Electric Vehicles

ELEC 5970/6970/6970-D01

Vehicle   Mechanics

References:

- Iqbal Husain, ”Electric and Hybrid Vehicles, Design Fundamentals,” Third Edition, March 2021, CRC Press, Taylor & Francis Group, ISBN: 978-0429-49092-7

Occupations

- Chemical engineers apply the principles of chemistry to design or improve equipment or to devise processes for manufacturing chemicals and products. Because the batteries of electric vehicles store power through chemical processes, chemical engineers are responsible for developing new battery designs and improving current battery technologies. They are also vital in designing equipment and processes for large-scale manufacturing and in planning and testing the methods of battery manufacturing.
- Electrical engineers design, develop, test, and supervise the manufacture of electrical components. They are responsible for designing the electrical circuitry that allows a gas engine to charge the battery and distribute the electricity from the battery to the electric motor. Electrical engineers also might work on the heating and air-conditioning systems, vehicle lighting, and visual displays.
- Electronics engineers design, develop, and test electronic components and systems for vehicles. These engineers are primarily focused on the control systems and additional electronic components for the vehicle. They are different from electrical engineers in that they do not focus on the generation and distribution of electricity.

Occupations

- Industrial engineers determine the most effective ways to use the basic factors of production—people, machines, materials, information, and energy—to manufacture vehicles. They are concerned primarily with increasing productivity through the management of people, use of technology, and improvement of production methods. Because many electric vehicles require original manufacturing plans, industrial engineers design innovative manufacturing processes and retool plants that formerly made different models of cars.
- Materials engineers are involved in the development, processing, and testing of materials used in electric vehicles. Many electric vehicles are made of newer materials that are lighter and stronger than those in traditional cars. Materials engineers may also incorporate environmentally friendly materials that are derived from plant-based materials or recycled materials.
- Mechanical engineers design, develop, and test the tools, engines, machines, and other mechanical devices in electric vehicles. These devices may be components of electric vehicles, or machines that are used in the manufacture or repair of these vehicles. These engineers may focus on engines, electric motors, or other mechanical devices, such as transmissions, drivetrains, or steering systems.

Occupations

https://www.bls.gov/green/electric_vehicles/

Selected design and development occupations in transportation equipment manufacturing	Median annual wages, 2010[1]
Chemical engineers	$97,480
Electrical engineers	$87,580
Electronics engineers, except computer	$100,450
Industrial engineers	$77,160
Materials engineers	$89,000
Mechanical engineers	$81,290
Mechanical engineering technicians	$52,950
Mechanical drafters	$53,840
Software developers, applications	$94,680
Commercial and industrial designers	$67,790

[1] Occupational Employment Statistics data are available at www.bls.gov/oes. The data do not include benefits.

Vehicle Mechanics

Vehicle Design Steps

➢ Power and energy requirement from the propulsion unit is determined from a given set of vehicle cruising and acceleration specifications.

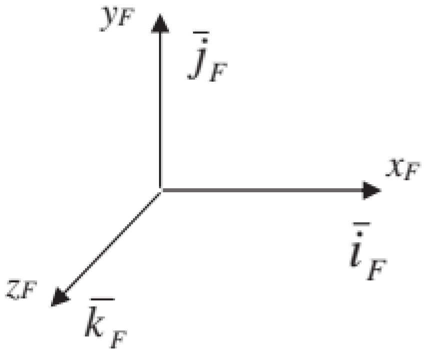
➢ Component level design:

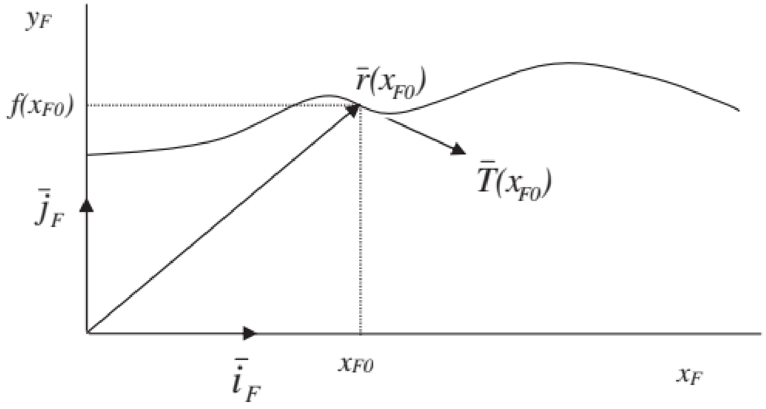
- Electrical and Mechanical engineers design the electric motor for EV or the combination of electric motor and internal combustion engine for HEVs.
- Power electronics engineers design the power conversion unit which links the energy source with the electric motor.
- Controls engineer working in conjunction with the power electronics engineer develops the propulsion control system.
- Electrochemists and Chemical engineers design the energy source based on the energy requirement and guidelines of the vehicle manufacturer.

➢ Vehicle design is an iterative process; several designers have to interact with each other to meet the design goals.

Roadway Fundamentals

- The road is considered to be straight, i.e. it is on the $x_F y_F$ plane of the fixed co-ordinate system; x_F is in the direction of the road, y_F is perpendicular to the road.
- The 2-dimentional roadway can be described as $y_F = f(x_F)$





i,j,k – cartesian coordinate
x,y,z – local coordinate wrt the moving vehicle

- The roadway position vector $r(x_F)$ & it's tangent vector are

$$\bar{r}(x_F) = x_F \bar{i}_F + f(x_F)\bar{j}_F \quad \text{for} \quad a \le x_F \le b.$$

$$\bar{T}(x_F) = \frac{d\bar{r}}{dx_F} = \bar{i}_F + \frac{df}{dx_F}\bar{j}_F$$

Roadway Percent Grade

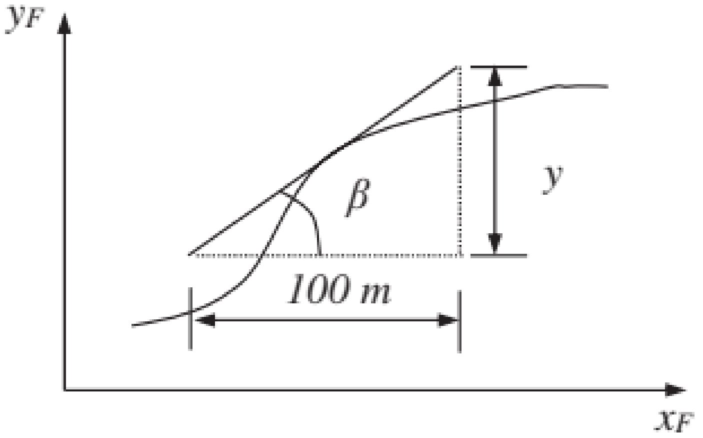


FIGURE 2.2 Grade of the roadway.

- The *roadway percent grade* is the vertical rise per 100 horizontal distance of roadway with both distances expressed in the same unit.
- The angle β of the roadway associated with the slope or grade is the angle between the tangent vector and the horizontal axis x_F .

$$\% \text{ grade} = \frac{\Delta y}{100 \text{ m}} 100\% = \Delta y\%.$$

$$\tan \beta = \frac{\Delta y}{100\ m}$$

Exercise 2.1

Roadway Percent Grade

A straight roadway has a profile in the $x_F y_F$ plane given by $f(x_F) = 3.9\sqrt{x_F}$ for $0 \le x_F \le 2$ miles. x_F and y_F are given in feet.

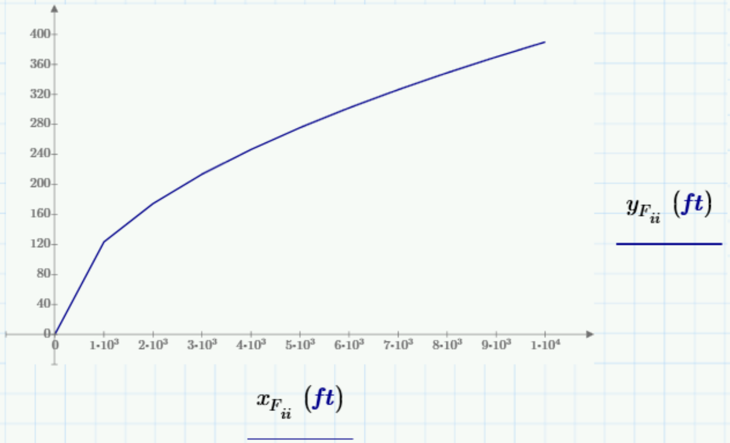
(a) Plot the roadway, (b) find $\beta(x_F)$, (c) find the percent grade at $x_F = 1$ mile and (d) find the tangential road length between 0 and 2 miles.

Ans. (b) $\tan^{-1}\dfrac{1.95}{\sqrt{x_F}}$; (c) 2.68% and (d) 10,580 ft.

$f(x_{FF}) := 3.9 \cdot \sqrt{x_{FF}}$ For $x_{F\_min} := 0 \cdot mi$ $x_{F\_max} := 2 \cdot mi$

a) Plot the roadway $ii := 0, 1..10$ $x_{ii} := 0 + ii \cdot 1000$

$1\ mi = (5.28 \cdot 10^3)\ ft$ $y_{F_{ii}} := f(x_{ii}) \cdot ft$ $x_{F_{ii}} := x_{ii} \cdot ft$



b) find $\beta(x_{FF})$

$tan\beta(x_{FF}) := f(x_{FF})'$

$$\frac{d}{dx_{FF}}\left(3.9 \cdot \sqrt{x_{FF}}\right) \rightarrow \frac{1.95}{\sqrt{x_{FF}}} \qquad tan\beta(x_{FF}) := \frac{1.95}{\sqrt{x_{FF}}} \cdot \left(\sqrt{ft}\right)$$

c) find percent grade at 1 mile $x_{Fo} := 1 \cdot mi = (5.28 \cdot 10^3)\ ft$

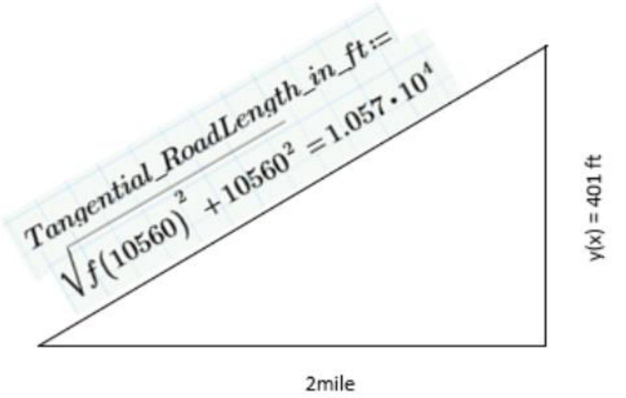
$pctGrade(x_{FF}) := tan\beta(x_{FF}) \cdot 100$ $pctGrade(x_{Fo}) = 2.684$

d) Find the tangential road length between 0 and 2 miles

$2\ mi = (1.056 \cdot 10^4)\ ft$

$f(10560) = 400.771$ $f(10560) \cdot ft = 400.771\ ft$

$Tangential\_RoadLength\_in\_ft := \sqrt{f(10560)^2 + 10560^2} = 1.057 \cdot 10^4$



Newton's Second Law of Motion

Fundamentals of a vehicle design are embedded in Newton's second law of motion:
"The acceleration of an object is proportional to the net force exerted on it."

$$\sum_i \overline{F}_i = m\overline{a}$$

The law is applied to the vehicle which is considered to be a particle mass located at the center of gravity (cg) of the vehicle

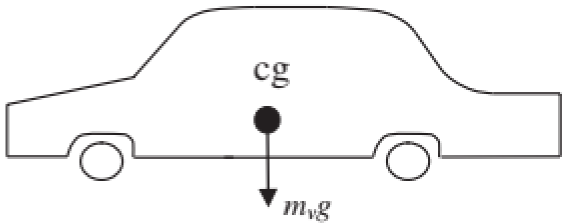


FIGURE 2.3 Center of gravity (cg) of a vehicle.

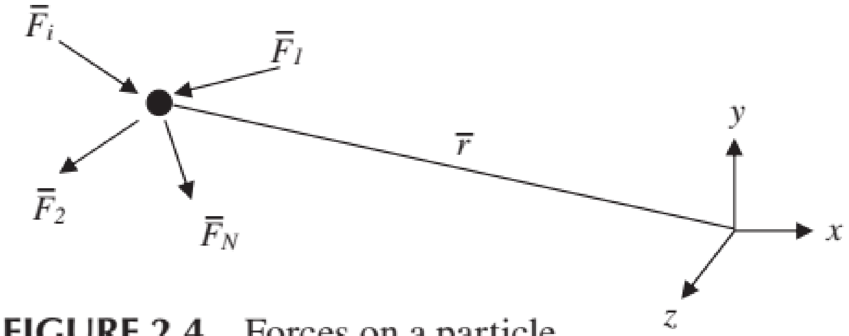
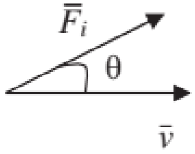


FIGURE 2.4 Forces on a particle.



For the position vector *r,* the velocity *v,* acceleration *a* and power input to the particle P_i for the *i*th force:

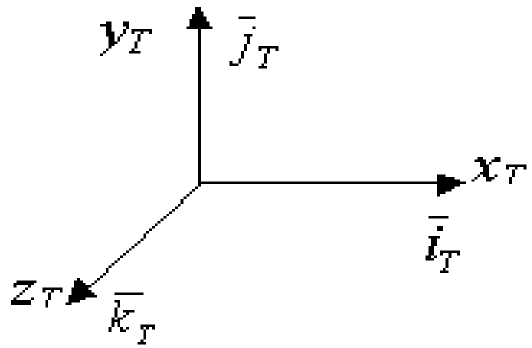
$$\overline{v} = \frac{d\overline{r}}{dt} \text{ and } \overline{a} = \frac{d\,\overline{v}}{dt}.$$

$$P_i = \overline{F}_i \cdot \overline{v} = \left|\overline{F}_i\right|\left|\overline{v}\right| \cos\theta$$

Vehicle Kinetics

• A tangential co-ordinate system is defined so that the forces acting on the vehicle can be defined in through a one-dimensional equation; here the *x*- and *y*- direction vectors are constantly changing with the slope of the roadway, while the z- direction vector remains the same as before.

• Newton's second law of motion is now applied to the cg of EV in the tangential co-ordinate system as



$$\sum \bar{F}_T = m\bar{a}_T = m\frac{d\,\bar{v}_T}{dt}$$

where *m* is the total vehicle mass. In terms of the components,

component tangent to the road

$$\sum \bar{F}_{xT} = m\frac{d\bar{v}_{xT}}{dt}$$

component normal to the road

$$\sum \bar{F}_{yT} = m\frac{d\bar{v}_{yT}}{dt}$$

since motion is in the *x-y* plane

$$\sum \bar{F}_{zT} = m\frac{d\bar{v}_{zT}}{dt} = 0$$

- v_{xT} is the vehicle tangential velocity.
- The normal velocity v_{yT} = 0, since gravitational force in the normal direction is balanced by the road reaction force.
- Therefore, a one-directional analysis can be used for vehicle propulsion in the x_T-*d*irection.
- The propulsion unit exerts a *tractive force* F_{TR} to propel the vehicle forward at a desired velocity. F_{TR} must overcome the opposing forces, viz. F_{gxT} the gravitational force, F_{roll} rolling resistance of the tires and F_{AD} the aerodynamic drag force; all summed together as the *road load force* F_{RL}.

$$F_{RL} = F_{gxT} + F_{roll} + F_{AD} \qquad (2.1)$$

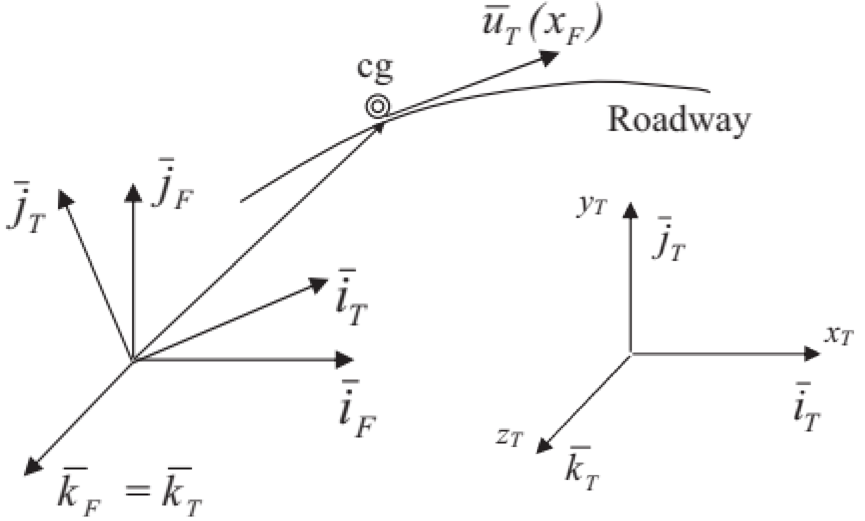


FIGURE 2.6 Tangential co-ordinate system and the unit tangent vector on a roadway.

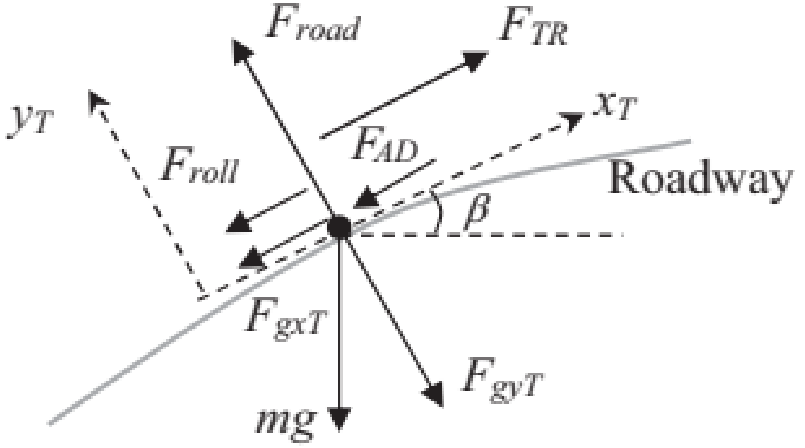


FIGURE 2.7 Forces acting on a vehicle.

Gravitational Force F_{gxT}

- The gravitational force, F_{gxT} depends on the slope of the roadway; it is positive when climbing a grade and is negative when descending a downgrade roadway.
- It can be expressed as,

$$F_{gxT} = m\, g \sin\beta. \qquad (2.2)$$

where m is the total mass of the vehicle,
g is the gravitational acceleration constant and
β is the grade angle with respect to the horizon.

Rolling Resistance Force F_{roll}

- The rolling resistance, F_{roll} is caused by flattening of the tire at the contact surface with the roadway, when the instantaneous center of rotation at the wheel moves forward from beneath the axle towards the direction of motion.
- This results in misalignment of the weight on the wheel and the road normal force and thus forms a couple that exerts a tangential retarding force F_{roll} on the wheel.
- The ratio of the rolling resistance force and the vertical load on the wheel is known as the *coefficient of rolling resistance* C_0. Typically $0.004<C_0<.02$ (unitless).

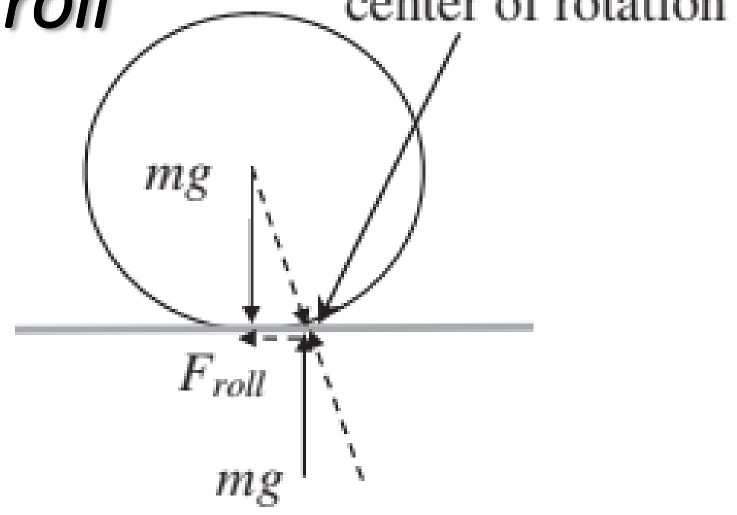


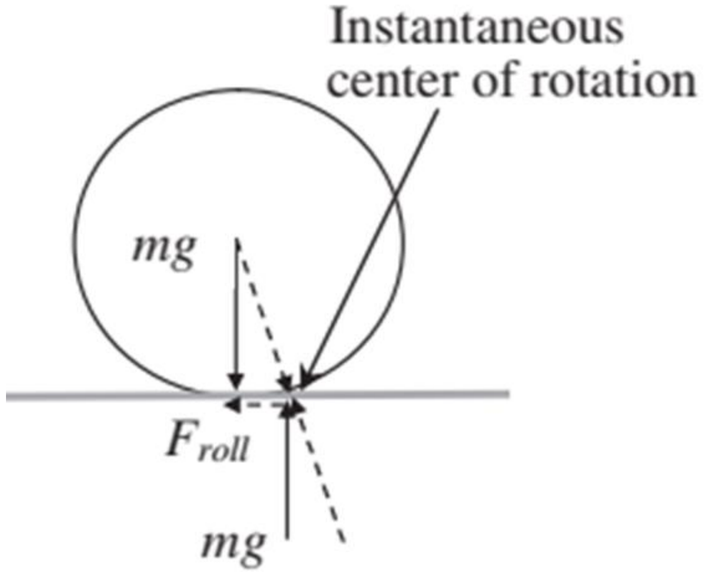
FIGURE 2.8 Rolling resistance force of wheels.

- The rolling resistance force is given by

$$F_{roll} = \begin{cases} \operatorname{sgn}[v_{xT}] mg \cos\beta \left(C_0 + C_1 v_{xT}{}^2\right) & \text{if } v_{xT} \neq 0 \\ (F_{TR} - F_{gxT}) & \text{if } v_{xT} = 0 \text{ and } \left|F_{TR} - F_{gxT}\right| \leq C_0 mg \cos\beta \\ \operatorname{sgn}[F_{TR} - F_{gxT}](C_0 mg \cos\beta) & \text{if } v_{xT} = 0 \text{ and } \left|F_{TR} - F_{gxT}\right| > C_0 mg \cos\beta \end{cases} \tag{2.3}$$

$C_o\,mg$ is the maximum rolling resistance at standstill. The *sgn[v_{xT}]* *is* the signum function given as

$$\operatorname{sgn}[v_{xT}] = \begin{cases} 1 & \text{if } v_{xT} \geq 0 \\ -1 & \text{if } v_{xT} < 0 \end{cases}$$



Aerodynamic Drag Force F_{AD}

- The aerodynamic drag force, F_{AD} is the viscous resistance of the air against the motion.

$$F_{AD} = \mathrm{sgn}[v_{xT}]\{0.5\rho C_D A_F (v_{xT} + v_0)^2\} \tag{2.4}$$

ρ : Air density (kg/m^3)
C_D : Aerodynamic drag coefficient ($0.2 < C_D < 0.4$)
A_F : Equivalent frontal area of the vehicle (in m^2)
v_0 : Head-wind velocity (in m/s)

Dynamics of Vehicle Motion

The dynamic equation of motion in the tangential direction

$$k_m m \frac{dv_{xT}}{dt} = F_{TR} - F_{RL} \qquad (2.5)$$

where

k_m is the rotational inertia coefficient to compensate for the apparent increase in the vehicle's mass due to the onboard rotating mass.

- Typically, 1.08< k_m< 1.1.

$$\boxed{k_m = 1 + \frac{4J_w}{m_v r_{wh}^2} + \frac{J_{eng}\xi_{eng}^2\xi_{FD}^2}{m_v r_{wh}^2} + \frac{J_{em}\xi_{em}^2\xi_{FD}^2}{m_{cv} r_{wh}^2}}$$

Vehicle Dynamics Simulation

The dynamic equations can be represented in the state space form for simulation of an EV/HEV system.

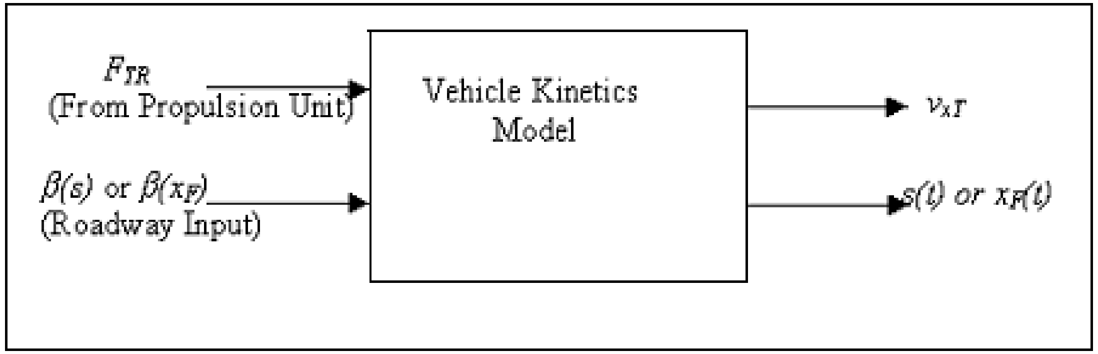
- One of the state variables is v_{xT}

- The second equation needed for simulation is the velocity equation where either *s* or x_F can be used as the state variable.

- $$\frac{ds}{dt} = v_{xT} \qquad (2.6)$$ or $$\frac{dx_F}{dt} = \frac{v_{xT}}{\sqrt{1+\left[\frac{df}{dx_F}\right]^2}} \qquad (2.7)$$

  The choice depends on whether roadway slope is given as $\beta = \beta(s)$ *or* $\beta = \beta(x_F)$.

Vehicle Dynamics Simulation Model



Inputs to the simulation model:

- Roadway slope β
- Propulsion Force $\boldsymbol{F_{TR}}$
- Road Load Force $\boldsymbol{F_{RL}}$

Outputs:

- Vehicle velocity v_{xT}
- Distance traversed s or x_F

End of Lecture 5

Propulsion Power

- Torque at the vehicle wheels is obtained from the power relation

$$\text{Power} = T_{TR} \cdot \omega_{wh} = F_{TR} \cdot v_{xT} \ [\text{W}]$$

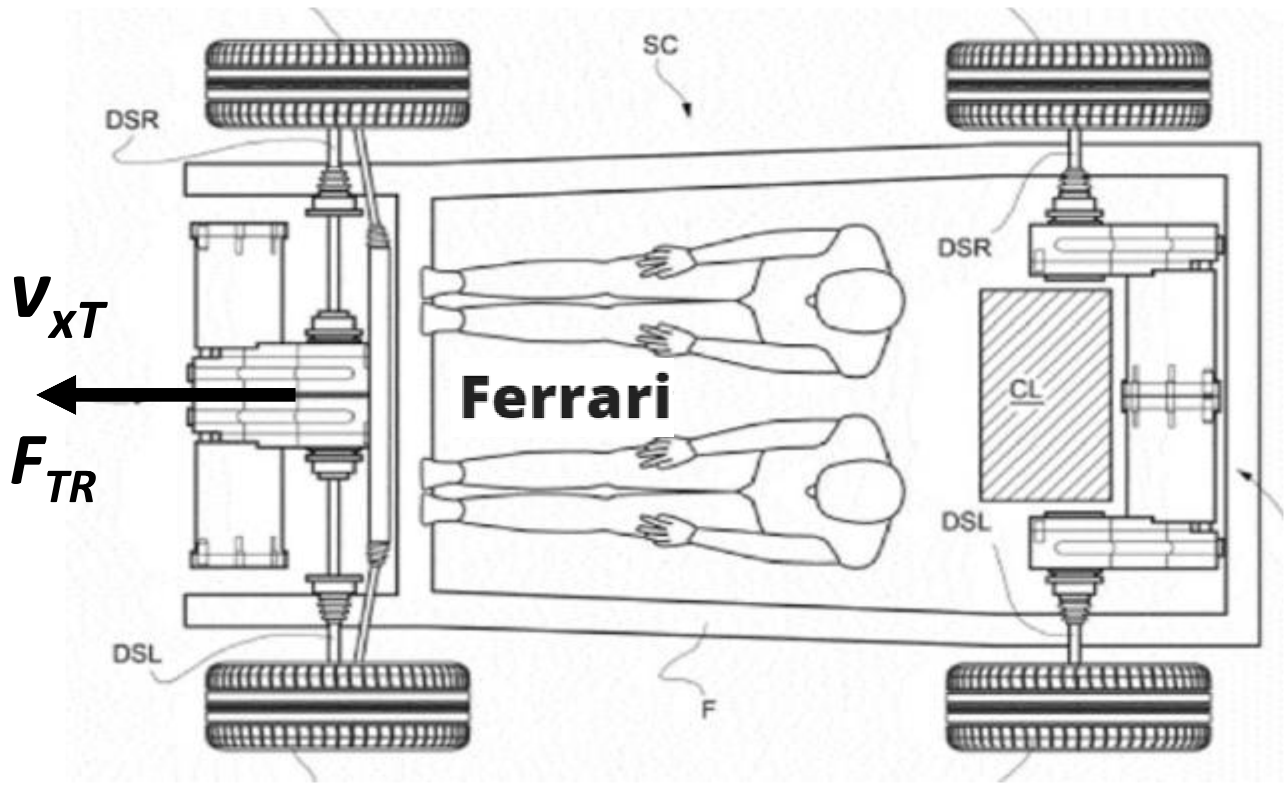
where

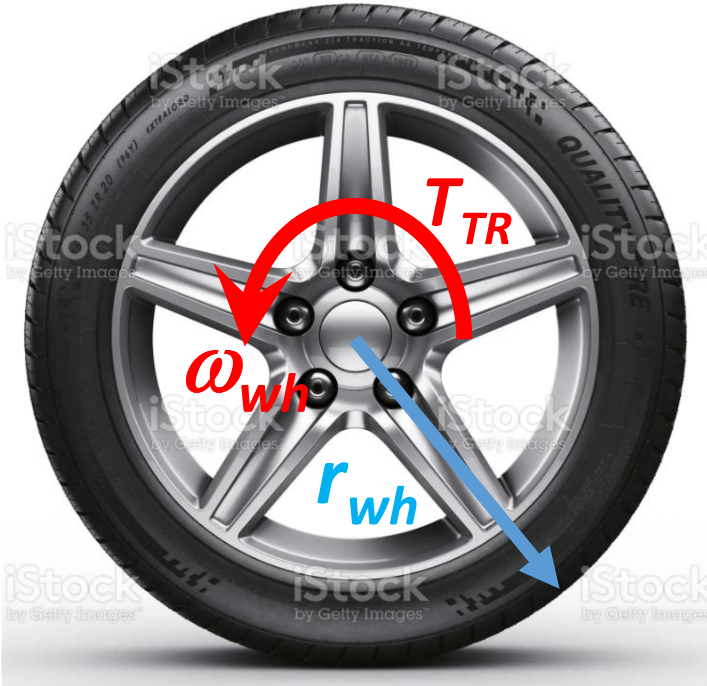
T_{TR} is the tractive torque in *N-m,*
ω_{wh} is the angular velocity in *rads/sec*,
F_{TR} is in *N* and
v_{xT} is in *m/sec*.

- The angular velocity and the vehicle speed is related by

$$v_{xT} = \omega_{wh} \cdot r_{wh}$$

$\boldsymbol{r_{wh}}$ is the radius of the wheel





$$v_{xT} = \omega_{wh}\, r_{wh}$$

r_{wh} is the radius of the wheel

Advantage of Electric Propulsion

- The wide speed range operation of electric motors enabled by power electronics control makes it possible to use a single gear-ratio transmission, eliminating multiple-gears

  $\Rightarrow$ Great advantage in EV propulsion system

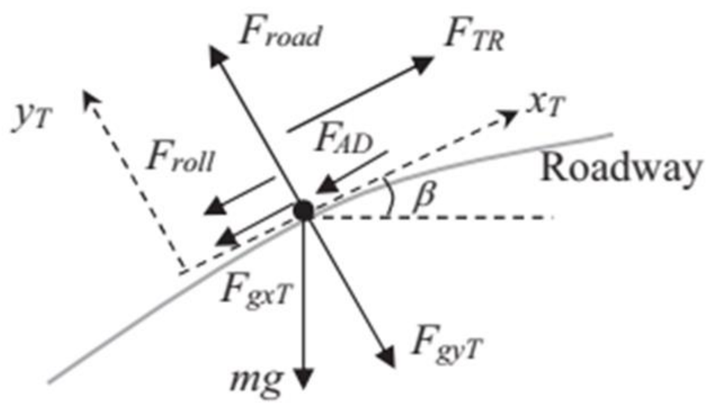
- The gear ratio depends on the maximum motor speed and higher motor speed is desired for higher power density of motor

  $\Rightarrow$ A compromise is necessary between the maximum motor speed and the gear-ratio to optimize the cost.

Force-Velocity Characteristics

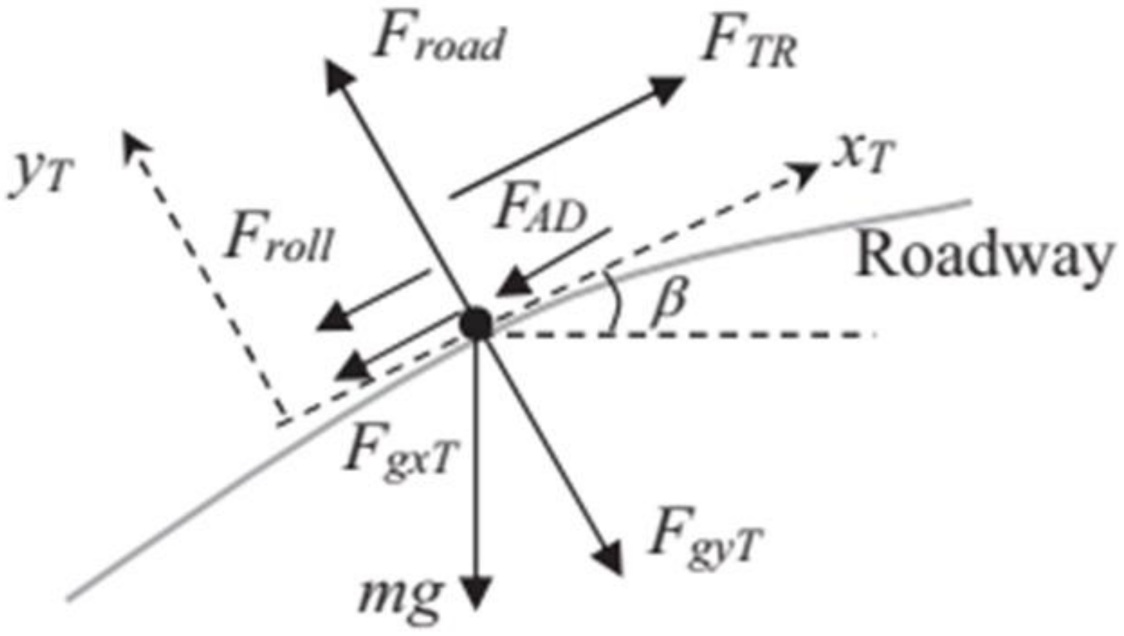
- For an efficient design of the propulsion unit, the designer must know the force needed :
  - to accelerate the vehicle to a cruising speed
  - within a certain time and
  - then to propel the vehicle forward at the rated steady- state speed
  - and at the maximum speed on a specified slope.
- A useful design information is contained in the vehicle speed versus time and the steady state tractive force versus constant velocity (accel = 0) characteristics.

$F_{ROLL} = \text{sgn}(V)\, mg(C_0 + C_1 V^2)$



V is the steady-state velocity

Force-Velocity Characteristics



F_{ROLL} = sgn(V) mg(C_0 + C_1 V^2)

The steady state tractive force versus constant velocity (accel = 0)

$$F_{TR} - F_{AD} - F_{ROLL} - F_{gxT} = 0$$
$$\Rightarrow F_{TR} = \underbrace{mg\left[\sin\beta + \boxed{C_0\,\mathrm{sgn}(V)\right] + \mathrm{sgn}(V)\left[mgC_1}\right.}_{} + \tfrac{\rho}{2}C_D A_F\Big]\,V^2.$$

F_{gxT} $\qquad$ F_{ROLL} $\qquad$ F_{AD}

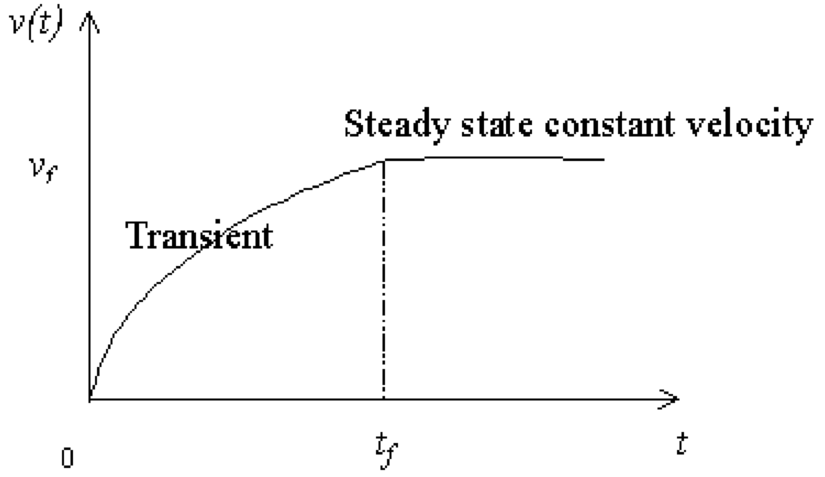
V is the steady-state velocity

Force-Velocity Characteristics

The tractive force vs. steady-state velocity characteristics can be obtained form the equation of motion, with zero acceleration

$$F_{TR} - F_{AD} - F_{ROLL} - F_{gxT} = 0$$
$$\Rightarrow F_{TR} = mg\left[\sin\beta + C_0\,\mathrm{sgn}(V)\right] + \mathrm{sgn}(V)\left[mgC_1 + \tfrac{\rho}{2}C_D A_F\right]V^2.$$

F_{gxT} *Constant ≠ f(V)*   *It varies with v = f(v)* F_{AD}

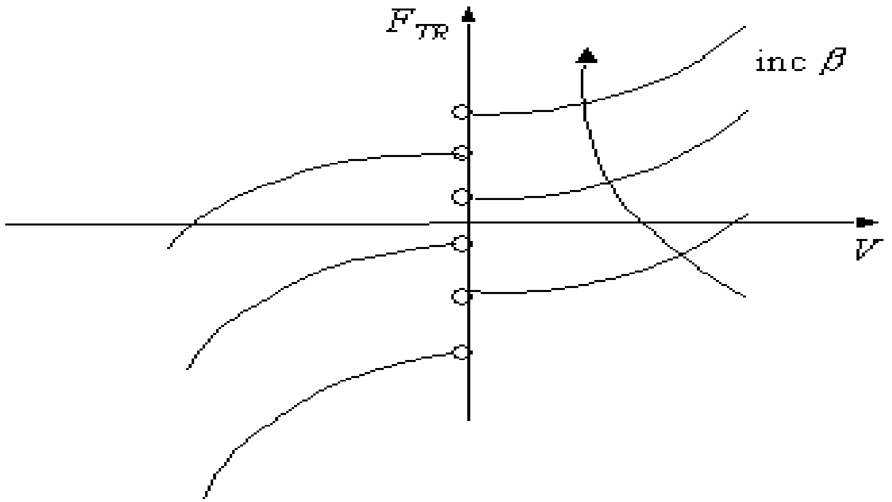


$$\frac{dF_{TR}}{dV} = 2V\,\mathrm{sgn}(V)\left(\frac{\rho C_D A_F}{2} + mgC_1\right) > 0 \quad \forall\, V$$

$\Rightarrow$ Slope of F_{TR} is always positive

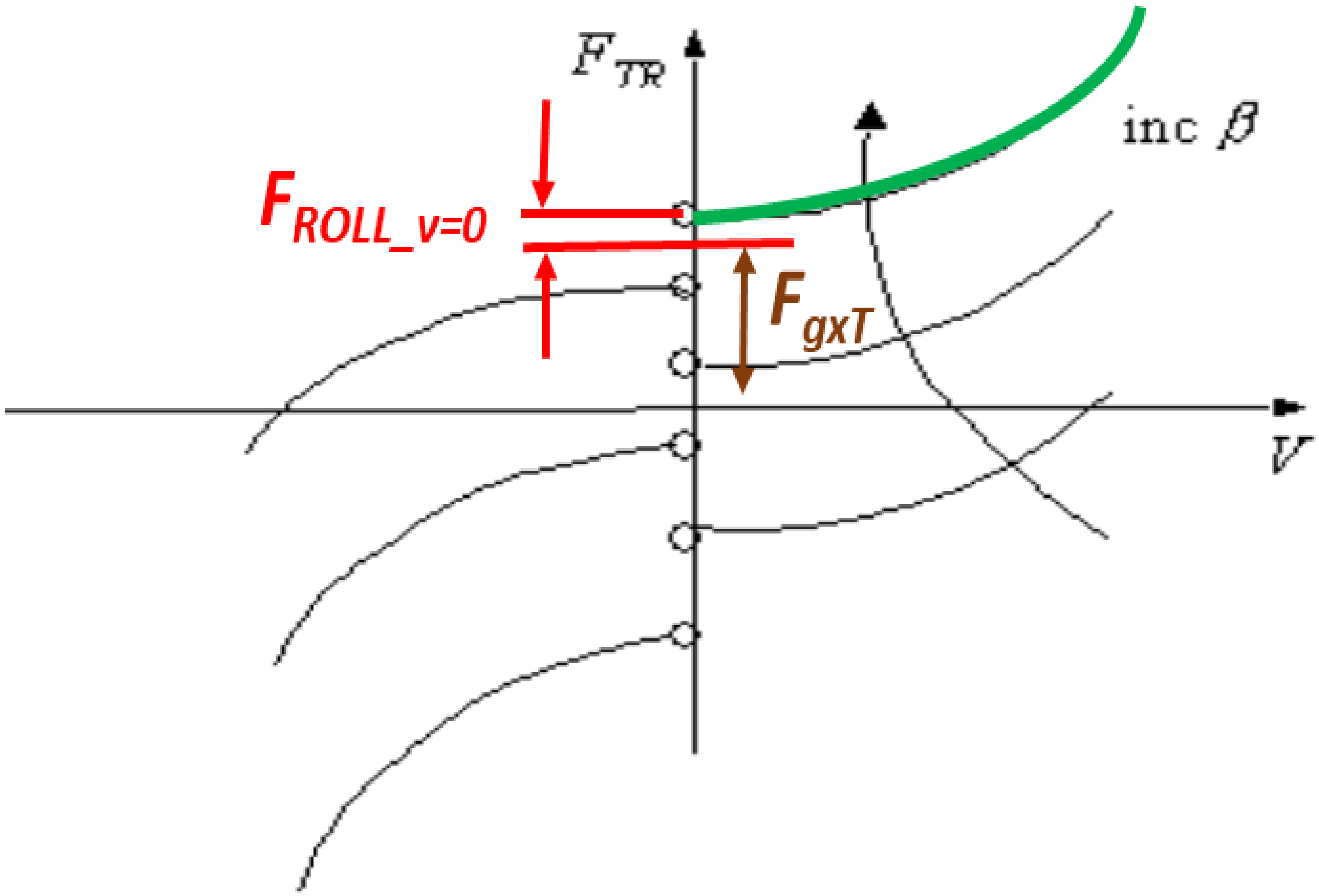
$$\lim_{V\to 0^+} F_{TR} \neq \lim_{V\to 0^-} F_{TR}$$

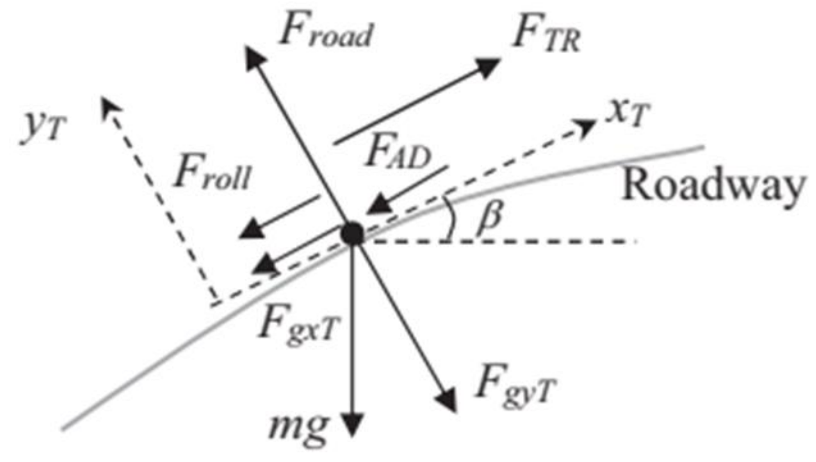
$\Rightarrow$ Discontinuity at zero velocity is due to rolling resistance



Force-Velocity Characteristics

The tractive force vs. steady-state velocity characteristics can be obtained form the equation of motion, with zero acceleration





$$F_{TR} - F_{AD} - F_{ROLL} - F_{gxT} = 0$$

$$\Rightarrow F_{TR} = mg\left[\sin\beta + \boxed{C_0 \operatorname{sgn}(V)\right] + \operatorname{sgn}(V)\left[mgC_1} + \tfrac{\rho}{2} C_D A_F\right] V^2.$$

F_{gxT} F_{ROLL} F_{AD}

$F_{ROLL} = F_{ROLL\_v=0} + F_{ROLL\_v \neq 0}$

$F_{ROLL\_v=0} = sgn(V)\ mgC_0$

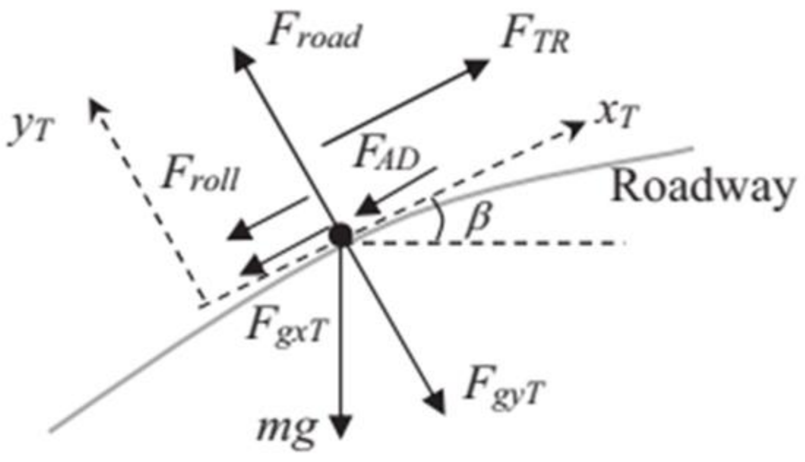
$F_{ROLL\_v \neq 0} = sgn(V)\ mg\ C_1 V^2$

$F_{gxT} = mg \sin \beta$

$$F_{AD} = \frac{\rho}{2} C_D A_F V^2$$

Maximum Gradability

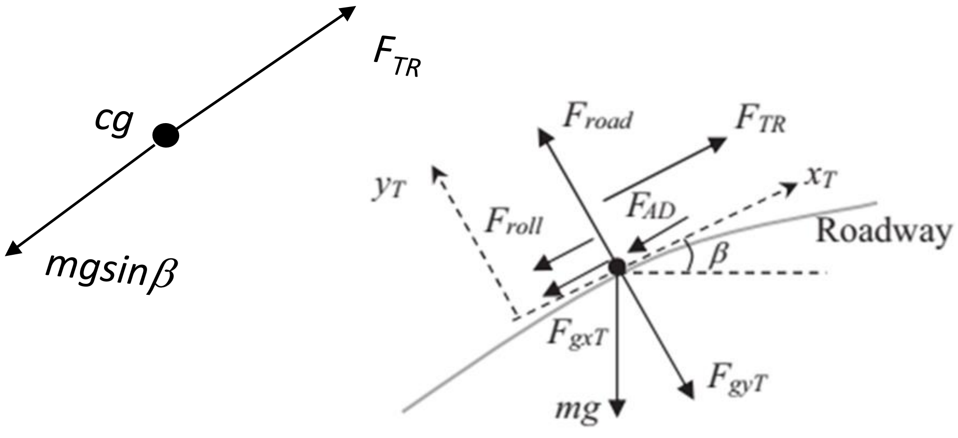
- The maximum grade that a vehicle will be able to overcome with the maximum force available from the propulsion unit is an important design criterion as well as performance measure.
- The vehicle is expected to move forward very slowly when climbing a steep slope, and hence, the following assumptions for maximum gradeability are made:
  - The vehicle moves very slowly $\Rightarrow v \cong 0$.
  - $F_{AD,}$ F_{roll} are negligible.
  - The vehicle is not accelerating, i.e. $dv/dt = 0.$
  - F_{TR} is the maximum tractive force delivered by motor at or near zero speed.



Maximum Gradability

With the assumptions, at near stall conditions

$$\Sigma F = 0 \quad \Rightarrow \quad F_{TR} - F_{gxT} = 0 \quad \Rightarrow \quad F_{TR} = mg \sin\beta$$

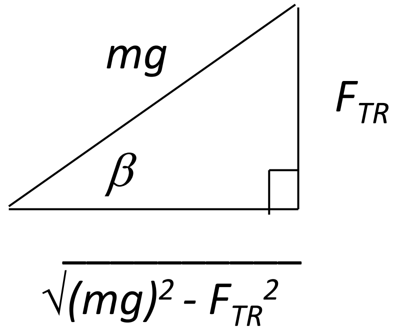


Force Diagram to determine maximum gradability

The maximum percent grade is

$$\text{max \% grade} = 100 \tan\beta$$

$$\Rightarrow \text{max \% grade} = \frac{100\, F_{TR}}{\sqrt{(mg)^2 - F_{TR}^2}}$$



Forces w. r. t. grade

Velocity and Acceleration

- The vehicles are typically designed with a certain objective, such as maximum acceleration on a given roadway slope on a typical weather condition.
- Energy required from propulsion unit depends on acceleration and road load force
- Maximum acceleration is limited by maximum tractive power and roadway condition
- Road load condition is unknown in a real-world scenario
- However, significant insights about vehicle velocity profile and energy requirement can be obtained by considering simplified scenarios.

Scenario 1: Constant F_{TR}, Level Road

- Constant F_{TR}, Level Road:
  - The level road condition implies that $\beta(s)=0$; $F_{gxT}=0$.
  - EV is assumed to be at rest initially; also the initial FTR is assumed to be capable of overcoming the initial rolling resistance.

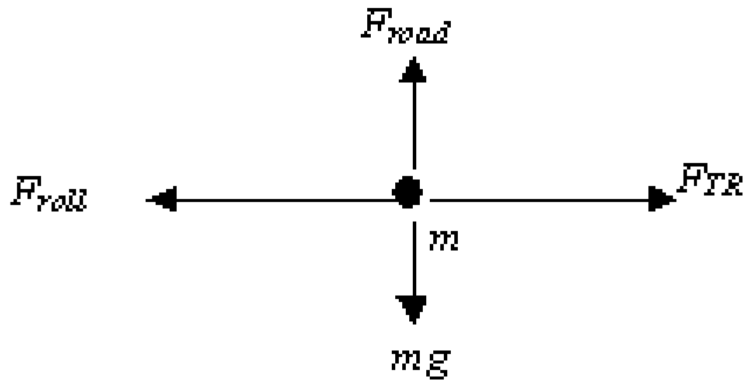
At $t > 0 \Rightarrow$

$$\Sigma F = m\frac{dv}{dt} \Rightarrow F_{TR} - F_{AD} - F_{roll} = m\frac{dv}{dt}$$

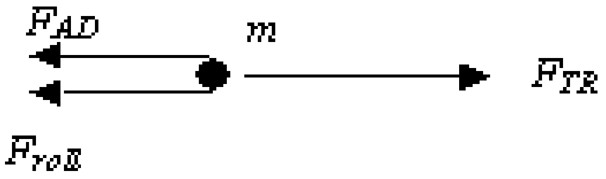
$$\Rightarrow F_{TR} - \mathrm{sgn}\,[v(t)]\frac{\rho}{2}C_D A_F v^2(t) - \mathrm{sgn}[v(t)]mg\left(C_0 + C_1 v^2(t)\right) = m\frac{dv}{dt}$$

$$\boxed{\frac{dv}{dt} = \left(\frac{F_{TR}}{m} - gC_0\right) - \left[\frac{\rho}{2m}C_D A_F + gC_1\right]v^2}$$

K_1 K_2



(a) Free body diagram at $t=0$.



(b) Forces on the vehicle at $t>0$.

Solving for acceleration, dv/dt

$$\frac{dv}{dt}=\left(\frac{F_{TR}}{m}-gC_0\right)-\left[\frac{\rho}{2m}C_DA_F+gC_1\right]v^2$$

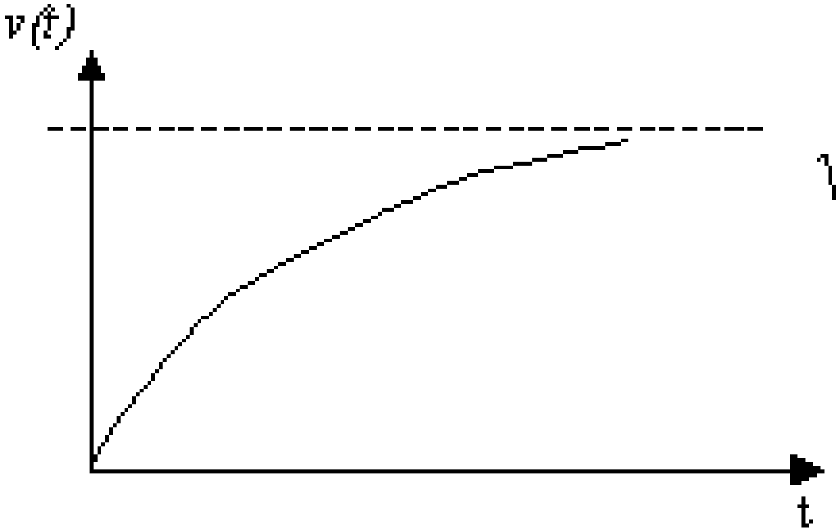
$$\frac{dv}{dt}=K_1-K_2v^2$$

$$K_1=\frac{F_{TR}}{m}-gC_0>0$$
$$K_2=\frac{\rho}{2m}C_DA_F+gC_1>0$$

The velocity profile:

$$v(t)=\sqrt{\frac{K_1}{K_2}}\tanh\left(\sqrt{K_1K_2}\,t\right)$$

$$V_T=\lim_{t\to\infty}v(t)=\sqrt{\frac{K_1}{K_2}}\Rightarrow\sqrt{K_1K_2}=K_2V_T$$



$\sqrt{\frac{K_1}{K_2}}$ = V_T

=(peak velocity)

Distance Traversed:

The speed is the derivative of the distance traversed $$\frac{ds(t)}{dt} = v(t) = V_T \tanh(K_2 V_T t)$$

$$s(t) = \int_{t_o}^{t_f} v(t)\, dt$$

$$s(t) = \frac{1}{K_2} \ln\left[\cosh(K_2 V_T t)\right]$$

The time to reach the desired velocity and distance traversed during that time is given by

$$t_f = \frac{1}{K_2 V_T} \cosh^{-1}\left[e^{(K_2 s_f)}\right]$$

$$s_f = \frac{1}{K_2} \ln\left[\cosh(K_2 V_T t_f)\right]$$

Tractive power:

The instantaneous tractive power delivered by the prop. unit is

$P_{TR}(t) = F_{TR}\, v(t).$

$$\Rightarrow \quad P_{TR}(t) = F_{TR} V_T \tanh\left(\sqrt{K_1 K_2}\, t\right) = P_T \tanh\left(\sqrt{K_1 K_2}\, t\right)$$

P_T =(power at V_T velocity)

The mean tractive power over the acceleration interval Δt *is*

$$\overline{P_{TR}} = \frac{1}{t_f} \int_0^{t_f} P_{TR}(t)\, dt$$

$$\Rightarrow \overline{P_{TR}} = \frac{P_T}{t_f} \frac{1}{\sqrt{K_1 K_2}} \ln\left[\cosh\left(\sqrt{K_1 K_2}\, t_f\right)\right]$$

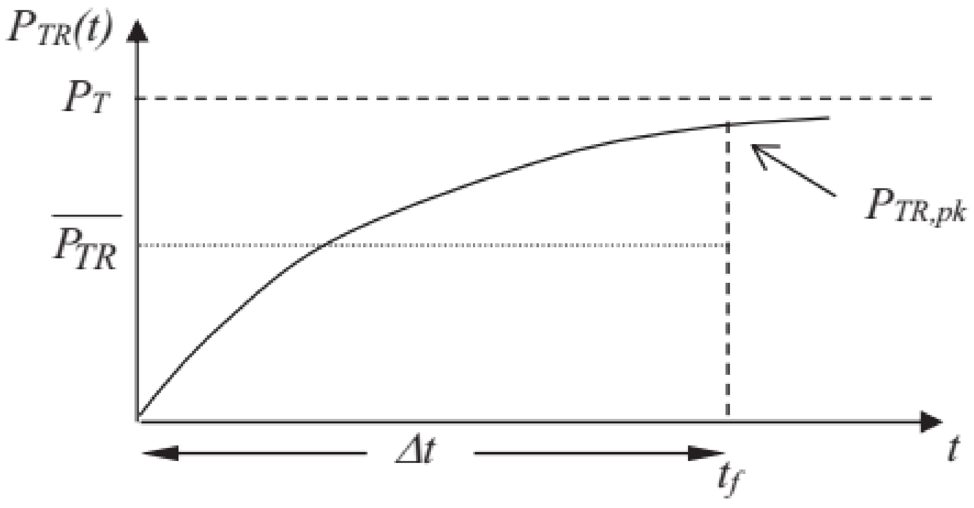


FIGURE 2.15 Acceleration interval $\Delta t = t_f - 0$.

Energy required during an interval of the vehicle can be obtained from the integration of the instantaneous power equation as

$$\int_{e_{TR}(0)}^{e_{TR}(t_f)} de_{TR} = \int_{t=0}^{t_f} P_{TR} dt$$

$$\Rightarrow \Delta e_{TR} = t_f \overline{P_{TR}}$$

Example 2.1

An EV has the following parameter values:

$$m = 800 \text{ kg}, C_D = 0.2, A_F = 2.2 \text{ m}^2, C_0 = 0.008, C_1 = 1.6 * 10^{-6} \text{ s}^2/\text{m}^2,$$

Also, take density of air ρ = 1.18 kg/m³, and acceleration due to gravity g = 9.81 m/s².

The vehicle is on level road. It accelerates from 0 to 65 mi/h in 10 s such that its velocity profile is given by

$$v(t) = 0.29055t^2 \quad \text{for } 0 \le t \le 10 \text{ s.}$$

a. Calculate $F_{TR}(t)$ for $0 \le t \le 10$ s.
b. Calculate $P_{TR}(t)$ for $0 \le t \le 10$ s.
c. Calculate the energy loss due to nonconservative forces E_{loss}.
d. Calculate Δe_{TR}.

Example 2.1

$$F_{TR} - F_{AD} - F_{ROLL} - F_{gxT} = m\,dv/dt$$

For a level road β=0 thus $F_{gxT} = 0$

$F_{AD} = 0.5\,\rho\,A_F\,C_D\,v^2$, $F_{gxT} = m\,g\,\sin(\beta) = 0$

$F_{roll\_0} = m\,g\,C_0$, $F_{roll\_1} = m\,g\,C_1 v^2$

An EV has the following parameter values:

$m := 800\,kg$ $\quad C_D := 0.2$ $\quad A_F := 2.2\,m^2$ $\quad C_o := 0.008$ $\quad C_1 := 1.6 \cdot 10^{-6} \left(\frac{s}{m}\right)^2$

$\rho_o := 1.18\,\frac{kg}{m^3}$ $\quad g_o := 9.81\,\frac{m}{s^2}$

The vehicle is on level road. It accelerates from 0 to 65 mph in 10 seconds, with its velocity profile is given by

$v(t) := 0.29055 \cdot t^2$ for $0 \le t \le 10$ s

a) Calculate $F_{TR}(t)$ for $0 \le t \le 10$ s

$$F_{TR} - F_{AD} - F_{roll} = m\frac{dv}{dt}$$

$$\frac{d}{dt}v(t) \rightarrow 0.5811 \cdot t$$

F_{AD}

$$m\,\frac{d}{dt}v(t) \rightarrow 464.88 \cdot t \qquad \frac{\rho_o}{2} \cdot C_D \cdot A_F \cdot v(t)^2 \rightarrow 0.021915250929 \cdot t^4$$

$$m \cdot g_o \cdot \left(C_o + C_1 \cdot v(t)^2\right) \rightarrow 0.001060036297632 \cdot t^4 + 62.784$$

F_{roll}

$$F_{TR}(t) = m\frac{dv}{dt} + \frac{\rho}{2}C_D A_F v^2 + mg(C_0 + C_1 v^2)$$

$$F_{TR}(t) := 464.88 \cdot t + 0.021915250929 \cdot t^4 + 0.001060036297632 \cdot t^4 + 62.784$$

$$F_{TR}(t) \rightarrow 464.88 \cdot t + 0.022975287226632 \cdot t^4 + 62.784$$

b) Calculate $P_{TR}(t)$ for $0 \le t \le 10$ s

$$P_{TR}(t) := F_{TR}(t) \cdot v(t)$$

$$P_{TR}(t) \rightarrow 0.29055 \cdot t^2 \cdot \left(464.88 \cdot t + 0.022975287226632 \cdot t^4 + 62.784\right)$$

Example 2.1

$$F_{TR} - F_{AD} - F_{ROLL} - F_{gxT} = m\,dv/dt$$

For a level road β=0 thus $F_{gxT} = 0$

$F_{AD} = 0.5\,\rho A_F C_D v^2$, $F_{gxT} = m\,g\,\sin(\beta) = 0$

$F_{roll\_0} = m\,g\,C_0$, $F_{roll\_1} = m\,g\,C_1 v^2$

An EV has the following parameter values:

$m := 800\,kg$ $\quad C_D := 0.2$ $\quad A_F := 2.2\,m^2$ $\quad C_o := 0.008$ $\quad C_1 := 1.6 \cdot 10^{-6}\left(\dfrac{s}{m}\right)^2$

$\rho_o := 1.18\,\dfrac{kg}{m^3}$ $\quad g_o := 9.81\,\dfrac{m}{s^2}$

The vehicle is on level road. It accelerates from 0 to 65 mph in 10 seconds, with its velocity profile is given by

$v(t) := 0.29055 \cdot t^2$ for $0 \le t \le 10$ s

c) Calculate the energy loss due to nonconservative forces E_{loss}

$$E_{loss} = \int_0^{10} \boxed{v(F_{AD} + F_{roll})}\,dt$$

$$P_{loss}(t) := v(t) \cdot \left(\left(0.021915250929 \cdot t^4\right) + \left(0.001060036297632 \cdot t^4 + 62.784\right)\right)$$

$$P_{loss}(t) \rightarrow 0.29055 \cdot t^2 \cdot \left(0.022975287226632 \cdot t^4 + 62.784\right)$$

$$E_{loss} := \int_0^{10} P_{loss}(t)\,dt = 1.562 \cdot 10^4\ Joule$$

d) The kinetic energy of the vehicle is

$$\Delta KE := \frac{1}{2} \cdot m \cdot \left(v(10)^2 - v(0)^2\right) = 3.377 \cdot 10^5\ \ Joule$$

Thus, the change in tractive energy is

$$\Delta e_{TR} := E_{loss} + \Delta KE = 3.533 \cdot 10^5\ \ Joule$$

Exercise 2.2

An EV has the following parameter values $\rho = 1.16\,\text{kg/m}^3$, $m = 692\,\text{kg}$, $C_D = 0.2$, $A_F = 2\,\text{m}^2$, $g = 9.81\,\text{m/s}^2$, $C_0 = 0.009$ and $C_1 = 1.75 \times 10^{-6}\,\text{s}^2/\text{m}^2$. The EV undergoes constant F_{TR} acceleration on a level road starting from rest at $t = 0$. The maximum continuous F_{TR} that the electric motor is capable of delivering to the wheels is 1,548 N.

a. Find $V_T\,(F_{TR})$ and plot it.
b. If $F_{TR} = 350\,\text{N}$, (i) find V_T, (ii) plot $v(t)$ for $t \geq 0$, (iii) find t_{VT}, (iv) calculate the time required to accelerate from 0 to 60 mph and (v) calculate P_{TRpk}, $\overline{P_{TR}}$, Δe_{TR} corresponding to acceleration to 0.98 V_T.

Ans. (a) $V_T\,(F_{TR}) = 53.2\sqrt{1.45 \times 10^{-3}\,F_{TR} - 0.0883}$ m/s, (b) (i) 34.4 m/s, (ii) $v(t) = 34.4\tanh(1.22 \times 10^{-2}\,t)$ m/s, (iii) 189 s, (iv) 85.6 s and (v) $P_{TRpk} = 11.8\,\text{kW}$, $\overline{P_{TR}} = 8.46\,\text{kW}$, $\Delta e_{TR} = 1.61$ MJ.

a) Find $V_T(F_{TR})$ and plot it

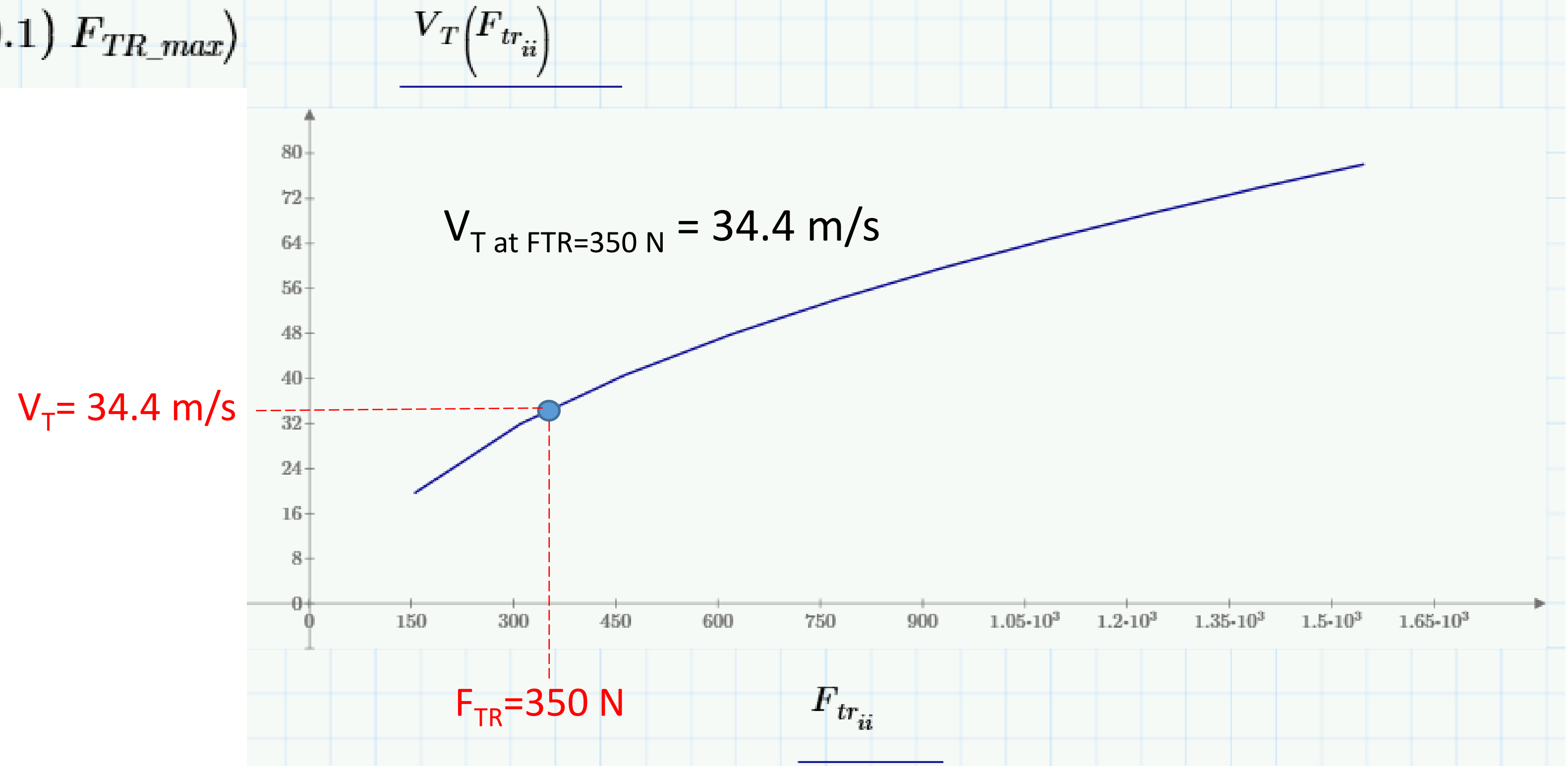
$$V_T = \lim_{t \to \infty} v(t) = \sqrt{\frac{K_1}{K_2}} \Rightarrow \sqrt{K_1 K_2} = K_2 V_T$$

$$K_1(F_{TR}) := \frac{F_{TR}}{m} - g_o \cdot C_o \qquad K_2 := \frac{\rho_o}{2 \cdot m} \cdot C_D \cdot A_F + g_o \cdot C_1 = 3.524 \cdot 10^{-4}$$

for $0 \le t \le 10$ s

$$V_T(F_{TR}) := \sqrt{\frac{K_1(F_{TR})}{K_2}} \to \sqrt{4.1003787478845635375 \cdot F_{TR} - 250.51952823830385539}$$

$$ii := 0, 1 .. 10 \qquad F_{tr_{ii}} := 0 + ii \cdot ((0.1)\ F_{TR\_max})$$



b) If $F_{TR\_o} := 350\,\text{Newton}$, (i) find V_T

$$V_T(F_{TR\_o}) = 34.418 \quad \frac{m}{s}$$

$F_{TR\_o} := 350\,\text{Newton}$,
(ii) plot v(t) for t ≥ 0 s

$$v(t) = \sqrt{\frac{K_1}{K_2}} \tanh\left(\sqrt{K_1 K_2}\, t\right)$$

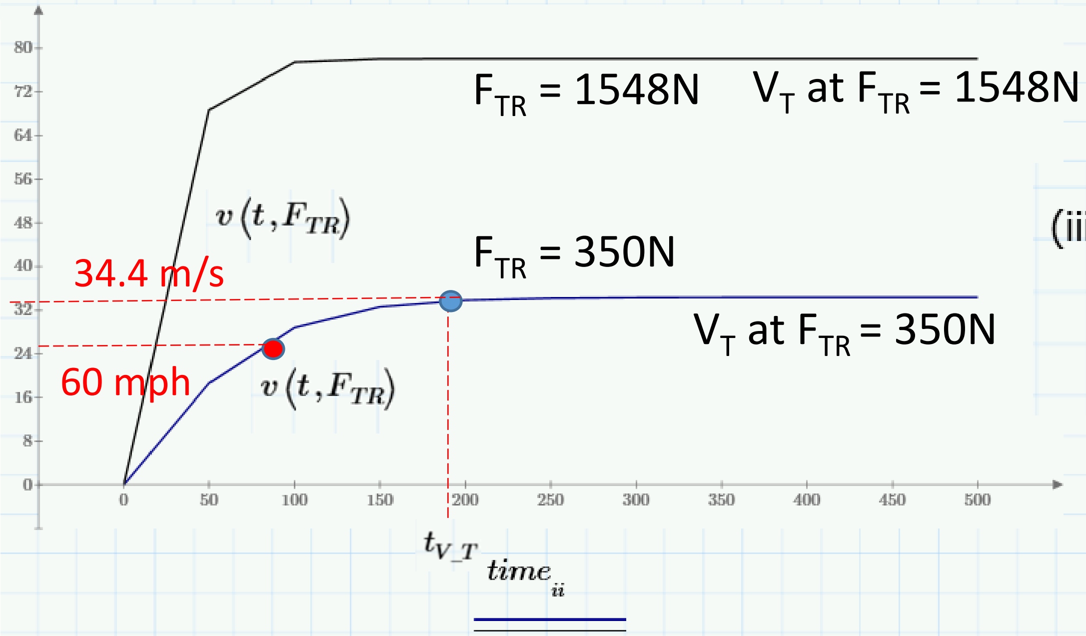
$$time_{ii} := 0 + ii \cdot 50$$

$$10^5\ s = 27.778\ hr$$

$$v(t, F_{TR}) := V_T(F_{TR}) \cdot \tanh\left(\sqrt{K_1(F_{TR}) \cdot K_2} \cdot t\right)$$

$$K_1(F_{TR\_o}) = 0.417 \qquad K_2 = 3.524 \cdot 10^{-4} \qquad \sqrt{K_1(F_{TR\_o}) \cdot K_2} = 0.012$$

$$v(t, F_{TR\_o}) \rightarrow 34.418207877826721365 \cdot \tanh(0.012129926935846345977 8 \cdot t)$$



(iii) find t_{V_T} (time to reach V_T)

$$t_{V_T} = \frac{2.3}{\sqrt{K_1 K_2}} \text{ or } \frac{2.3}{K_2 V_T}.$$

$$t_{V\_T} := \frac{2.3}{K_2 \cdot V_T(F_{TR\_o})} = 189.614 \quad \text{seconds}$$

(iv) find the time required to accelerate from 0 to 60 mph

$$60\ \frac{mi}{hr} = 26.822\ \frac{m}{s} \qquad v_o(t) := 34.42 \cdot \tanh(0.0122 \cdot t)$$

$$t_{60mph} := \frac{1}{0.0122} \cdot \operatorname{atanh}\left(\frac{26.822}{34.418}\right) \qquad t_{60mph} = 85.54 \quad \text{seconds}$$

(v) calculate P_{TRpk} $\quad P_{TR}(t) = F_{TR} V_T \tanh\left(\sqrt{K_1 K_2} t\right) = P_T \tanh\left(\sqrt{K_1 K_2} t\right)$

$P_{TR}(t) = F_{TR}\, v(t)$

$$P_T := F_{TR\_o} \cdot V_T(F_{TR\_o}) = 1.205 \cdot 10^4$$

$$P_T \cdot \tanh\left(\sqrt{K_1(F_{TR\_o}) \cdot K_2} \cdot t_{V\_T}\right) = 1.181 \cdot 10^4 \quad \text{watt}$$

or you can use this equation

$$P_{TR}(P_T, t) := P_T \cdot \tanh\left(\sqrt{K_1(F_{TR\_o}) \cdot K_2} \cdot t\right)$$

$$P_{TRpk} := P_{TR}(P_T, t_{V\_T}) = 1.181 \cdot 10^4 \quad \text{watt}$$

Calculate average traction power

$$\overline{P_{TR}} = \frac{P_T}{t_f}\frac{1}{\sqrt{K_1K_2}}\ln\left[\cosh\left(\sqrt{K_1K_2}\,t_f\right)\right]$$

$$P_{TR\_ave}(P_T, t_f) := \frac{P_T}{t_f}\cdot\frac{1}{\sqrt{K_1(F_{TR\_o})\cdot K_2}}\cdot\ln\left(\cosh\left(\sqrt{K_1(F_{TR\_o})\cdot K_2}\cdot t_f\right)\right)$$

$$P_{TR\_ave}(P_T, t_{V\_T}) = 8.468\cdot 10^3 \quad watt$$

Calculate energy required to achieve the steady state speed V_T at time t_f

$$P_{TR\_average} := P_{TR\_ave}(P_T, t_{V\_T})$$

$$\Delta e_{TR}(t_f, P_{TR\_average}) := t_f\cdot P_{TR\_average}$$

$$\Delta e_{TR}(t_{V\_T}, P_{TR\_average}) = 1.606\cdot 10^6 \quad J$$

End of Lecture 6