City Yield Formulae and Considerations

We start with two considerations: To balance tall and wide play, we want to make one tall city yield as much as a given number of small cities. This gives us the first equation, which is the ratio of one city with three times the population of another. Since population yields scale exponentially, once this break-even point is reached, the tall play-style begins to outpace the wide play-style. The second consideration is how much one wide city should yield in absolute terms. Together, these two considerations result in two equations, where the flat and exponential yield per pop represent the unknown.

y ... Yield of an n-sized city with.
d ... Number of n-sized cities required to match one $3 \cdot n$-sized city.
n ... Pop size used as basis for comparison and calculation.
a ... Linear yield per pop.
b ... Exponential yield per pop, also known as yield per pop per pop.
c ... Constant yield from districts and buildings.
s ... Yield from specialists, provided all specialist slots are filled.

$$A := d = \frac{3 \cdot n \cdot a + (3 \cdot n)^2 \cdot b + c + s}{n \cdot a + n^2 \cdot b + c + s} \xrightarrow{solve,a} \frac{(-d+1) \cdot s + (9 \cdot b - b \cdot d) \cdot n^2 + (c - c \cdot d)}{(d-3) \cdot n}$$

$$B := y = n \cdot a + n^2 \cdot b + c + s \xrightarrow{solve,a} \frac{y - (s + b \cdot n^2 + c)}{n}$$

$$A = B \xrightarrow{solve,b} \frac{(d-3) \cdot y + 2 \cdot s + 2 \cdot c}{6 \cdot n^2}$$

Resubstituting, this gives us formulae for a and b:

$$a(y, n, d, c, s) := \frac{-(8 \cdot s) + (9 \cdot y - 8 \cdot c - y \cdot d)}{6 \cdot n}$$

$$b(y, n, d, c, s) := \frac{2 \cdot s + y \cdot d + (2 \cdot c - 3 \cdot y)}{6 \cdot n^2}$$

For purposes of creating value tables, we also define a formula for the overall resulting yield of a city:

$$Y(n, a, b, c, s) := n \cdot a + n^2 \cdot b + c + s$$

Science

Tier 3:

$y := 83 \qquad n := 10 \qquad d := 5 \qquad c := 19 \qquad s := 18$

$A := a(y, n, d, c, s) = 0.6$
$B := b(y, n, d, c, s) = 0.4$

$Y(5, A, B, c, s) = 50 \qquad Y(5, A, B, c, 0) = 32$
$Y(10, A, B, c, s) = 83 \qquad Y(10, A, B, c, 0) = 65$
$Y(15, A, B, c, s) = 136 \qquad Y(15, A, B, c, 0) = 118$
$Y(20, A, B, c, s) = 209 \qquad Y(20, A, B, c, 0) = 191$
$Y(25, A, B, c, s) = 302 \qquad Y(25, A, B, c, 0) = 284$
$Y(30, A, B, c, s) = 415 \qquad Y(30, A, B, c, 0) = 397$
$Y(40, A, B, c, s) = 701 \qquad Y(40, A, B, c, 0) = 683$
$Y(50, A, B, c, s) = 1067 \qquad Y(50, A, B, c, 0) = 1049$

Tier 2:

$y := 44 \qquad n := 10 \qquad d := 5 \qquad c := 10 \qquad s := 9$

$A := a(y, n, d, c, s) = 0.4$
$B := b(y, n, d, c, s) = 0.21$

$Y(5, A, B, c, s) = 26.25 \qquad Y(5, A, B, c, 0) = 17.25$
$Y(10, A, B, c, s) = 44 \qquad Y(10, A, B, c, 0) = 35$
$Y(15, A, B, c, s) = 72.25 \qquad Y(15, A, B, c, 0) = 63.25$
$Y(20, A, B, c, s) = 111 \qquad Y(20, A, B, c, 0) = 102$
$Y(25, A, B, c, s) = 160.25 \qquad Y(25, A, B, c, 0) = 151.25$
$Y(30, A, B, c, s) = 220 \qquad Y(30, A, B, c, 0) = 211$
$Y(40, A, B, c, s) = 371 \qquad Y(40, A, B, c, 0) = 362$
$Y(50, A, B, c, s) = 564 \qquad Y(50, A, B, c, 0) = 555$

Tier 1:

$y := 19 \qquad n := 10 \qquad d := 5 \qquad c := 5 \qquad s := 3$

$A := a(y, n, d, c, s) = 0.2$
$B := b(y, n, d, c, s) = 0.09$

$Y(5, A, B, c, s) = 11.25 \qquad Y(5, A, B, c, 0) = 8.25$
$Y(10, A, B, c, s) = 19 \qquad Y(10, A, B, c, 0) = 16$
$Y(15, A, B, c, s) = 31.25 \qquad Y(15, A, B, c, 0) = 28.25$
$Y(20, A, B, c, s) = 48 \qquad Y(20, A, B, c, 0) = 45$
$Y(25, A, B, c, s) = 69.25 \qquad Y(25, A, B, c, 0) = 66.25$
$Y(30, A, B, c, s) = 95 \qquad Y(30, A, B, c, 0) = 92$
$Y(40, A, B, c, s) = 160 \qquad Y(40, A, B, c, 0) = 157$
$Y(50, A, B, c, s) = 243 \qquad Y(50, A, B, c, 0) = 240$

Tier 0:

$y := 9 \qquad n := 10 \qquad d := 5 \qquad c := 3 \qquad s := 0$

$A := a(y, n, d, c, s) = 0.2$
$B := b(y, n, d, c, s) = 0.04$

$Y(5, A, B, c, s) = 5$
$Y(10, A, B, c, s) = 9$
$Y(15, A, B, c, s) = 15$
$Y(20, A, B, c, s) = 23$
$Y(25, A, B, c, s) = 33$
$Y(30, A, B, c, s) = 45$
$Y(40, A, B, c, s) = 75$
$Y(50, A, B, c, s) = 113$

$Y(5, A, B, c, 0) = 5$
$Y(10, A, B, c, 0) = 9$
$Y(15, A, B, c, 0) = 15$
$Y(20, A, B, c, 0) = 23$
$Y(25, A, B, c, 0) = 33$
$Y(30, A, B, c, 0) = 45$
$Y(40, A, B, c, 0) = 75$
$Y(50, A, B, c, 0) = 113$

Culture

Tier 3:

$y := 101 \qquad n := 10 \qquad d := 5 \qquad c := 19 \qquad s := 18$

$A := a(y, n, d, c, s) = 1.8$
$B := b(y, n, d, c, s) = 0.46$

$Y(5, A, B, c, s) = 57.5 \qquad Y(5, A, B, c, 0) = 39.5$
$Y(10, A, B, c, s) = 101 \qquad Y(10, A, B, c, 0) = 83$
$Y(15, A, B, c, s) = 167.5 \qquad Y(15, A, B, c, 0) = 149.5$
$Y(20, A, B, c, s) = 257 \qquad Y(20, A, B, c, 0) = 239$
$Y(25, A, B, c, s) = 369.5 \qquad Y(25, A, B, c, 0) = 351.5$
$Y(30, A, B, c, s) = 505 \qquad Y(30, A, B, c, 0) = 487$
$Y(40, A, B, c, s) = 845 \qquad Y(40, A, B, c, 0) = 827$
$Y(50, A, B, c, s) = 1277 \qquad Y(50, A, B, c, 0) = 1259$

Tier 2:

$y := 53 \qquad n := 10 \qquad d := 5 \qquad c := 10 \qquad s := 9$

$A := a(y, n, d, c, s) = 1$
$B := b(y, n, d, c, s) = 0.24$

$Y(5, A, B, c, s) = 30 \qquad Y(5, A, B, c, 0) = 21$
$Y(10, A, B, c, s) = 53 \qquad Y(10, A, B, c, 0) = 44$
$Y(15, A, B, c, s) = 88 \qquad Y(15, A, B, c, 0) = 79$
$Y(20, A, B, c, s) = 135 \qquad Y(20, A, B, c, 0) = 126$
$Y(25, A, B, c, s) = 194 \qquad Y(25, A, B, c, 0) = 185$
$Y(30, A, B, c, s) = 265 \qquad Y(30, A, B, c, 0) = 256$
$Y(40, A, B, c, s) = 443 \qquad Y(40, A, B, c, 0) = 434$
$Y(50, A, B, c, s) = 669 \qquad Y(50, A, B, c, 0) = 660$

Tier 1:

$y := 25 \qquad n := 10 \qquad d := 5 \qquad c := 5 \qquad s := 3$

$A := a(y, n, d, c, s) = 0.6$
$B := b(y, n, d, c, s) = 0.11$

$Y(5, A, B, c, s) = 13.75 \qquad Y(5, A, B, c, 0) = 10.75$
$Y(10, A, B, c, s) = 25 \qquad Y(10, A, B, c, 0) = 22$
$Y(15, A, B, c, s) = 41.75 \qquad Y(15, A, B, c, 0) = 38.75$
$Y(20, A, B, c, s) = 64 \qquad Y(20, A, B, c, 0) = 61$
$Y(25, A, B, c, s) = 91.75 \qquad Y(25, A, B, c, 0) = 88.75$
$Y(30, A, B, c, s) = 125 \qquad Y(30, A, B, c, 0) = 122$
$Y(40, A, B, c, s) = 208 \qquad Y(40, A, B, c, 0) = 205$
$Y(50, A, B, c, s) = 313 \qquad Y(50, A, B, c, 0) = 310$

Tier 0:

$y := 15 \qquad n := 10 \qquad d := 5 \qquad c := 3 \qquad s := 0$

$A := a(y, n, d, c, s) = 0.6$
$B := b(y, n, d, c, s) = 0.06$

$Y(5, A, B, c, s) = 7.5$ $\qquad$ $Y(5, A, B, c, 0) = 7.5$
$Y(10, A, B, c, s) = 15$ $\qquad$ $Y(10, A, B, c, 0) = 15$
$Y(15, A, B, c, s) = 25.5$ $\qquad$ $Y(15, A, B, c, 0) = 25.5$
$Y(20, A, B, c, s) = 39$ $\qquad$ $Y(20, A, B, c, 0) = 39$
$Y(25, A, B, c, s) = 55.5$ $\qquad$ $Y(25, A, B, c, 0) = 55.5$
$Y(30, A, B, c, s) = 75$ $\qquad$ $Y(30, A, B, c, 0) = 75$
$Y(40, A, B, c, s) = 123$ $\qquad$ $Y(40, A, B, c, 0) = 123$
$Y(50, A, B, c, s) = 183$ $\qquad$ $Y(50, A, B, c, 0) = 183$

Faith

Tier 3:

$y := 74 \qquad n := 10 \qquad d := 5 \qquad c := 19 \qquad s := 18$

$A := a(y, n, d, c, s) = 0$
$B := b(y, n, d, c, s) = 0.37$

$Y(5, A, B, c, s) = 46.25 \qquad Y(5, A, B, c, 0) = 28.25$
$Y(10, A, B, c, s) = 74 \qquad Y(10, A, B, c, 0) = 56$
$Y(15, A, B, c, s) = 120.25 \qquad Y(15, A, B, c, 0) = 102.25$
$Y(20, A, B, c, s) = 185 \qquad Y(20, A, B, c, 0) = 167$
$Y(25, A, B, c, s) = 268.25 \qquad Y(25, A, B, c, 0) = 250.25$
$Y(30, A, B, c, s) = 370 \qquad Y(30, A, B, c, 0) = 352$
$Y(40, A, B, c, s) = 629 \qquad Y(40, A, B, c, 0) = 611$
$Y(50, A, B, c, s) = 962 \qquad Y(50, A, B, c, 0) = 944$

Tier 2:

$y := 38 \qquad n := 10 \qquad d := 5 \qquad c := 10 \qquad s := 9$

$A := a(y, n, d, c, s) = 0$
$B := b(y, n, d, c, s) = 0.19$

$Y(5, A, B, c, s) = 23.75 \qquad Y(5, A, B, c, 0) = 14.75$
$Y(10, A, B, c, s) = 38 \qquad Y(10, A, B, c, 0) = 29$
$Y(15, A, B, c, s) = 61.75 \qquad Y(15, A, B, c, 0) = 52.75$
$Y(20, A, B, c, s) = 95 \qquad Y(20, A, B, c, 0) = 86$
$Y(25, A, B, c, s) = 137.75 \qquad Y(25, A, B, c, 0) = 128.75$
$Y(30, A, B, c, s) = 190 \qquad Y(30, A, B, c, 0) = 181$
$Y(40, A, B, c, s) = 323 \qquad Y(40, A, B, c, 0) = 314$
$Y(50, A, B, c, s) = 494 \qquad Y(50, A, B, c, 0) = 485$

Tier 1:

$y := 16 \qquad n := 10 \qquad d := 5 \qquad c := 5 \qquad s := 3$

$A := a(y, n, d, c, s) = 0$
$B := b(y, n, d, c, s) = 0.08$

$Y(5, A, B, c, s) = 10 \qquad Y(5, A, B, c, 0) = 7$
$Y(10, A, B, c, s) = 16 \qquad Y(10, A, B, c, 0) = 13$
$Y(15, A, B, c, s) = 26 \qquad Y(15, A, B, c, 0) = 23$
$Y(20, A, B, c, s) = 40 \qquad Y(20, A, B, c, 0) = 37$
$Y(25, A, B, c, s) = 58 \qquad Y(25, A, B, c, 0) = 55$
$Y(30, A, B, c, s) = 80 \qquad Y(30, A, B, c, 0) = 77$
$Y(40, A, B, c, s) = 136 \qquad Y(40, A, B, c, 0) = 133$
$Y(50, A, B, c, s) = 208 \qquad Y(50, A, B, c, 0) = 205$

Tier 0:

$y := 6 \qquad n := 10 \qquad d := 5 \qquad c := 3 \qquad s := 0$

$A := a(y, n, d, c, s) = 0$
$B := b(y, n, d, c, s) = 0.03$

$Y(5, A, B, c, s) = 3.75$ $\qquad$ $Y(5, A, B, c, 0) = 3.75$
$Y(10, A, B, c, s) = 6$ $\qquad$ $Y(10, A, B, c, 0) = 6$
$Y(15, A, B, c, s) = 9.75$ $\qquad$ $Y(15, A, B, c, 0) = 9.75$
$Y(20, A, B, c, s) = 15$ $\qquad$ $Y(20, A, B, c, 0) = 15$
$Y(25, A, B, c, s) = 21.75$ $\qquad$ $Y(25, A, B, c, 0) = 21.75$
$Y(30, A, B, c, s) = 30$ $\qquad$ $Y(30, A, B, c, 0) = 30$
$Y(40, A, B, c, s) = 51$ $\qquad$ $Y(40, A, B, c, 0) = 51$
$Y(50, A, B, c, s) = 78$ $\qquad$ $Y(50, A, B, c, 0) = 78$

Gold

Tier 3:

$y := 106 \qquad n := 10 \qquad d := 4 \qquad c := 19 \qquad s := 18$

$A := a(y, n, d, c, s) = 3.9$
$B := b(y, n, d, c, s) = 0.3$

$Y(5, A, B, c, s) = 64$	$Y(5, A, B, c, 0) = 46$
$Y(10, A, B, c, s) = 106$	$Y(10, A, B, c, 0) = 88$
$Y(15, A, B, c, s) = 163$	$Y(15, A, B, c, 0) = 145$
$Y(20, A, B, c, s) = 235$	$Y(20, A, B, c, 0) = 217$
$Y(25, A, B, c, s) = 322$	$Y(25, A, B, c, 0) = 304$
$Y(30, A, B, c, s) = 424$	$Y(30, A, B, c, 0) = 406$
$Y(40, A, B, c, s) = 673$	$Y(40, A, B, c, 0) = 655$
$Y(50, A, B, c, s) = 982$	$Y(50, A, B, c, 0) = 964$

Tier 2:

$y := 58 \qquad n := 10 \qquad d := 4 \qquad c := 10 \qquad s := 9$

$A := a(y, n, d, c, s) = 2.3$
$B := b(y, n, d, c, s) = 0.16$

$Y(5, A, B, c, s) = 34.5$	$Y(5, A, B, c, 0) = 25.5$
$Y(10, A, B, c, s) = 58$	$Y(10, A, B, c, 0) = 49$
$Y(15, A, B, c, s) = 89.5$	$Y(15, A, B, c, 0) = 80.5$
$Y(20, A, B, c, s) = 129$	$Y(20, A, B, c, 0) = 120$
$Y(25, A, B, c, s) = 176.5$	$Y(25, A, B, c, 0) = 167.5$
$Y(30, A, B, c, s) = 232$	$Y(30, A, B, c, 0) = 223$
$Y(40, A, B, c, s) = 367$	$Y(40, A, B, c, 0) = 358$
$Y(50, A, B, c, s) = 534$	$Y(50, A, B, c, 0) = 525$

Tier 1:

$y := 29 \qquad n := 10 \qquad d := 4 \qquad c := 5 \qquad s := 3$

$A := a(y, n, d, c, s) = 1.35$
$B := b(y, n, d, c, s) = 0.08$

$Y(5, A, B, c, s) = 16.63$	$Y(5, A, B, c, 0) = 13.63$
$Y(10, A, B, c, s) = 29$	$Y(10, A, B, c, 0) = 26$
$Y(15, A, B, c, s) = 45.13$	$Y(15, A, B, c, 0) = 42.13$
$Y(20, A, B, c, s) = 65$	$Y(20, A, B, c, 0) = 62$
$Y(25, A, B, c, s) = 88.63$	$Y(25, A, B, c, 0) = 85.63$
$Y(30, A, B, c, s) = 116$	$Y(30, A, B, c, 0) = 113$
$Y(40, A, B, c, s) = 182$	$Y(40, A, B, c, 0) = 179$
$Y(50, A, B, c, s) = 263$	$Y(50, A, B, c, 0) = 260$

Tier 0:

$y := 18 \qquad n := 10 \qquad d := 4 \qquad c := 3 \qquad s := 0$

$A := a(y, n, d, c, s) = 1.1$
$B := b(y, n, d, c, s) = 0.04$

$Y(5, A, B, c, s) = 9.5$ $\qquad$ $Y(5, A, B, c, 0) = 9.5$
$Y(10, A, B, c, s) = 18$ $\qquad$ $Y(10, A, B, c, 0) = 18$
$Y(15, A, B, c, s) = 28.5$ $\qquad$ $Y(15, A, B, c, 0) = 28.5$
$Y(20, A, B, c, s) = 41$ $\qquad$ $Y(20, A, B, c, 0) = 41$
$Y(25, A, B, c, s) = 55.5$ $\qquad$ $Y(25, A, B, c, 0) = 55.5$
$Y(30, A, B, c, s) = 72$ $\qquad$ $Y(30, A, B, c, 0) = 72$
$Y(40, A, B, c, s) = 111$ $\qquad$ $Y(40, A, B, c, 0) = 111$
$Y(50, A, B, c, s) = 158$ $\qquad$ $Y(50, A, B, c, 0) = 158$

Production

Tier 3:

$y := 50 \qquad n := 10 \qquad d := 2 \qquad c := 19 \qquad s := 18$

$A := a(y, n, d, c, s) = 0.9$
$B := b(y, n, d, c, s) = 0.04$

$Y(5, A, B, c, s) = 42.5 \qquad Y(5, A, B, c, 0) = 24.5$
$Y(10, A, B, c, s) = 50 \qquad Y(10, A, B, c, 0) = 32$
$Y(15, A, B, c, s) = 59.5 \qquad Y(15, A, B, c, 0) = 41.5$
$Y(20, A, B, c, s) = 71 \qquad Y(20, A, B, c, 0) = 53$
$Y(25, A, B, c, s) = 84.5 \qquad Y(25, A, B, c, 0) = 66.5$
$Y(30, A, B, c, s) = 100 \qquad Y(30, A, B, c, 0) = 82$
$Y(40, A, B, c, s) = 137 \qquad Y(40, A, B, c, 0) = 119$
$Y(50, A, B, c, s) = 182 \qquad Y(50, A, B, c, 0) = 164$

Tier 2:

$y := 26 \qquad n := 10 \qquad d := 2 \qquad c := 10 \qquad s := 9$

$A := a(y, n, d, c, s) = 0.5$
$B := b(y, n, d, c, s) = 0.02$

$Y(5, A, B, c, s) = 22 \qquad Y(5, A, B, c, 0) = 13$
$Y(10, A, B, c, s) = 26 \qquad Y(10, A, B, c, 0) = 17$
$Y(15, A, B, c, s) = 31 \qquad Y(15, A, B, c, 0) = 22$
$Y(20, A, B, c, s) = 37 \qquad Y(20, A, B, c, 0) = 28$
$Y(25, A, B, c, s) = 44 \qquad Y(25, A, B, c, 0) = 35$
$Y(30, A, B, c, s) = 52 \qquad Y(30, A, B, c, 0) = 43$
$Y(40, A, B, c, s) = 71 \qquad Y(40, A, B, c, 0) = 62$
$Y(50, A, B, c, s) = 94 \qquad Y(50, A, B, c, 0) = 85$

Tier 1:

$y := 13 \qquad n := 10 \qquad d := 2 \qquad c := 5 \qquad s := 3$

$A := a(y, n, d, c, s) = 0.45$
$B := b(y, n, d, c, s) = 0.01$

$Y(5, A, B, c, s) = 10.38 \qquad Y(5, A, B, c, 0) = 7.38$
$Y(10, A, B, c, s) = 13 \qquad Y(10, A, B, c, 0) = 10$
$Y(15, A, B, c, s) = 15.88 \qquad Y(15, A, B, c, 0) = 12.88$
$Y(20, A, B, c, s) = 19 \qquad Y(20, A, B, c, 0) = 16$
$Y(25, A, B, c, s) = 22.38 \qquad Y(25, A, B, c, 0) = 19.38$
$Y(30, A, B, c, s) = 26 \qquad Y(30, A, B, c, 0) = 23$
$Y(40, A, B, c, s) = 34 \qquad Y(40, A, B, c, 0) = 31$
$Y(50, A, B, c, s) = 43 \qquad Y(50, A, B, c, 0) = 40$

Tier 0:

$y := 6 \qquad n := 10 \qquad d := 2 \qquad c := 3 \qquad s := 0$

$A := a(y, n, d, c, s) = 0.3$
$B := b(y, n, d, c, s) = 0$

$Y(5, A, B, c, s) = 4.5$ $\qquad$ $Y(5, A, B, c, 0) = 4.5$
$Y(10, A, B, c, s) = 6$ $\qquad$ $Y(10, A, B, c, 0) = 6$
$Y(15, A, B, c, s) = 7.5$ $\qquad$ $Y(15, A, B, c, 0) = 7.5$
$Y(20, A, B, c, s) = 9$ $\qquad$ $Y(20, A, B, c, 0) = 9$
$Y(25, A, B, c, s) = 10.5$ $\qquad$ $Y(25, A, B, c, 0) = 10.5$
$Y(30, A, B, c, s) = 12$ $\qquad$ $Y(30, A, B, c, 0) = 12$
$Y(40, A, B, c, s) = 15$ $\qquad$ $Y(40, A, B, c, 0) = 15$
$Y(50, A, B, c, s) = 18$ $\qquad$ $Y(50, A, B, c, 0) = 18$