



# Electric Vehicles

ELEC 5970/6970/6970-D002

## Exam 1

### References:

- Iqbal Husain, "Electric and Hybrid Vehicles, Design Fundamentals," Third Edition, March 2021, CRC Press, Taylor & Francis Group, ISBN: 978-0429-49092-7

# EV Overview – Smart Grid Perspectives

Potential participation of EV in the Smart Grids by providing Ancillary Services to the grid via EV aggregator

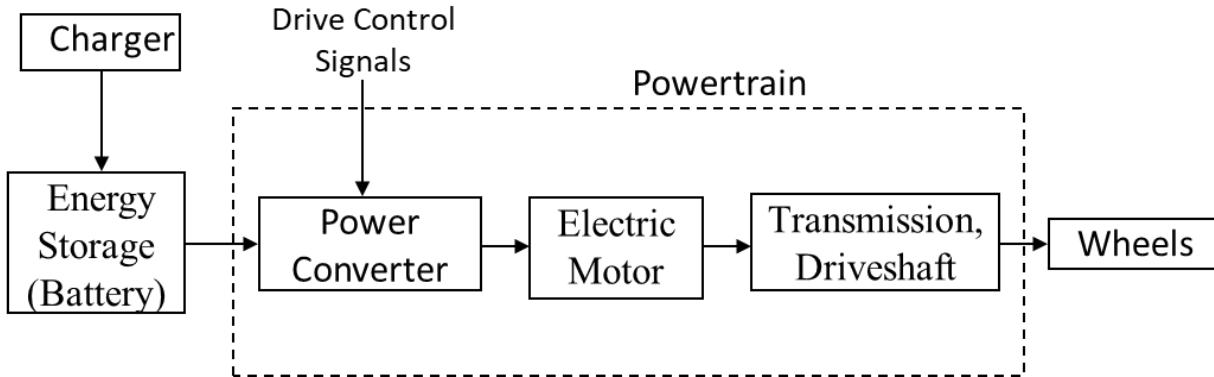
- Demand response participation: controllable loads
- Energy arbitrage: buy low – sell high, providing incentive to the PV owners
- Frequency regulation: provide real power boost when the frequency decreases and absorb reactive power when the frequency is high
- Energy storage provision: provide grid support during islanding within microgrid

# Electric Vehicles

- What is an (Battery) Electric Vehicle?

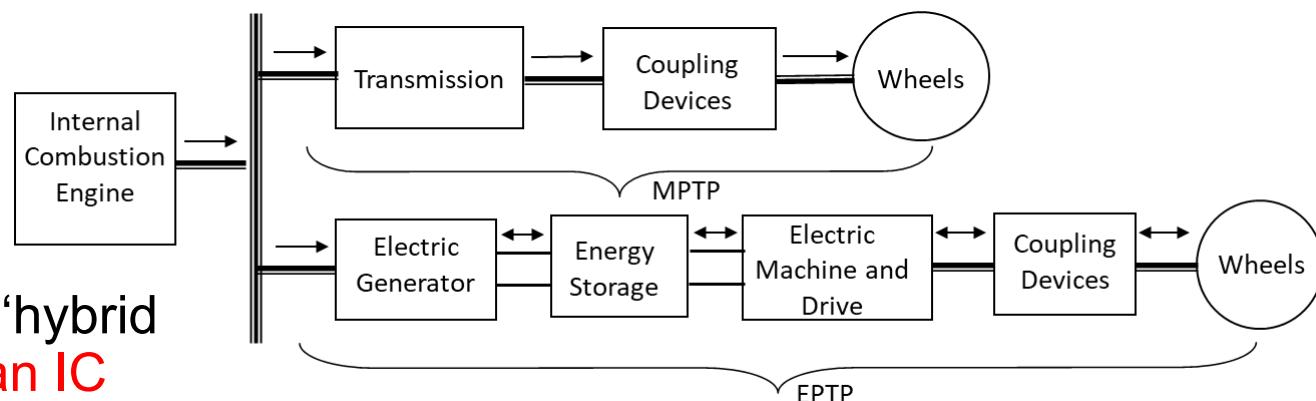
Definition: An Electric Vehicle has two primary features:

- The energy source is **portable** and electrochemical or electromechanical in nature.
- **Traction** effort is supplied **only** by an electric motor.
- The drive train of the electric vehicle is the **electromechanical system between the vehicle energy source and the road**.



- What is a Hybrid Electric Vehicle?

Definition: 'Hybrid electric vehicle' or simply 'hybrid vehicle' generally refers to vehicles that use **an IC engine** and one or more **electric machines** for propulsion.



# Vehicle Mass and Performance

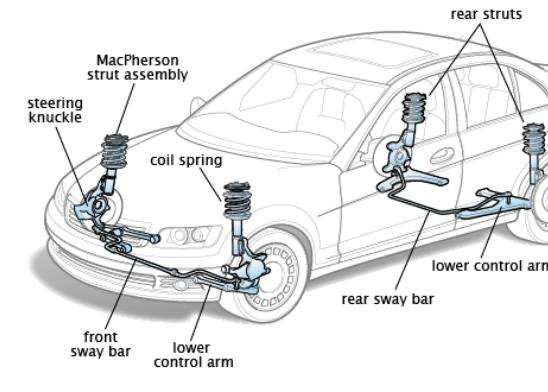
- Vehicle curb mass  $m_v$  is the total mass of the vehicle with all standard equipment, components, lubricants, full tank of gas but **without any passenger or cargo**.
- Vehicle gross vehicle mass  $m_{gv}$  is the curb mass **plus the passengers and cargo**.
- Vehicle maximum gross vehicle mass is the curb mass plus the maximum number of passengers and the maximum mass of the cargo that the vehicle is designed for.

# Vehicle Mass and Performance

- The sprung mass is the fraction of the **vehicle curb mass that is supported by suspension** including suspension members that are in motion.
- The unsprung mass is **the remaining fraction of vehicle curb mass that is carried by the wheels and moving with it.**
- A 10:1 ratio of sprung to unsprung mass is a desirable target, although a slightly lower ratio can be used for hybrid vehicles that may have more unsprung components.

$$m_{eq} = k_m m_v + N_p m_p$$

$$k_m = 1 + \frac{4J_w}{m_v r_{wh}^2} + \frac{J_{eng} \xi_{eng}^2 \xi_{FD}^2}{m_v r_{wh}^2} + \frac{J_{em} \xi_{em}^2 \xi_{FD}^2}{m_{cv} r_{wh}^2}$$



$$k_m \sim 1.08 \text{ to } 1.1$$



# Well-to-Wheel Analysis

Measure of the overall efficiency analysis of a vehicle starting from the extraction of raw fuel to the wheels including the efficiencies of energy conversion, transport and delivery at each stage.

## WTW Analysis using GREET

	SI ICEV (Baseline CG and RFG)	Plug-in SI HEV (Gasoline and Electricity)	Battery EV (Electricity)
Total Energy (Wh)	257,551 <b>Oil</b>	526,261 <b>Oil-Coa</b>	1,632,131 <b>Coal</b>
WTT Efficiency	79.5%	66.5% <b>I</b>	38.0%
TTW Efficiency	21.9%	23% <b>I</b>	48.51%
WTW Efficiency	17.41%	15.29% <b>I</b>	18.43%
CO <sub>2</sub> (grams/ Million BTU)	17,495 <b>Oil</b>	57,024 <b>Oil-Coa</b>	219,704 <b>Coal</b>
CH <sub>4</sub> (grams/ Million BTU)	109.120 <b>Oil</b>	145.658 <b>Oil-Coa</b>	296.031 <b>Coal</b>
N <sub>2</sub> O (grams/ Million BTU)	1.152 <b>Oil</b>	1.535 <b>I</b>	3.111 <b>Coal</b>
VOC(volatile organic compound): Total (grams/ Million BTU)	27.077	25.630 <b>I</b>	19.679
CO: Total (grams/ Million BTU)	15.074 <b>Oil</b>	23.553 <b>Oil-Coa</b>	58.448 <b>Coal</b>
NOx: Total (grams/ Million BTU)	50.052 <b>Oil</b>	87.100 <b>Oil-Coa</b>	239.571 <b>Coal</b>

# Electric Vehicles

ELEC 5970/6970/6970-D01

## Vehicle Mechanics

### References:

- Iqbal Husain, "Electric and Hybrid Vehicles, Design Fundamentals," Third Edition, March 2021, CRC Press, Taylor & Francis Group, ISBN: 978-0429-49092-7

## Exercise 2.1

# Roadway Percent Grade

A straight roadway has a profile in the  $x_Fy_F$  plane given by  $f(x_F) = 3.9\sqrt{x_F}$  for  $0 \leq x_F \leq 2$  miles.  $x_F$  and  $y_F$  are given in feet.

(a) Plot the roadway, (b) find  $\beta(x_F)$ , (c) find the percent grade at  $x_F = 1$  mile and (d) find the tangential road length between 0 and 2 miles.

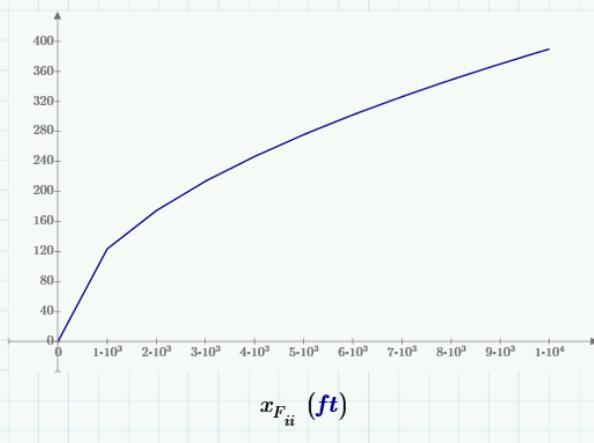
Ans. (b)  $\tan^{-1} \frac{1.95}{\sqrt{x_F}}$ ; (c) 2.68% and (d) 10,580 ft.

$$f(x_{FF}) := 3.9 \cdot \sqrt{x_{FF}} \quad \text{For} \quad x_{F\_min} := 0 \cdot \text{mi} \quad x_{F\_max} := 2 \cdot \text{mi}$$

$$\text{a) Plot the roadway} \quad ii := 0, 1..10 \quad x_{ii} := 0 + ii \cdot 1000$$

$$1 \text{ mi} = (5.28 \cdot 10^3) \text{ ft} \quad y_{F_{ii}} := f(x_{ii}) \cdot \text{ft} \quad x_{F_{ii}} := x_{ii} \cdot \text{ft}$$

b) find  $\beta(x_{FF})$



$$\frac{d}{dx_{FF}} (3.9 \cdot \sqrt{x_{FF}}) \rightarrow \frac{1.95}{\sqrt{x_{FF}}}$$

c) find percent grade at 1 mile

$$\text{pctGrade}(x_{FF}) := \tan\beta(x_{FF}) \cdot 100$$

$$\tan\beta(x_{FF}) := \frac{1.95}{\sqrt{x_{FF}}} \cdot (\sqrt{\text{ft}})$$

$$x_{Fo} := 1 \cdot \text{mi} = (5.28 \cdot 10^3) \text{ ft}$$

$$\text{pctGrade}(x_{Fo}) = 2.684$$

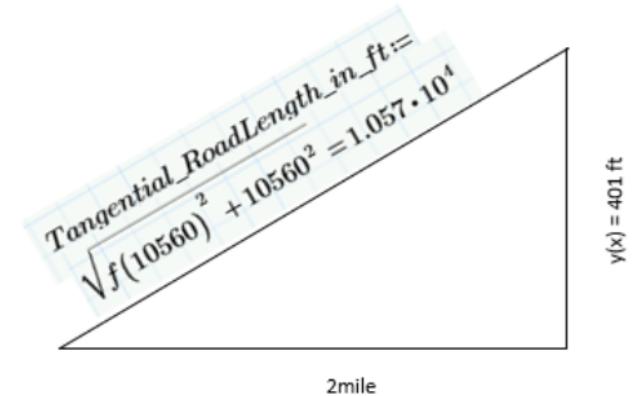
d) Find the tangential road length between 0 and 2 miles

$$2 \text{ mi} = (1.056 \cdot 10^4) \text{ ft}$$

$$f(10560) = 400.771$$

$$f(10560) \cdot \text{ft} = 400.771 \text{ ft}$$

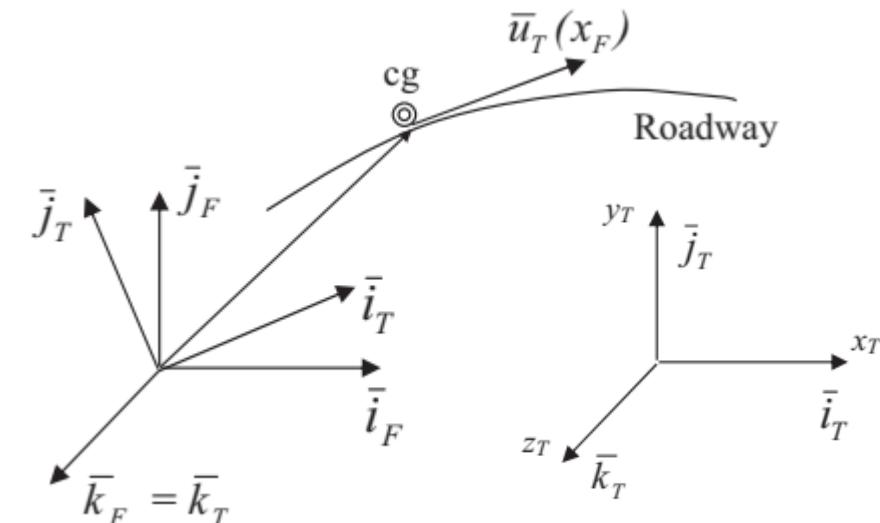
$$\text{Tangential_RoadLength\_in\_ft} := \sqrt{f(10560)^2 + 10560^2} = 1.057 \cdot 10^4$$



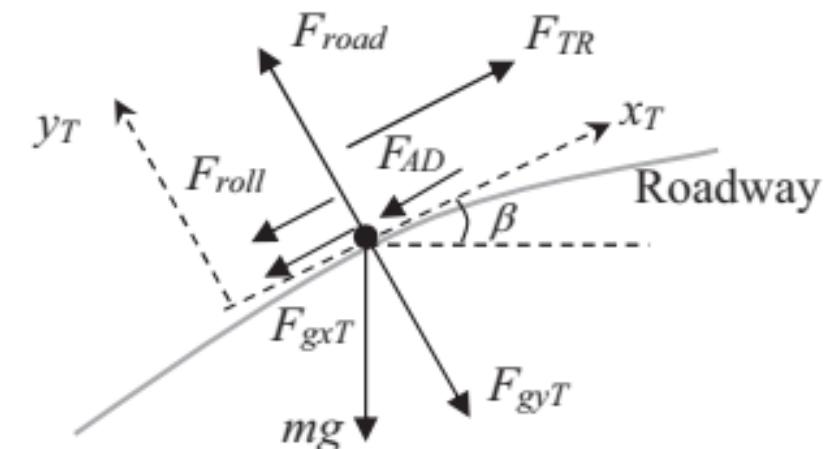
$$y(x) = 401 \text{ ft}$$

- $v_{xT}$  is the vehicle tangential velocity.
- The normal velocity  $v_{yT} = 0$ , since gravitational force in the normal direction is balanced by the road reaction force.
- Therefore, a one-directional analysis can be used for vehicle propulsion in the  $x_T$ -direction.
- The propulsion unit exerts a *tractive force*  $F_{TR}$  to propel the vehicle forward at a desired velocity.  $F_{TR}$  must overcome the opposing forces, viz.  $F_{gxT}$  the gravitational force,  $F_{roll}$  rolling resistance of the tires and  $F_{AD}$  the aerodynamic drag force; all summed together as the *road load force*  $F_{RL}$ .

$$F_{RL} = F_{gxT} + F_{roll} + F_{AD} \quad (2.1)$$

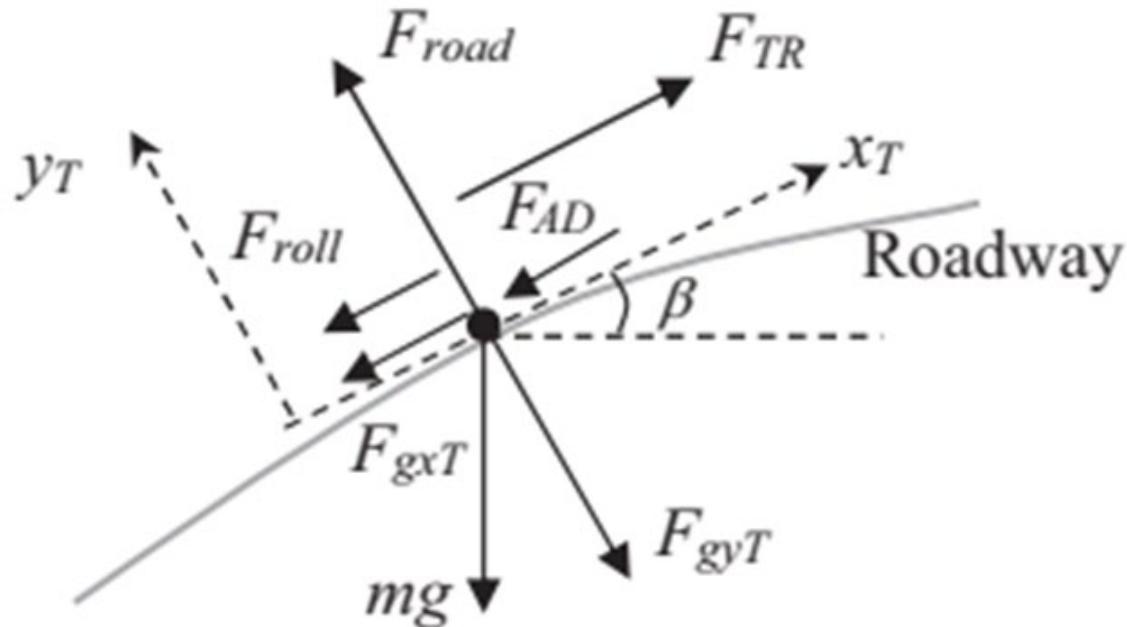


**FIGURE 2.6** Tangential co-ordinate system and the unit tangent vector on a roadway.



**FIGURE 2.7** Forces acting on a vehicle.

# Force-Velocity Characteristics



$$F_{ROLL} = \text{sgn}(V) mg(C_0 + C_1 V^2)$$

The steady state tractive force versus constant velocity (accel = 0)

$$\begin{aligned} F_{TR} - F_{AD} - F_{ROLL} - F_{gxT} &= 0 \\ \Rightarrow F_{TR} &= mg [\sin \beta + [C_0 \text{sgn}(V)] + \text{sgn}(V) [mgC_1] + \frac{\rho}{2} C_D A_F] V^2. \end{aligned}$$

$V$  is the  
steady-state  
velocity

$F_{gxT}$

$F_{ROLL}$

$F_{AD}$

# Propulsion Power

- Torque at the vehicle wheels is obtained from the power relation

$$\text{Power} = T_{TR} \cdot \omega_{wh} = F_{TR} \cdot v_{xT} [\text{W}]$$

where

$T_{TR}$  is the tractive torque in  $\text{N}\cdot\text{m}$ ,

$\omega_{wh}$  is the angular velocity in  $\text{rads/sec}$ ,

$F_{TR}$  is in  $\text{N}$  and

$v_{xT}$  is in  $\text{m/sec}$ .

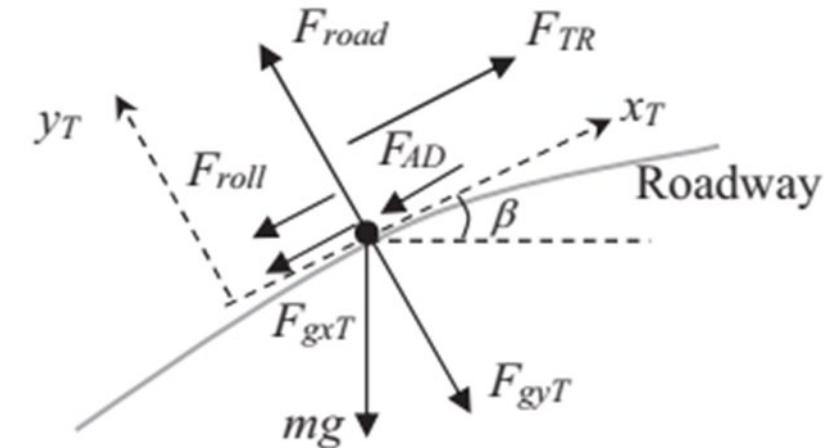
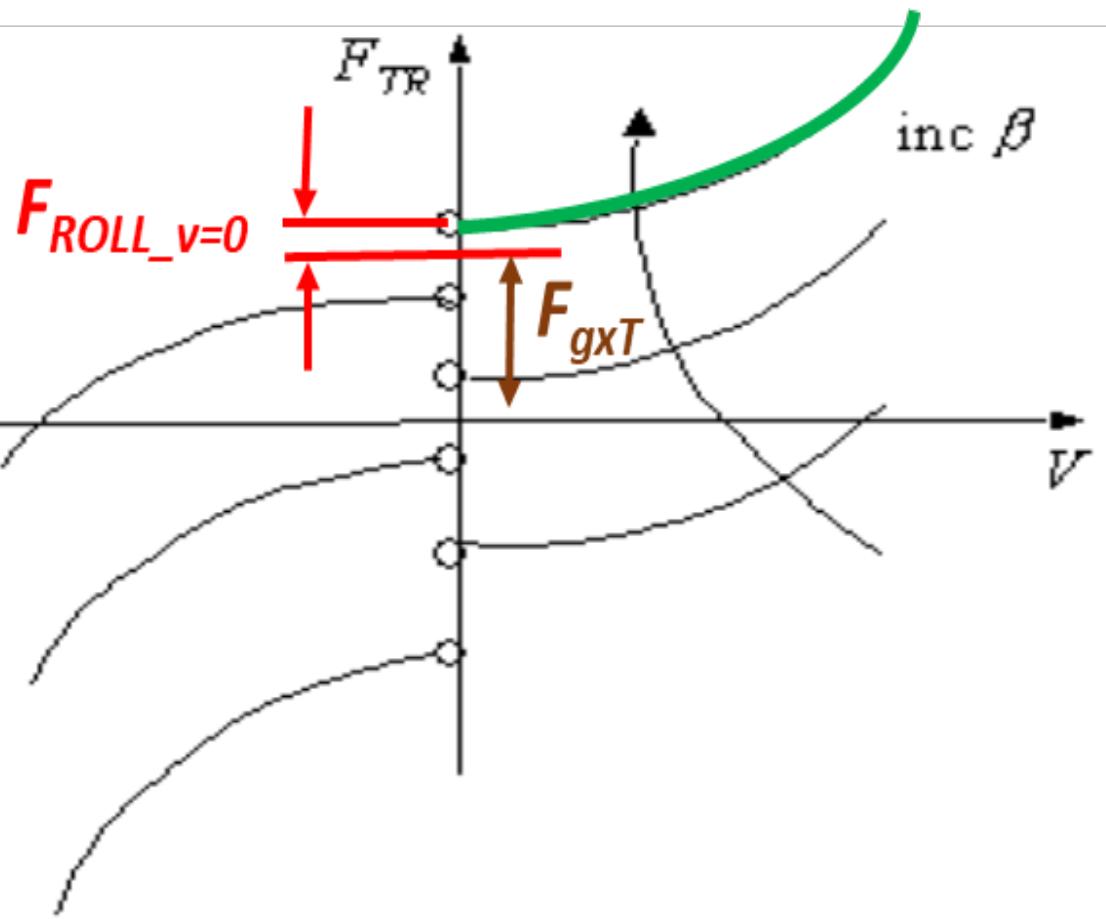
- The angular velocity and the vehicle speed is related by

$$v_{xT} = \omega_{wh} \cdot r_{wh}$$

$r_{wh}$  is the radius of the wheel

# Force-Velocity Characteristics

The tractive force vs. steady-state velocity characteristics can be obtained from the equation of motion, with zero acceleration



$$F_{TR} - F_{AD} - F_{ROLL} - F_{gxT} = 0$$

$$\Rightarrow F_{TR} = mg [\sin \beta + C_0 \operatorname{sgn}(V)] + \operatorname{sgn}(V) [mgC_1 + \frac{\rho}{2} C_D A_F] V^2.$$

$$F_{ROLL} = F_{ROLL\_{v=0}} + F_{ROLL\_{v \neq 0}} \quad F_{gxT} = mg \sin \beta$$

$$F_{ROLL\_{v=0}} = \operatorname{sgn}(V) mg C_0$$

$$F_{ROLL\_{v \neq 0}} = \operatorname{sgn}(V) mg C_1 V^2$$

$$F_{AD} = \frac{\rho}{2} C_D A_F V^2$$

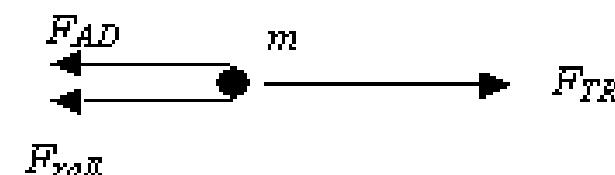
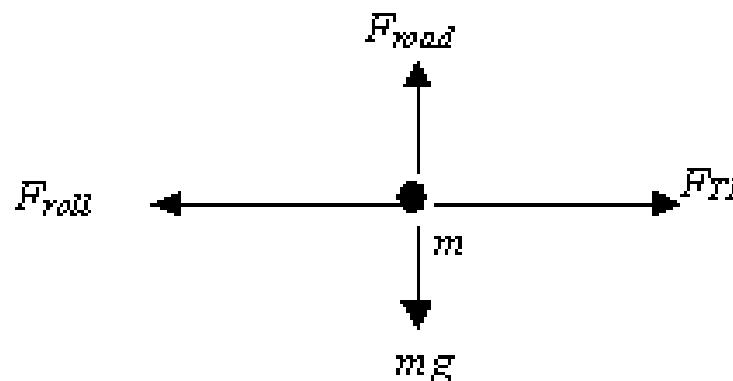
# Scenario 1: Constant $F_{TR}$ , Level Road

- Constant  $F_{TR}$ , Level Road:

- The level road condition implies that  $\beta(s)=0$ ;  $F_{gxT}=0$ .
- EV is assumed to be at rest initially; also the initial FTR is assumed to be capable of overcoming the initial rolling resistance.

$$\text{At } t > 0 \Rightarrow \Sigma F = m \frac{dv}{dt} \Rightarrow F_{TR} - F_{AD} - F_{roll} = m \frac{dv}{dt}$$

$$\Rightarrow F_{TR} - \text{sgn}[v(t)] \frac{\rho}{2} C_D A_F v^2(t) - \text{sgn}[v(t)] mg(C_0 + C_1 v^2(t)) = m \frac{dv}{dt}$$



$$\frac{dv}{dt} = \left( \frac{F_{TR}}{m} - gC_0 \right) - \left[ \frac{\rho}{2m} C_D A_F + gC_1 \right] v^2$$

$K_1$

$K_2$

(a) Free body diagram at  $t=0$ . (b) Forces on the vehicle at  $t>0$ .

Solving for acceleration,  $dv/dt$

$$\frac{dv}{dt} = \left( \frac{F_{TR}}{m} - gC_0 \right) - \left[ \frac{\rho}{2m} C_D A_F + gC_1 \right] v^2$$

$$\frac{dv}{dt} = K_1 - K_2 v^2$$

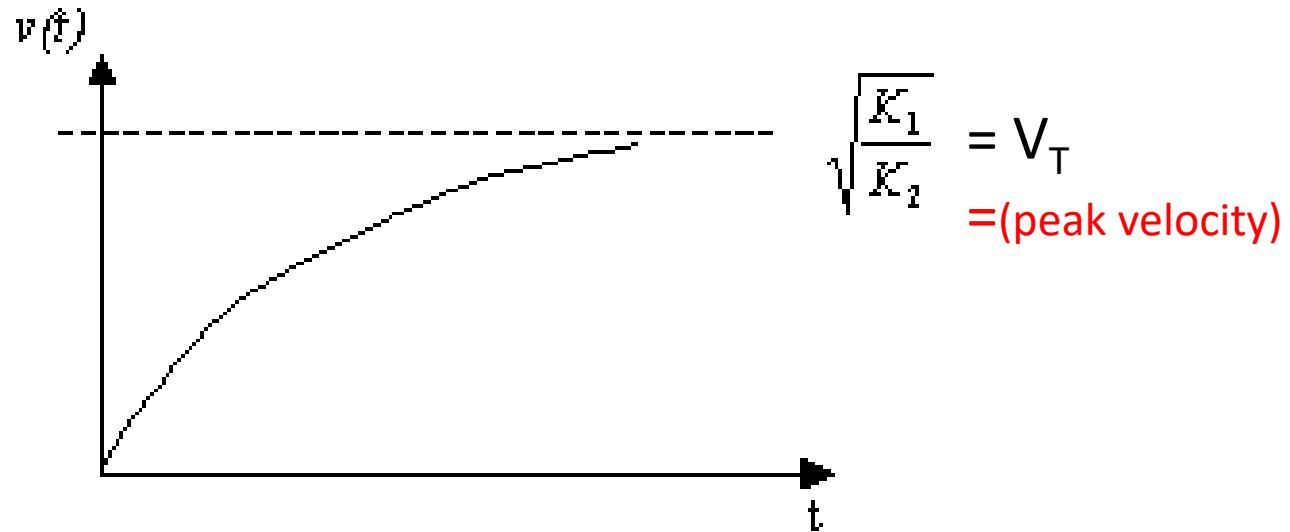
$$K_1 = \frac{F_{TR}}{m} - gC_0 > 0$$

$$K_2 = \frac{\rho}{2m} C_D A_F + gC_1 > 0$$

The velocity profile:

$$v(t) = \sqrt{\frac{K_1}{K_2}} \tanh(\sqrt{K_1 K_2} t)$$

$$V_T = \lim_{t \rightarrow \infty} v(t) = \sqrt{\frac{K_1}{K_2}} \Rightarrow \sqrt{K_1 K_2} = K_2 V_T$$



## Distance Traversed:

The speed is the derivative of the distance traversed

$$\frac{ds(t)}{dt} = v(t) = V_T \tanh(K_2 V_T t)$$

$$s(t) = \int_{t_o}^{t_f} v(t) dt$$

$$s(t) = \frac{1}{K_2} \ln [\cosh(K_2 V_T t)]$$

The time to reach the desired velocity and distance traversed during that time is given by

$$t_f = \frac{1}{K_2 V_T} \cosh^{-1} [e^{(K_2 s_f)}]$$

$$s_f = \frac{1}{K_2} \ln [\cosh(K_2 V_T t_f)]$$



## Tractive power:

The instantaneous tractive power delivered by the prop. unit is

$$P_{TR}(t) = F_{TR} v(t).$$

$$\Rightarrow P_{TR}(t) = F_{TR} V_T \tanh(\sqrt{K_1 K_2} t) = P_T \tanh(\sqrt{K_1 K_2} t)$$

*P<sub>T</sub> = (power at V<sub>T</sub> velocity)*

The mean tractive power over the acceleration interval  $\Delta t$  is

$$\overline{P}_{TR} = \frac{1}{t_f} \int_0^{t_f} P_{TR}(t) dt$$

$$\Rightarrow \overline{P}_{TR} = \frac{P_T}{t_f} \frac{1}{\sqrt{K_1 K_2}} \ln \left[ \cosh(\sqrt{K_1 K_2} t_f) \right]$$

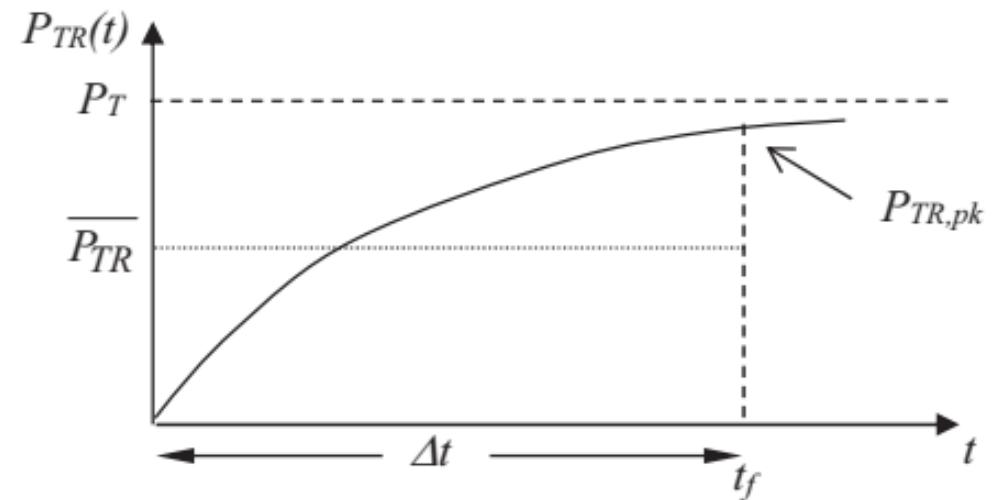
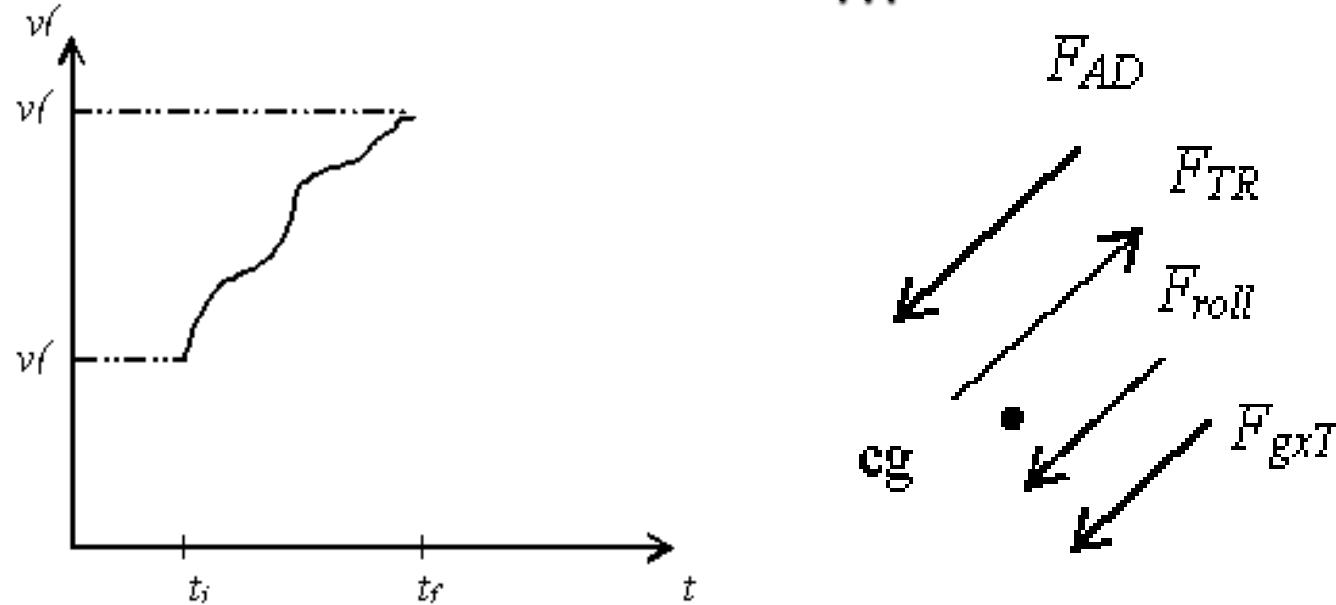


FIGURE 2.15 Acceleration interval  $\Delta t = t_f - 0$ .

Energy required during an interval of the vehicle can be obtained from the integration of the instantaneous power equation as

$$\int_{e_{TR}(0)}^{e_{TR}(t_f)} de_{TR} = \int_{t=0}^{t_f} P_{TR} dt$$
$$\Rightarrow \Delta e_{TR} = t_f \overline{P_{TR}}$$

# Scenario 2: Non-constant $F_{TR}$ , General Acceleration



**Arbitrary velocity profile**

**In the general case,**

$$\begin{aligned}\Sigma F &= m \frac{dv}{dt} \\ \Rightarrow F_{TR} - F_{AD} - F_{roll} - F_{gxT} &= m \frac{dv}{dt} \\ \Rightarrow F_{TR} &= m \frac{dv}{dt} + mg \sin \beta + F_{AD} + F_{roll} \\ &= m \frac{dv}{dt} + mg \sin \beta + [mgC_1 + \frac{\rho}{2} A_F C_D] v^2 + mgC_0\end{aligned}$$

The instantaneous tractive power  $P_{TR}(t)$  is

$$P_{TR}(t) = F_{TR}(t)v(t)$$
$$= mv \frac{dv}{dt} + v(F_{gxT} + F_{AD} + F_{roll})$$

The change in tractive energy during an interval

$$\Delta e_{TR} = \int_{t_i}^{t_f} P_{TR}(t)dt$$
$$= m \int_{v(t_i)}^{v(t_f)} vdv + \int_{t_i}^{t_f} (v)F_{gxT}dt + \int_{t_i}^{t_f} (v)(F_{AD} + F_{roll})dt$$

kinetic energy      potential energy      loss term

- The 1<sup>st</sup> & 2<sup>nd</sup> terms represent kinetic and potential energy;
- 3<sup>rd</sup> & 4<sup>th</sup> terms represent the loss energy needed to overcome the non-constructive forces including the rolling resistance and the aerodynamic drag force. These two are known as loss term.

- 1<sup>st</sup> term:

$$m \int_{v(t_i)}^{v(t_f)} v dv = \frac{1}{2} m [v^2(t_f) - v^2(t_i)] = \Delta(\text{Kinetic Energy})$$

- 2<sup>nd</sup> term:

$$\begin{aligned} \int_{t_i}^{t_f} (v) F_{gxt} dt &= mg \int_{t_i}^{t_f} v \sin \beta dt = mg \int_{s(t_i)}^{s(t_f)} \sin \beta ds = mg \int_{f(t_i)}^{f(t_f)} df \\ &= mg [f(t_f) - f(t_i)] \\ &= \Delta(\text{Potential Energy}) \end{aligned}$$

- 3<sup>rd</sup> term:

Let

$$\int_{t_i}^{t_f} (v) (F_{AD} + F_{roll}) dt = E_{loss}$$

$$K_3 = mgC_0, \quad K_4 = mgC_1 + \frac{\rho}{2} C_D A_F$$

then

$$\begin{aligned} E_{loss} &= K_3 \int_{t_i}^{t_f} \frac{ds}{dt} dt + K_4 \int_{t_i}^{t_f} v^3 dt; \\ &= K_3 \Delta s + K_4 \int_{t_i}^{t_f} v^3 dt. \end{aligned}$$

### **Exercise 2.3**

The vehicle with parameters given in Exercise 2.2 accelerates from 0 to 60 mph in 13.0 s for the following two acceleration types: (i) constant  $F_{TR}$  and (ii) uniform acceleration.

- a. Plot on the same graph, the velocity profile of each acceleration type.
- b. Calculate and compare the tractive energy required for the two types of acceleration.

$$F_{TR} = \text{const.} = 1,548\text{N.}$$

Ans. (b)  $\Delta e_{TR} = 0.2752 \text{ MJ}$  for constant  $F_{TR}$  and  $\Delta e_{TR} = 0.2744 \text{ MJ}$  for uniform acceleration.

### Exercise 2.3

An EV has the following parameter values:

$$m := 692 \text{ kg} \quad C_D := 0.2 \quad A_F := 2 \text{ m}^2 \quad C_o := 0.009 \quad C_1 := 1.75 \cdot 10^{-6} \quad \left(\frac{s}{m}\right)^2$$
$$\rho_o := 1.16 \frac{\text{kg}}{\text{m}^3} \quad g_o := 9.81 \frac{\text{m}}{\text{s}^2}$$

The EV accelerates from 0 to 60 mph in 13 s for two acceleration types:  $60 \text{ mph} = 26.822 \frac{\text{m}}{\text{s}}$

(i) constant  $F_{TR}$  (ii) uniform acceleration (constant a)

a) Plot on the same graph, the velocity profile of each acceleration type

b) Calculate and compare the tractive energy required for the two types of acceleration

$$F_{TR\_max} := 1548 \text{ Newton}$$

Ans. (b)  $\Delta e_{TR} = 0.2752 \text{ MJ}$  for constant  $F_{TR}$  and  $\Delta e_{TR} = 0.2744 \text{ MJ}$  for uniform acceleration.

(i) Constant  $F_{TR}$ ;  $F_{TR\_o} := 1548 \text{ Newton}$ , we will use the result from Exercise 2-2

$$V_T = \lim_{t \rightarrow \infty} v(t) = \sqrt{\frac{K_1}{K_2}} \Rightarrow \sqrt{K_1 K_2} = K_2 V_T$$

$$K_1(F_{TR}) := \frac{F_{TR}}{m} - g_o \cdot C_o \quad K_2 := \frac{\rho_o}{2 \cdot m} \cdot C_D \cdot A_F + g_o \cdot C_1 = 3.524 \cdot 10^{-4}$$

for  $0 \leq t \leq 13 \text{ s}$

$$V_T(F_{TR}) := \sqrt{\frac{K_1(F_{TR})}{K_2}} \rightarrow \sqrt{4.1003787478845635375 \cdot F_{TR} - 250.51952823830385539}$$

## From the previous Lecture

The velocity profile:

$$v(t) = \sqrt{\frac{K_1}{K_2}} \tanh(\sqrt{K_1 K_2} t)$$

$$V_T = \lim_{t \rightarrow \infty} v(t) = \sqrt{\frac{K_1}{K_2}} \Rightarrow \sqrt{K_1 K_2} = K_2 V_T$$



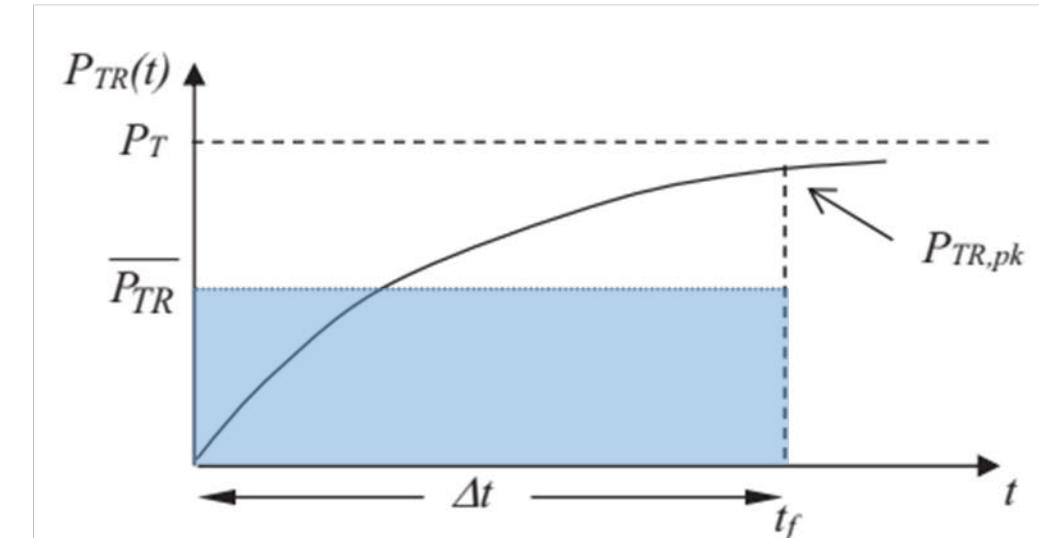
$$\sqrt{\frac{K_1}{K_2}} = V_T$$

=(peak velocity)

The mean tractive power over the acceleration interval  $\Delta t$  is

$$\overline{P}_{TR} = \frac{1}{t_f} \int_0^{t_f} P_{TR}(t) dt$$

$$\Rightarrow \overline{P}_{TR} = \frac{P_T}{t_f} \frac{1}{\sqrt{K_1 K_2}} \ln \left[ \cosh \left( \sqrt{K_1 K_2} t_f \right) \right]$$



**FIGURE 2.15** Acceleration interval  $\Delta t = t_f - 0$ .

$$F_{TR\_o} = 1.548 \cdot 10^3 \quad N$$

$ii := 0, 1..13$

$$v(t) = \sqrt{\frac{K_1}{K_2}} \tanh(\sqrt{K_1 K_2} t)$$

$$time_{ii} := 0 + ii \cdot 1 \qquad \qquad \qquad 10^5 \ s = 27.778 \ hr$$

$$v_{const\_FTR}(t, F_{TR}) := \sqrt{\frac{K_1(F_{TR})}{K_2}} \cdot \tanh(\sqrt{K_1(F_{TR}) \cdot K_2} \cdot t)$$

$$K_1(F_{TR\_o}) = 2.149 \qquad \qquad K_2 = 3.524 \cdot 10^{-4} \qquad \qquad \sqrt{K_1(F_{TR\_o}) \cdot K_2} = 0.028$$

$$v_{const\_FTR}(t, F_{TR\_o}) \rightarrow 78.082435755341293662 \cdot \tanh(0.027518406654036728097 \cdot t)$$

Find the time required to accelerate from 0 to 60 mph

$$60 \frac{\text{mi}}{\text{hr}} = 26.822 \frac{\text{m}}{\text{s}} \quad v_o(t) := 78.08 \cdot \tanh(0.0275 \cdot t)$$

$$t_{60\text{mph}} := \frac{1}{0.0275} \cdot \text{atanh}\left(\frac{26.822}{78.08}\right) \quad t_{60\text{mph}} = 13.021 \text{ seconds}$$

Calculate  $P_{TR}$   $P_{TR}(t) = F_{TR} V_T \tanh(\sqrt{K_1 K_2} t) = P_T \tanh(\sqrt{K_1 K_2} t)$

$$P_T := F_{TR\_o} \cdot V_T (F_{TR\_o}) = 1.209 \cdot 10^5 \text{ watt}$$

Calculate average traction power

$$\overline{P}_{TR} = \frac{P_T}{t_f} \frac{1}{\sqrt{K_1 K_2}} \ln \left[ \cosh \left( \sqrt{K_1 K_2} t_f \right) \right]$$

$$P_{TR\_ave}(P_T, t_f) := \frac{P_T}{t_f} \cdot \frac{1}{\sqrt{K_1(F_{TR\_o}) \cdot K_2}} \cdot \ln \left( \cosh \left( \sqrt{K_1(F_{TR\_o}) \cdot K_2} \cdot t_f \right) \right)$$

$$P_{TR\_ave}(P_T, t_{60\text{mph}}) = 2.121 \cdot 10^4 \text{ watt}$$

Calculate energy required to achieve the steady state speed  $V_T$  at time  $t_f$

$$P_{TR\_average} := P_{TR\_ave}(P_T, t_{60mph})$$

$$\Delta e_{TR}(t_f, P_{TR\_average}) := t_f \cdot P_{TR\_average}$$

$$\Delta e_{TR}(t_{60mph}, P_{TR\_average}) = 2.761 \cdot 10^5 \text{ Joule}$$

Another way to compute  $\Delta e_{TR}$

$$P_{TR\_const\_FTR}(t) := F_{TR\_o} \cdot (v_{const\_FTR}(t, F_{TR\_o}))$$

$$P_{TR\_const\_FTR}(t) \rightarrow 120871.6105492683225888 \cdot \tanh(0.027518406654036728097 \cdot t)$$

$$\Delta e_{TR\_const\_a}(t_f) := \int_0^{t_f} P_{TR\_const\_FTR}(t) dt$$

$$\Delta e_{TR\_const\_a}(t_f) \rightarrow 4.392391320793910614455 \cdot 10^6 \cdot \ln(\cosh(0.02751840665403672809697 \cdot t_f))$$

$$\Delta e_{TR\_const\_a}(t_{60mph}) = 2.761 \cdot 10^5 \text{ Joule}$$

(ii) Constant acceleration  $a$  ;  $a_{TR\_o} := \frac{26.8}{13} = 2.062 \text{ m/s}$  (60 mph in 13 seconds)

$$v_{const\_a}(t) := a_{TR\_o} \cdot t \quad \text{Note: } \beta = 0 \quad \sin(\beta) = 0$$

$$F_{TR} - F_{AD} - F_{roll} - F_{gxT} = m \frac{dv}{dt}$$

$$F_{TR} = m \frac{dv}{dt} + mg \sin \beta + F_{AD} + F_{roll}$$

$$m \frac{dv}{dt} + mg \sin \beta + [mgC_1 + \frac{\rho_o}{2} A_F C_D] v^2 + mgC_0$$

$$F_{TR}(a_{TR}, t) := m \cdot a_{TR} + \left( m \cdot g_o \cdot C_1 + \frac{\rho_o}{2} \cdot A_F \cdot C_D \right) \cdot (a_{TR} \cdot t)^2 + m \cdot g_o \cdot C_o$$

$$P_{TR\_const\_a}(a_{TR}, t) := F_{TR}(a_{TR}, t) \cdot (a_{TR} \cdot t)$$

$$P_{TR\_const\_a}(a_{TR}, t) \rightarrow a_{TR} \cdot t \cdot (692 \cdot a_{TR} + 0.24387991 \cdot a_{TR}^2 \cdot t^2 + 61.09668)$$

$$\Delta e_{TR\_const\_a}(t_f) := \int_0^{t_f} P_{TR\_const\_a}(a_{TR\_o}, t) dt$$

$$\Delta e_{TR\_const\_a}(t_f) \rightarrow 0.53418336547167976633 \cdot t_f^4 + 1533.4561044733731622 \cdot t_f^2$$

$$\Delta e_{TR\_const\_a}(t_{60mph}) = 2.753 \cdot 10^5 \text{ Joule}$$

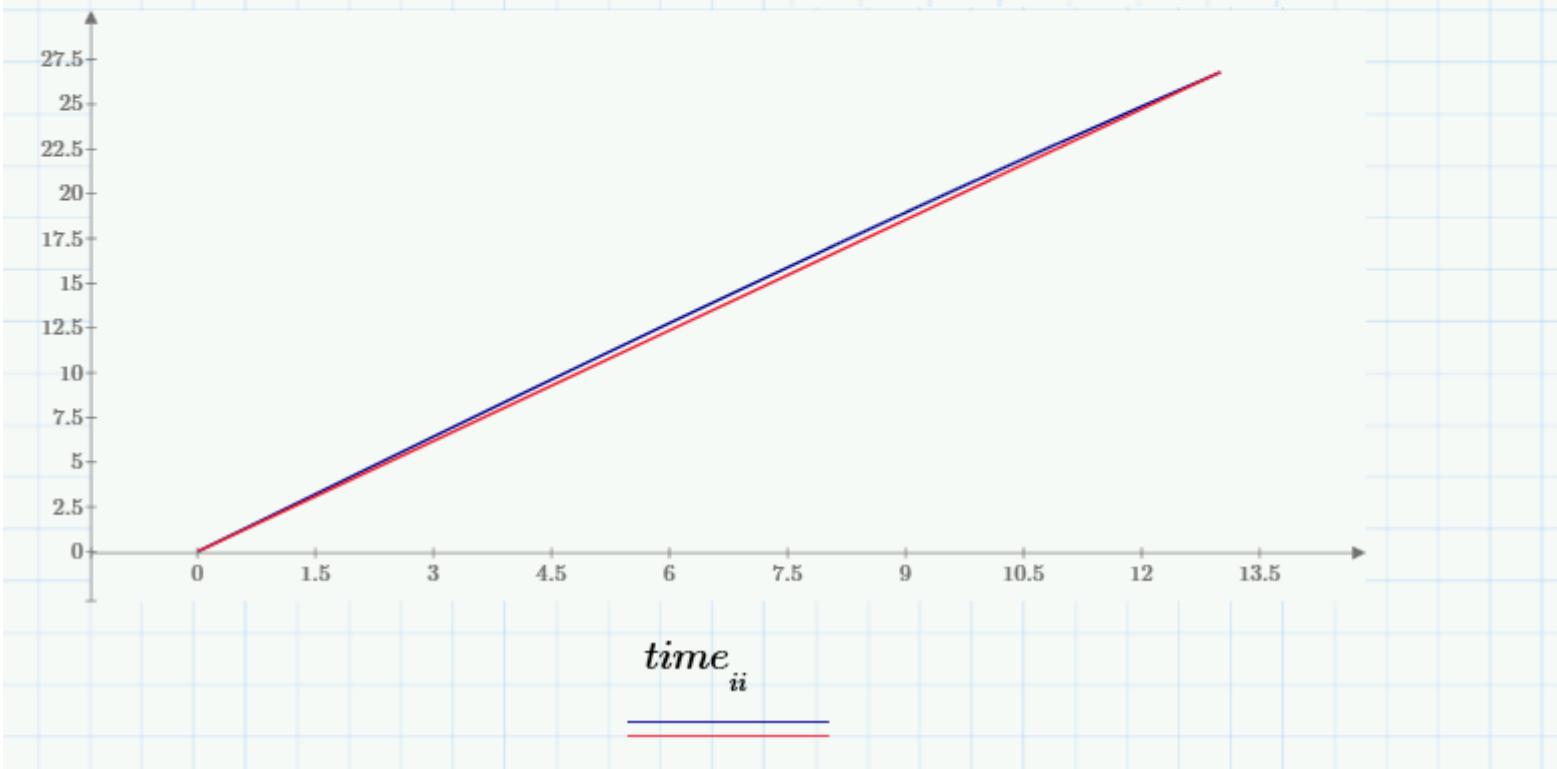
a) Plot on the same graph, the velocity profile of each acceleration type

$$v_{const\_FTR}(time_{ii}, F_{TR\_o})$$

$$\underline{v_{const\_a}(time_{ii})}$$

$$v_{const\_FTR}(t, F_{TR}) := \sqrt{\frac{K_1(F_{TR})}{K_2}} \cdot \tanh\left(\sqrt{K_1(F_{TR}) \cdot K_2} \cdot t\right)$$

$$v_{const\_a}(t) := a_{TR\_o} \cdot t$$



(iii) This example is extended to compute the Kinetic Energy and Potential energy gained for Uniform Acceleration (constant  $a$ ).

$$\Delta e_{TR} = \int_{t_i}^{t_f} P_{TR}(t) dt$$
$$= m \int_{v(t_i)}^{v(t_f)} v dv + \int_{t_i}^{t_f} (v) F_{gxT} dt + \int_{t_i}^{t_f} (v) (F_{AD} + F_{roll}) dt$$

v(t<sub>i</sub>) kinetic energy      t<sub>i</sub> potential energy      t<sub>f</sub> loss term

iiia) Compute the kinetic energy gained from 0 - 60 mph at constant acceleration in 13 s

$$a_{TR\_o} := \frac{26.8}{13} = 2.062 \frac{m}{s^2}$$

$$\Delta Kinetic\_Energy(t_i, t_f) := \frac{1}{2} \cdot m \cdot \left( v_{const\_a}(t_f)^2 - v_{const\_a}(t_i)^2 \right)$$

$$\Delta Kinetic\_Energy(0, 13) = 2.485 \cdot 10^5 Joule = 248.5 \text{ kJoule}$$

iiib) Compute the potential energy gained from 0 - 60 mph at constant acceleration in 13 s

$$\begin{aligned}\int_{t_i}^{t_f} (v) F_{g\alpha T} dt &= mg \int_{t_i}^{t_f} v \sin \beta dt = mg \int_{s(t_i)}^{s(t_f)} \sin \beta ds = mg \int_{f(t_i)}^{f(t_f)} df \\ &= mg [f(t_f) - f(t_i)] \\ &= \Delta(\text{Potential Energy})\end{aligned}$$

Note:  $\beta = 0$ ; thus  $\sin(\beta) = 0$

Level road - Thus no altitude gain:  
 $\Delta\text{Potential\_Energy} = 0$

$$\Delta\text{Potential\_Energy}(t_i, t_f, \beta_o, a_{TR\_o}) := m \cdot g_o \int_{t_i}^{t_f} a_{TR\_o} \cdot t \cdot \sin(\beta_o) dt$$

$$\Delta\text{Potential\_Energy}(t_i, t_f, \beta_o, a_{TR\_o}) \rightarrow -6997.3975384615393493 \cdot \sin(\beta_o) \cdot (1.0 \cdot t_i^2 - t_f^2)$$

$$\Delta\text{Potential\_Energy}(0, 13, 0, a_{TR\_o}) = 0 \quad Joule$$

iiic) Compute the energy loss from 0 - 60 mph at constant acceleration in 13 s

$$E_{loss} = K_3 \int_{t_i}^{t_f} \frac{ds}{dt} dt + K_4 \int_{t_i}^{t_f} v^3 dt; \quad \text{for } 0 \leq t \leq 13 \text{ s}$$

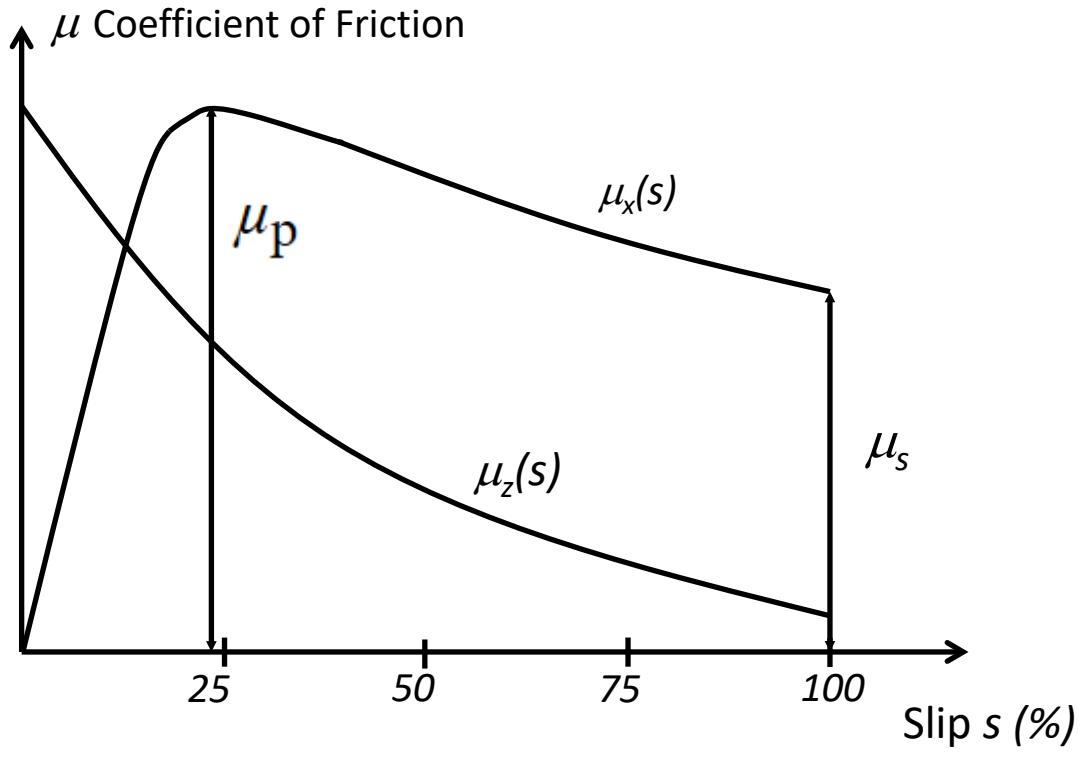
$$K_3 := m \cdot g_o \cdot C_1$$

$$= K_3 \Delta s + K_4 \int_{t_i}^{t_f} v^3 dt. \quad K_4 := m \cdot g_o \cdot C_1 + \frac{\rho_o}{2} \cdot C_D \cdot A_F = 0.244$$

$$E_{loss}(t_i, t_f, a_{TR\_o}) := K_3 \cdot \int_{t_i}^{t_f} a_{TR\_o} \cdot t \, dt + K_4 \cdot \int_{t_i}^{t_f} (a_{TR\_o} \cdot t)^3 \, dt$$

$$E_{loss}(0, 13, a_{TR\_o}) \rightarrow 15258.880581558644556 \text{ Joule} = 15.258 \text{ kJoule}$$

# Traction Limit and Control



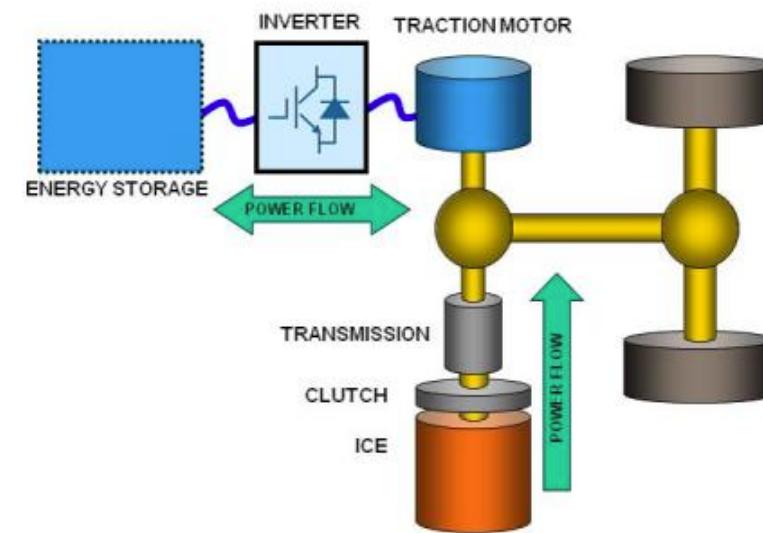
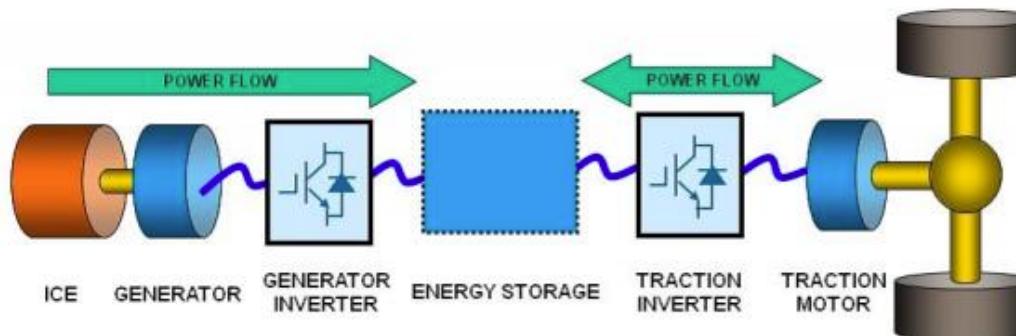
$$F_{roll}(s) = \mu(s) F_{gyT}$$

The friction coefficient when the slip reaches 100% is known as sliding coefficient  $\mu_s$

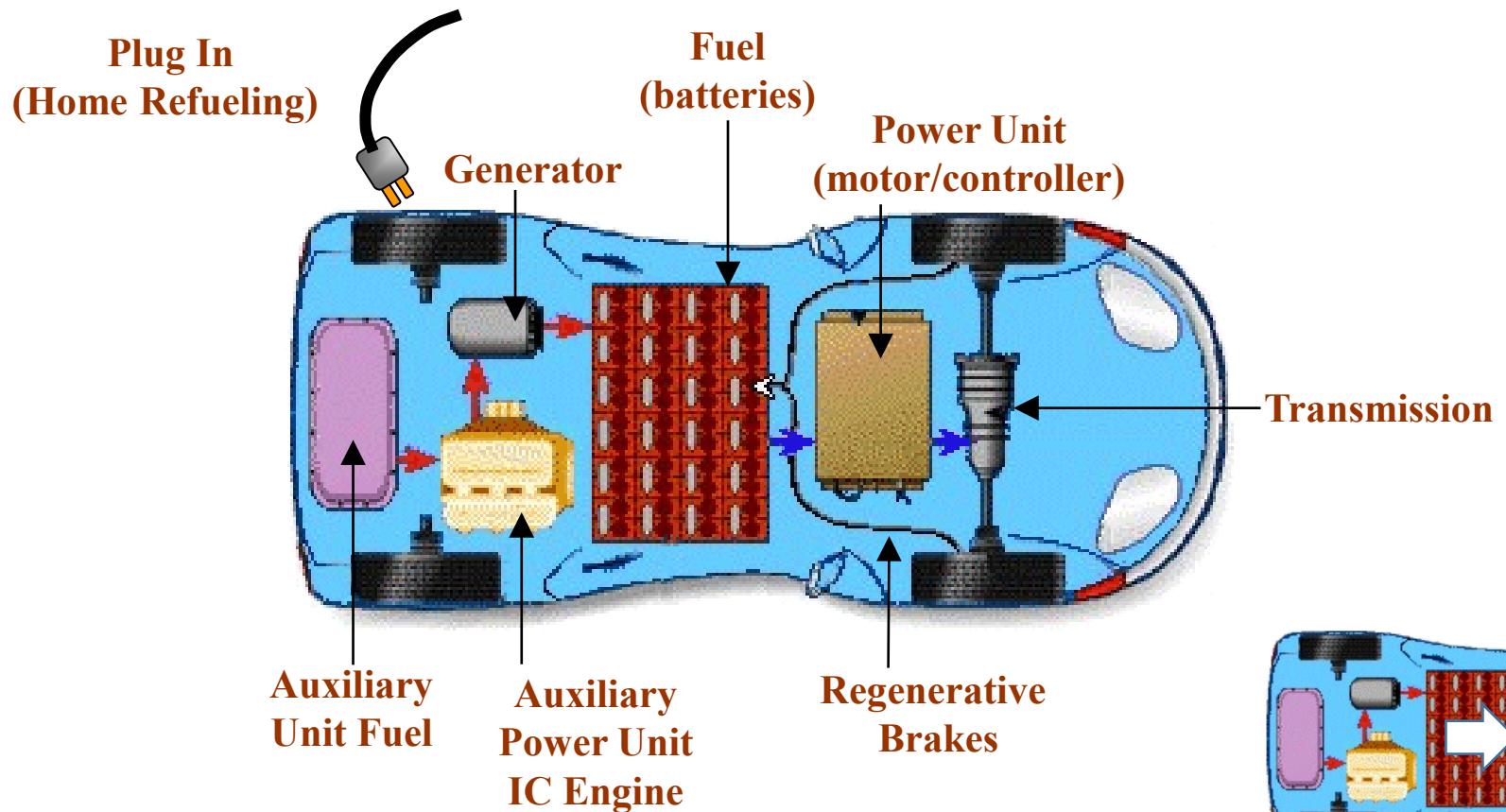
Surface	Peak coefficient, $\mu_p$	Sliding coefficient, $\mu_s$
Ashphalt and concrete (dry)	0.9	0.75
Concrete (wet)	0.8	0.7
Ashphalt (wet)	0.6	0.5
Gravel	0.6	0.55
Snow	0.2	0.15
Ice	0.1	0.07

## 3.5.2.2 Plug-in Hybrid (PHEV)

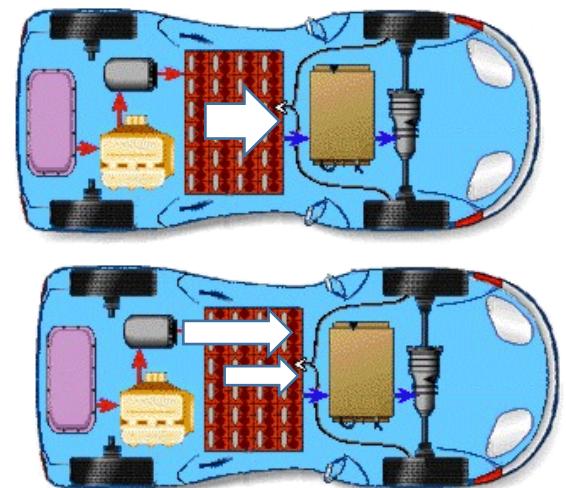
- **Series hybrid systems** - use two power sources linked together, with only one source directly connected to the vehicle's transmission. A small ICE is used to power a generator that converts the energy to provide electric power to the vehicle's wheels and auxiliary devices as well as to a battery system and/or capacitor(s).
- **Parallel hybrid systems** - provide a dual power supply that is physically connected to the vehicle's driving wheels. Either the ICE or the electric motor – or both – can power the vehicle's wheels.



# Fundamentals of Plug-In HEV

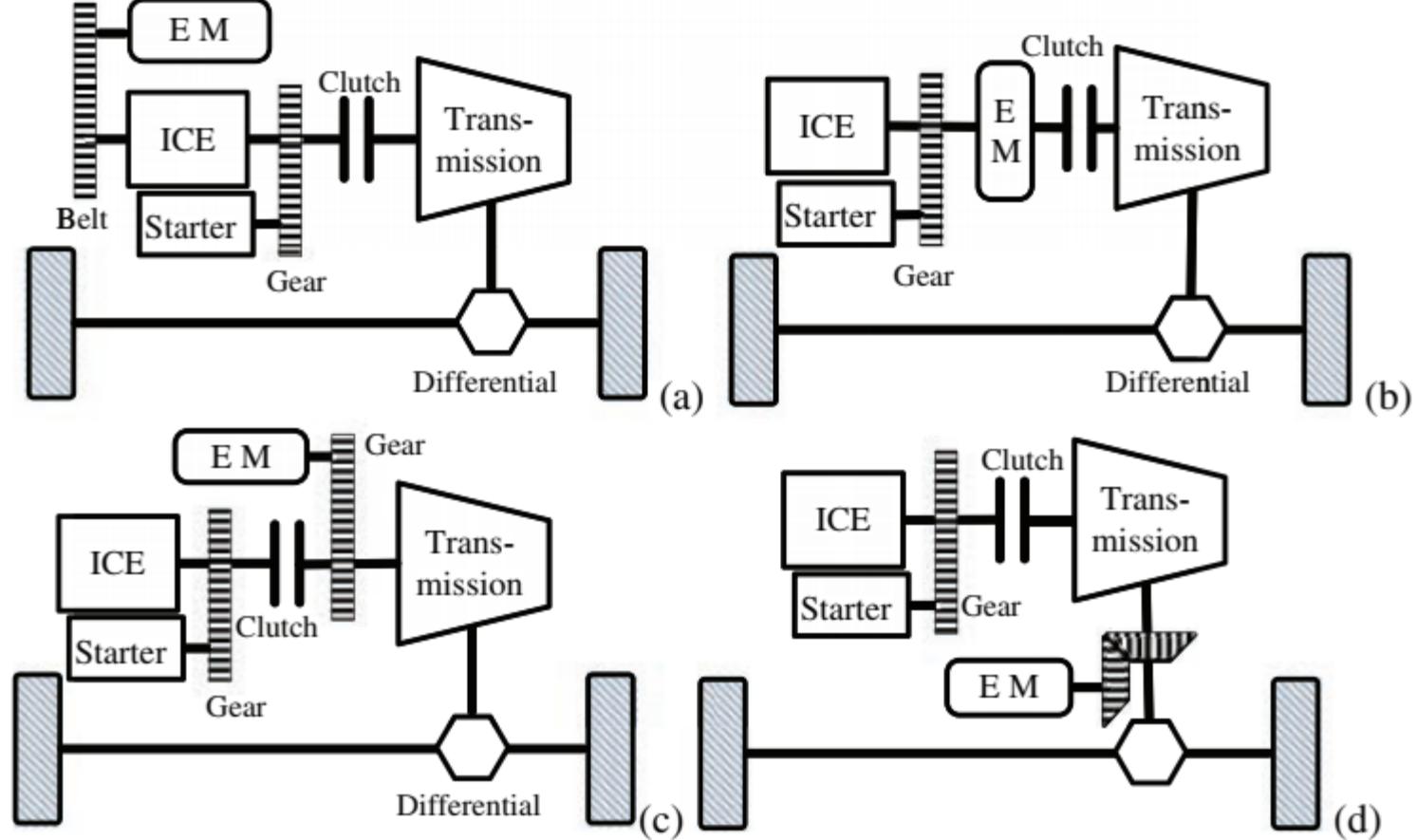


- ❖ PHEV runs as electric vehicle **when the battery charge is high**.
- ❖ Switches to hybrid mode when **battery power is low**, and when **driver requires more power**.

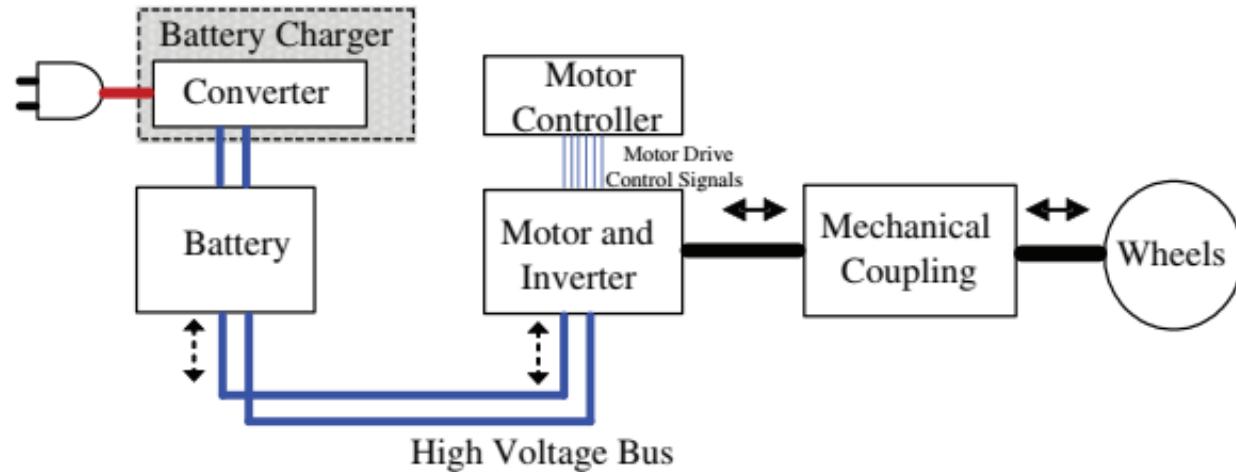


The P0, P1 and P2 architectures are pre-transmission type, while P3 and P4 architectures are the post-transmission type.

FIGURE 3.8 HEV P0 to P4 architectures:  
 (a) P0 architecture with belt-starter generator;  
 (b) P1 architecture (Integrated Starter Generator) ISG;  
 (c) P2 architecture and  
 (d) P3 or P4 (post transmission) architecture.

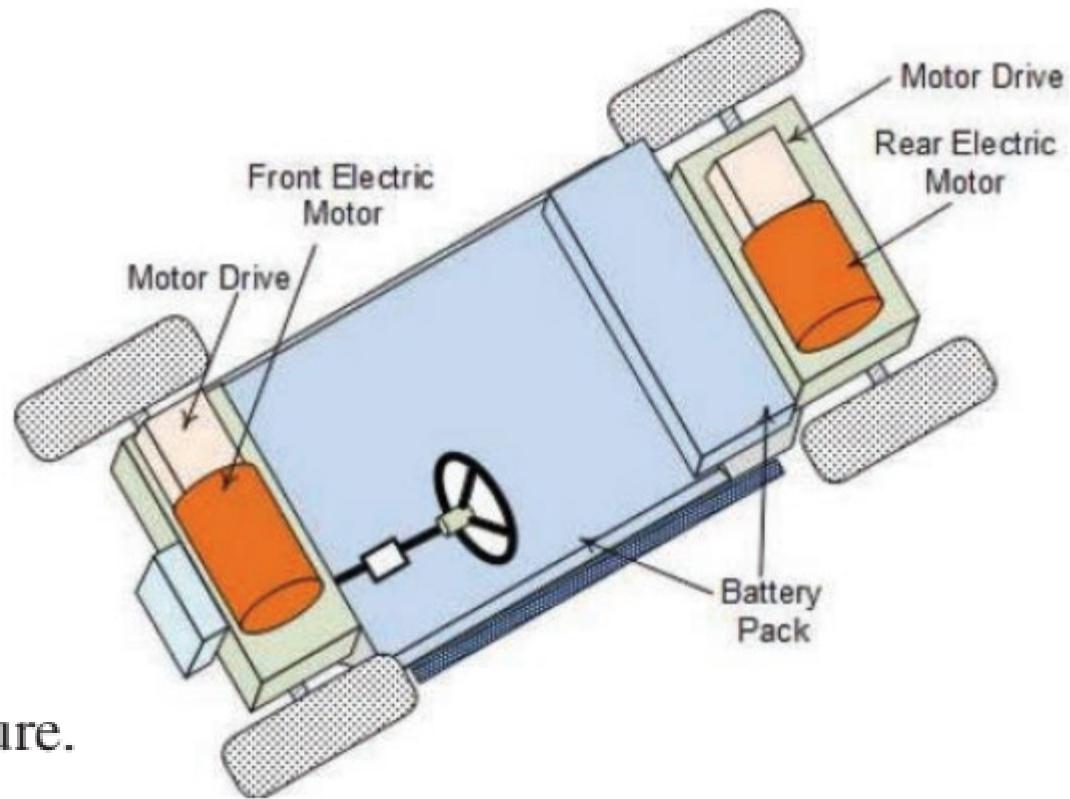


# Electric Vehicles: Skateboard Chassis

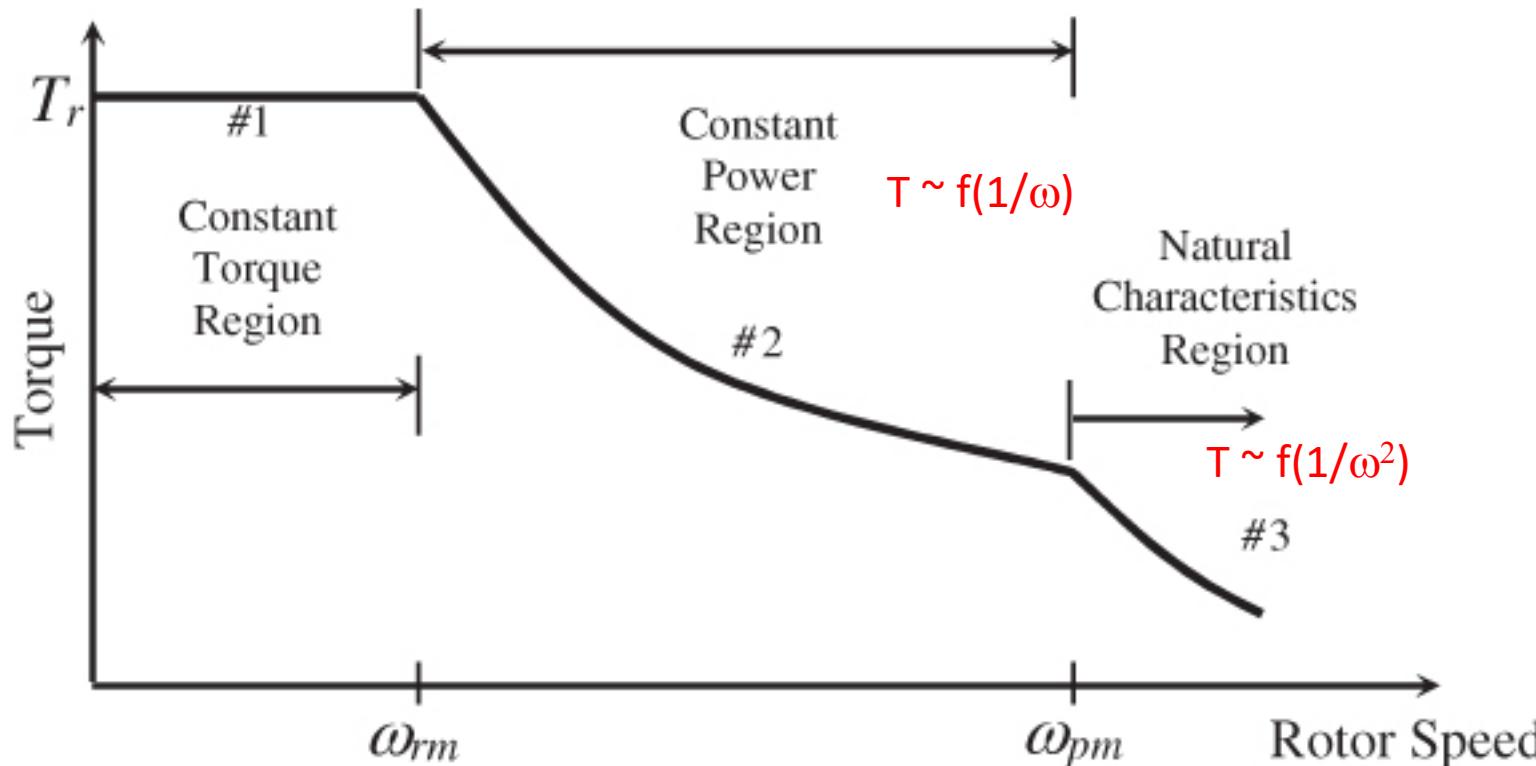


**FIGURE 3.10** EV equivalent powertrain architecture.

**FIGURE 3.11** Skateboard chassis layout for EVs.



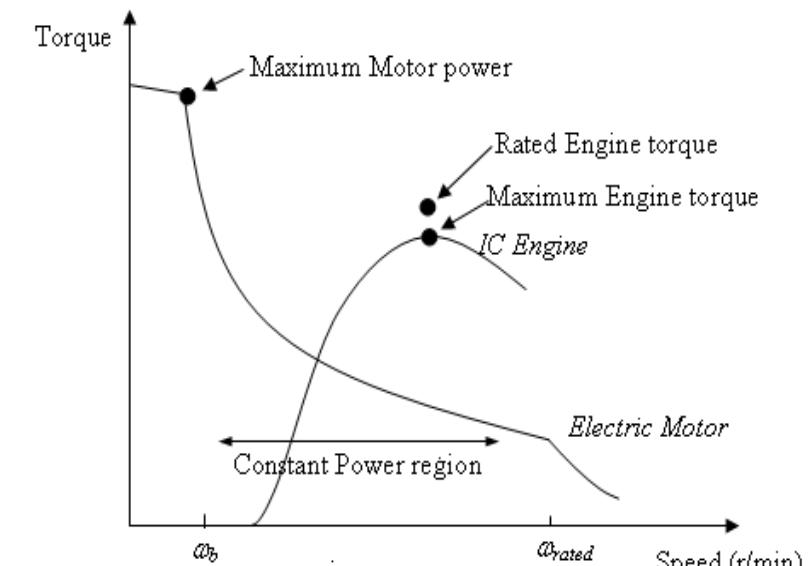
# EV Motor Sizing



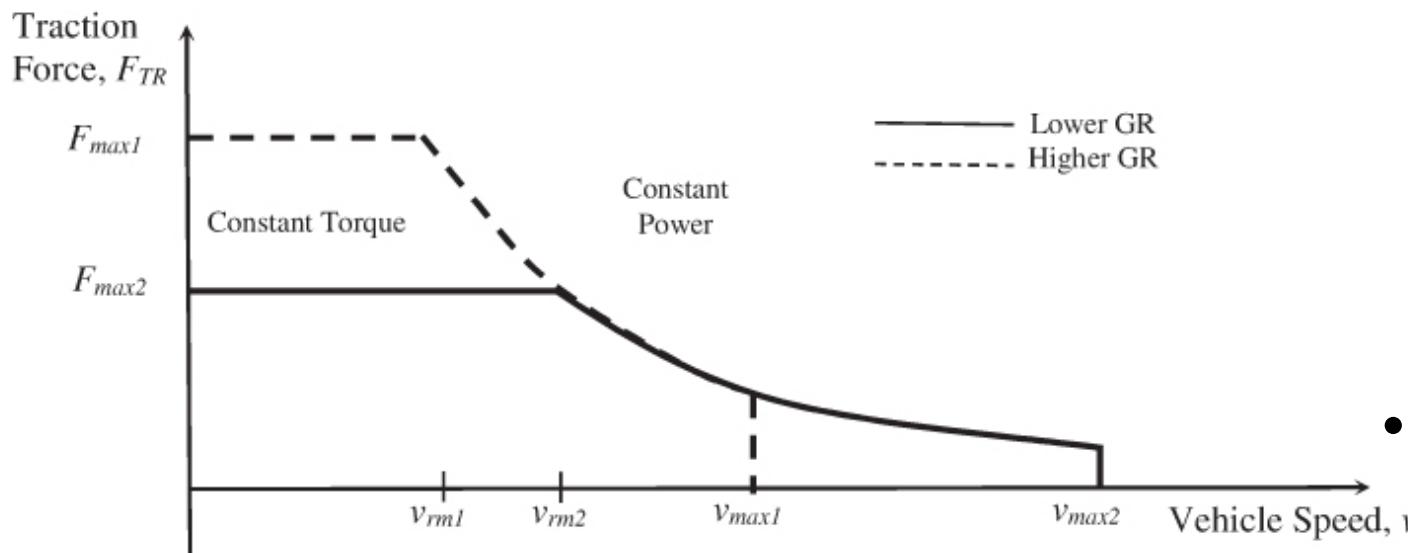
*Electric motor torque-speed envelope*

$$GearRatio(w_{engine}, w_{wheel}) := \frac{w_{engine}}{w_{wheel}}$$

$$(\omega_{wheel}, R_{tire}) := \omega_{wheel} \cdot R_{tire}$$



# EV Motor Sizing



***Electric Motor torque-speed characteristics in terms of traction force and vehicle speed for two gear ratios***

- Gear ratio depends on
  - motor rated speed
  - Vehicle maximum speed
  - Wheel radius
  - Maximum gradability
- Higher GR means larger gear size
- GR and electric motor rated speed to be selected simultaneously to optimize the overall size and performance requirements

## Exam 1: 20 problems (5 points/problem)

ELEC 5670/6970/6970-D02

Electric Vehicles

Fall 2022

Exam 1 (Online Via Canvas)

NAME: \_\_\_\_\_

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