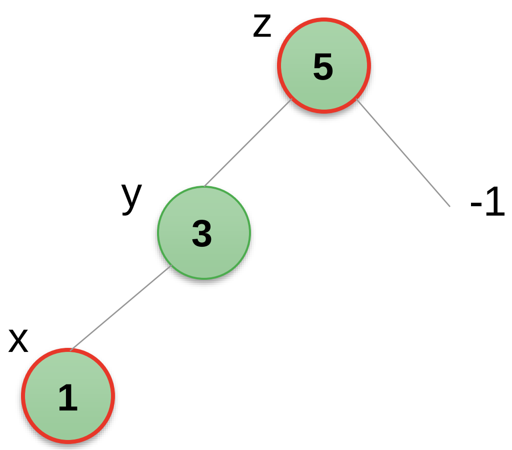

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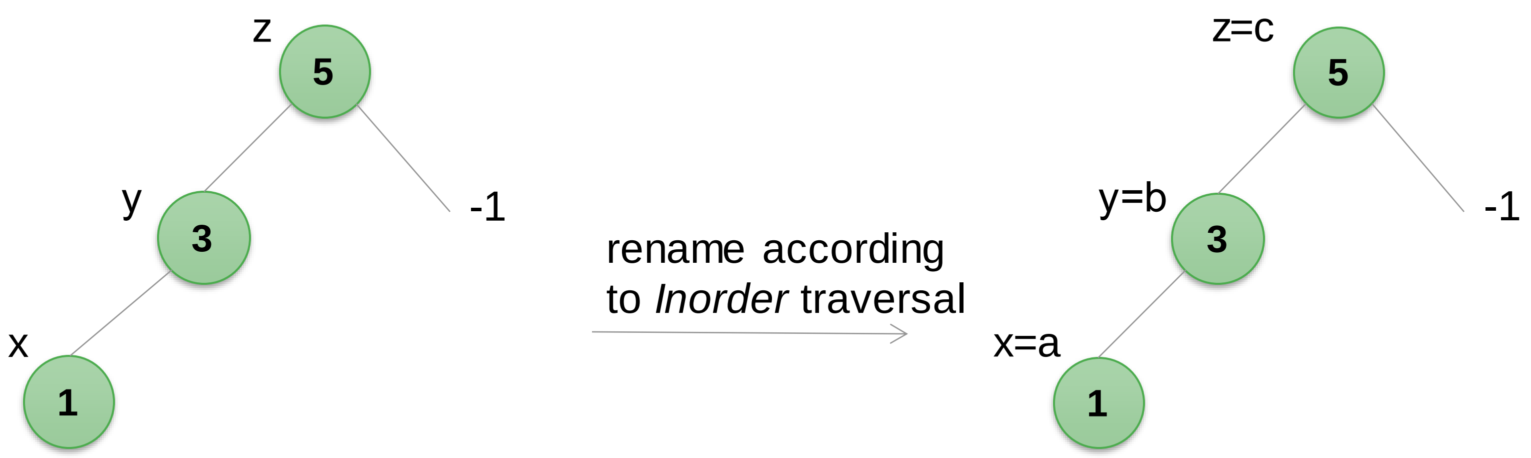

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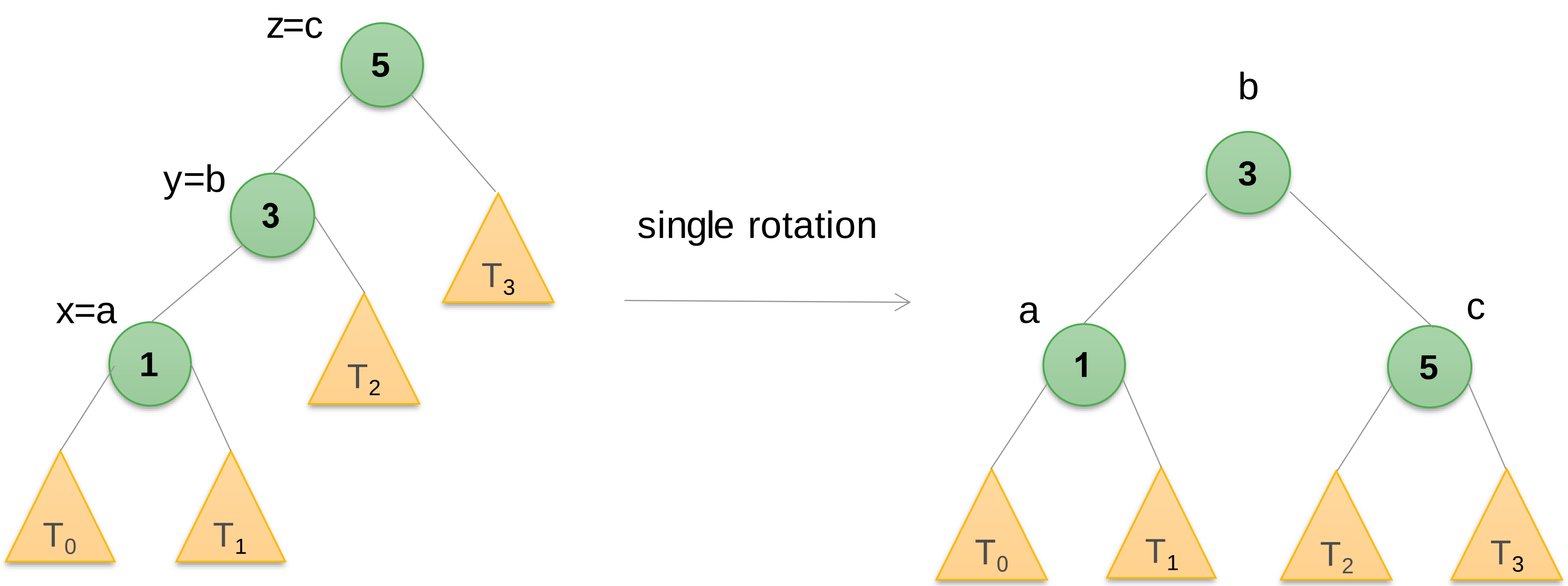

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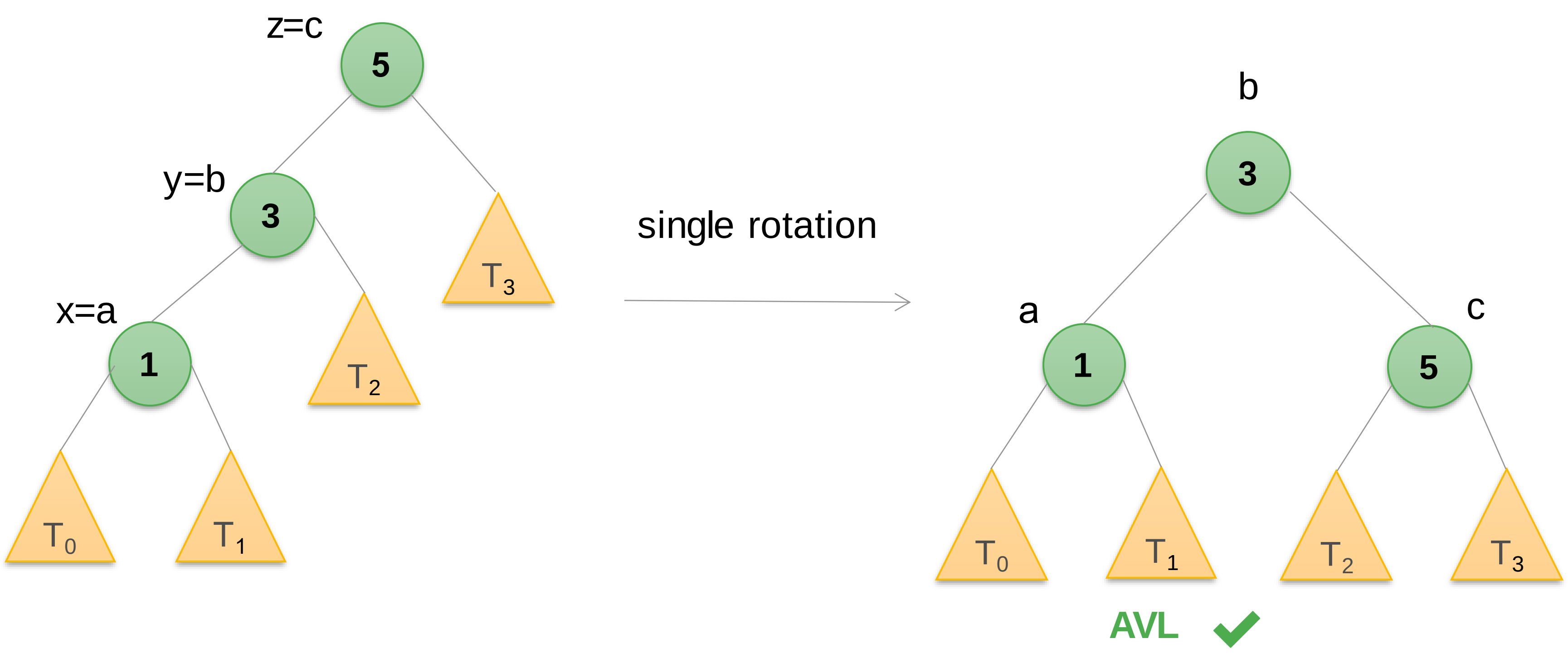
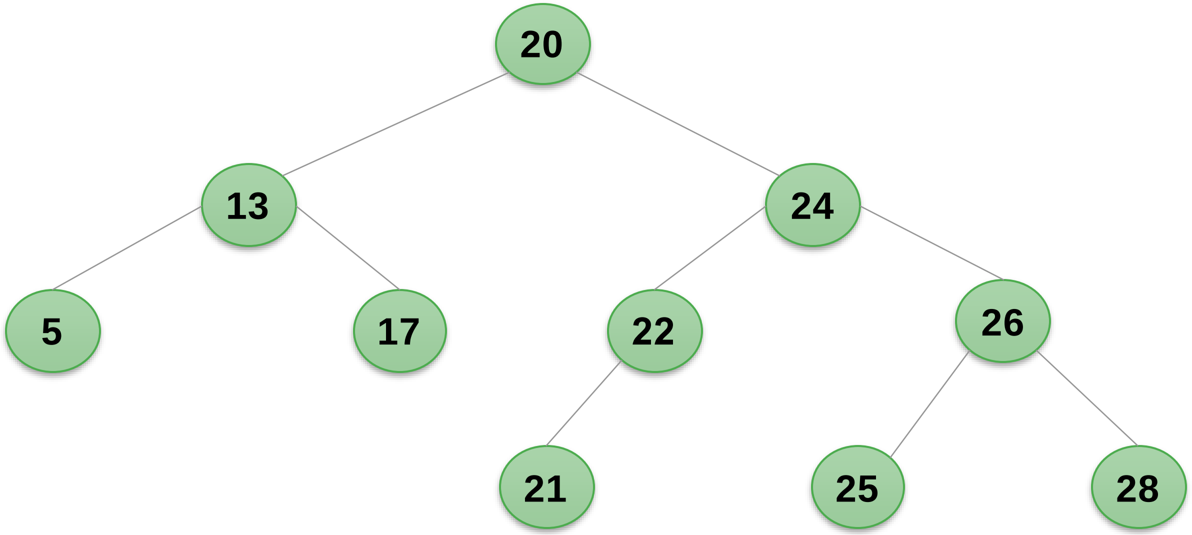


AVL TREE :: SINGLE ROTATION EXAMPLE



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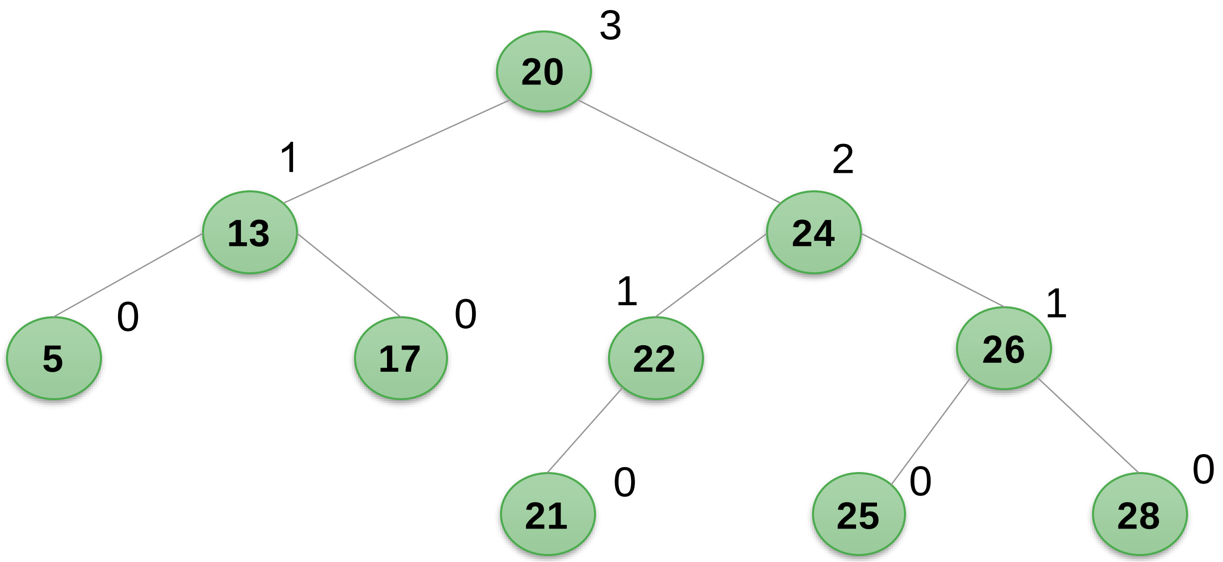

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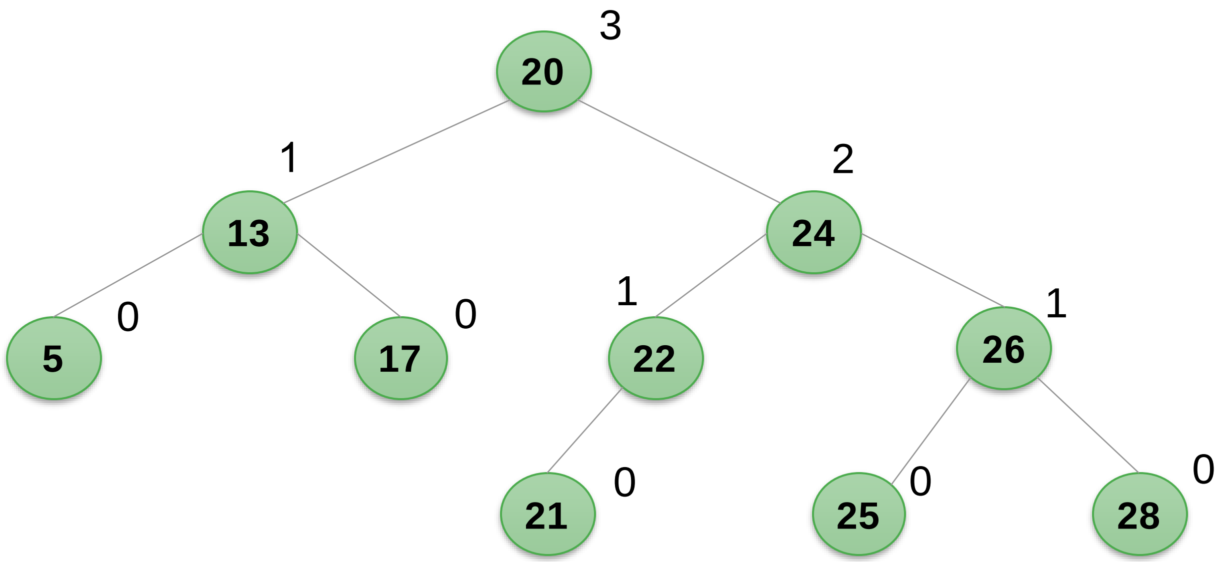

insert ‘30’

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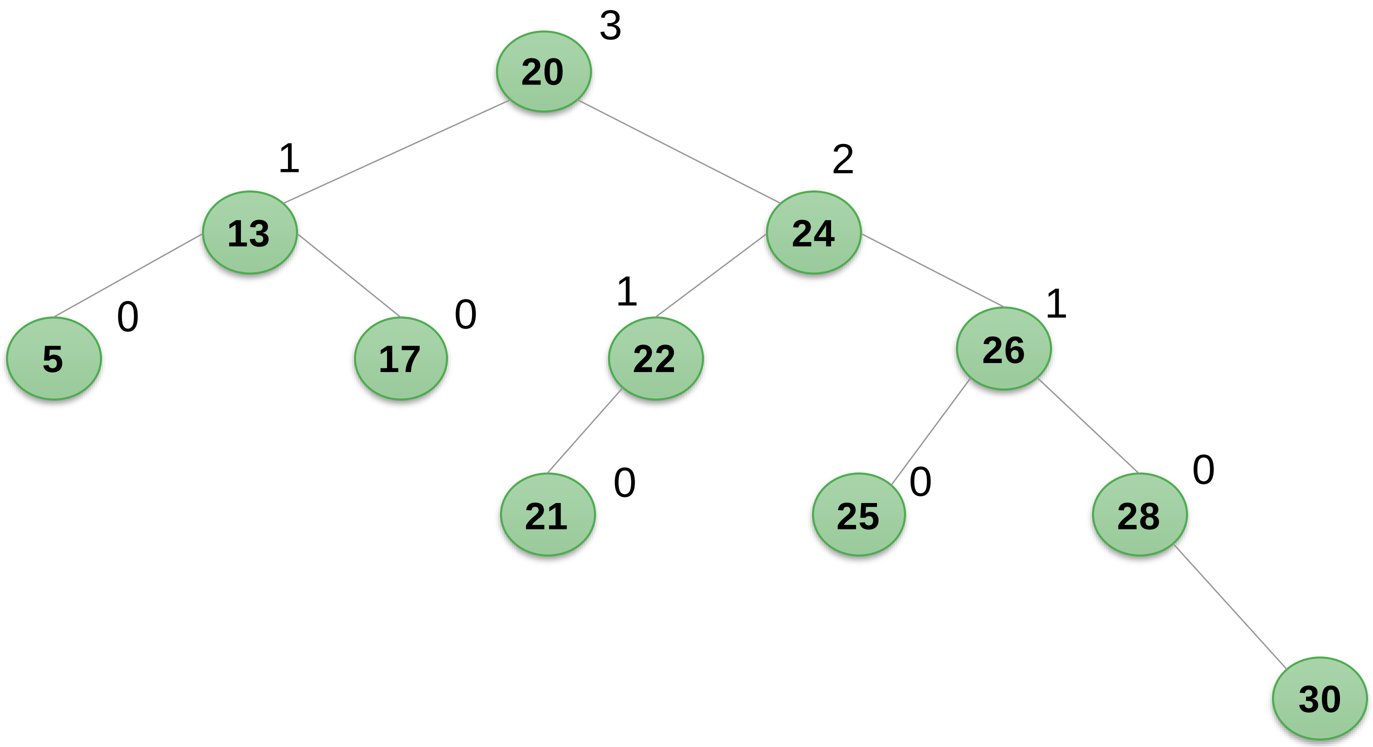

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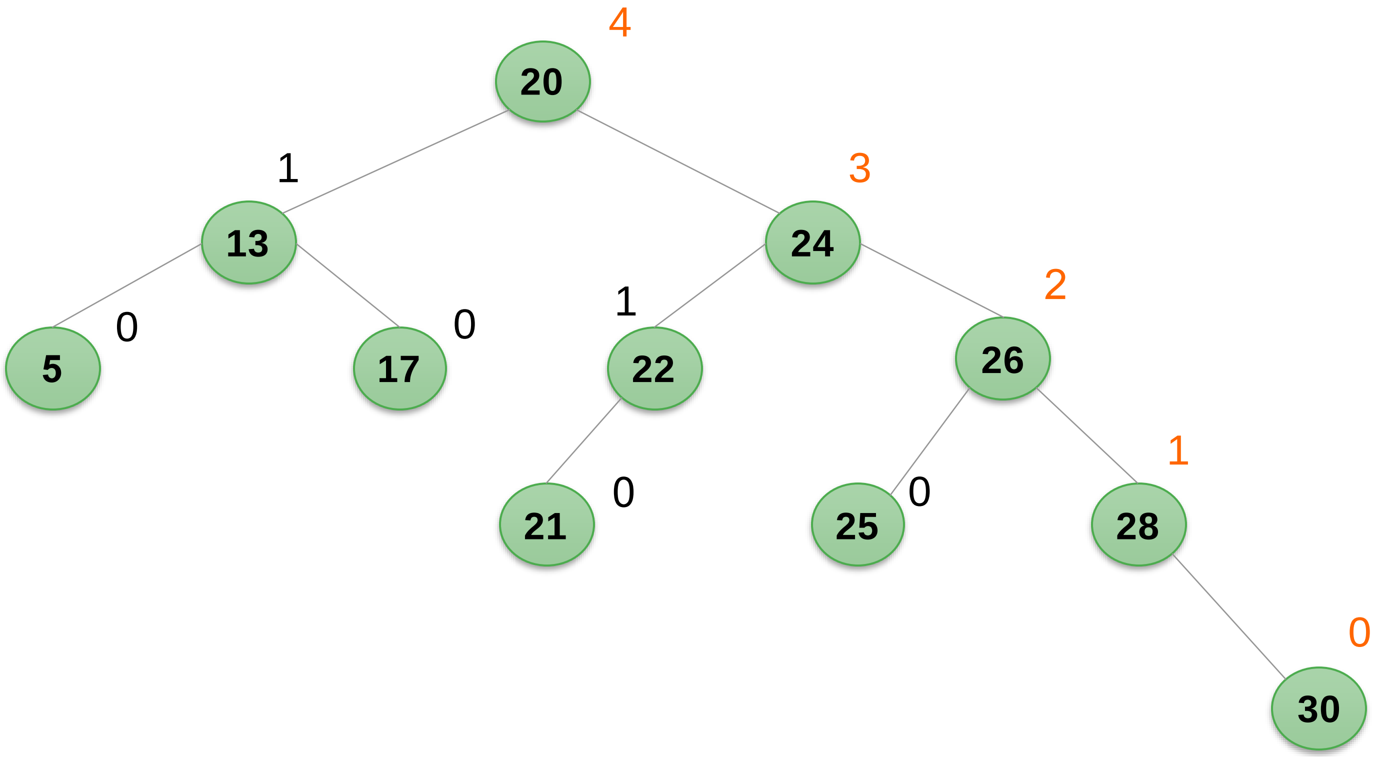

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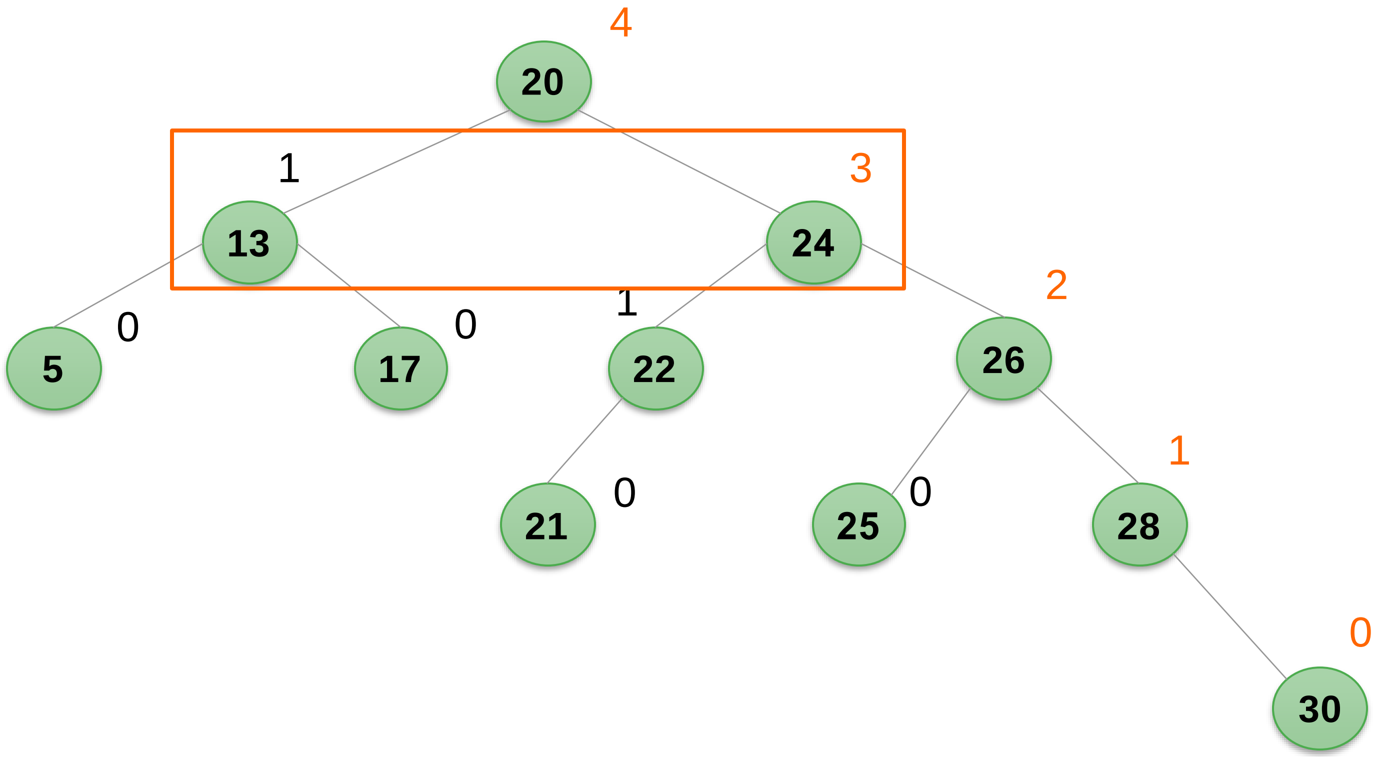

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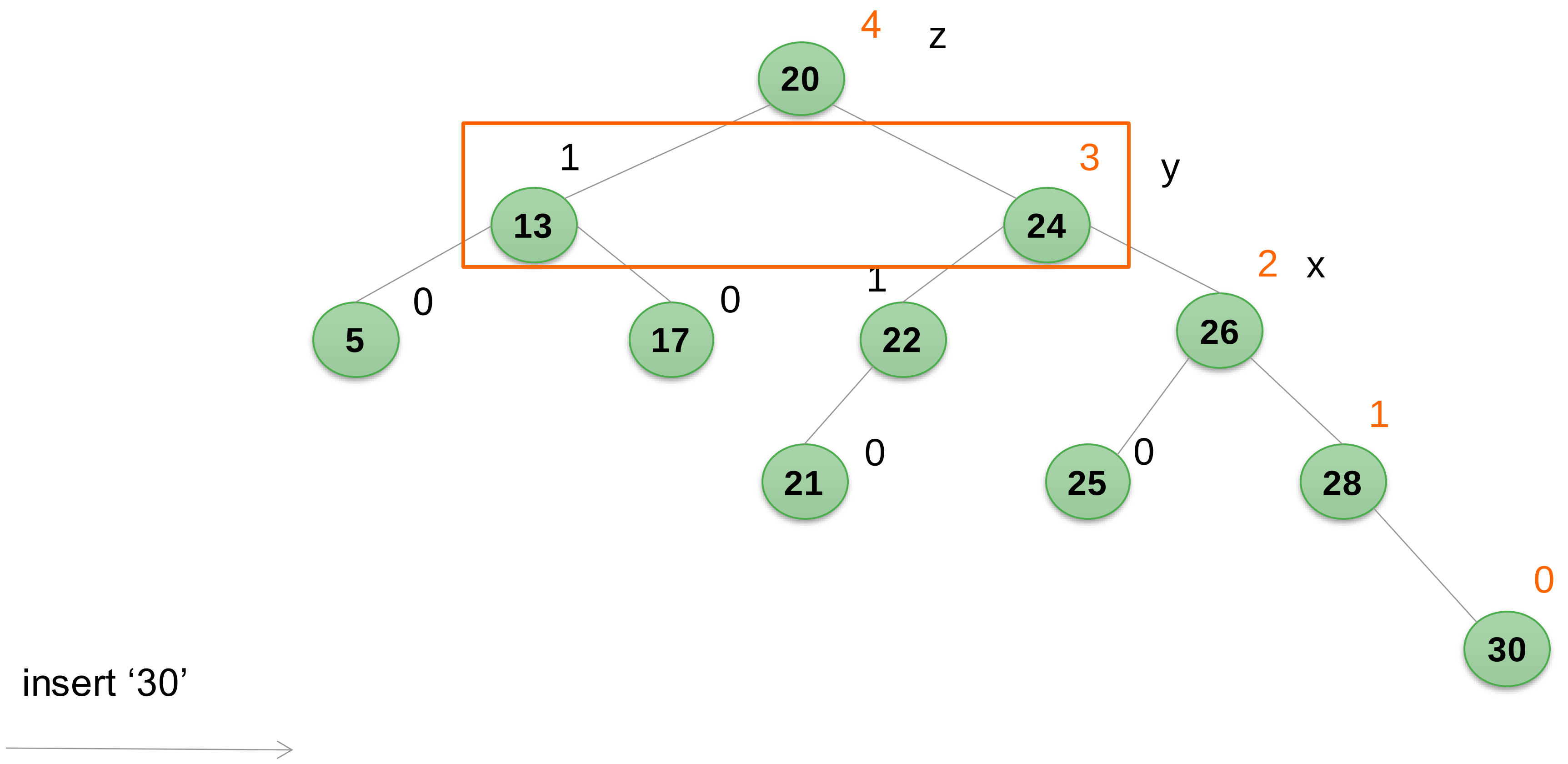

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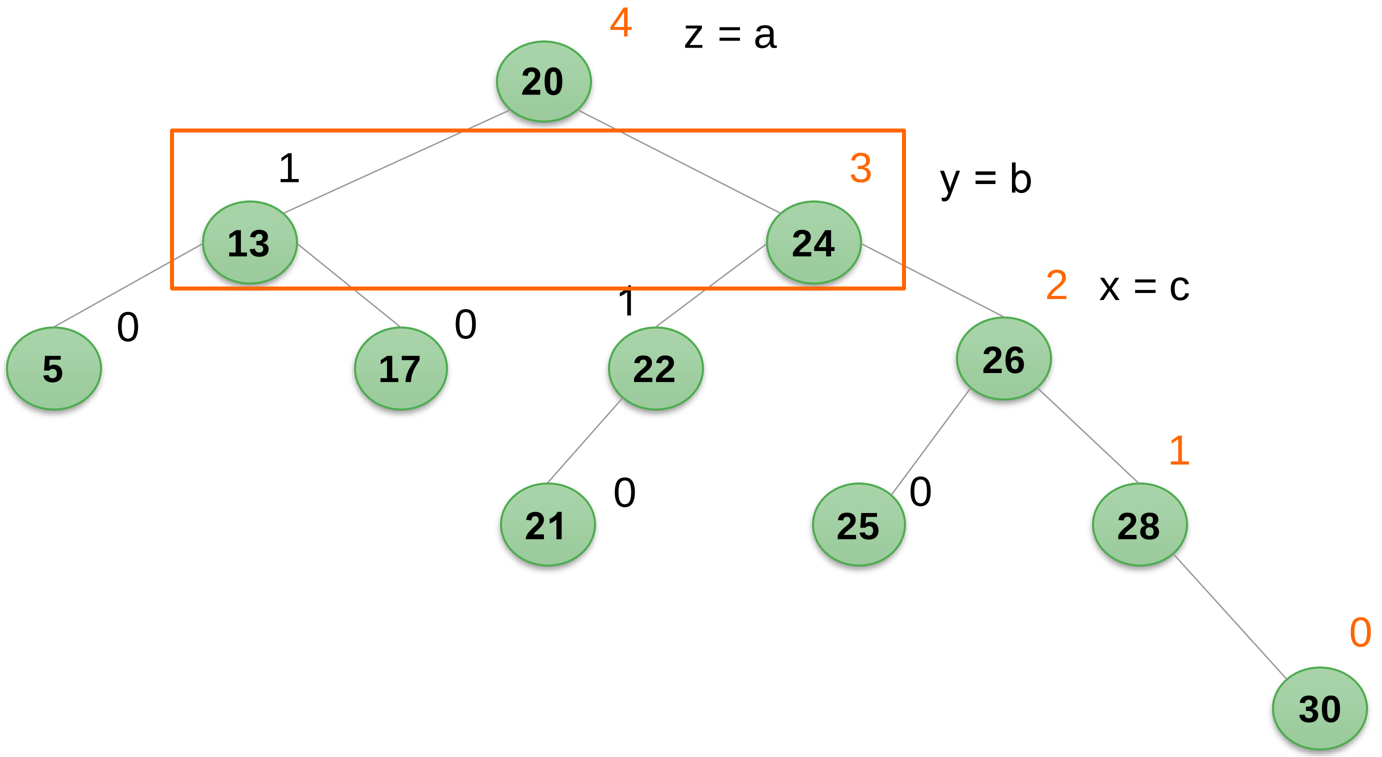

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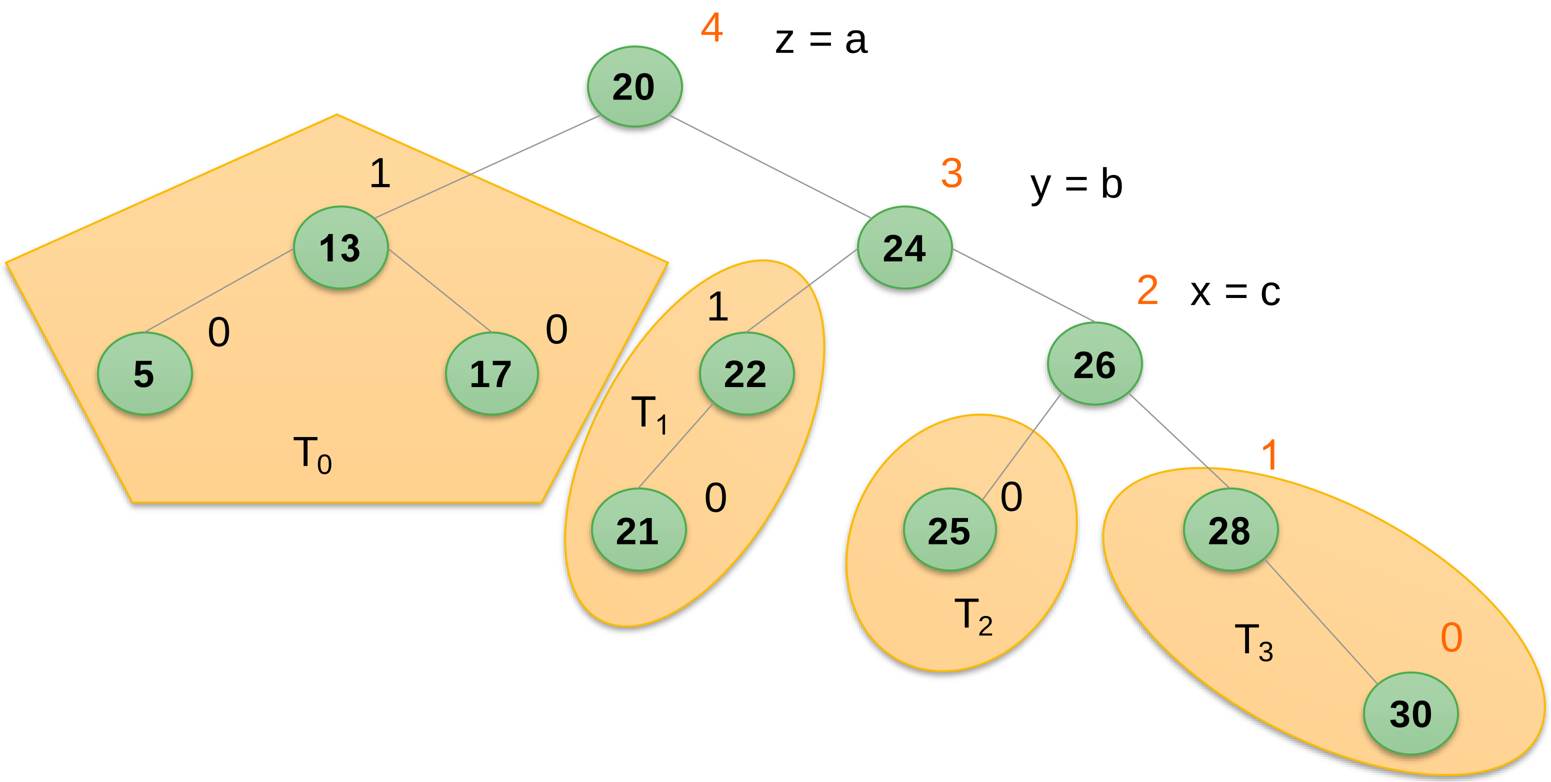

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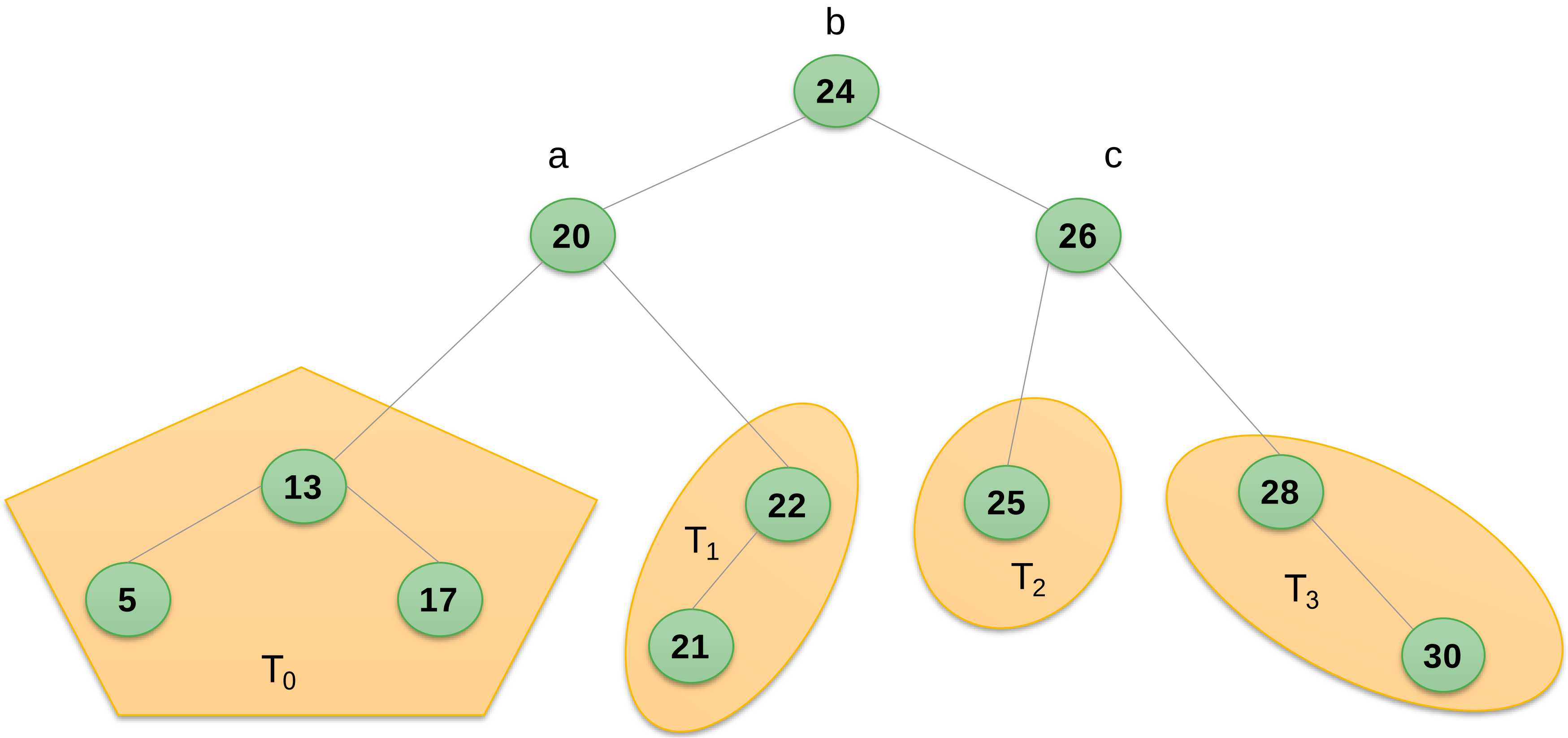

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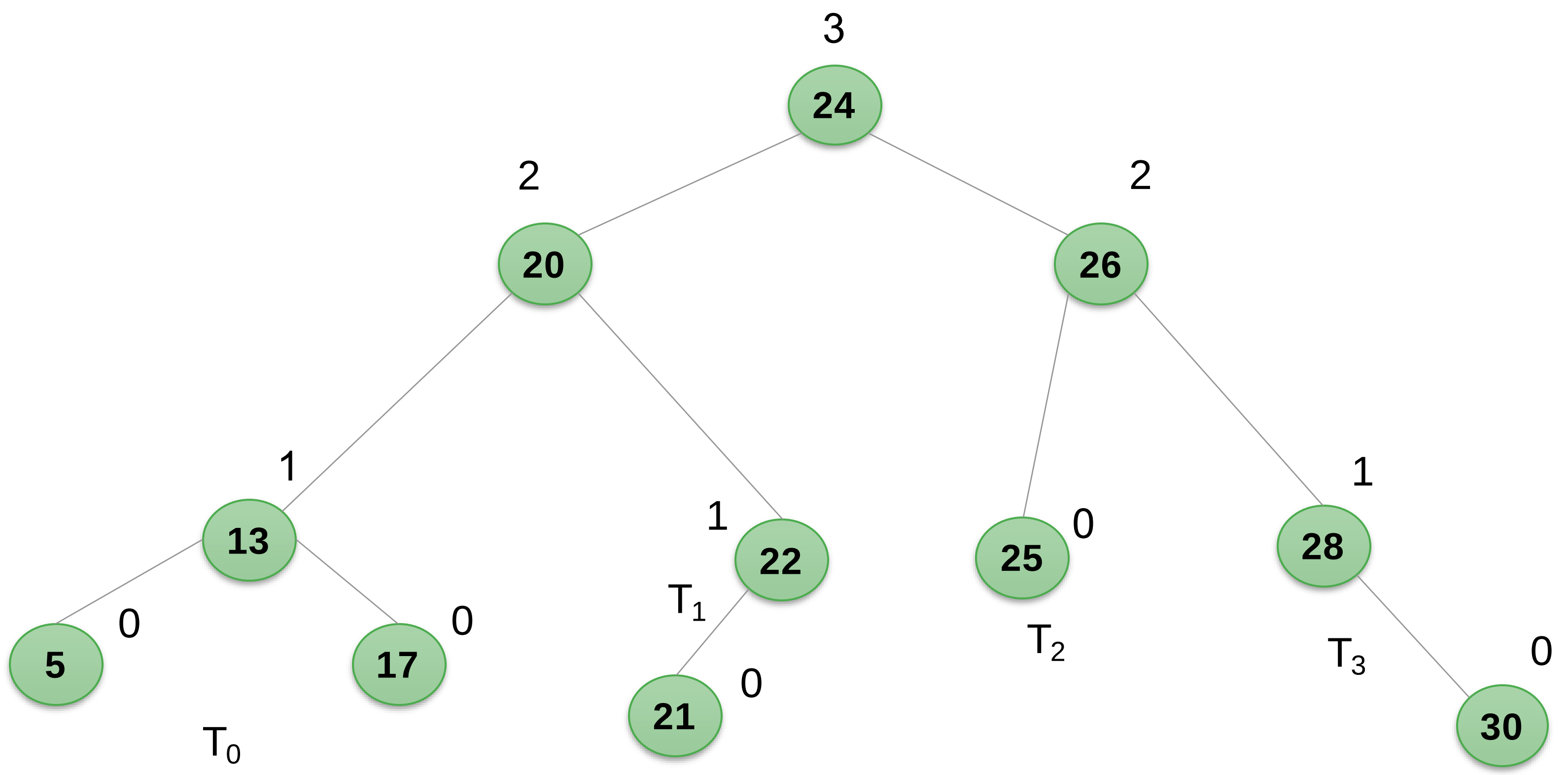

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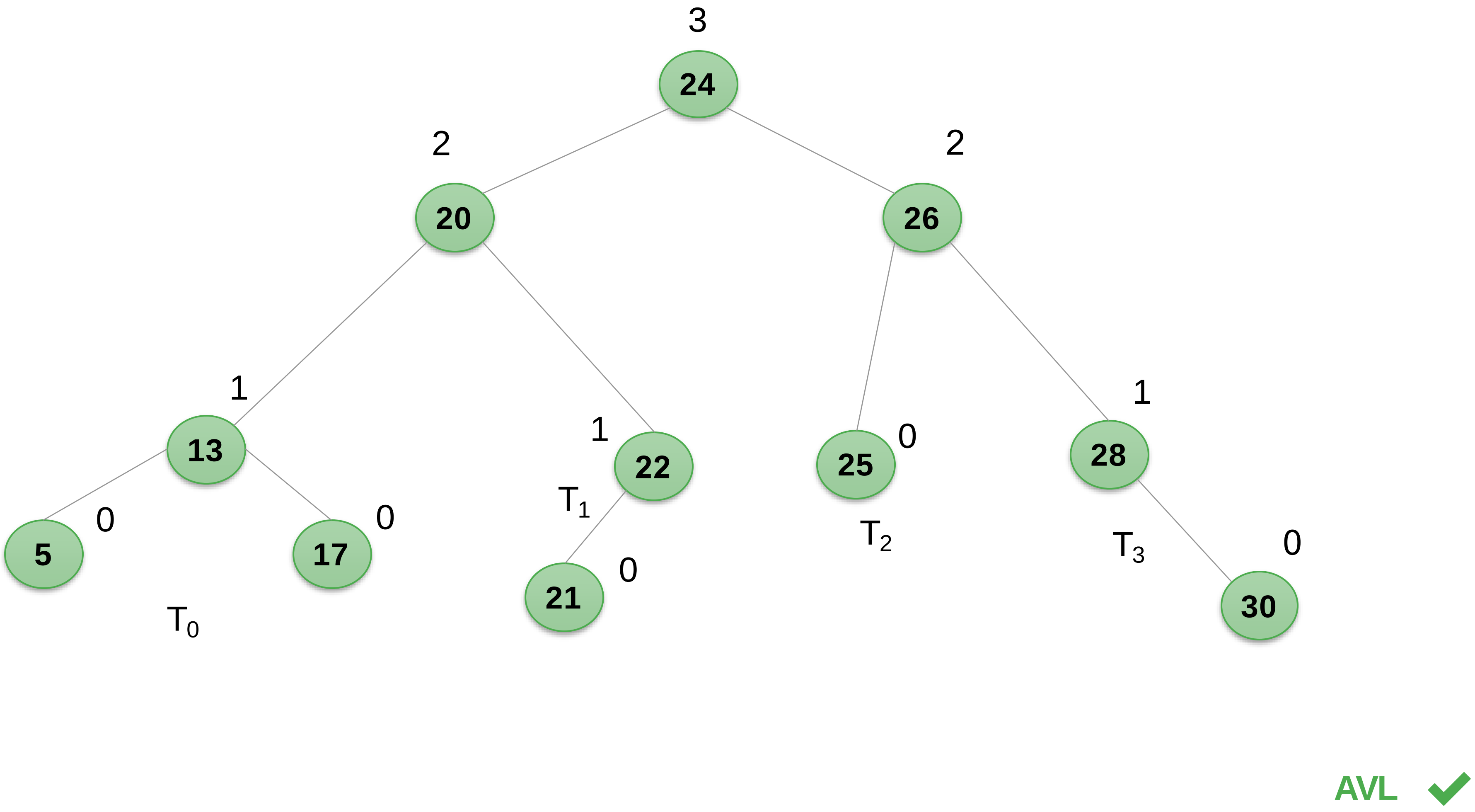


AVL TREES :: CUT & LINK RESTRUCTURING

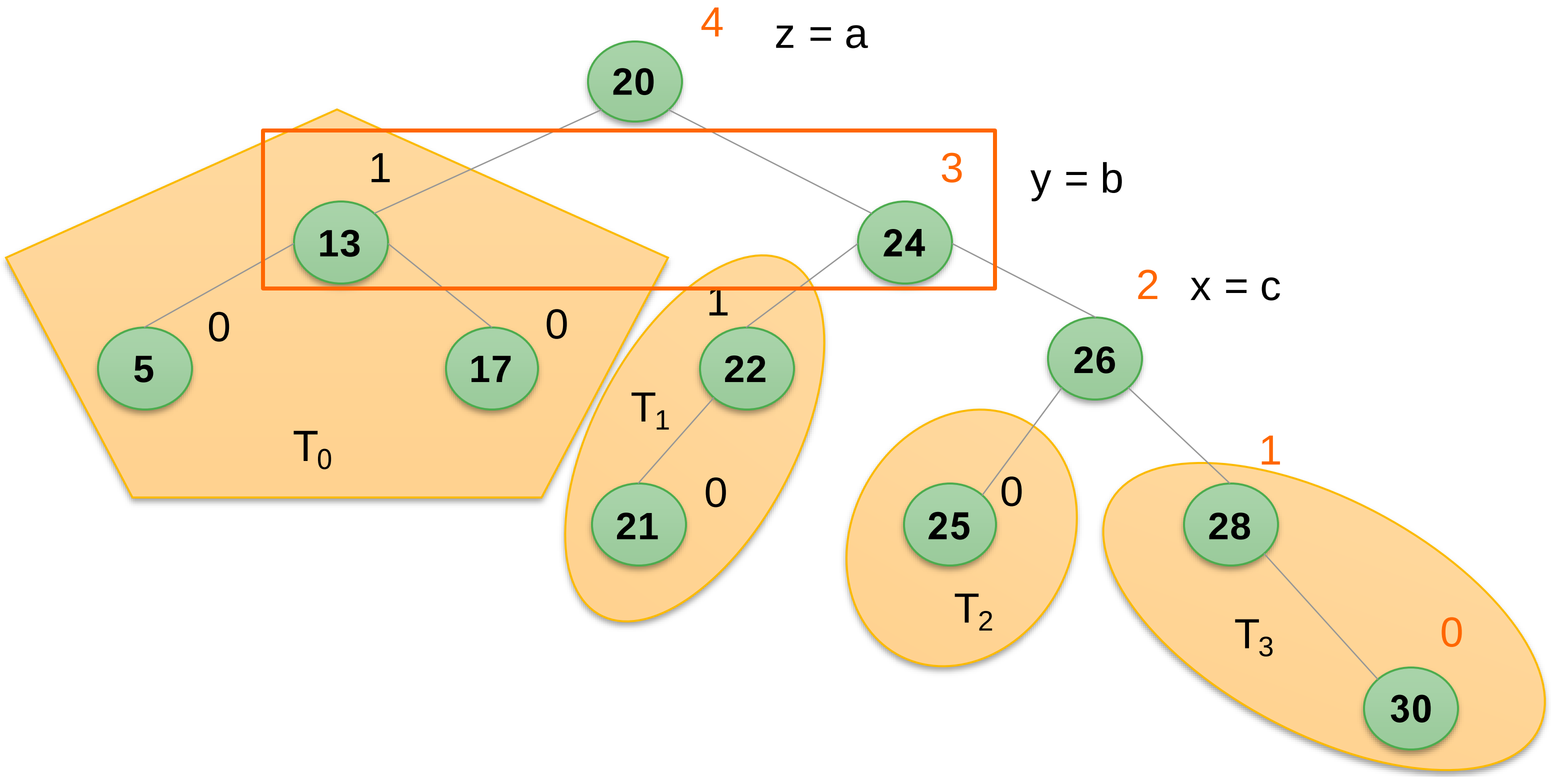
Procedure

1. Number 7 parts according to Inorder traversal

2. Create an array with the indices 0..6, „*cut*“ the 4 subtrees as well as the nodes x, y and z out and put them into the array according to their numbering.

3. (Re)Link the subtrees by setting the element on position 3 as root, those on position 1 and 5 as left and right child of 3, and finally 0, 2, 4 and 6 as children of 1 and 5.

AVL TREE :: SINGLE ROTATION EXAMPLE



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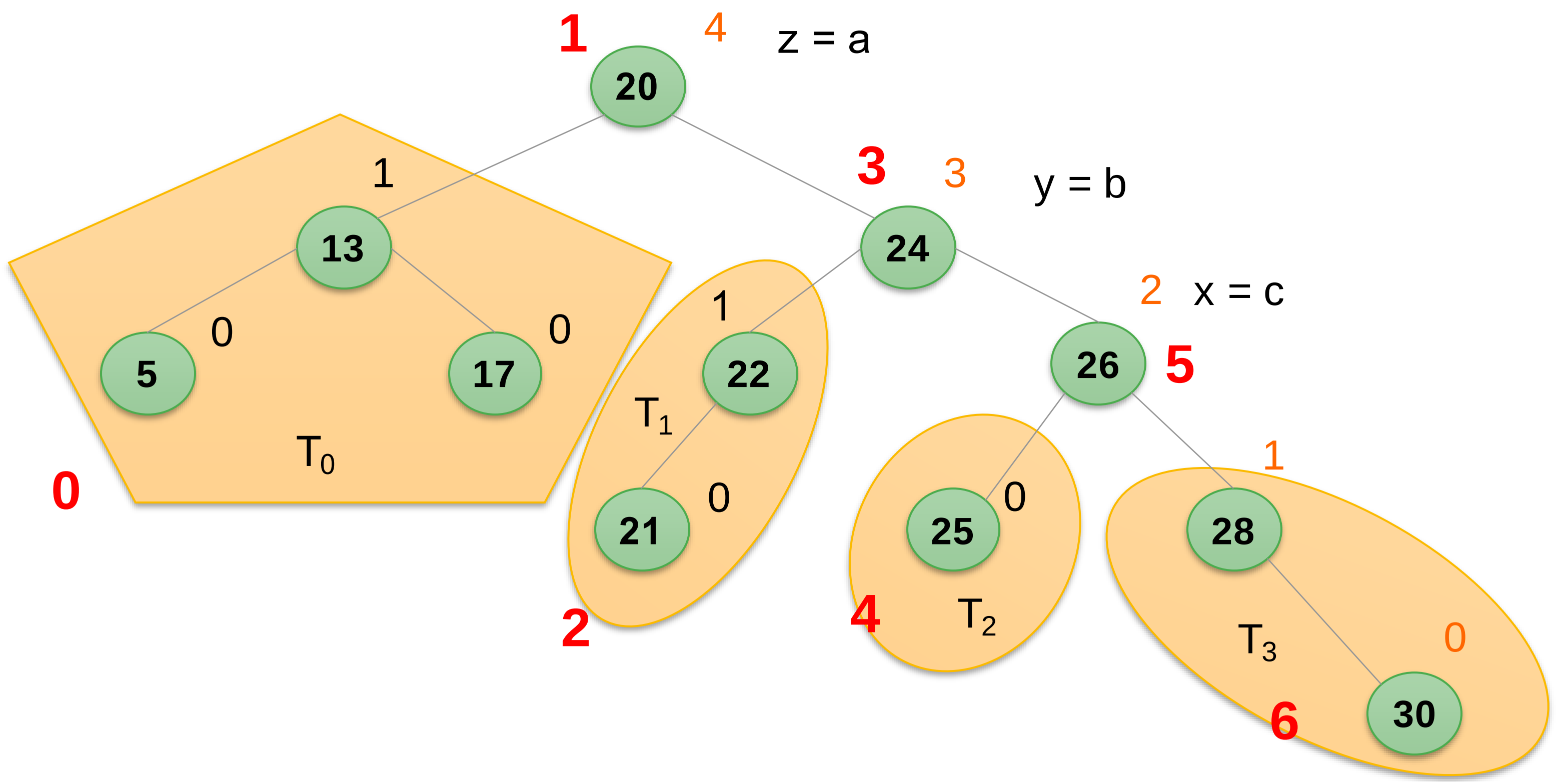

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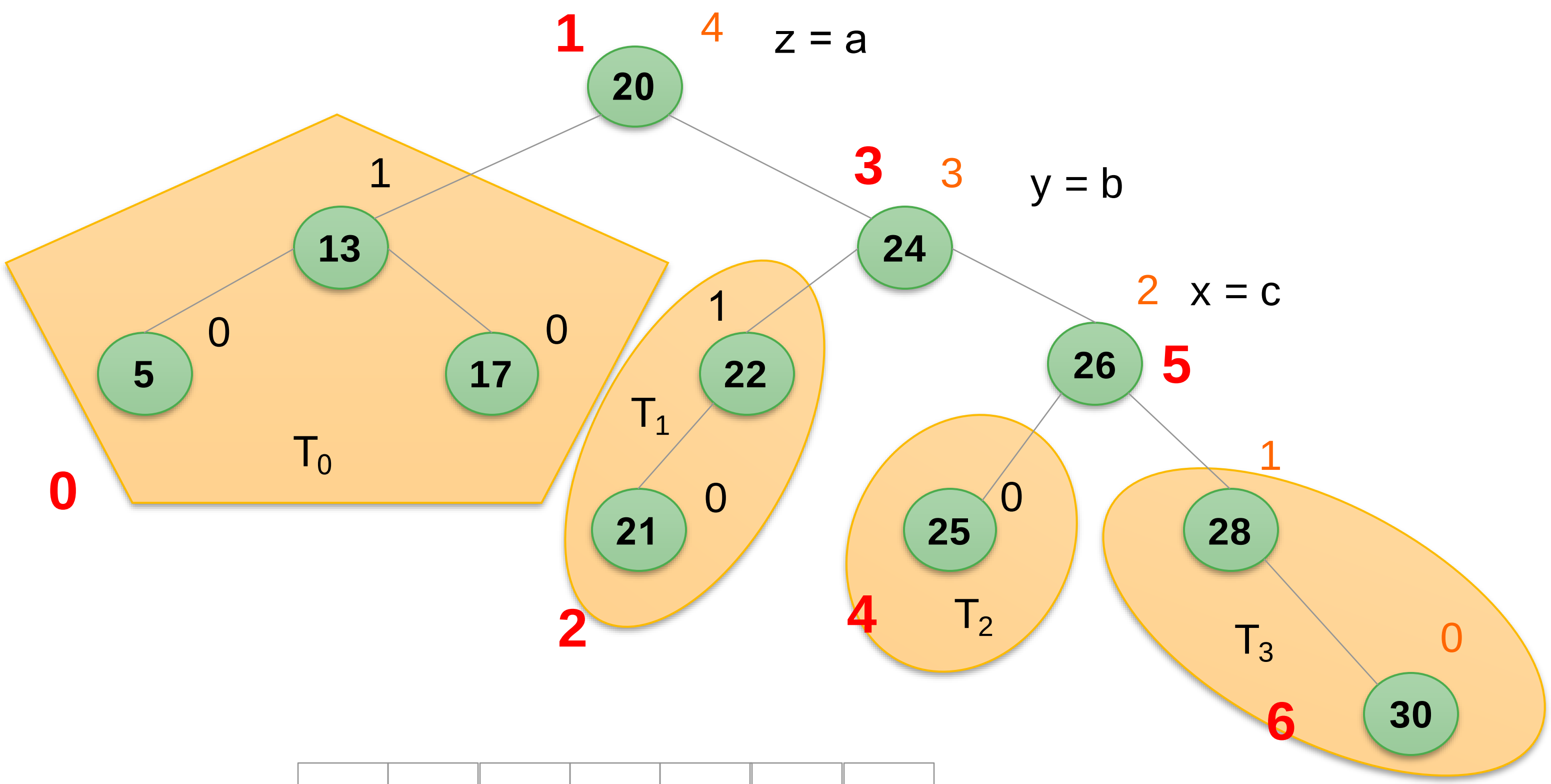
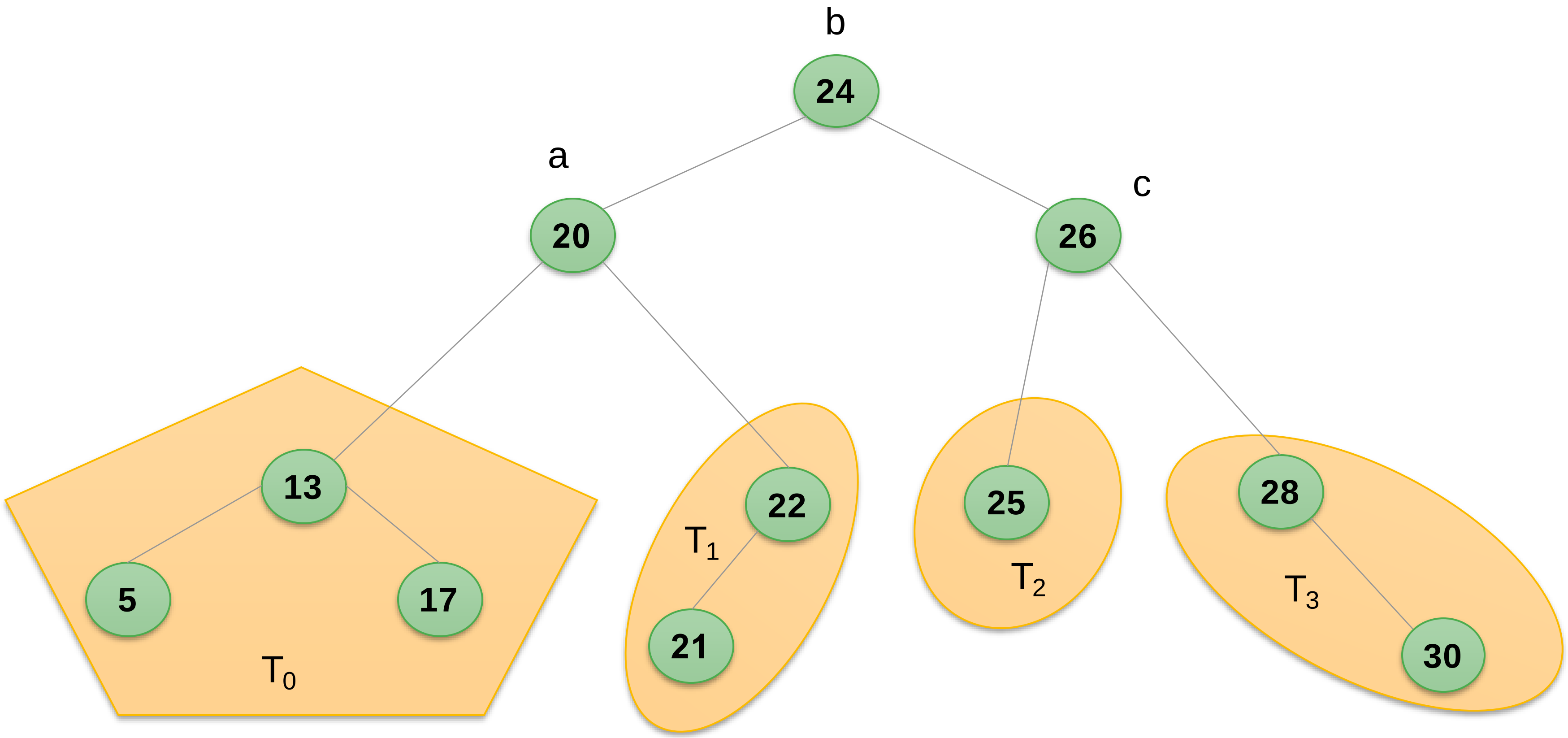


0	1	2	3	4	5	6
T_0	z=a	T_1	y=b	T_2	x=c	T_3

AVL TREE :: SINGLE ROTATION EXAMPLE



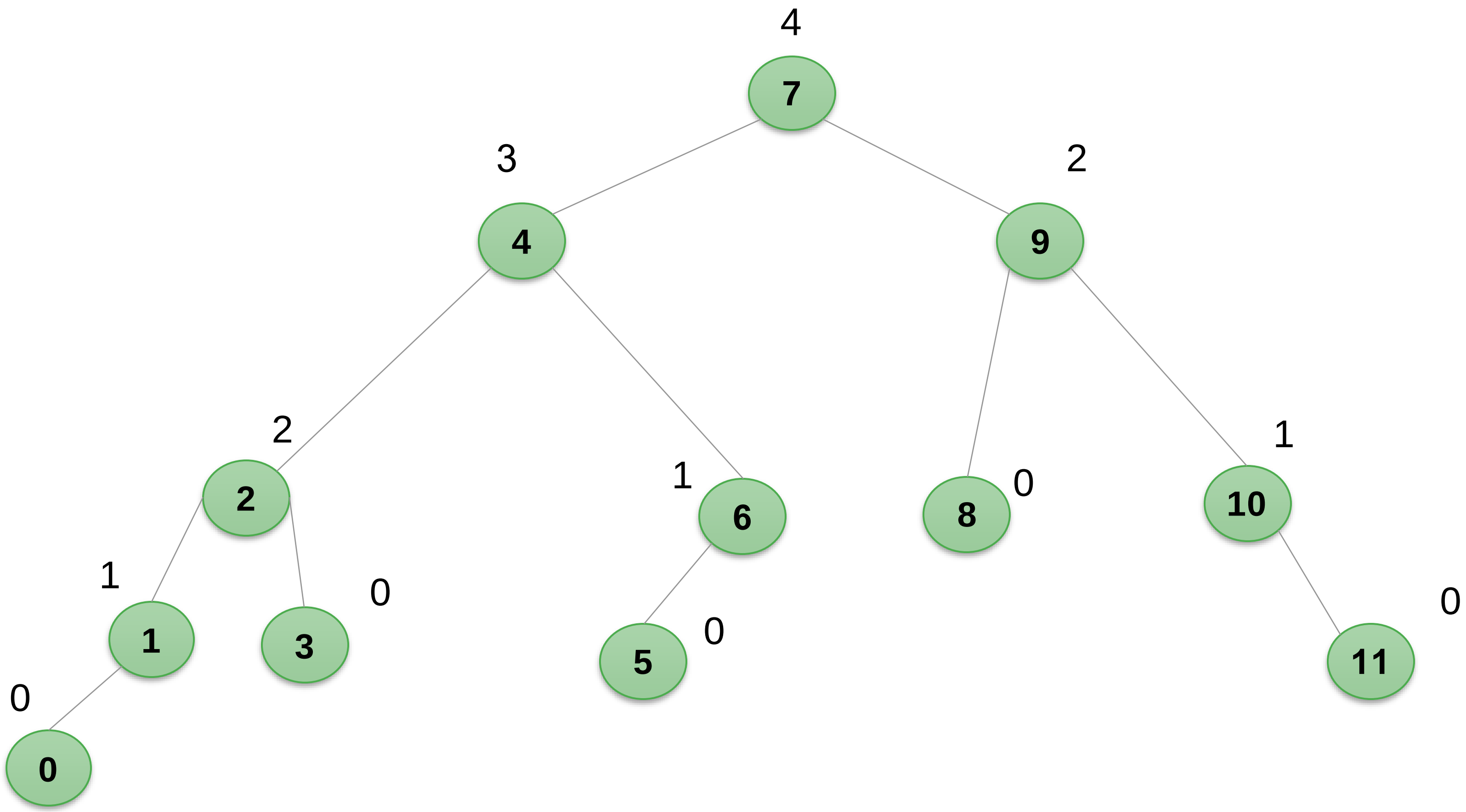
AVL TREES :: REMOVE

- Remove as in binary search tree
- Check the balance starting from the parent node of the removed **Inorder** successor to the root.
- **Restructure** if necessary

Procedure

1. Search for the 1. unbalanced node z
2. Put y on child of z with greatest height
3. Put x on child of y with greatest height

AVL TREES :: REMOVE



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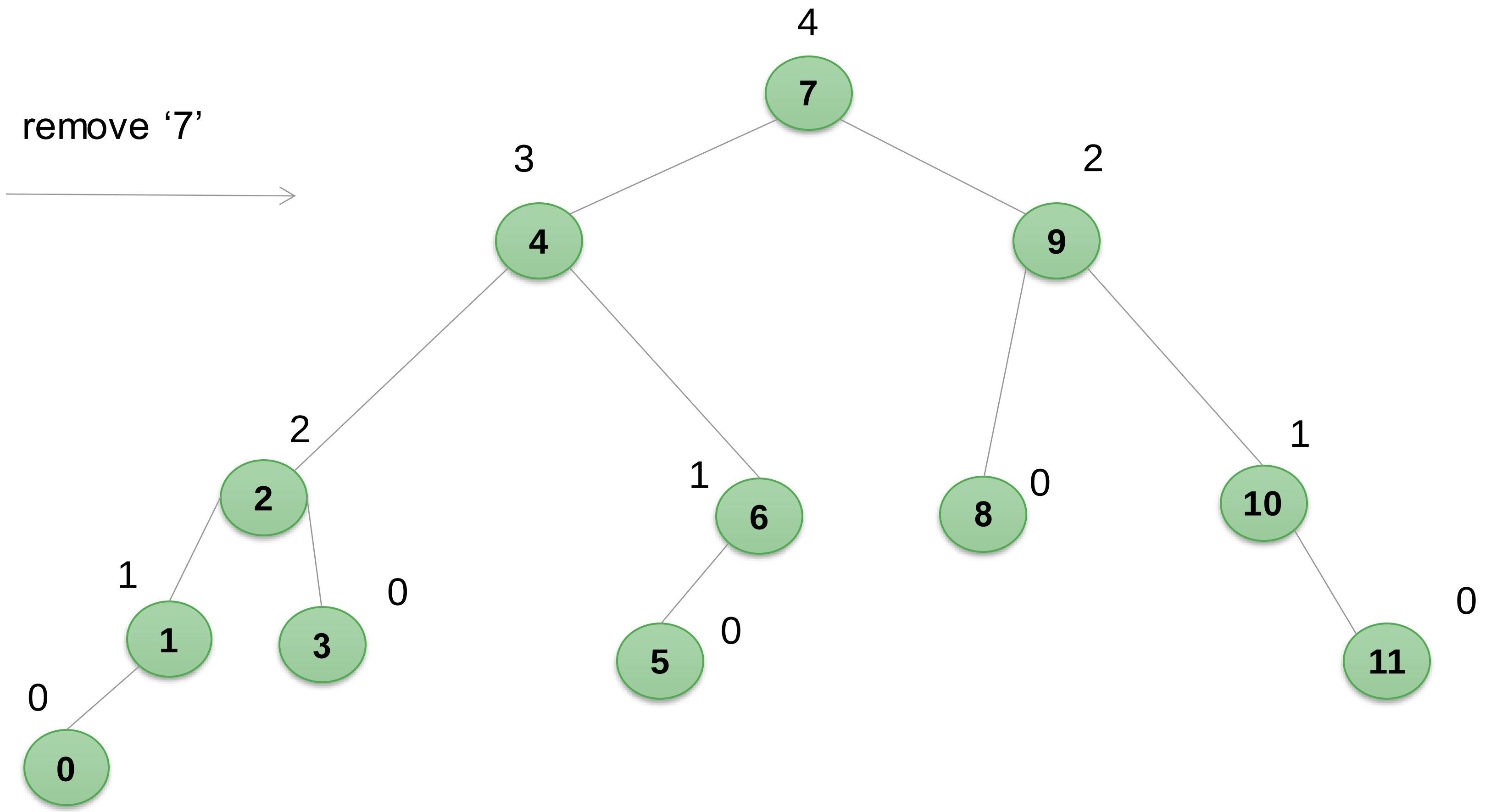
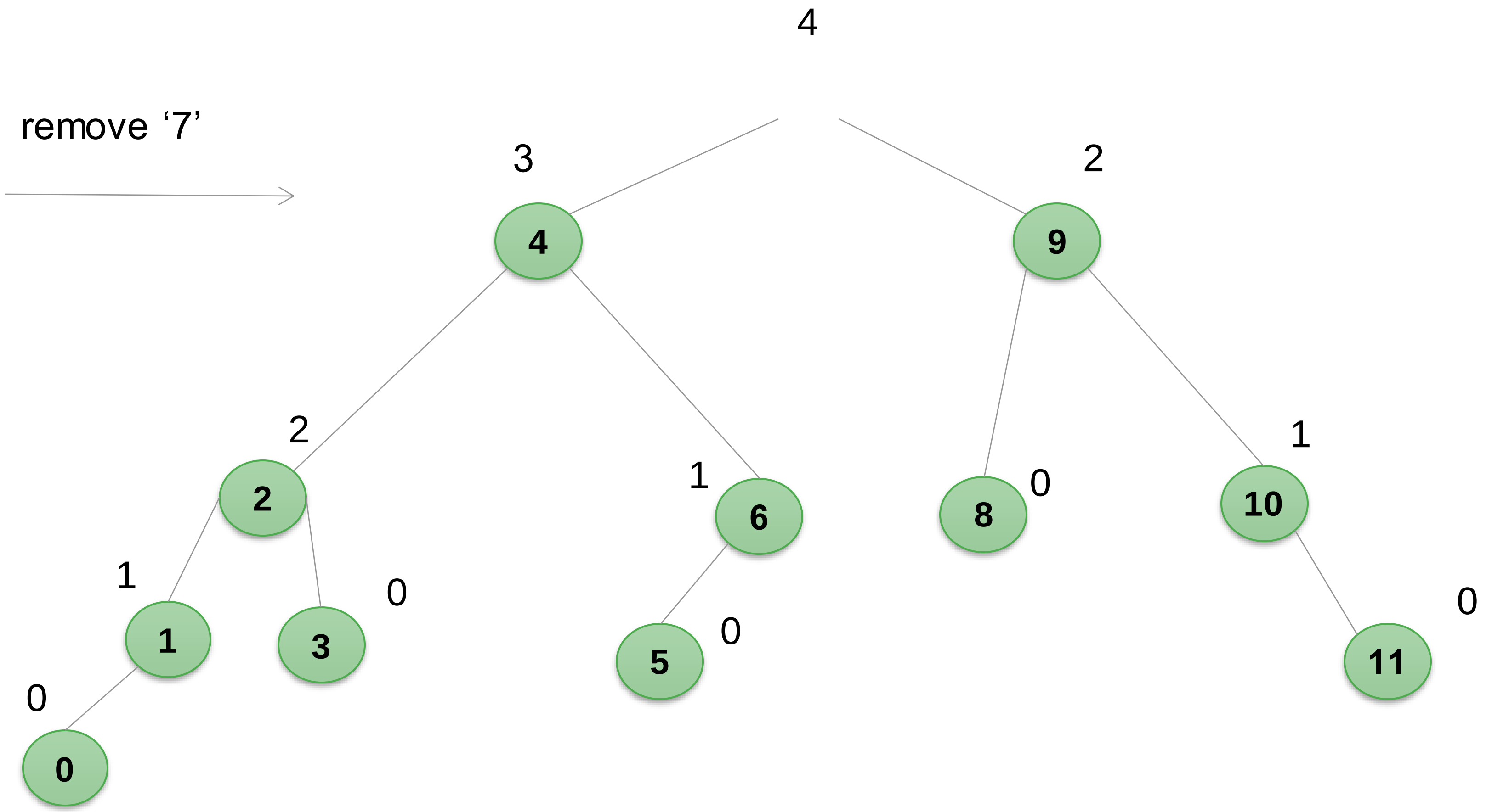


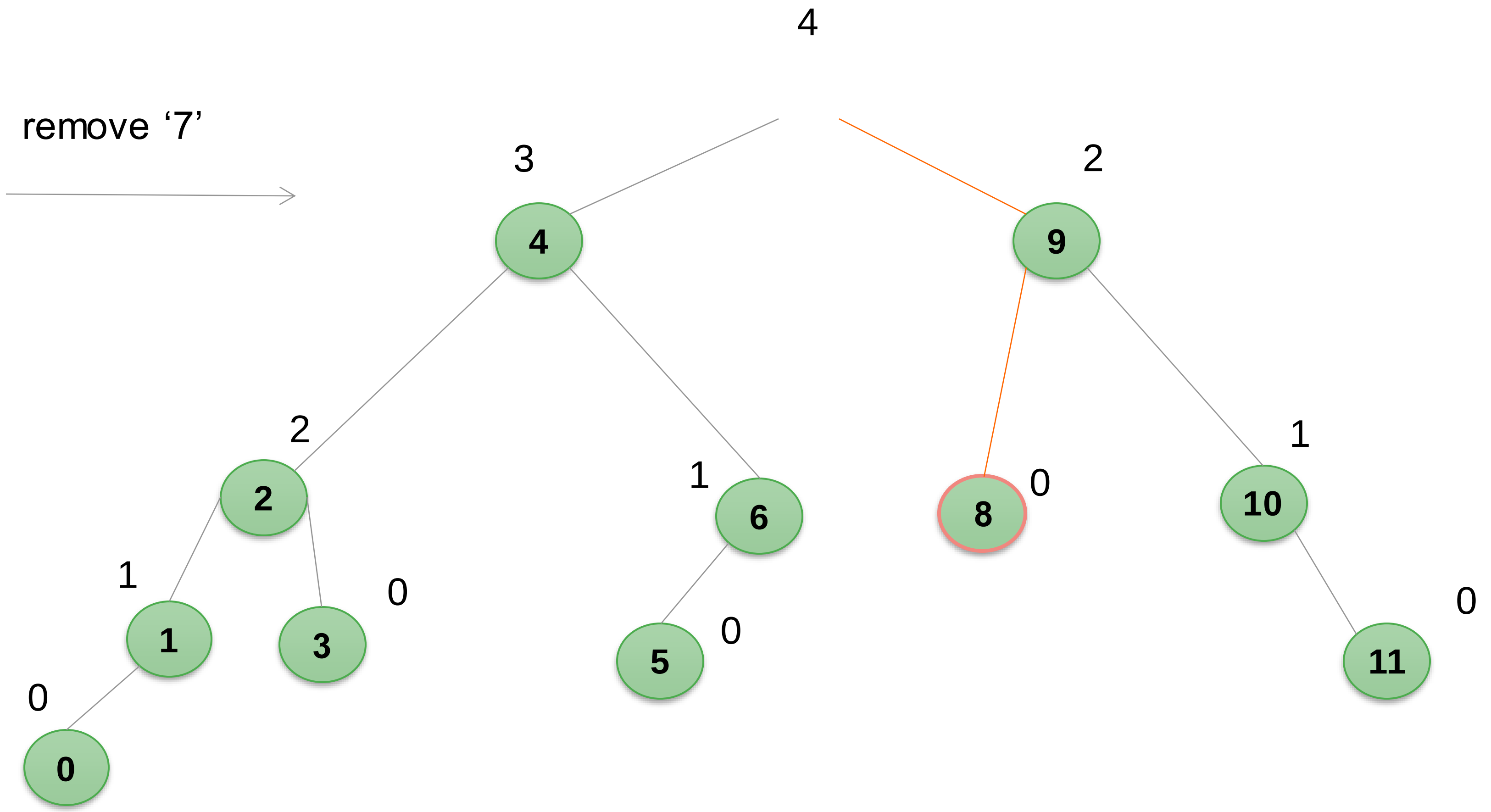
AVL TREES :: REMOVE

remove ‘7’



AVL TREES :: REMOVE

remove '7'



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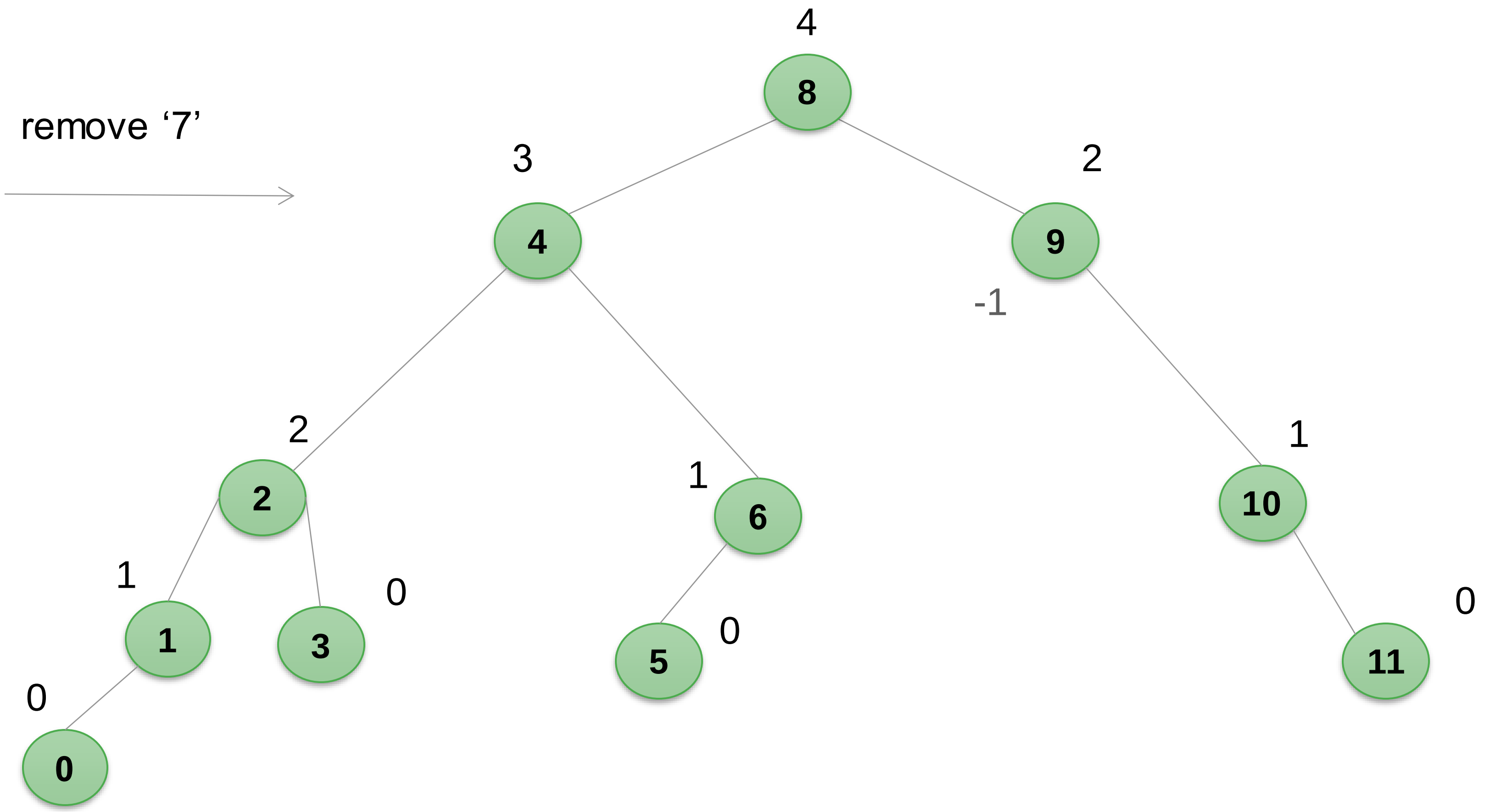

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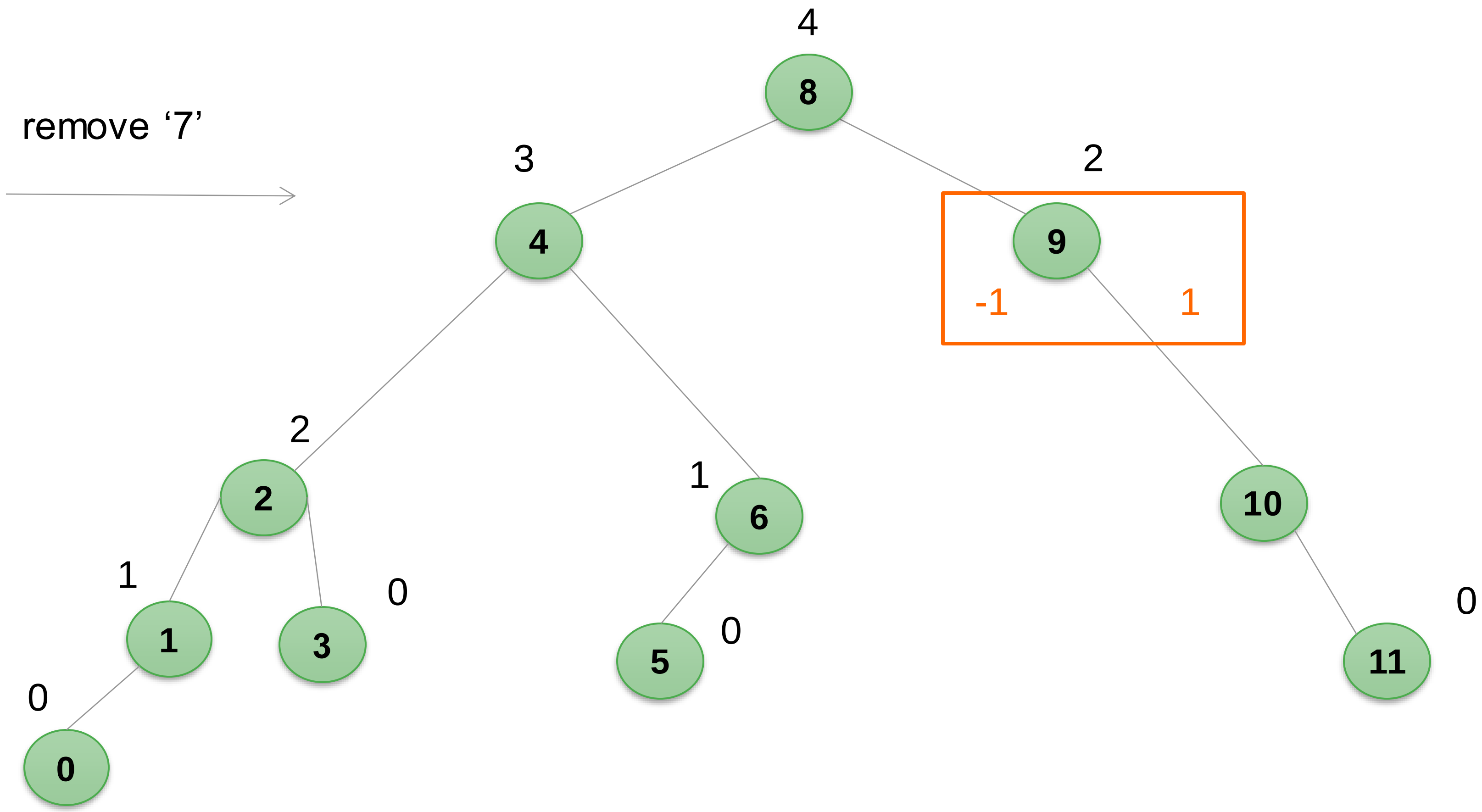

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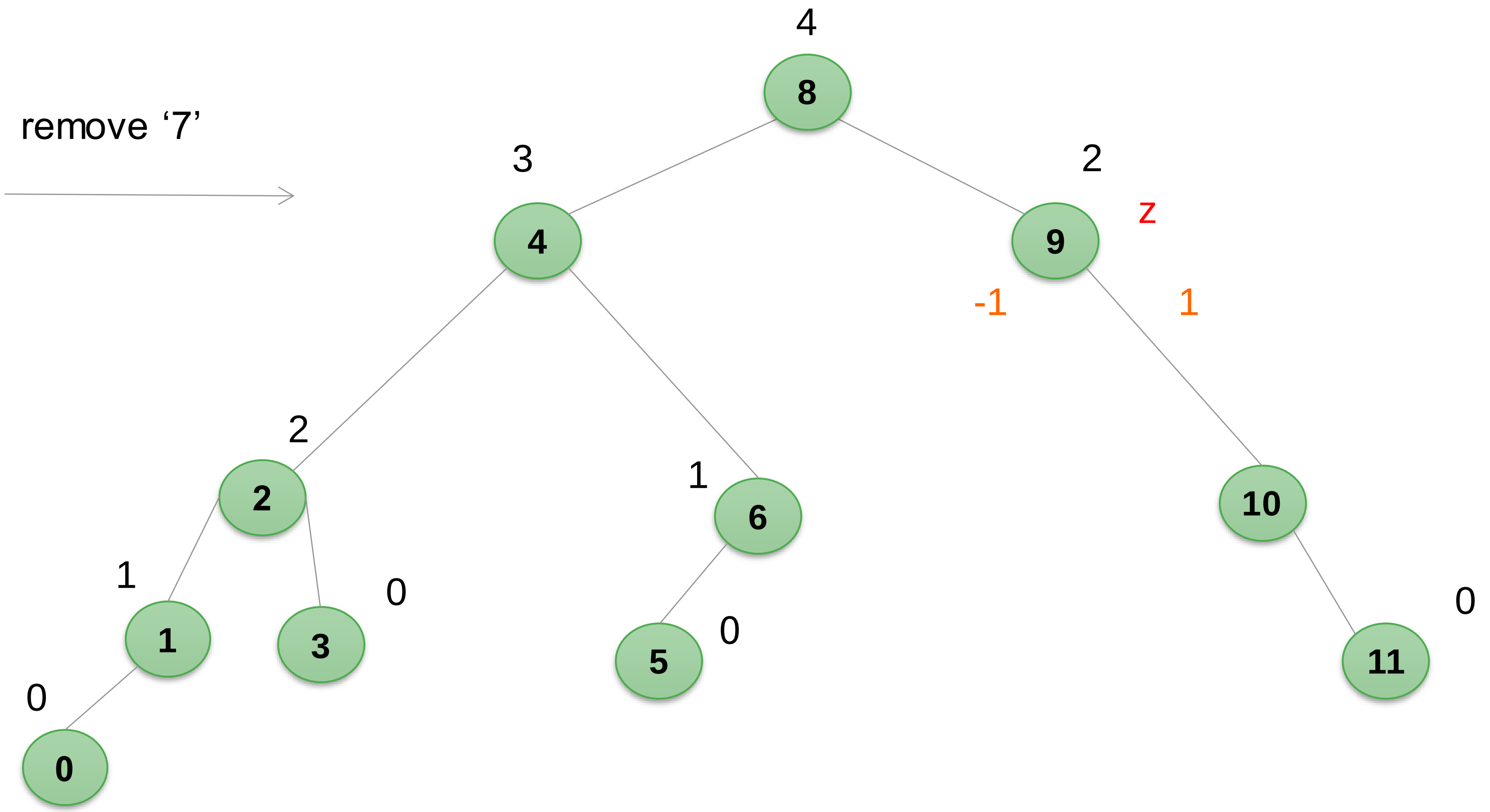

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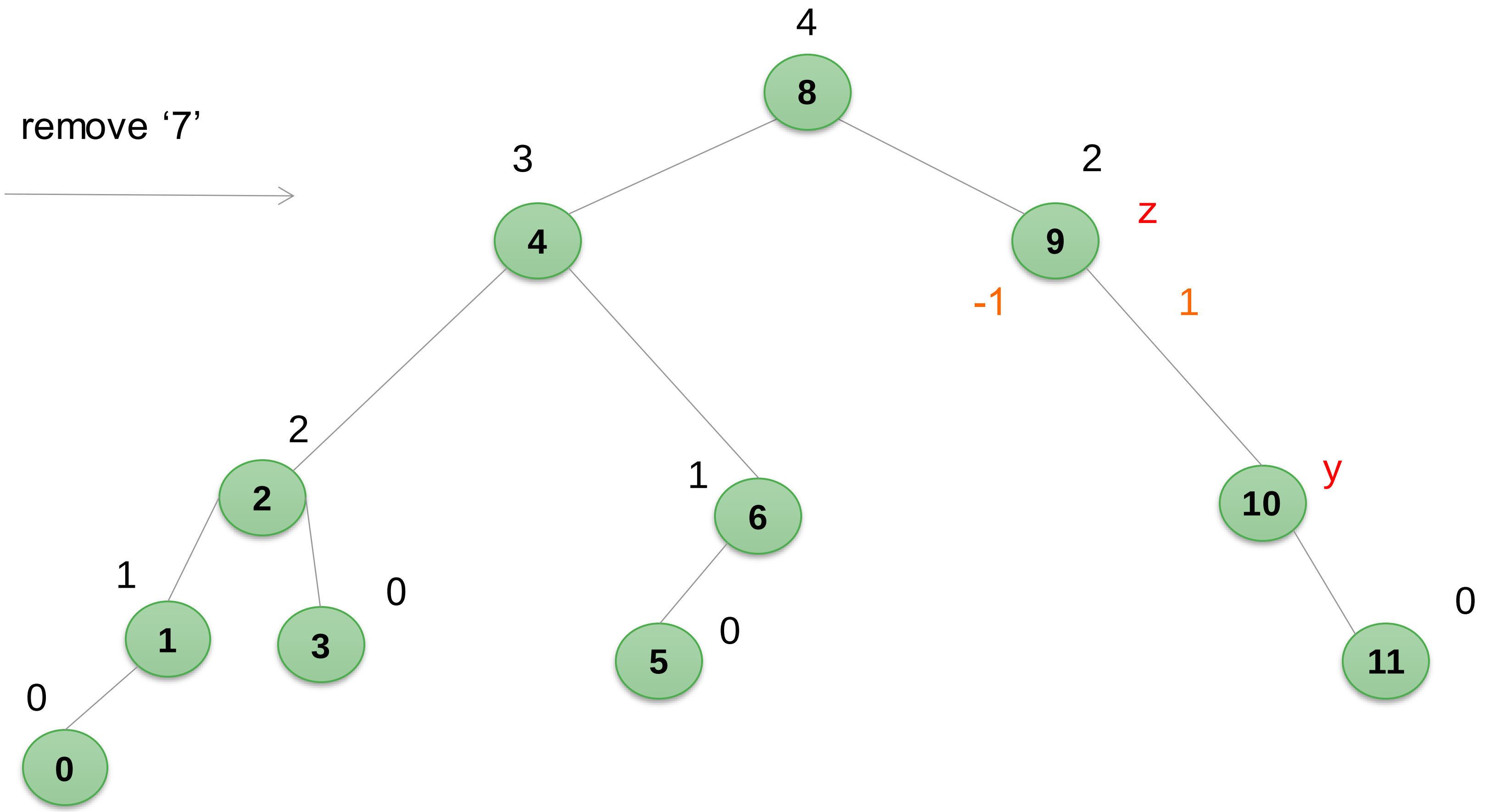

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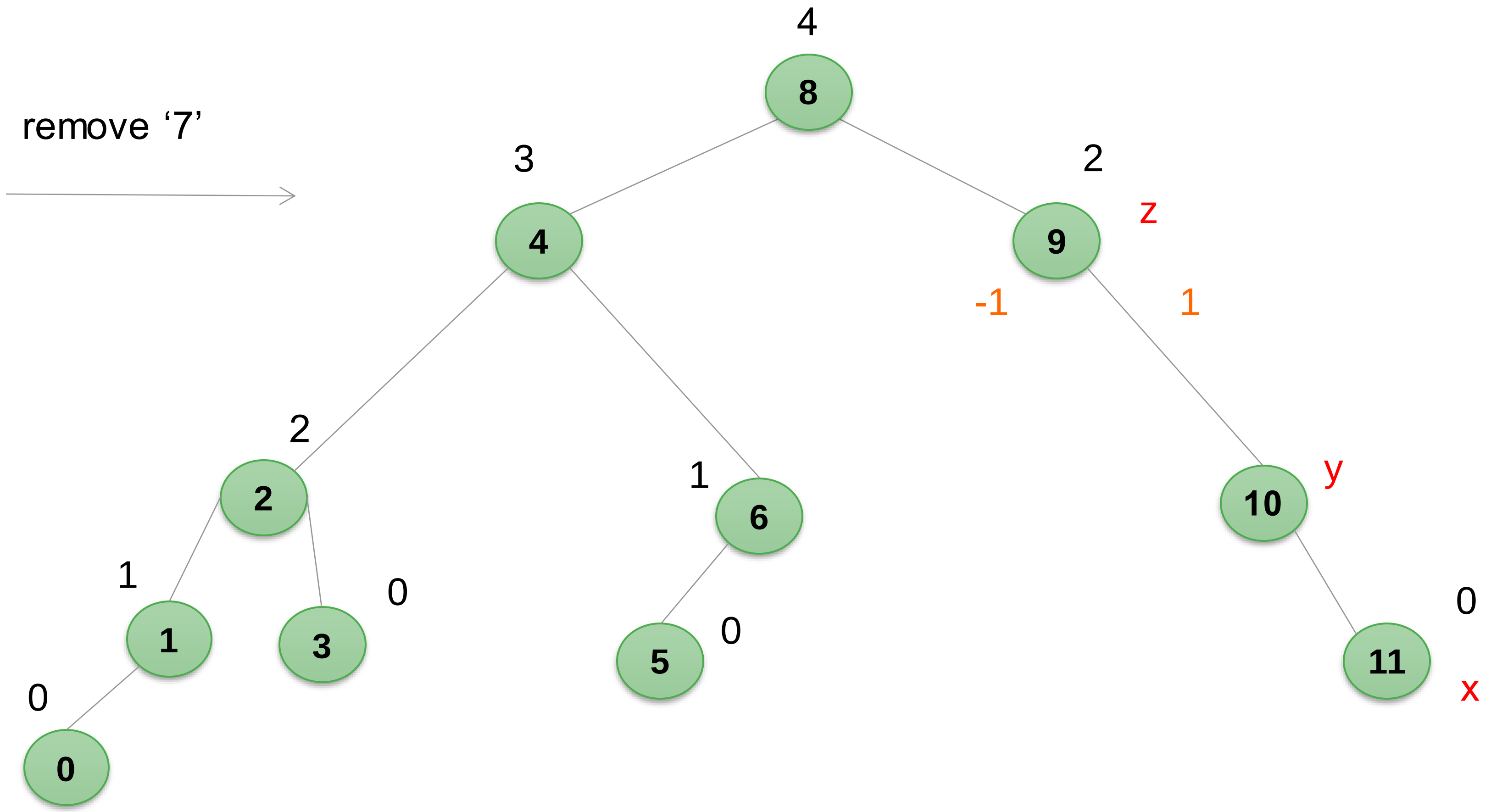

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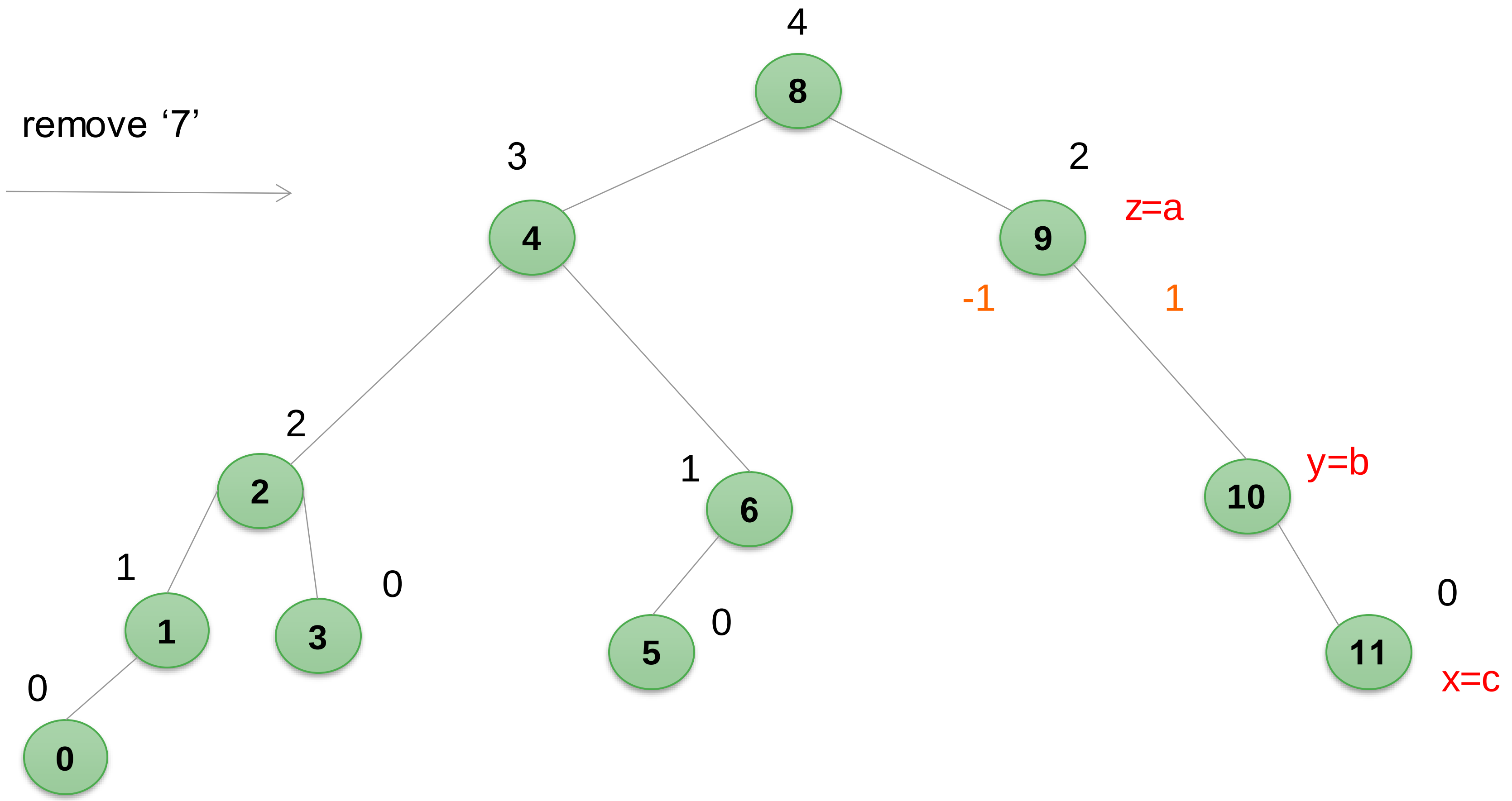

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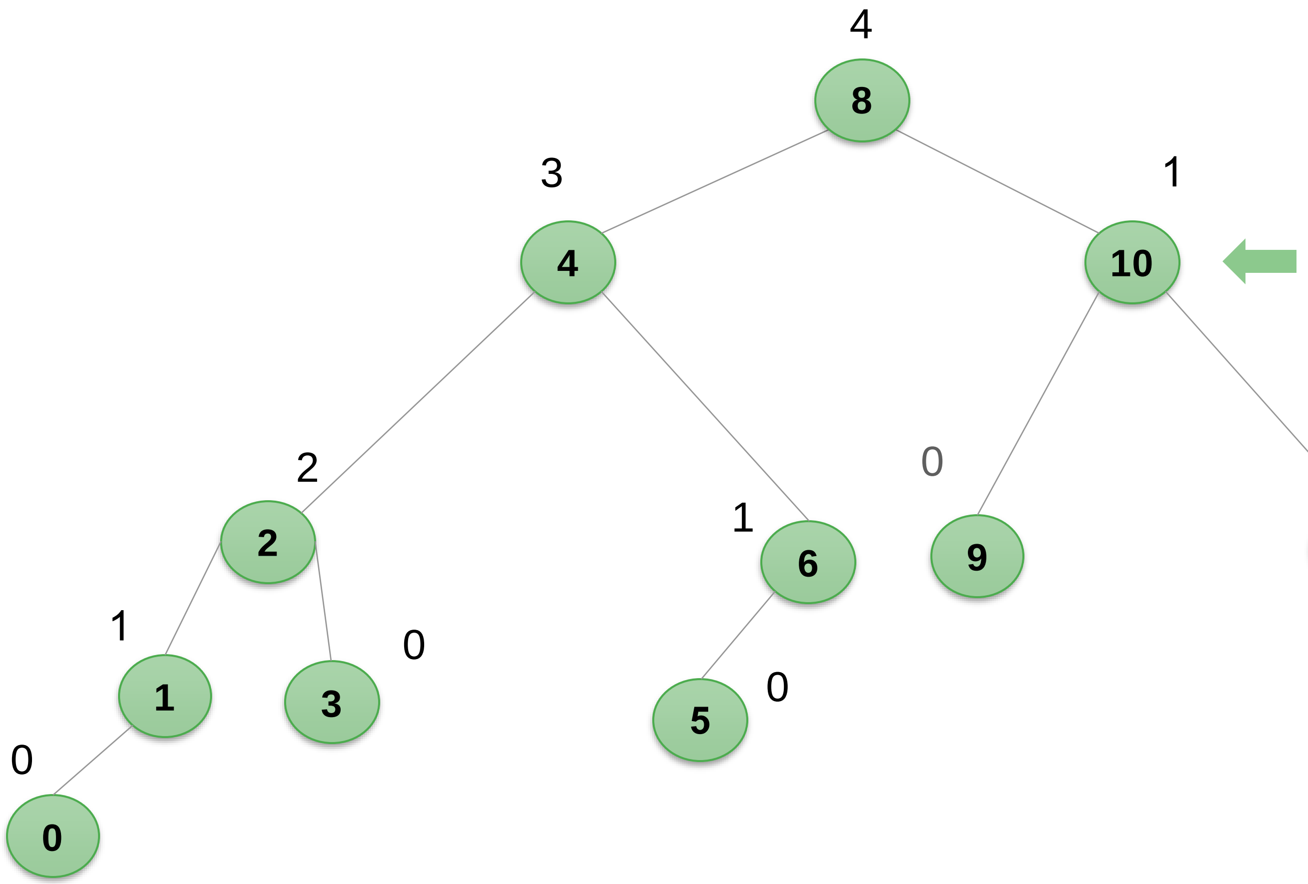

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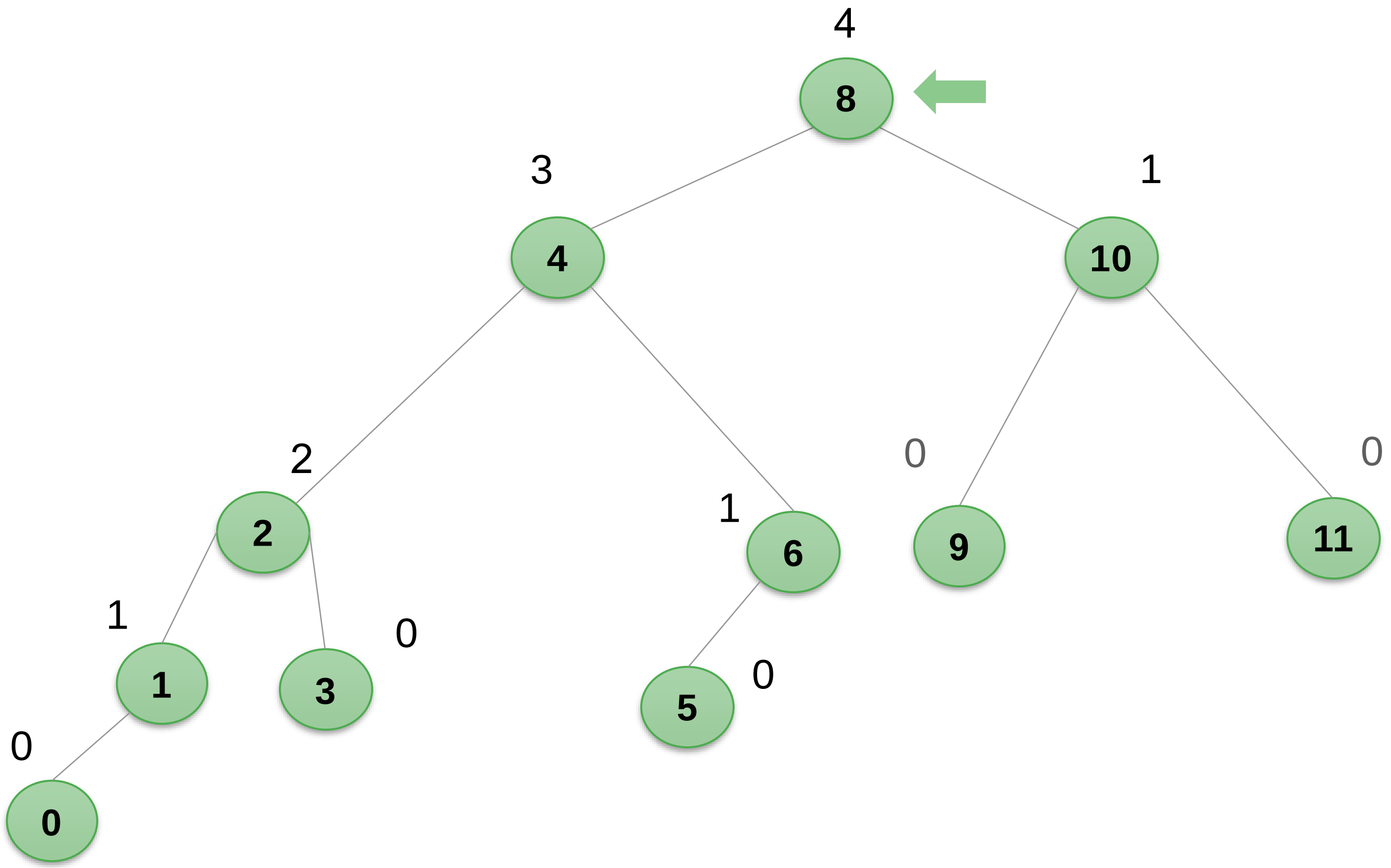

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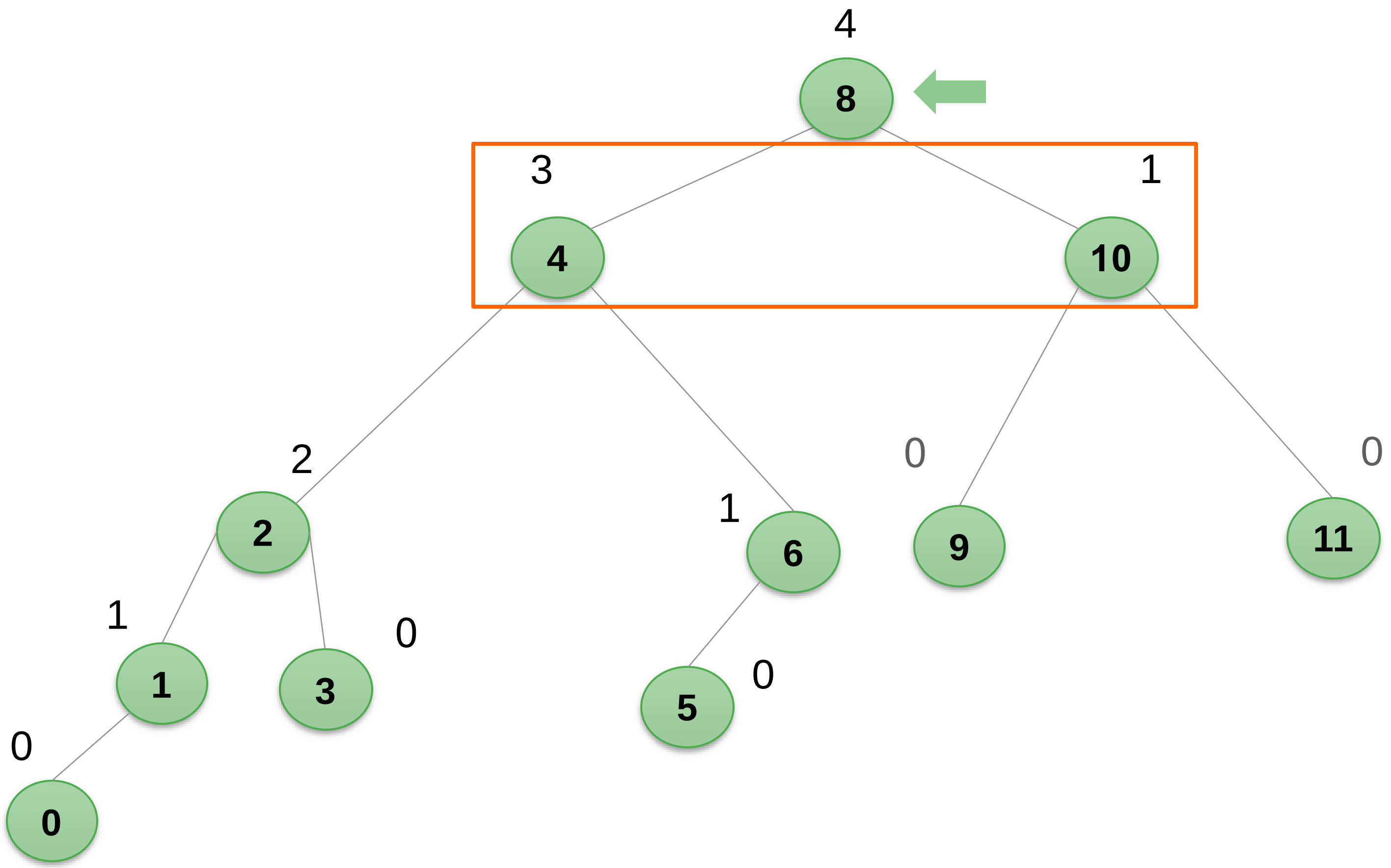

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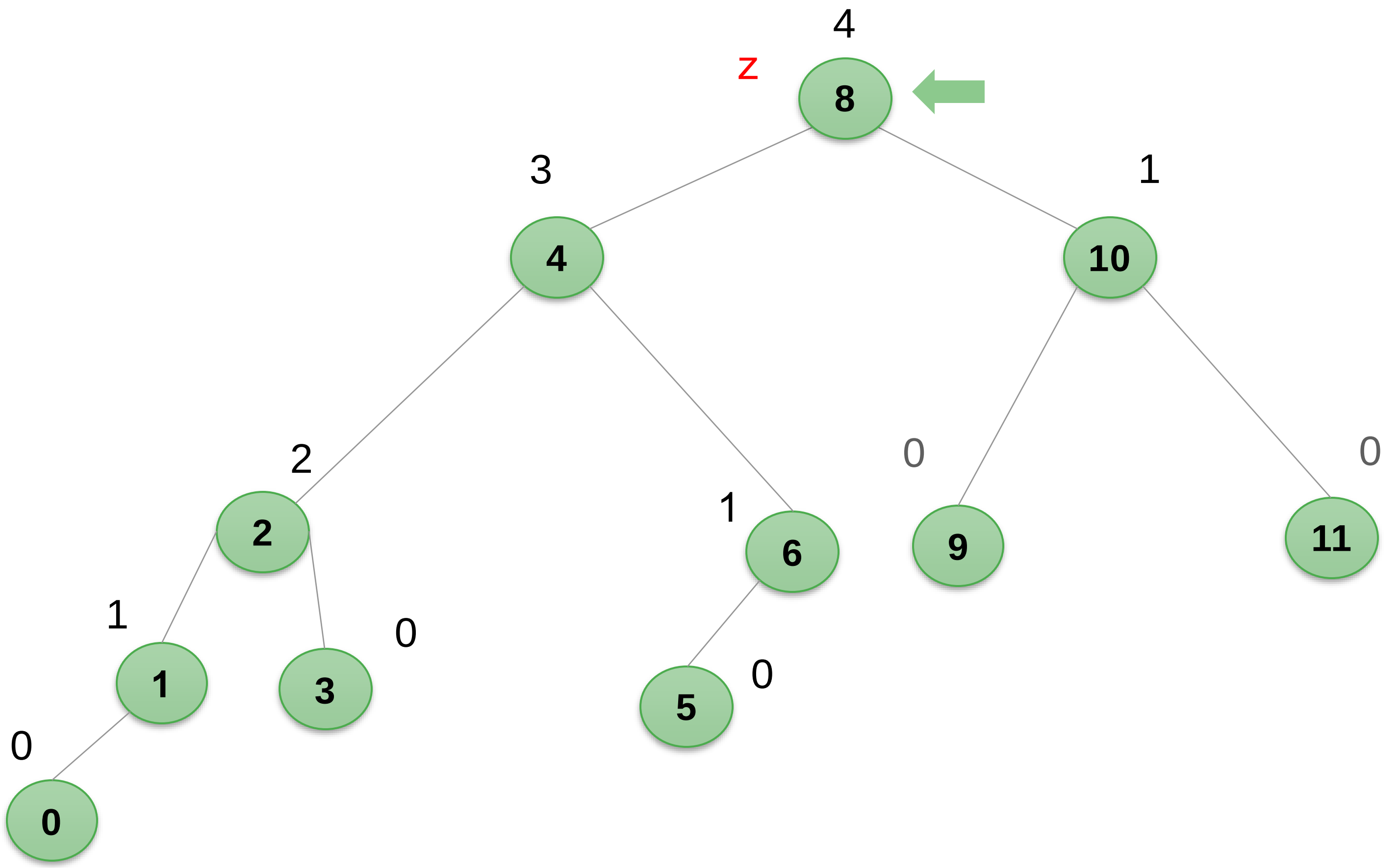

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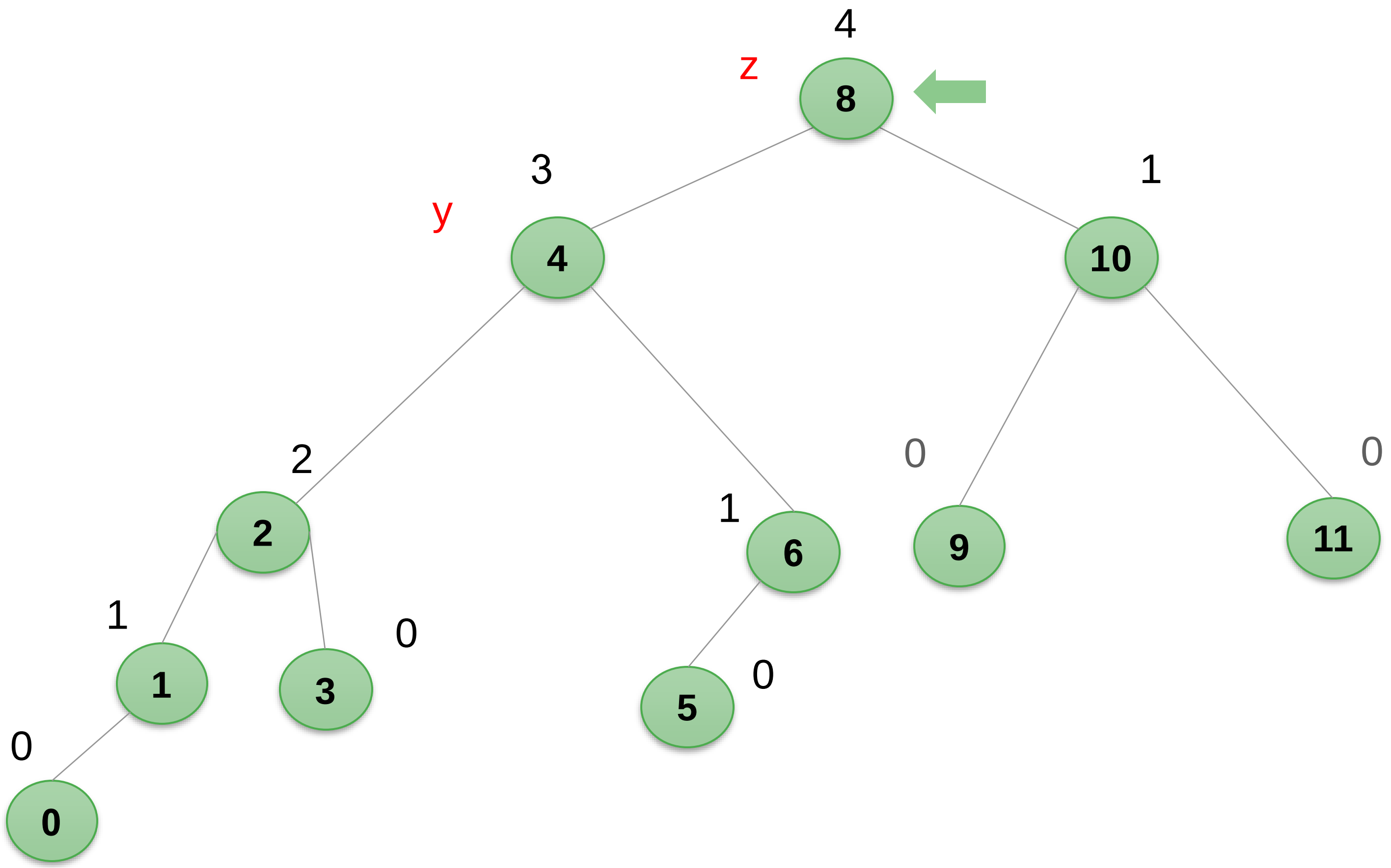

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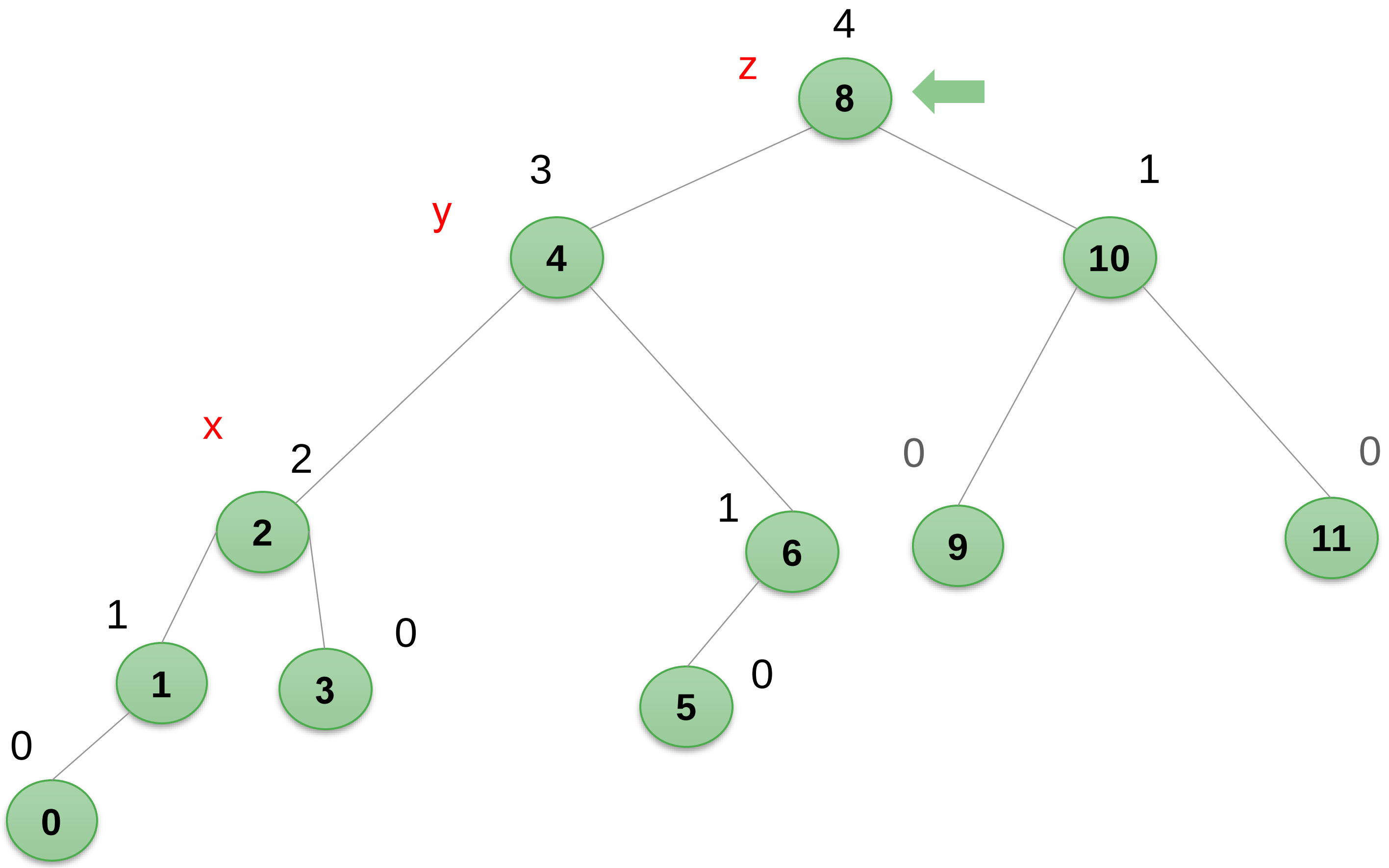

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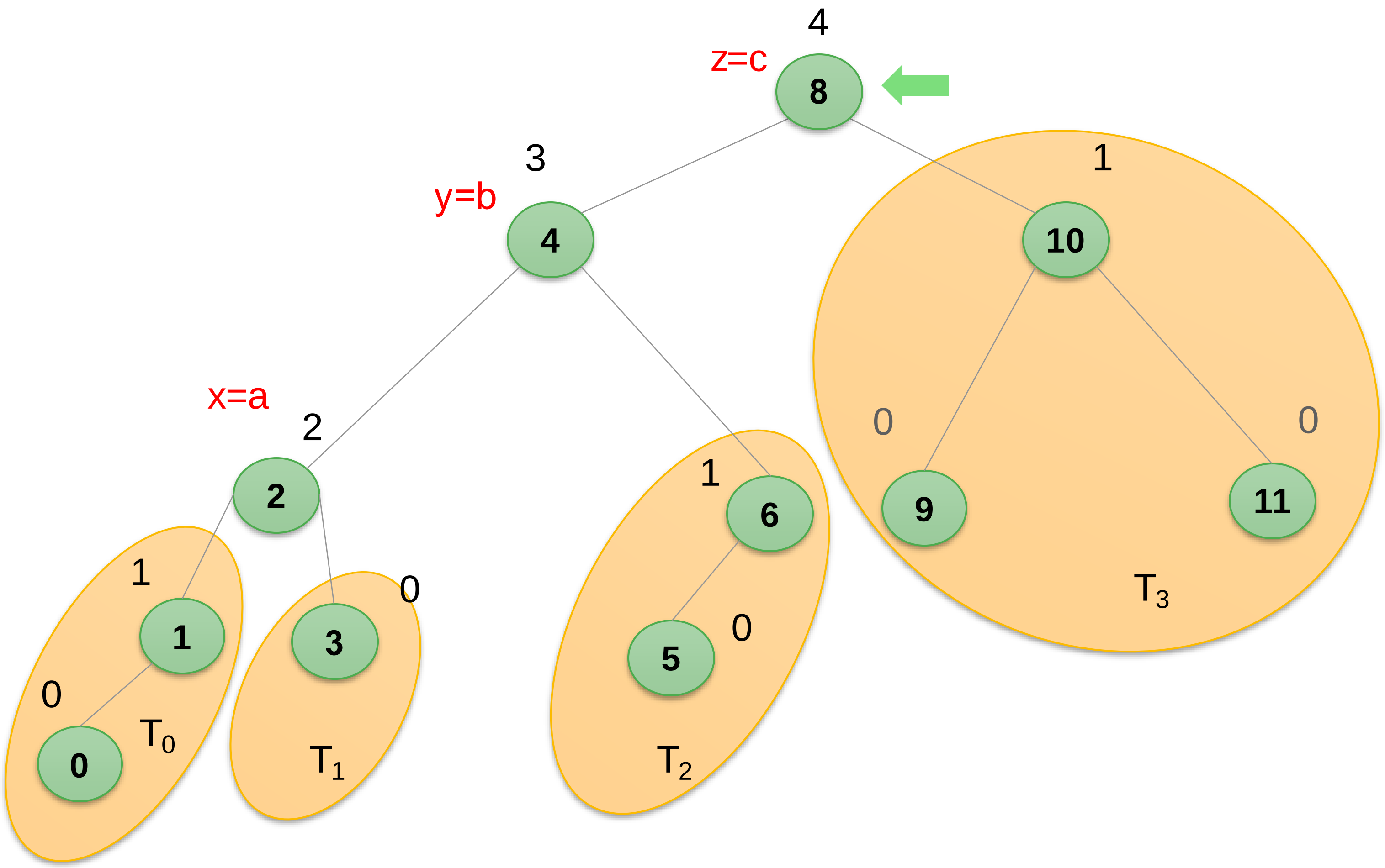

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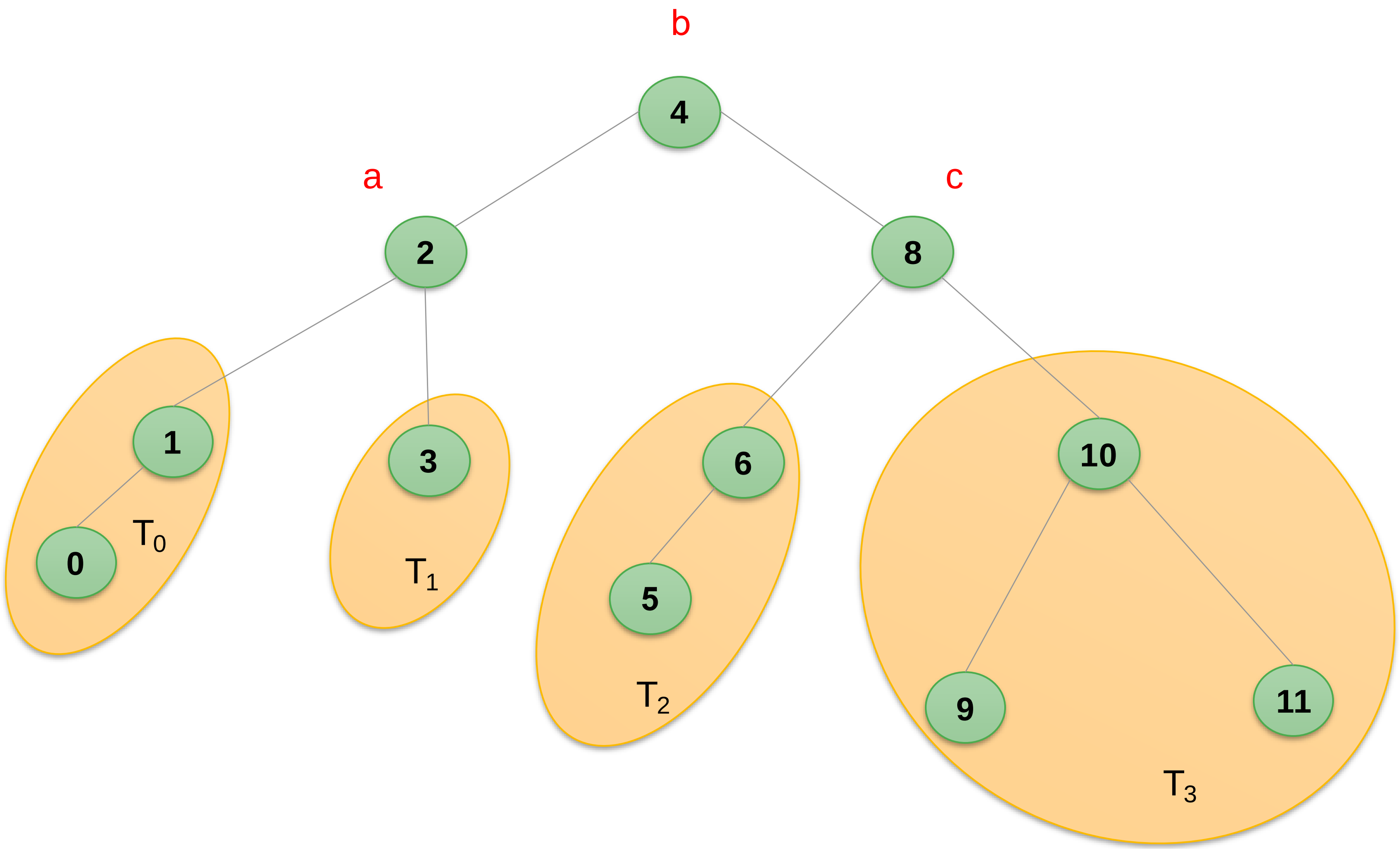

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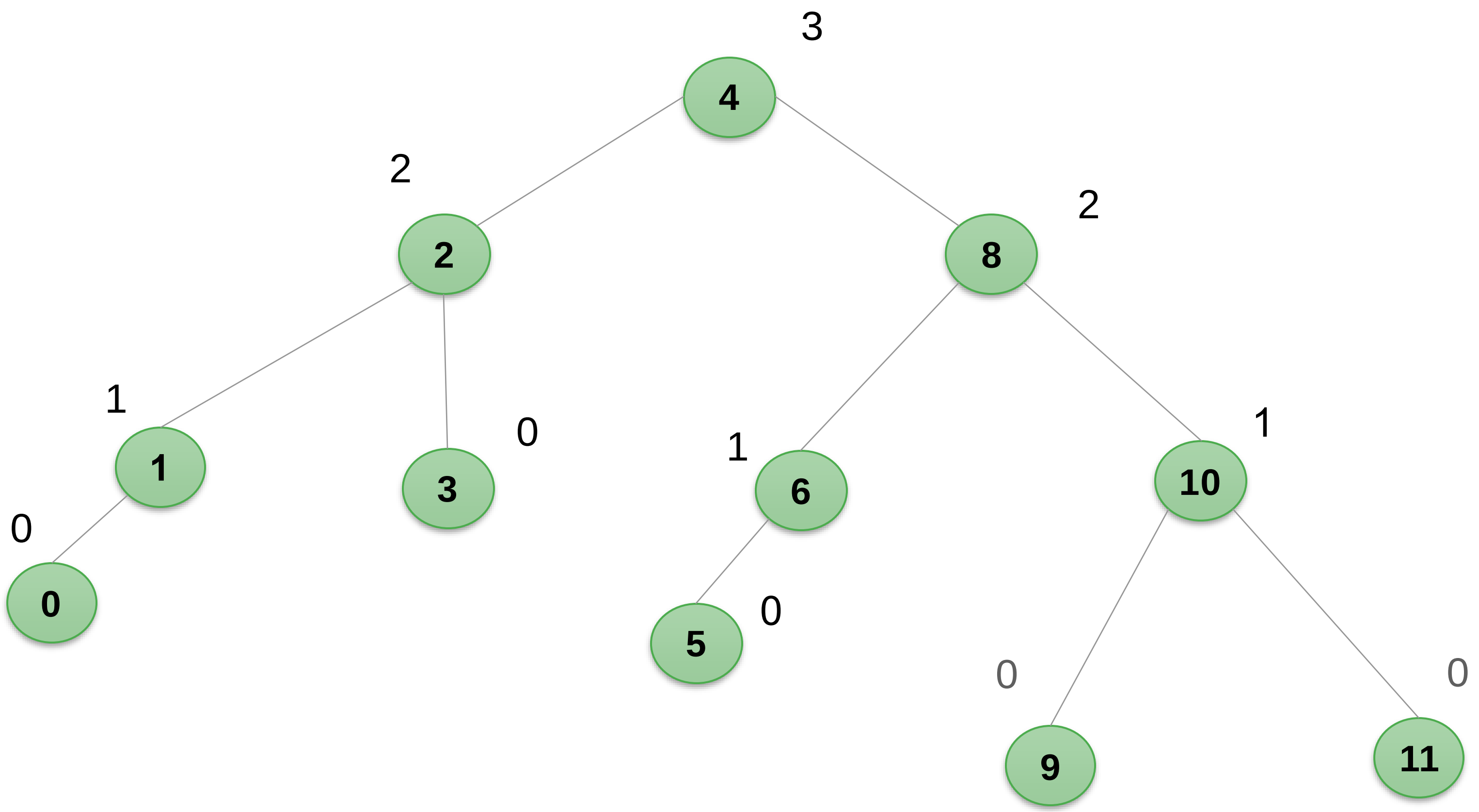

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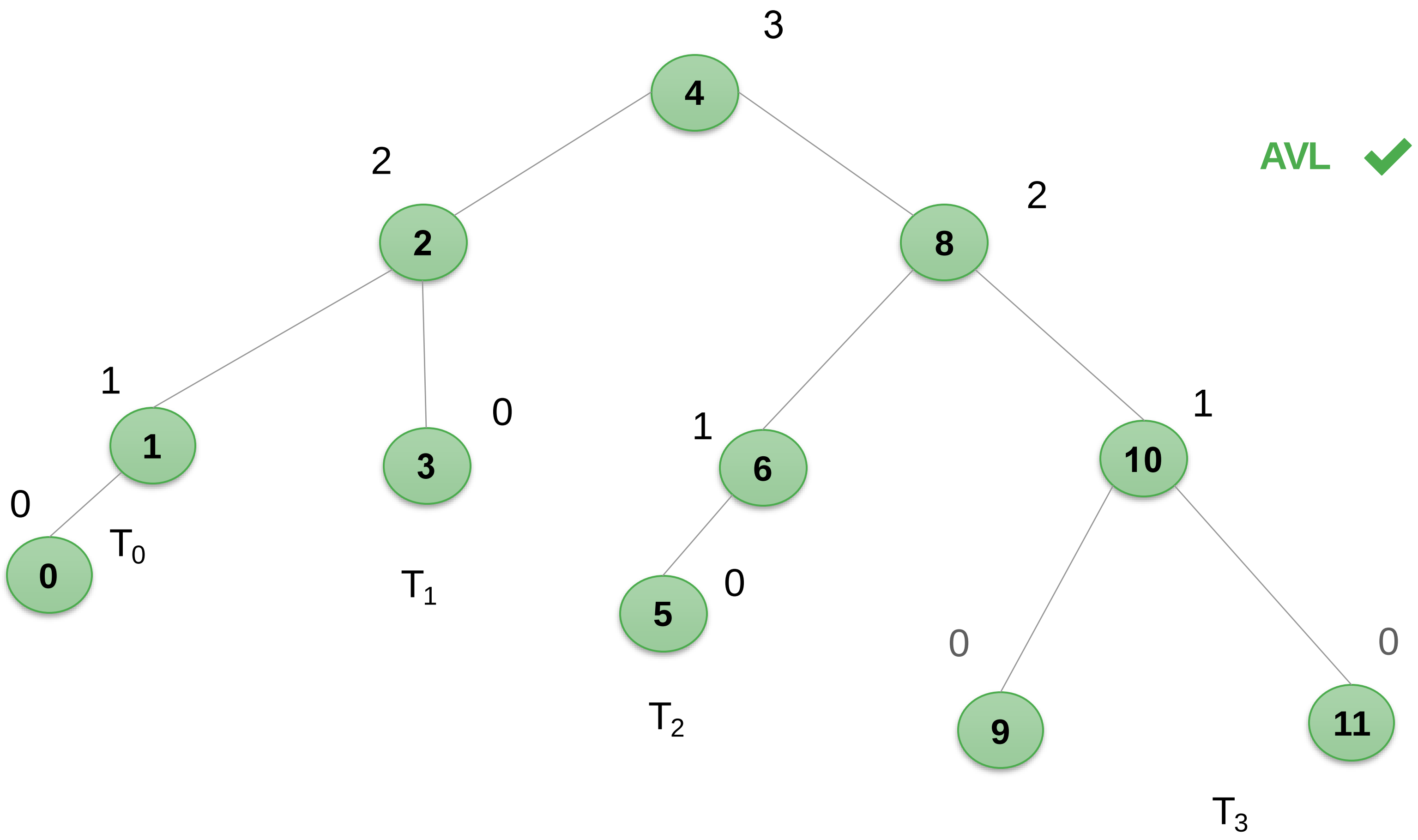

ASSIGNMENT 02

FAST SEARCHING / BALANCED TREES

Algorithms and Data Structures 2
Exercise – 2021W