Strongly Solving 7 × 6 ConnectFour on Consumer Grade Hardware

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Abstract. While the game ConnectFour has been solved mathematically and the best move can be effectively computed with search based methods, a strong solution in the form of a look-up table was believed to be infeasible. In this paper, we revisit a symbolic search method based on binary decision diagrams to produce strong solutions. With our efficient implementation we were able to produce a 89.6 GB large look-up table in 47 hours on a single CPU core with 128 GB main memory for the standard 7×6 board size. In addition to this win-draw-loss evaluation, we include an alpha-beta search in our open source artifact to find the move which achieves the fastest win or slowest loss.

Keywords: Symbolic search · Binary Decision Diagrams · Two Player Games · ConnectFour

1 Introduction

ConnectFour is a game in which two players take turns placing colored discs into a two-dimensional board. A players win if they manage to form a horizontal, vertical, or diagonal line of four of their own disks. The game is a draw if the board is fully filled and no such lines were achieved. Piece placement is restricted as players are only allowed to place their disk in the lowest available cell within the column. In the real world, this restriction is enforced by suspending the board vertically such that the disk fall straight down. The most commonly-used board size is 7 columns × 6 rows.

In 1988 Allen [1] was the first to solve ConnectFour by describing winning strategies. Published only 15 days later, Allis [2] independently came up with a knowledge-based approach to solve the game. With the advancements of computer hardware, explicit search-based approaches alá alpha-beta pruning [11] were used by Tromp [14] to build a database which stores the win-draw-loss evaluation for each 8-ply position. This effectively strongly solved 7×6 ConnectFour as solving positions deeper than eight plies can be done in seconds with explicit search. Later, Tromp [15] weakly solved ConnectFour instances with width + height < 16. Recently, Steininger [13] extended these solutions for board sizes where width + height = 16.

While explicit search traverses one state at a time, so called symbolic search methods handle sets of states at once. Edelkamp and Kissmann [7] pioneered the use of binary decision diagrams (BDDs) to encode sets of states where a single state is represented by a conjunction of binary variables. Kissmann and Edelkamp [10] developed a layered approach in which all game positions at a given ply are encoded by a BDD. Thus, at the end of the so-called forward pass we have 42 disjoint BDDs encoding all 7×6 ConnectFour positions. The approach then proceeds by performing a retrograde analysis starting at all fully filled positions at ply 42, splitting them into win-, draw-, and lost-BDDs, and propagating the classification back ply-by-ply to the empty root position.

The algorithmic complexity of BDDs depends heavily on the state encoding. Edelkamp and Kissmann [7] proved the unfortunate fact that for a natural encoding the number of nodes required to encode all positions scales exponentially in the number of columns or rows. As a consequence, they were only able to solve the 6×6 board configuration with 64 GB RAM in 22:38 hours. To the best of our knowledge Edelkamp et al. [8] constitutes their last published effort to solve the standard 7×6 board configuration. Their hybrid between explicit and symbolic search trades of memory with computational requirements was estimated to produce a solution in 93 days on a 192 GB machine.

In this paper, we revisit the approach of Kissmann and Edelkamp [10]. With a different state encoding and minor algorithmic improvements, we were able to solve the 7×6 board configuration in 47 hours on a single core of the AMD Ryzen 5950x CPU with 128 GB main memory. By writing the solution as BDDs to disk, they take only 89.6 GB storage. We were also able to reproduce the counting of unique positions of Edelkamp and Kissmann [7] in 2:15 hours and the solution of the 6×6 board configuration in 2:13 hours, both using 32 GB RAM. Lastly, we implemented a rudimentary alpha-beta search on top of our win-draw-loss solution to find the move which achieves the fastest win or slowest loss. The source code to reproduce our results and to query the win-draw-loss table is openly available at https://github.com/markus7800/Connect4-Strong-Solver.

2 Background

2.1 Binary Decision Diagrams

Binary Decision Diagrams (BDDs) are a data structure to represent Boolean functions over variables x_i as a rooted, directed, acyclic graph [3,4]. Each BDD contains two terminal nodes labeled 0 (`false`) and 1 (`true`). Each internal node is a decision node associated with one Boolean variable x_i and two child nodes called low and high child. The edges to the low and high child represent the assignment of the variable x_i to `false` and `true`, respectively. Thus, a path from the root node to the 1 terminal node, represents a possibly partial variable assignment for which the encoded function is `true`. Likewise, a path to the 0 terminal corresponds to a `false`-assignment. Importantly, BDDs allow the direct application of Boolean operations. If the ordering of variables is the same for each path from root to terminal, then the BDD is called ordered. If the graph is unique up to isomorphism, the BDD is called reduced. In this work, we work with these reduced ordered binary decision diagrams (ROBDDs).

2.2 BDDs for Symbolic Search for Two-Player Games

In symbolic search, we represent *sets of states* as Boolean functions f over some set of variables S [8,10]. We have $f(S) = \texttt{true}$ if the state corresponding to the variable assignments S belongs to the state of sets f, otherwise $f(S) = \texttt{false}$. The way we encode a ConnectFour state in terms of Boolean variables is described in Section 3.1, but not important for this section. Computationally, the Boolean functions are represented as BDDs and for which we can directly apply operations like $\wedge, \vee, \neg$, etc.

To encode transitions from one set of states to another following the game rules, we need a copy of the variables S' and a transition relation $\mathrm{trans}(S, S')$. It typically has the form $\mathrm{trans}(S, S') = \bigvee_a \mathrm{pre}_a(S) \wedge \mathrm{eff}_a(S') \wedge \mathrm{frame}_a(S, S')$, where for each game action a we have a pre-condition, effect, and a frame that determines the variables that remain unchanged when applying a. With the transition relation, we can define the image and pre-image operations:

$$\mathrm{image}(f) = \exists S\colon f(S) \wedge \mathrm{trans}(S, S'), \quad \text{pre-image}(f) = \exists S'\colon \mathrm{trans}(S, S') \wedge f(S')$$

To solve a game with win/draw/loss outcome, we first perform a *forward pass*, where we compute states_i – the set of all positions at a given ply i. We start at the initial game state $\mathrm{states}_0 = \mathrm{initial\_state}$ and compute recursively

$$\mathrm{states}_{i+1} \leftarrow \mathrm{image}(\mathrm{states}_i \wedge \neg\mathrm{terminal})[S' \to S],$$

where terminal is the Boolean function representing the set of all terminal states. To keep representing the states in terms of variables S, we have to replace the variables S' with S after performing the image operation, denoted by $[S' \to S]$.

After computing the set of states at each ply, we propagate the game outcome backwards with following rules

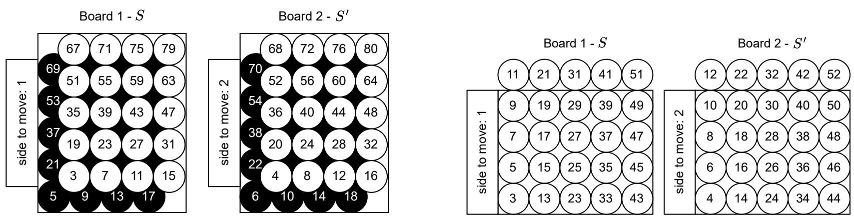
$$\begin{aligned}
\mathrm{win}_i &\leftarrow \text{pre-image}(\mathrm{lost}_{i+1}[S \to S']) \wedge (\mathrm{states}_i \wedge \neg\mathrm{terminal}),\\
\mathrm{draw}_i &\leftarrow \text{pre-image}(\mathrm{draw}_{i+1}[S \to S']) \wedge (\mathrm{states}_i \wedge \neg\mathrm{terminal} \wedge \neg\mathrm{win}_i),\\
\mathrm{lost}_i &\leftarrow (\mathrm{states}_i \wedge \neg\mathrm{terminal} \wedge \neg\mathrm{win}_i \wedge \neg\mathrm{draw}_i) \vee (\mathrm{states}_i \wedge \mathrm{terminal}).
\end{aligned}$$

Here, we leverage the fact that a win for one player is a loss for the other. These Boolean functions are from the perspective of the player to move. Thus, at the maximum ply N, the current player cannot move and thus we have a draw on non-terminal and a loss on terminal states and initialise $\mathrm{win}_N = \mathrm{false}$, $\mathrm{draw}_N = \mathrm{states}_N \wedge \neg\mathrm{terminal}$, and $\mathrm{lost}_N = \mathrm{states}_N \wedge \mathrm{terminal}$.

3 Implementation Details

3.1 State Encoding

There are multiple ways to encode a ConnectFour position with Boolean variables. Edelkamp and Kissmann [7] use $2 \cdot (\mathrm{width} \cdot \mathrm{height}) + 1$ variables: one variable denotes the side-to-move and there are two variables per board cell indicating whether it is empty, occupied by the first player, or occupied by the second player. For this cell-wise encoding, Edelkamp and Kissmann [7] proved that the algorithmic complexity of counting all unique positions is exponential in min(width, height), independent of variable ordering. We have implemented their encoding for both a row-wise and column-wise order. While they have not explicitly mentioned how to order the variables belonging to the same cell we take following approach: player-1-board-1, player-1-board-2, player-2-board-1, player-2-board-2, as illustrated in Figure 1a. Here board-1 corresponds to the variable set S and board 2 to the variable set S' as explained in Section 2.2.



(a) Standard row-wise encoding: first-player white, second-player black

(b) Compressed column-wise encoding: an additional row is needed

Fig. 1: Variable ordering for different ways of encoding ConnectFour boards. 4×5 board on the left, 5×4 board on the right.

Inspired by bitboard representations [15,9], we have also implemented a more compressed encoding with only width $\cdot$ (height $+ 1) + 1$ variables, illustrated in Figure 1b. In this encoding, each cell has only one variable. The lowest available cell per column is always `true`. All cells below the lowest available cell are occupied and `true` if the disc belongs to the first player else `false` (the disc belongs to the second player). To facilitate this logic, we need an additional row. This encoding outperforms the standard encoding for the 7×6 board configuration, but performs poorly if the number of rows is large.

3.2 BDD Implementation

We have implemented a minimal BDD library in C which allows us to implement the computations described in Section 2.2. No special improvements in the implementation of the primitive BDD operations were made. The garbage collection is typically a challenging aspect of BDD libraries. In our implementation, the user specifies the number of nodes desired and the library pre-allocates all nodes. With a form of reference counting, the user has to manually free nodes to make them available for reuse. This makes our library fast with a lot of control over memory consumption, but also a bit tedious to work with.

The termination criteria in ConnectFour is a conjunction of many Boolean formula each representing the condition that there are four discs of the same color in a row (horizontally, vertically, or diagonally) for a different set of cells. A lot of nodes would be necessary to encode the terminal BDD as a whole. Instead, in the implementation of $\text{states}_i \wedge \neg\text{terminal}$, we iteratively subtract the individual termination condition. We handle $\text{states}_i \wedge \text{terminal}$ in a similar fashion.

Lastly, we avoid renaming variables, $[S \to S']$, by computing the mirrored transition relation $\text{trans}'(S', S)$ which switches the roles of variable sets S and S'. We also implemented a special sat-counting routine to count the number of states encoded in a BDD with respect to only S *or* S' instead of $S \cup S'$.

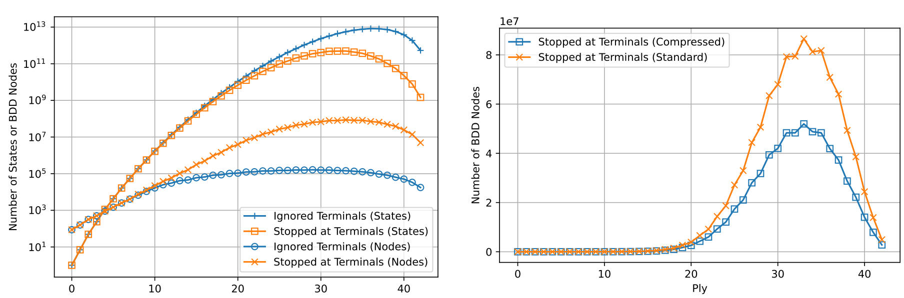
3.3 Alpha-Beta Search

In addition to the win-draw-loss evaluation of a position, we like to also find the move that gives the fastest win or prolongs the game as long as possible in the case of a loss. To this end, we implemented a rudimentary alpha-beta principal variation search [11]. It uses the the usual bitboard representation to facilitate fast move and position generation [12,15,9]. We leverage the vertical symmetry of the position evaluation in a transposition table of size 2^{28} that always overwrites old entries in case of collision. Moves are sorted by their distance to the central board column and by the number of threats they generate. In this context, a threat is an alignment of three discs on the board waiting to be completed to connect four discs and win the game. We prune some positions where we can statically detect a win, draw, or loss. For example, a player cannot prevent two threats at the same time, thus losing the game. Lastly, in the lower depths, we probe the win-draw-loss table to avoid searching losing moves in a won position. For a state-of-the-art ConnectFour search engine see Steininger [13].

4 Results

4.1 Counting Unique Positions

Before presenting the ConnectFour solution, we briefly discuss the number of unique game positions computed by the forward pass.



(a) Number of states and BDD nodes at each ply for 7×6 ConnectFour with and without termination criterion

(b) BDD nodes required to represent all states at each ply: compressed versus standard state encoding.

Fig. 2: Investigation of BDD sizes for the 7 × 6 board configuration.

In Figure 2a, you can see the number of positions and the size of the BDD to encode them at each ply for the 7×6 board computed in 3:34 hours with 32 GB RAM. In addition, you can see the same quantities if we remove the termination condition. This graph effectively reproduces the results presented in Edelkamp and Kissmann [7]. Presumably due to slightly different variable ordering, we find that it takes 95,124,612 nodes to encode all positions into a single BDD instead of the 84,088,763 reported Edelkamp and Kissmann [6]. As can be seen in Figure 2b, with the compressed encoding we can lower this number to 59,853,336 and computation takes only 2:15 hours. With our implementations, we can confirm all unique positions counts of Tromp [16] and produce novel counts included in the Appendix. For example, it took almost two days to compute that the 7×7 game has 161,965,120,344,045 unique positions.

4.2 Solving ConnectFour

With our implementation and the compressed encoding, we were able to confirm the solution for the 6×6 game Edelkamp et al. [8] in 2:13 hours with 32 GB RAM. We were able to produce the 89.6 GB win-draw-table for the 7×6 game in 47 hours with 128 GB RAM. The memory was not sufficient for solving with the standard encoding. In the Appendix, we list the number of won, drawn, lost, and terminal position at each ply. This table agrees with the partial solution presented in Edelkamp et al. [8]. With over 1.1 billion nodes and 10.2 GB, ply 27 takes the most storage to store. However, since draw $= \neg$lost $\wedge \neg$win, we do not need to store the draw BDD reducing the amount of storage required significantly.

By leveraging the win-draw-loss evaluation in the alpha-beta search, we can confirm in 9.2 seconds that the first player wins in 41 plies by playing in the center. Not using the table increases the number of explored positions by a factor of 6 and takes 128 seconds. To facilitate almost instantaneous position evaluation, we generated an openingbook by evaluating all 184,275 8-ply positions with the alpha-beta search making use of the win-draw-loss. Using all 16 cores, this took 5:11 hours. The full win-draw-loss solution and the openingbook can be found in Böck [5].

5 Conclusion

In this work, we revisited symbolic search for solving 7×6 ConnectFour. Due to exponential algorithmic complexity, it was believed that this method cannot be scaled to this board configuration. With our open-source implementation, we demonstrate that, in fact, this instance can be solved with moderate hardware requirements. By storing the solution directly as binary decision diagrams, we are the first to produce a look-up-table-like solution. As a by-product, we effectively reproduced prior research concerned with counting unique game positions and present novel counts including the 7×7 board.

References

1. Allen, J.D.: The Complete Book of Connect 4: History, Strategy, Puzzles. Puzzle Wright Press (2010)
2. Allis, L.V.: A knowledge-based approach of connect-four. J. Int. Comput. Games Assoc. **11**(4), 165 (1988)
3. Andersen, H.R.: An introduction to binary decision diagrams. Lecture notes, available online, IT University of Copenhagen **5** (1997)
4. Bryant, R.E.: Graph-based algorithms for boolean function manipulation. Computers, IEEE Transactions on **100**(8), 677–691 (1986)
5. Böck, M.: Connect4 7 x 6 strong solution (2025). https://doi.org/10.5281/zenodo.14582823, https://doi.org/10.5281/zenodo.14582823
6. Edelkamp, S., Kissmann, P.: Symbolic classification of general two-player games. In: KI 2008: Advances in Artificial Intelligence: 31st Annual German Conference on AI, KI 2008, Kaiserslautern, Germany, September 23-26, 2008. Proceedings 31. pp. 185–192. Springer (2008)
7. Edelkamp, S., Kissmann, P.: On the complexity of bdds for state space search: A case study in connect four. In: Proceedings of the AAAI Conference on Artificial Intelligence. vol. 25, pp. 18–23 (2011)
8. Edelkamp, S., Kissmann, P., Rohte, M.: Symbolic and explicit search hybrid through perfect hash functions—a case study in connect four. In: Proceedings of the International Conference on Automated Planning and Scheduling. vol. 24, pp. 101–110 (2014)
9. Herzberg, D.: Bitboardc4 (2024), https://github.com/denkspuren/BitboardC4
10. Kissmann, P., Edelkamp, S.: Layer-abstraction for symbolically solving general two-player games. In: Proceedings of the International Symposium on Combinatorial Search. vol. 1, pp. 63–70 (2010)
11. Knuth, D.E., Moore, R.W.: An analysis of alpha-beta pruning. Artificial intelligence **6**(4), 293–326 (1975)
12. Pons, P.: Solving connect4: How to build a perfect ai (2019), http://blog.gamesolver.org/
13. Steininger, C.: c4 (2024), https://github.com/ChristopheSteininger/c4
14. Tromp, J.: Connect-4. UCI Machine Learning Repository (1995), DOI: https://doi.org/10.24432/C59P43
15. Tromp, J.: Solving connect-4 on medium board sizes. ICGA Journal **31**(2), 110–112 (2008)
16. Tromp, J.: John's connect four playground (2015), https://tromp.github.io/c4/c4.html

Appendix

Table 1: Number of positions for several $w \times h$ board configurations with computation time, percentage spent garbage collecting, RAM used, and maximum number of BDD nodes allocated at the same time.

w	h	# positions	time	GC	RAM	#nodes alloc.
1	1	2	0:00:01.06	54.41%	3.42GB	61
1	2	3	0:00:02.35	54.24%	3.42GB	222
1	3	4	0:00:03.25	48.33%	3.42GB	468
1	4	5	0:00:02.99	42.06%	3.42GB	825
1	5	6	0:00:03.12	40.28%	3.42GB	1,306
1	6	7	0:00:03.17	43.47%	3.42GB	1,923
1	7	8	0:00:03.87	38.74%	3.42GB	2,688
1	8	9	0:00:04.21	35.62%	3.42GB	3,613
1	9	10	0:00:06.06	51.84%	3.42GB	4,710
1	10	11	0:00:08.95	58.61%	3.42GB	5,991
1	11	12	0:00:09.10	60.28%	3.42GB	7,468
1	12	13	0:00:10.46	66.53%	3.42GB	9,153
1	13	14	0:00:13.70	71.95%	3.42GB	11,058
2	1	5	0:00:01.93	58.09%	3.42GB	203
2	2	18	0:00:02.86	46.32%	3.42GB	800
2	3	58	0:00:03.68	38.29%	3.42GB	1,957
2	4	179	0:00:04.29	33.52%	3.42GB	3,777
2	5	537	0:00:08.48	54.39%	3.42GB	6,330
2	6	1,571	0:00:10.42	66.42%	3.42GB	9,700
2	7	4,587	0:00:14.47	69.97%	3.42GB	13,971
2	8	13,343	0:00:13.47	71.62%	3.42GB	19,227
2	9	38,943	0:00:23.26	76.53%	3.42GB	25,552
2	10	113,835	0:00:19.88	75.92%	3.42GB	33,030
2	11	333,745	0:00:26.56	76.63%	3.42GB	41,745
2	12	980,684	0:00:22.12	76.01%	3.42GB	51,781
2	13	2,888,780	0:00:20.44	78.30%	3.42GB	63,222
3	1	13	0:00:03.35	46.73%	3.42GB	426
3	2	116	0:00:03.08	39.53%	3.42GB	1,836
3	3	869	0:00:05.32	40.88%	3.42GB	4,567
3	4	6,000	0:00:11.00	67.31%	3.42GB	10,781
3	5	38,310	0:00:19.85	70.88%	3.42GB	18,520
3	6	235,781	0:00:23.68	76.04%	3.42GB	29,974
3	7	1,417,322	0:00:20.59	74.19%	3.42GB	75,162
3	8	8,424,616	0:00:28.97	72.81%	3.42GB	172,756
3	9	49,867,996	0:00:26.03	59.16%	3.42GB	422,983
3	10	294,664,010	0:00:35.54	40.98%	3.42GB	1,010,556
3	11	1,741,288,730	0:01:10.14	28.39%	3.42GB	2,381,296
3	12	10,300,852,227	0:04:03.01	25.93%	3.42GB	5,427,670
3	13	61,028,884,959	0:09:06.17	18.47%	3.42GB	13,049,070
4	1	35	0:00:03.03	46.03%	3.42GB	745
4	2	741	0:00:04.17	33.98%	3.42GB	3,426
4	3	12,031	0:00:10.67	66.98%	3.42GB	8,853
4	4	161,029	0:00:27.73	85.17%	3.42GB	17,799
4	5	1,706,255	0:01:01.09	86.01%	3.42GB	129,589
4	6	15,835,683	0:01:07.39	74.44%	3.42GB	561,346
4	7	135,385,909	0:01:18.26	44.51%	3.42GB	1,648,551
4	8	1,104,642,469	0:03:01.01	27.01%	3.42GB	4,307,162
4	9	8,754,703,921	0:09:25.69	20.52%	3.42GB	11,478,742
4	10	67,916,896,758	0:19:22.84	16.40%	13.69GB	29,923,914
4	11	519,325,538,608	1:06:08.86	12.89%	13.69GB	80,053,135
4	12	3,928,940,117,357	3:36:00.93	9.23%	13.69GB	220,438,835
4	13	29,499,214,177,403	9:42:44.88	9.78%	54.76GB	608,399,867

w	h	# positions	time	GC	RAM	#nodes alloc.
5	1	96	0:00:03.32	43.38%	3.42GB	1,172
5	2	4,688	0:00:06.98	58.01%	3.42GB	5,666
5	3	158,911	0:00:12.97	73.18%	3.42GB	15,062
5	4	3,945,711	0:01:00.14	86.09%	3.42GB	129,415
5	5	69,763,700	0:01:21.73	64.22%	3.42GB	1,230,084
5	6	1,044,334,437	0:04:52.75	21.07%	3.42GB	8,235,134
5	7	14,171,315,454	0:25:17.41	15.38%	3.42GB	25,600,589
5	8	182,795,971,462	0:41:32.27	14.91%	27.38GB	69,165,125
5	9	2,284,654,770,108	2:32:23.19	11.96%	54.76GB	201,469,478
6	1	267	0:00:03.00	42.70%	3.42GB	1,719
6	2	29,737	0:00:10.52	66.09%	3.42GB	8,652
6	3	2,087,325	0:00:24.93	75.60%	3.42GB	32,107
6	4	94,910,577	0:01:10.13	73.27%	3.42GB	706,477
6	5	2,818,972,642	0:04:11.02	23.74%	3.42GB	7,852,100
6	6	69,173,028,785	0:23:07.92	14.52%	6.85GB	61,634,539
6	7	1,523,281,696,228	4:58:47.33	9.16%	27.38GB	365,099,251
6	8	31,936,554,362,084	17:39:00.27	9.35%	109.52GB	1,023,026,899
7	1	750	0:00:03.15	38.79%	3.42GB	2,398
7	2	189,648	0:00:15.09	71.34%	3.42GB	12,480
7	3	27,441,956	0:00:20.13	71.29%	3.42GB	97,803
7	4	2,265,792,710	0:01:54.27	45.32%	3.42GB	2,418,545
7	5	112,829,665,923	0:28:21.81	16.11%	3.42GB	27,594,037
7	6	4,531,985,219,092	3:22:58.57	13.07%	27.38GB	238,538,432
7	7	161,965,120,344,045	44:41:45.37	10.20%	109.52GB	1,797,440,896
8	1	2,118	0:00:04.13	35.78%	3.42GB	3,221
8	2	1,216,721	0:00:14.79	74.06%	3.42GB	17,246
8	3	362,940,958	0:00:19.49	64.98%	3.42GB	238,318
8	4	54,233,186,631	0:04:51.88	27.88%	3.42GB	5,820,061
8	5	4,499,431,376,127	0:53:32.50	18.26%	27.38GB	70,093,438
8	6	290,433,534,225,566	16:00:43.57	11.45%	54.76GB	625,763,115
9	1	6,010	0:00:06.07	47.99%	3.42GB	4,200
9	2	7,844,298	0:00:19.03	72.86%	3.42GB	23,046
9	3	4,816,325,017	0:00:28.97	56.19%	3.42GB	478,748
9	4	1,295,362,125,552	0:09:11.61	29.47%	13.69GB	11,398,845
9	5	178,942,601,291,926	2:37:32.12	17.09%	54.76GB	163,165,045
10	1	17,120	0:00:07.15	57.45%	3.42GB	5,347
10	2	50,780,523	0:00:31.32	76.73%	3.42GB	36,276
10	3	64,137,689,503	0:00:26.48	35.96%	3.42GB	840,337
10	4	30,932,968,221,097	0:20:38.81	23.53%	13.69GB	24,226,909
11	1	48,930	0:00:09.47	66.06%	3.42GB	6,674
11	2	329,842,064	0:00:24.87	75.23%	3.42GB	57,846
11	3	856,653,299,180	0:00:42.25	26.89%	3.42GB	1,615,299
11	4	738,548,749,700,312	0:44:45.79	21.18%	13.69GB	48,646,573
12	1	140,243	0:00:11.16	67.08%	3.42GB	8,193
12	2	2,148,495,091	0:00:23.60	73.27%	3.42GB	92,737
12	3	11,470,572,730,124	0:01:58.76	32.64%	3.42GB	2,910,928
12	4	17,631,656,694,578,591	1:25:25.41	18.07%	13.69GB	88,481,565
13	1	402,956	0:00:12.29	68.79%	3.42GB	9,916
13	2	14,027,829,516	0:00:24.42	72.20%	3.42GB	147,712
13	3	153,906,772,806,519	0:02:57.83	26.38%	3.42GB	4,927,508
13	4	420,788,402,285,901,831	2:03:51.22	19.22%	27.38GB	150,330,130

Table 2: Number of won, drawn, and lost positions for 7×6 ConnectFour from the perspective of the first player. If ply is odd, then terminal positions are won otherwise lost for the first player.

ply	won	drawn	lost	total	terminal
0	1	0	0	1	0
1	1	2	4	7	0
2	27	12	10	49	0
3	35	58	145	238	0
4	690	200	230	1,120	0
5	1,080	697	2,486	4,263	0
6	10,889	1,943	3,590	16,422	0
7	17,507	5,944	31,408	54,859	728
8	124,624	14,676	44,975	184,275	1,892
9	197,749	42,896	317,541	558,186	19,412
10	1,122,696	97,532	442,395	1,662,623	44,225
11	1,734,122	255,780	2,578,781	4,568,683	273,261
12	8,191,645	541,825	3,502,631	12,236,101	573,323
13	12,333,735	1,286,746	17,308,630	30,929,111	2,720,636
14	49,756,539	2,583,292	23,097,764	75,437,595	5,349,954
15	73,263,172	5,596,074	97,682,013	176,541,259	20,975,690
16	255,117,922	10,681,110	128,792,359	394,591,391	38,918,821
17	369,230,362	21,226,658	467,761,723	858,218,743	130,632,515
18	1,112,643,249	38,582,237	612,658,408	1,763,883,894	229,031,670
19	1,589,752,959	70,754,712	1,907,752,131	3,568,259,802	670,491,437
20	4,132,585,341	122,495,056	2,491,075,548	6,746,155,945	1,108,210,254
21	5,849,074,428	208,240,707	6,616,029,910	12,673,345,045	2,858,601,535
22	13,031,002,559	342,506,047	8,637,315,382	22,010,823,988	4,434,627,684
23	18,317,405,077	543,074,854	19,402,748,258	38,263,228,189	10,130,180,393
24	34,623,818,387	845,872,717	25,361,122,355	60,830,813,459	14,654,767,176
25	48,376,711,901	1,256,717,558	47,632,685,500	97,266,114,959	29,672,303,474
26	76,568,242,258	1,846,266,966	62,314,059,815	140,728,569,039	39,696,898,910
27	106,274,173,915	2,578,399,088	96,436,935,052	205,289,508,055	71,042,927,249
28	138,476,323,812	3,567,644,646	126,013,643,486	268,057,611,944	86,949,129,149
29	190,301,585,678	4,687,144,532	157,638,115,456	352,626,845,666	136,563,138,602
30	199,698,237,436	6,071,049,190	204,609,218,821	410,378,505,447	150,692,335,491
31	269,818,663,336	7,481,813,611	201,906,000,786	479,206,477,733	205,243,451,746
32	221,858,140,210	9,048,082,187	258,000,224,786	488,906,447,183	200,299,011,722
33	291,549,830,422	10,381,952,902	194,705,107,378	496,636,890,702	232,494,602,432
34	180,530,409,295	11,668,229,290	241,273,091,751	433,471,730,336	195,427,938,799
35	226,007,657,501	12,225,240,861	132,714,989,361	370,947,887,723	188,065,840,647
36	98,839,977,654	12,431,825,174	155,042,098,394	266,313,901,222	131,014,104,050
37	114,359,332,473	11,509,102,126	57,747,247,782	183,615,682,381	100,184,819,358
38	32,161,409,500	10,220,085,105	61,622,970,744	104,004,465,349	54,716,901,301
39	33,666,235,957	7,792,641,079	13,697,133,737	55,156,010,773	31,270,711,562
40	4,831,822,472	5,153,271,363	12,710,802,660	22,695,896,495	11,972,173,842
41	4,282,128,782	2,496,557,393	1,033,139,763	7,811,825,938	4,282,128,782
42	0	713,298,878	746,034,021	1,459,332,899	746,034,021