AUBURN UNIVERSITY

Electric Vehicles

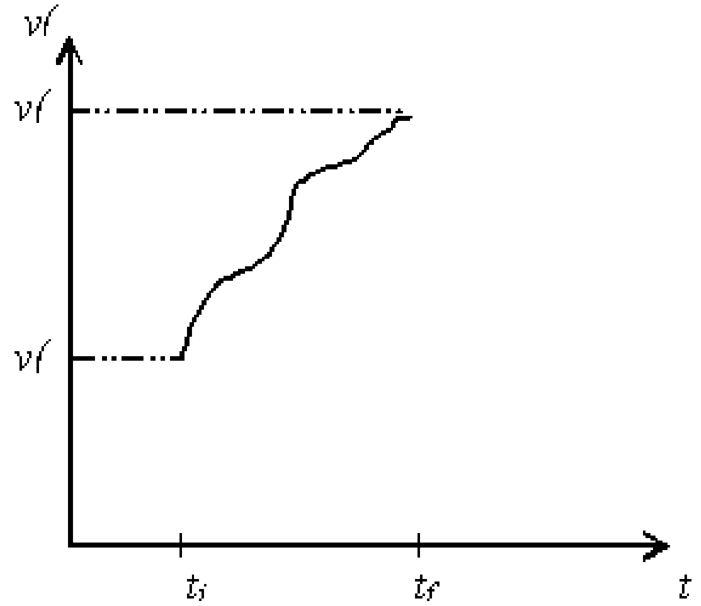
ELEC 5970/6970/6970-D01

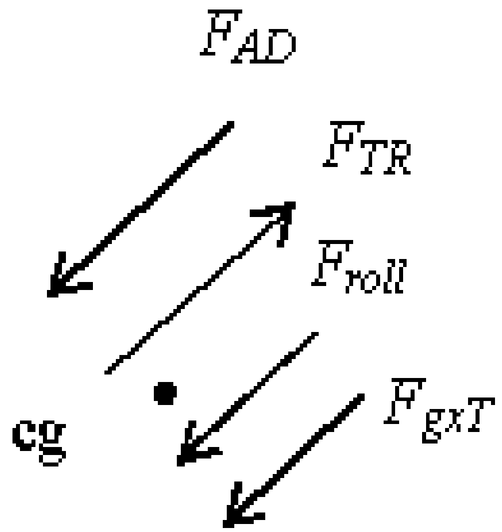
Vehicle Mechanics

References:

- Iqbal Husain, "Electric and Hybrid Vehicles, Design Fundamentals," Third Edition, March 2021, CRC Press, Taylor & Francis Group, ISBN: 978-0429-49092-7

Scenario 2: Non-constant F_{TR}, General Acceleration





Arbitrary velocity profile

In the general case,

$$\Sigma F = m\frac{dv}{dt}$$

$$\Rightarrow F_{TR} - F_{AD} - F_{roll} - F_{gxT} = m\frac{dv}{dt}$$

$$\Rightarrow F_{TR} = m\frac{dv}{dt} + mg\sin\beta + F_{AD} + F_{roll}$$

$$= m\frac{dv}{dt} + mg\sin\beta + \left[mgC_1 + \frac{\rho}{2}A_F C_D\right]v^2 + mgC_0$$

The instantaneous tractive power $P_{TR}(t)$ *is*

$$P_{TR}(t) = F_{TR}(t)v(t)$$
$$= mv\frac{dv}{dt} + v\left(F_{gxT} + F_{AD} + F_{roll}\right)$$

The change in tractive energy during an interval

$$\Delta e_{TR} = \int_{t_i}^{t_f} P_{TR}(t)dt$$
$$= \underbrace{m\int_{v(t_i)}^{v(t_f)} vdv}_{\text{kinetic energy}} + \underbrace{\int_{t_i}^{t_f}(v)F_{gxT}dt}_{\text{potential energy}} + \underbrace{\int_{t_i}^{t_f}(v)\left(F_{AD} + F_{roll}\right)dt}_{\text{loss term}}$$

- The 1st & 2nd terms represent kinetic and potential energy;
- 3rd & 4th terms represent the loss energy needed to overcome the non-constructive forces including the rolling resistance and the aerodynamic drag force. These two are known as loss term.

- 1st term:

$$m\int_{v(t_i)}^{v(t_f)} v\,dv = \frac{1}{2}m\left[v^2(t_f) - v^2(t_i)\right] = \Delta(\text{Kinteic Energy})$$

- 2nd term:

$$\int_{t_i}^{t_f}(v)F_{gxT}\,dt = mg\int_{t_i}^{t_f} v\sin\beta\,dt = mg\int_{s(t_i)}^{s(t_f)}\sin\beta\,ds = mg\int_{f(t_i)}^{f(t_f)} df$$

$$= mg\left[f(t_f) - f(t_i)\right]$$

$$= \Delta(\text{Potential Energy})$$

- 3rd term:

Let

$$\int_{f}^{t_f}(v)(F_{AD} + F_{roll})\,dt = E_{loss}$$

$$K_3 = mgC_0,\ \ K_4 = mgC_1 + \frac{\rho}{2}C_D A_F$$

then

$$E_{loss} = K_3\int_{t_i}^{t_f}\frac{ds}{dt}dt + K_4\int_{t_i}^{t_f} v^3\,dt;$$

$$= K_3\Delta s + K_4\int_{t_i}^{t_f} v^3\,dt.$$

Exercise 2.3

The vehicle with parameters given in Exercise 2.2 accelerates from 0 to 60 mph in 13.0 s for the following two acceleration types: (i) constant F_{TR} and (ii) uniform acceleration.

a. Plot on the same graph, the velocity profile of each acceleration type.

b. Calculate and compare the tractive energy required for the two types of acceleration.

$$F_{TR} = \text{const.} = 1,548\text{N}.$$

Ans. (b) $\Delta e_{\mathrm{TR}} = 0.2752$ MJ for constant F_{TR} and $\Delta e_{\mathrm{TR}} = 0.2744$ MJ for uniform acceleration.

Exercise 2.3

An EV has the following parameter values:

$m := 692\ kg \qquad C_D := 0.2 \qquad A_F := 2\ \ m^2 \qquad C_o := 0.009 \qquad C_1 := 1.75 \cdot 10^{-6}\ \left(\frac{s}{m}\right)^2$

$\rho_o := 1.16 \frac{kg}{m^3} \qquad g_o := 9.81 \frac{m}{s^2}$

The EV accellerates from 0 to 60 mph in 13 s for two accelleration types: $60\ mph = 26.822\ \frac{m}{s}$
(i) constant F_{TR} (ii) uniform acceleration (constant a)
a) Plot on the same graph, the velocity profile of each acceleration type
b) Calculate and compare the tractive energy required forr the two types of acceleration
$F_{TR\_max} := 1548$ Newton

Ans. (b) $\Delta e_{TR} = 0.2752$ MJ for constant F_{TR} and $\Delta e_{TR} = 0.2744$ MJ for uniform acceleration.

(i) Constant F_{TR} ; $F_{TR\_o} := 1548$ Newton, we will use the result from Excerice 2-2

$$V_T = \lim_{t \to \infty} v(t) = \sqrt{\frac{K_1}{K_2}} \Rightarrow \sqrt{K_1 K_2} = K_2 V_T$$

$K_1(F_{TR}) := \frac{F_{TR}}{m} - g_o \cdot C_o \qquad K_2 := \frac{\rho_o}{2 \cdot m} \cdot C_D \cdot A_F + g_o \cdot C_1 = 3.524 \cdot 10^{-4}$

for $0 \le t \le 13$ s

$$V_T(F_{TR}) := \sqrt{\frac{K_1(F_{TR})}{K_2}} \rightarrow \sqrt{4.1003787478845635375 \cdot F_{TR} - 250.51952823830385539}$$

$F_{TR\_o} = 1.548 \cdot 10^3$ N

$ii := 0, 1..13$

$$v(t) = \sqrt{\frac{K_1}{K_2}} \tanh\left(\sqrt{K_1 K_2} t\right)$$

$time_{ii} := 0 + ii \cdot 1$ $10^5\ s = 27.778\ hr$

$$v_{const\_FTR}(t, F_{TR}) := \sqrt{\frac{K_1(F_{TR})}{K_2}} \cdot \tanh\left(\sqrt{K_1(F_{TR}) \cdot K_2} \cdot t\right)$$

$K_1(F_{TR\_o}) = 2.149$ $K_2 = 3.524 \cdot 10^{-4}$ $\sqrt{K_1(F_{TR\_o}) \cdot K_2} = 0.028$

$$v_{const\_FTR}(t, F_{TR\_o}) \rightarrow 78.0824357553412936 62 \cdot \tanh(0.027518406654036728097 \cdot t)$$

Find the time required to accelerate from 0 to 60 mph

$$60\ \frac{mi}{hr} = 26.822\ \frac{m}{s} \qquad v_o(t) := 78.08 \cdot \tanh(0.0275 \cdot t)$$

$$t_{60mph} := \frac{1}{0.0275} \cdot \operatorname{atanh}\left(\frac{26.822}{78.08}\right) \qquad t_{60mph} = 13.021$$ seconds

Calculate P_{TR} $\quad P_{TR}(t) = F_{TR} V_T \tanh\left(\sqrt{K_1 K_2}\,t\right) = P_T \tanh\left(\sqrt{K_1 K_2}\,t\right)$

$$P_T := F_{TR\_o} \cdot V_T\left(F_{TR\_o}\right) = 1.209 \cdot 10^5$$ watt

Calculate average traction power

$$\overline{P_{TR}} = \frac{P_T}{t_f} \frac{1}{\sqrt{K_1 K_2}} \ln\left[\cosh\left(\sqrt{K_1 K_2}\,t_f\right)\right]$$

$$P_{TR\_ave}\left(P_T, t_f\right) := \frac{P_T}{t_f} \cdot \frac{1}{\sqrt{K_1\left(F_{TR\_o}\right) \cdot K_2}} \cdot \ln\left(\cosh\left(\sqrt{K_1\left(F_{TR\_o}\right) \cdot K_2} \cdot t_f\right)\right)$$

$$P_{TR\_ave}\left(P_T, t_{60mph}\right) = 2.121 \cdot 10^4\ watt$$

Calculate energy required to achieve the steady state speed V_T at time t_f

$$P_{TR\_average} := P_{TR\_ave}\left(P_T, t_{60mph}\right)$$

$$\Delta e_{TR}\left(t_f, P_{TR\_average}\right) := t_f \cdot P_{TR\_average}$$

$$\Delta e_{TR}\left(t_{60mph}, P_{TR\_average}\right) = 2.761 \cdot 10^5 \text{ Joule}$$

Another way to compute Δe_{TR}

$$P_{TR\_const\_FTR}(t) := F_{TR\_o} \cdot \left(v_{const\_FTR}\left(t, F_{TR\_o}\right)\right)$$

$$P_{TR\_const\_FTR}(t) \rightarrow 120871.6105492683225888 \cdot \tanh\left(0.027518406654036728097 \cdot t\right)$$

$$\Delta e_{TR\_const\_a}\left(t_f\right) := \int_0^{t_f} P_{TR\_const\_FTR}(t)\, \mathrm{d}t$$

$$\Delta e_{TR\_const\_a}\left(t_f\right) \rightarrow 4.39239132079391061445 \cdot 10^6 \cdot \ln\left(\cosh\left(0.027518406654036728097 \cdot t_f\right)\right)$$

$$\Delta e_{TR\_const\_a}\left(t_{60mph}\right) = 2.761 \cdot 10^5 \text{ Joule}$$

(ii) Constant accelleration a ; $a_{TR\_o} := \frac{26.8}{13} = 2.062$ m/s (60 mph in 13 seconds)

$v_{const\_a}(t) := a_{TR\_o} \cdot t$

Note: $\beta = 0$ $\sin(\beta) = 0$

$$F_{TR} - F_{AD} - F_{roll} - F_{gxT} = m\frac{dv}{dt}$$

$$F_{TR} = m\frac{dv}{dt} + mg\sin\beta + F_{AD} + F_{roll}$$

$$m\frac{dv}{dt} + mg\sin\beta + \left[mgC_1 + \frac{\rho}{2}A_F C_D\right]v^2 + mgC_0$$

$$F_{TR}(a_{TR}, t) := m \cdot a_{TR} + \left(m \cdot g_o \cdot C_1 + \frac{\rho_o}{2} \cdot A_F \cdot C_D\right) \cdot (a_{TR} \cdot t)^2 + m \cdot g_o \cdot C_o$$

$$P_{TR\_const\_a}(a_{TR}, t) := F_{TR}(a_{TR}, t) \cdot (a_{TR} \cdot t)$$

$$P_{TR\_const\_a}(a_{TR}, t) \rightarrow a_{TR} \cdot t \cdot \left(692 \cdot a_{TR} + 0.24387991 \cdot a_{TR}^2 \cdot t^2 + 61.09668\right)$$

$$\Delta e_{TR\_const\_a}(t_f) := \int_0^{t_f} P_{TR\_const\_a}(a_{TR\_o}, t)\, dt$$

$$\Delta e_{TR\_const\_a}(t_f) \rightarrow 0.53418336547167976633 \cdot t_f^4 + 1533.4561044733731622 \cdot t_f^2$$

$\Delta e_{TR\_const\_a}(t_{60mph}) = 2.753 \cdot 10^5$ Joule

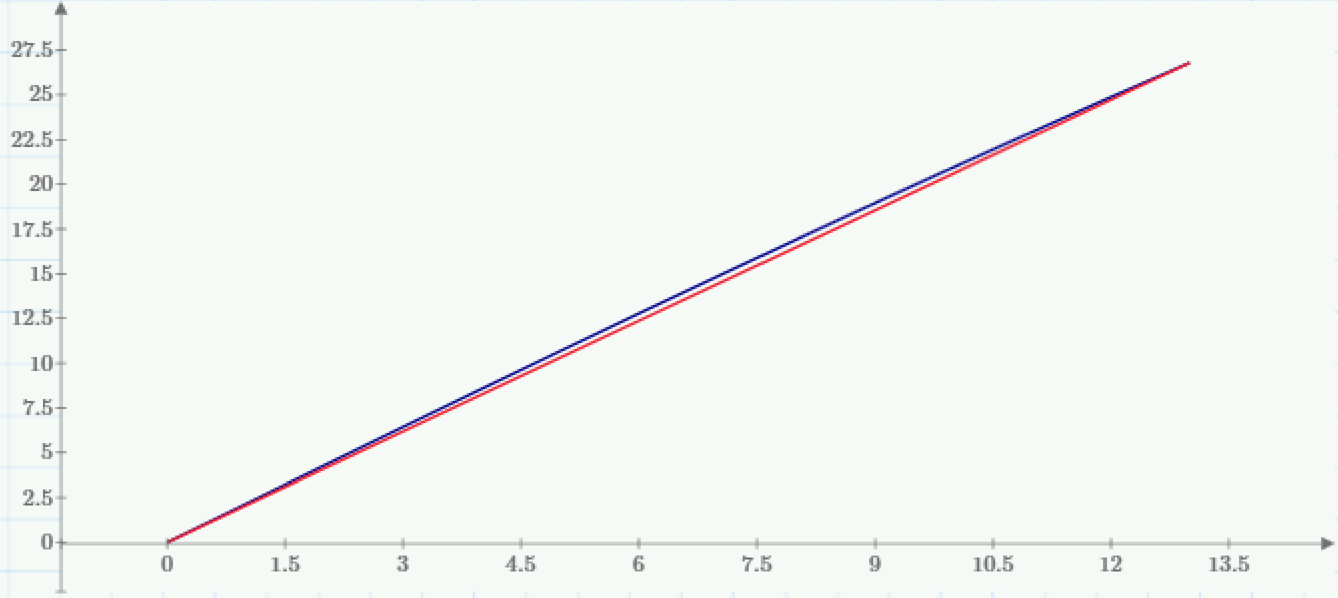
a) Plot on the same graph, the velocity profile of each acceleration type

$$v_{const\_FTR}(t, F_{TR}) := \sqrt{\frac{K_1(F_{TR})}{K_2}} \cdot \tanh\left(\sqrt{K_1(F_{TR}) \cdot K_2} \cdot t\right)$$

$$v_{const\_a}(t) := a_{TR\_o} \cdot t$$

$v_{const\_FTR}(time_{ii}, F_{TR\_o})$

$v_{const\_a}(time_{ii})$



$time_{ii}$

(iii) This example is extended to compute the Kinetic Energy and Potential energy gained for Uniform Acceleration (constant a).

$$\Delta e_{TR} = \int_{t_i}^{t_f} P_{TR}(t)dt$$

$$= m \int_{v(t_i)}^{v(t_f)} v dv + \int_{t_i}^{t_f} (v) F_{gxT} dt + \int_{t_i}^{t_f} (v)\left(F_{AD} + F_{roll}\right) dt$$

kinetic energy — potential energy — loss term

iiia) Compute the kinetic energy gained from 0 - 60 mph at constant accelleration in 13 s

$$a_{TR\_o} := \frac{26.8}{13} = 2.062 \quad \frac{m}{s^2}$$

$$\Delta Kinetic\_Energy(t_i, t_f) := \frac{1}{2} \cdot m \cdot \left(v_{const\_a}(t_f)^2 - v_{const\_a}(t_i)^2\right)$$

$$\Delta Kinetic\_Energy(0, 13) = 2.485 \cdot 10^5 \quad Joule$$ = 248.5 kJoule

iiib) Compute the potential energy gained from 0 - 60 mph at constant accelleration in 13 s

$$\int_{t_i}^{t_f} (v) F_{gxT} dt = mg \int_{t_i}^{t_f} v \sin\beta dt = mg \int_{s(t_i)}^{s(t_f)} \sin\beta ds = mg \int_{f(t_i)}^{f(t_f)} df$$
$$= mg\left[f(t_f) - f(t_i)\right]$$
$$= \Delta(\text{Potential Energy})$$

Note: $\beta = 0$; thus $\sin(\beta)=0$

Level road - Thus no altitude gain:
$\Delta Potential\_Energy = 0$

$$\Delta Potential\_Energy\left(t_i, t_f, \beta_o, a_{TR\_o}\right) := m \cdot g_o \int_{t_i}^{t_f} a_{TR\_o} \cdot t \cdot \sin\left(\beta_o\right) \mathrm{d}t$$

$$\Delta Potential\_Energy\left(t_i, t_f, \beta_o, a_{TR\_o}\right) \rightarrow -6997.3975384615393493 \cdot \sin\left(\beta_o\right) \cdot \left(1.0 \cdot t_i^{\,2} - t_f^{\,2}\right)$$

$$\Delta Potential\_Energy\left(0, 13, 0, a_{TR\_o}\right) = 0 \qquad Joule$$

iiic) Compute the energy loss from 0 - 60 mph at constant accelleration in 13 s

$$E_{loss} = K_3 \int_{t_i}^{t_f} \frac{ds}{dt} dt + K_4 \int_{t_i}^{t_f} v^3 dt;$$

$$= K_3 \Delta s + K_4 \int_{t_i}^{t_f} v^3 dt.$$

for $0 \leq t \leq 13$ s

$$K_3 := m \cdot g_o \cdot C_1$$

$$K_4 := m \cdot g_o \cdot C_1 + \frac{\rho_o}{2} \cdot C_D \cdot A_F = 0.244$$

$$E_{loss}(t_i, t_f, a_{TR\_o}) := K_3 \cdot \int_{t_i}^{t_f} a_{TR\_o} \cdot t \, dt + K_4 \cdot \int_{t_i}^{t_f} (a_{TR\_o} \cdot t)^3 \, dt$$

$$E_{loss}(0, 13, a_{TR\_o}) \rightarrow 15258.880581558644556 \quad Joule$$ = 15.258 kJoule

Propulsion System Design

- The design requirements related to vehicle power typically specified by a customer are:
  - the initial acceleration
  - rated velocity on a given slope
  - maximum % grade
  - maximum steady state velocity.

- The complete design is a complex issue involving numerous variables, constraints, considerations and judgment, which is beyond the scope of this book. Some component design and sizing issues will be addressed in chapters 9 and 10.

Propulsion System Design

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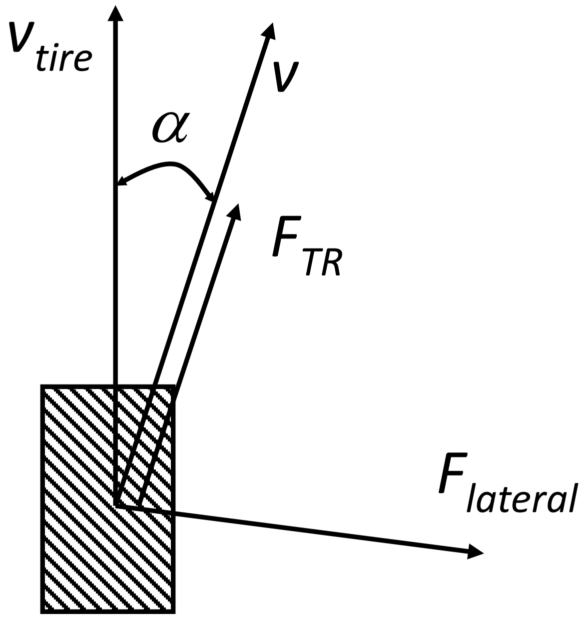
Tire-Road Force Mechanics

- A number of forces including the tractive force works at the tire-ground contact place of a moving vehicle.
- Pneumatic tire is designed to support vehicle weight and to provide traction for driving and braking
- The performance potential of a vehicle depends on the maximum traction (or braking) force that can be sustained at the tire-road interface.

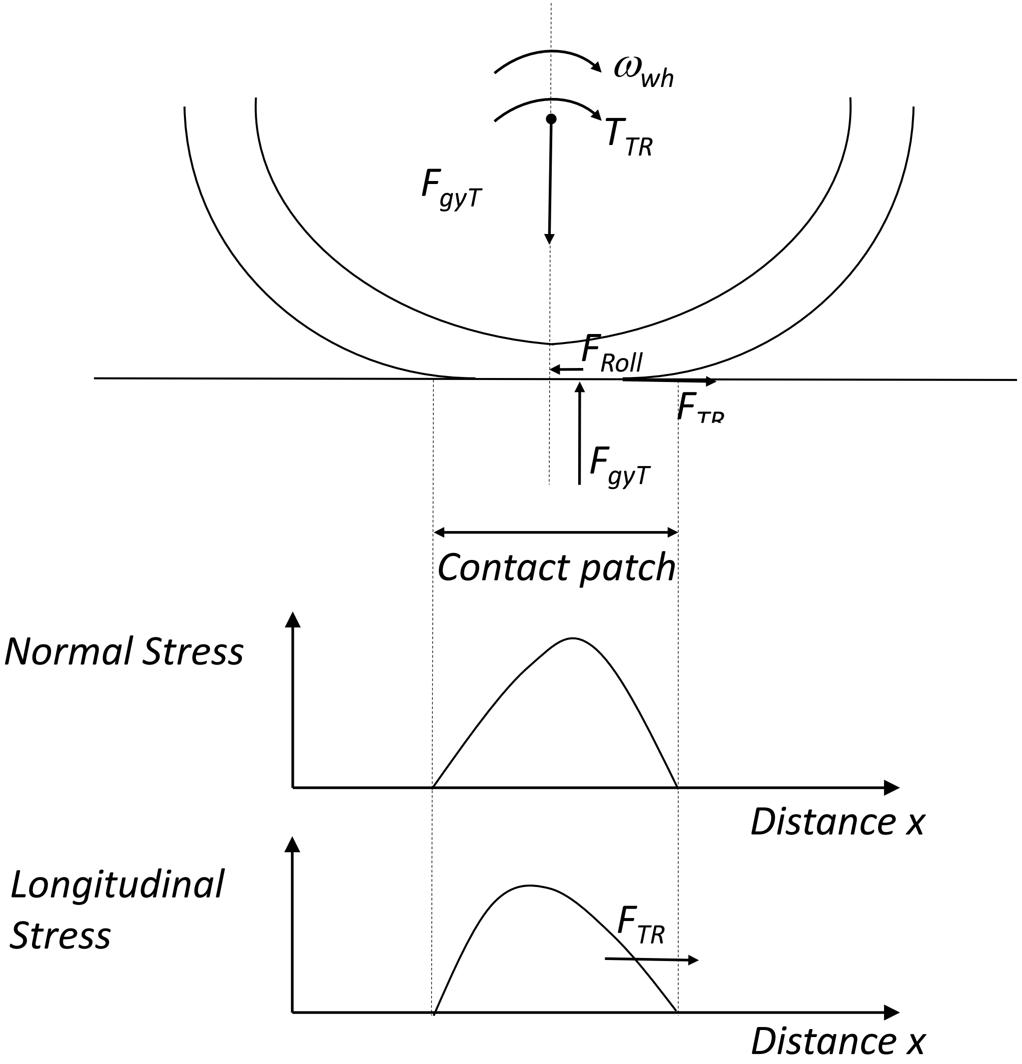
Vehicle Slip

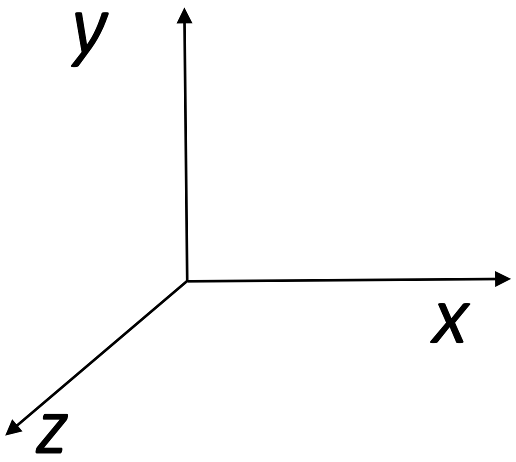
- The distance that the tire travels when subject to propulsion torque is less than that in a free rolling tire due to the compression. This results in a difference between the wheel angular speed and vehicle speed which is what is known as *longitudinal slip* or simply *slip*.
- Mathematically, slip *s* of a vehicle is given as

$$s = \left(1 - \frac{v}{\omega_{wh} r_{wh}}\right)$$



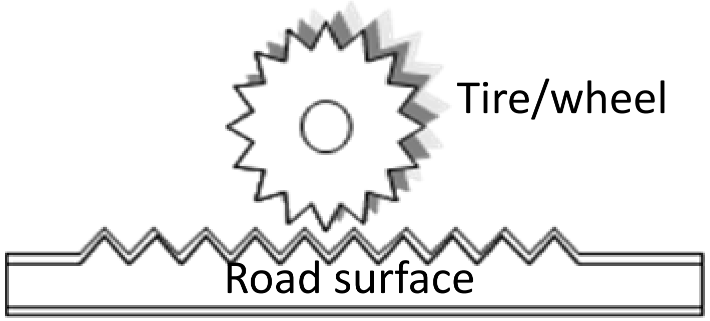
Force Transmission at Tire-Road Interface

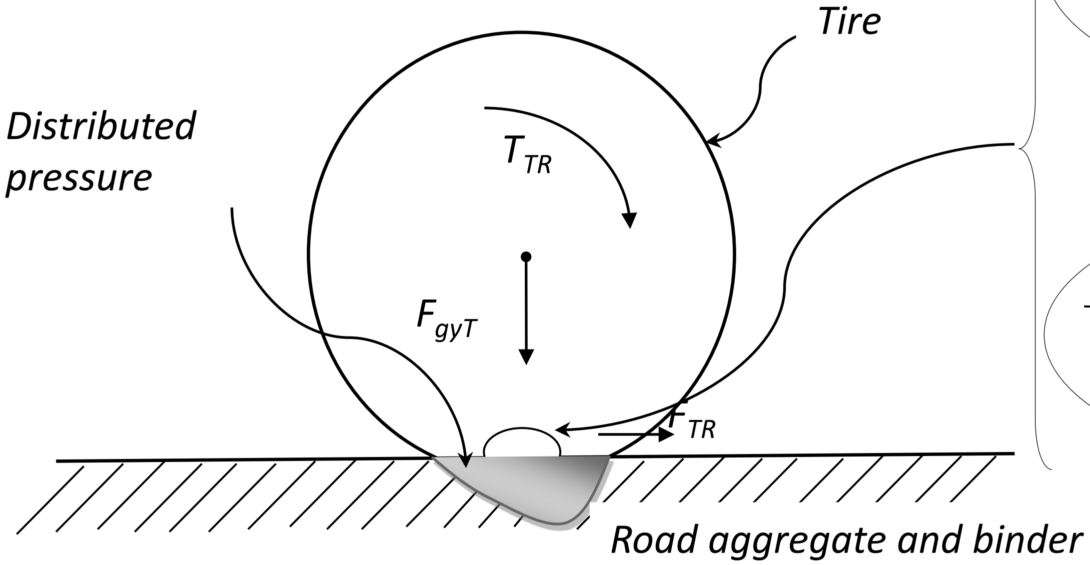


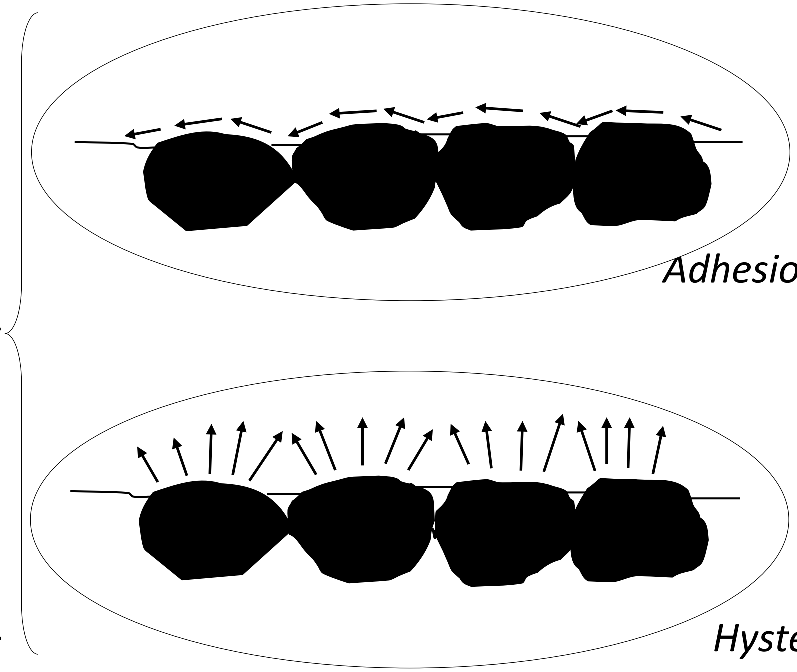


The longitudinal force is the traction force F_{TR} responsible for forward velocity of the vehicle.

Force Transmission at Tire-Road Interface







Adhesion

Hysteresis

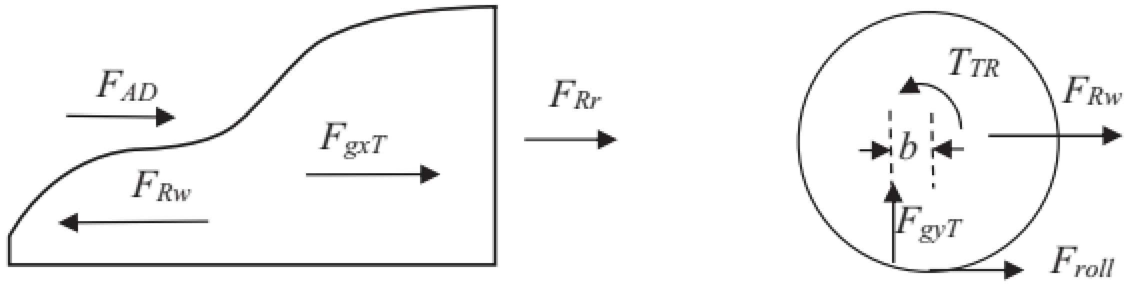
Adhesion:
- Dominate on dry surface
- Decreases on wet surface

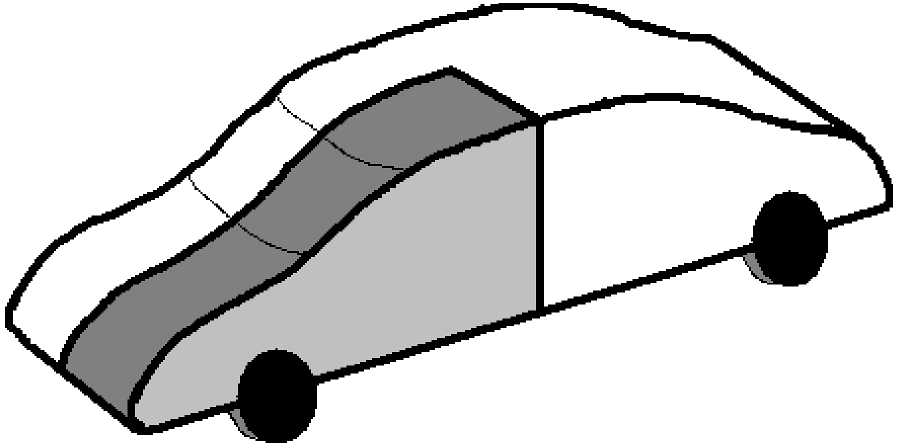
Hysteresis:
- Deformation of the rubber when sliding over road aggregates
- Not affected by wet surface

Quarter-car model

Per Quarter Basis:

$$F_{Rw} - F_{AD} - F_{gxT} - F_{Rr} = m\frac{dv}{dt}$$





The wheel dynamic can be achieved from the balance of forces and torque as

$$-F_{Rw} - F_{roll} = m_w\frac{dv}{dt}$$

$$T_{TR} - F_{gyT}b + F_{roll}r_{wh} = J_w\frac{d\omega_{wh}}{dt}$$

T_{TR} is the driving or tractive torque, m_w is the mass of the wheel and r_{wh} is the wheel radius.

Quarter-car model

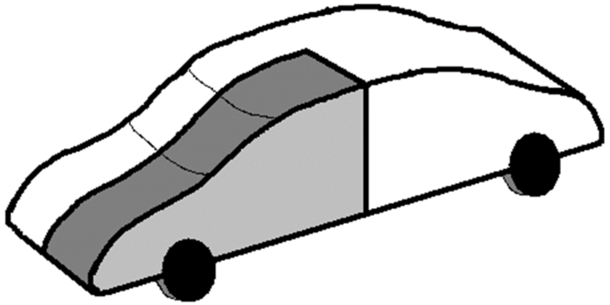
$P_{TR} = T_{TR}\ \omega_{wh} = F_{TR}\, v$ and $v = \omega_{wh}\, r_{wh}$

$$\frac{T_{TR}}{r_{wh}} - F_{gyT}\frac{b}{r_{wh}} - F_{AD} - F_{gxT} - F_{Rr} = (m + m_w)\frac{dv}{dt} + \frac{J_w}{r_{wh}}\frac{d\omega_{wh}}{dt}$$

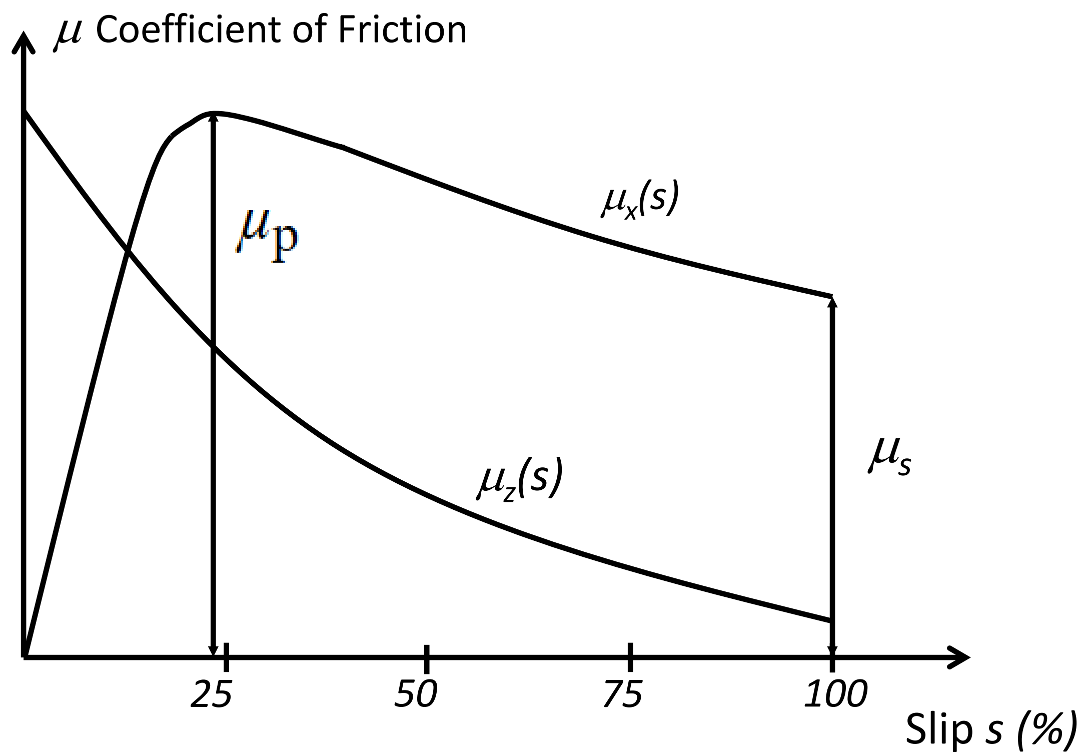
F_{TR}

$$(m + m_w)\frac{dv}{dt} + \frac{J_w}{r_{wh}}\frac{d\omega_{wh}}{dt} = k_m m \frac{dv}{dt}$$

The general form of the dynamic equation of motion in the tangential direction



Traction Limit and Control



$$F_{roll}(s) = \mu(s) F_{gyT}$$

The friction coefficient when the slip reaches 100% is known as sliding coefficient μ_s

Surface	Peak coefficient, μ_p	Sliding coefficient, μ_s
Ashphalt and concrete (dry)	0.9	0.75
Concrete (wet)	0.8	0.7
Ashphalt (wet)	0.6	0.5
Gravel	0.6	0.55
Snow	0.2	0.15
Ice	0.1	0.07

End of Lecture 7