Foundations of graphics

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2 d) The Euler-method is a quite bad approximation for the given function since the Runge Kutta-Method is a higer order method the truncation error is alot smaller resulting in a better approximation.

A3)

a) $\vec{v}(t) = \vec{v}(t_0) + \int_{t_0}^{t} \vec{a}(t)\,dt = \int_{t_0}^{t} \vec{a}(t)\,dt = \int_{t_0}^{t} r \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} dt =$

$r \left[\begin{matrix} \sin(t) \\ -\cos(t) \end{matrix} \right]_{t_0}^{t} = r \begin{pmatrix} \sin(t) & -\sin(t_0) \\ -\cos(t) & +\cos(t_0) \end{pmatrix}$

$\vec{x}(t) = \vec{x}(t_0) + \int_{t_0}^{t} \vec{v}(t)\,dt = \vec{x}(t_0) + \int_{t_0}^{t} r \begin{pmatrix} \sin(t) - \sin(t_0) \\ -\cos(t) + \cos(t_0) \end{pmatrix} dt$

$= \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + r \left[\begin{matrix} -\cos(t) & -\sin(t_0)\,t \\ -\sin(t) & +\cos(t_0)\,t \end{matrix} \right]_{t_0}^{t}$

$= \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + r \begin{pmatrix} -\cos(t) - \sin(t_0)\,t + \cos(t_0) + \sin(t_0)\,t_0 \\ -\sin(t) + \cos(t_0)\,t + \sin(t_0) - \cos(t_0)\,t_0 \end{pmatrix}$

b) Forward Euler Integration ... (fog Side 50)

... is about the simplest possible way to do nomerical integration. It works by iterating the linear slope of the derivative of a particular value as an approximation to the function at some nearby value

important equations:

$\vec{x}_{n+1} = x_n + \dot{x}_n \Delta t$

$\vec{v}_{n+1} = \vec{v}_n + \vec{a}_n \Delta t$

$\vec{x}_{n+1} = \vec{x}_n + v_{n+1} \Delta t$

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c) $\vec{x}(t_0+h) = \vec{x}(t_1) = \vec{x}(t_0) + h\,\vec{v}(t_1) =$

$$\begin{pmatrix}1\\0\end{pmatrix} + \pi\begin{pmatrix}\sin(t_1)-\sin(t_0)\\ -\cos(t_1)+\cos(t_0)\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} + \pi\begin{pmatrix}\sin(\pi)-\sin(0)\\ -\cos(\pi)+\cos(0)\end{pmatrix} = \begin{pmatrix}1\\2\pi\end{pmatrix}$$

d) $\vec{x}(t_0+h) = \vec{x}(t_1) = \vec{x}(t_0) + hv(t_1) = \begin{pmatrix}1\\0\end{pmatrix} + \frac{\pi}{4} r\begin{pmatrix}\sin(t_1)-\sin(t_0)\\ -\cos(t_1)+\cos(t_0)\end{pmatrix}$

$$= \begin{pmatrix}1\\0\end{pmatrix} + \frac{\pi}{4}\begin{pmatrix}\sin(\frac{\pi}{4})\\ -\cos(\frac{\pi}{4})+1\end{pmatrix}$$

$$\vec{x}(t_1+h) = \vec{x}(t_2) = \vec{x}(t_1) + hv(t_2) = \vec{x}(t_1) + \frac{\pi}{4} r\begin{pmatrix}\sin(t_2)-\sin(t_0)\\ -\cos(t_2)+\cos(t_0)\end{pmatrix}$$

$$= x(t_1) + \frac{\pi}{4}\begin{pmatrix}\sin(\frac{\pi}{2})\\ -\cos(\frac{\pi}{2})+1\end{pmatrix}$$

$$\vec{x}(t_2+h) = \vec{x}(t_3) = \vec{x}(t_2) + hv(t_3) = \vec{x}(t_2) + \frac{\pi}{4} r\begin{pmatrix}\sin(t_3)-\sin(t_0)\\ -\cos(t_3)+\cos(t_0)\end{pmatrix}$$

$$= x(t_2) + \frac{\pi}{4}\begin{pmatrix}\sin(\frac{3\pi}{4})\\ -\cos(\frac{3\pi}{4})+1\end{pmatrix}$$

$$\vec{x}(t_3+h) = \vec{x}(t_4) = \vec{x}(t_3) + hv(t_4) = \vec{x}(t_3) + \frac{\pi}{4} r\begin{pmatrix}\sin(t_4)-\sin(t_0)\\ -\cos(t_4)+\cos(t_0)\end{pmatrix}$$

$$= x(t_3) + \frac{\pi}{4}\begin{pmatrix}\sin(\pi)\\ -\cos(\pi)+1\end{pmatrix} = \vec{x}(t_3) + \frac{\pi}{4}\begin{pmatrix}0\\2\end{pmatrix}$$

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