

City Yield Formulae and Considerations

We start with two considerations: To balance tall and wide play, we want to make one tall city yield as much as a given number of small cities. This gives us the first equation, which is the ratio of one city with three times the population of another. Since population yields scale exponentially, once this break-even point is reached, the tall play-style begins to outpace the wide play-style. The second consideration is how much one wide city should yield in absolute terms. Together, these two considerations result in two equations, where the flat and exponential yield per pop represent the unknown.

y ... Yield of an n -sized city with.

d ... Number of n -sized cities required to match one $3 \cdot n$ -sized city.

n ... Pop size used as basis for comparison and calculation.

a ... Linear yield per pop.

b ... Exponential yield per pop, also known as yield per pop per pop.

c ... Constant yield from districts and buildings.

s ... Yield from specialists, provided all specialist slots are filled.

$$A := d = \frac{3 \cdot n \cdot a + (3 \cdot n)^2 \cdot b + c + s}{n \cdot a + n^2 \cdot b + c + s} \xrightarrow{\text{solve}, a} \frac{(-d+1) \cdot s + (9 \cdot b - b \cdot d) \cdot n^2 + (c - c \cdot d)}{(d-3) \cdot n}$$

$$B := y = \frac{n \cdot a + n^2 \cdot b + c + s}{n} \xrightarrow{\text{solve}, a} \frac{y - (s + b \cdot n^2 + c)}{n}$$

$$A = B \xrightarrow{\text{solve}, b} \frac{(d-3) \cdot y + 2 \cdot s + 2 \cdot c}{6 \cdot n^2}$$

Resubstituting, this gives us formulae for a and b :

$$a(y, n, d, c, s) := \frac{-(8 \cdot s) + (9 \cdot y - 8 \cdot c - y \cdot d)}{6 \cdot n}$$

$$b(y, n, d, c, s) := \frac{2 \cdot s + y \cdot d + (2 \cdot c - 3 \cdot y)}{6 \cdot n^2}$$

For purposes of creating value tables, we also define a formula for the overall resulting yield of a city:

$$Y(n, a, b, c, s) := n \cdot a + n^2 \cdot b + c + s$$

Science

Tier 3:

$$y := 83 \quad n := 10 \quad d := 5 \quad c := 19 \quad s := 18$$

$$A := a(y, n, d, c, s) = 0.6$$

$$B := b(y, n, d, c, s) = 0.4$$

$$Y(5, A, B, c, s) = 50$$

$$Y(10, A, B, c, s) = 83$$

$$Y(15, A, B, c, s) = 136$$

$$Y(20, A, B, c, s) = 209$$

$$Y(25, A, B, c, s) = 302$$

$$Y(30, A, B, c, s) = 415$$

$$Y(40, A, B, c, s) = 701$$

$$Y(50, A, B, c, s) = 1067$$

$$Y(5, A, B, c, 0) = 32$$

$$Y(10, A, B, c, 0) = 65$$

$$Y(15, A, B, c, 0) = 118$$

$$Y(20, A, B, c, 0) = 191$$

$$Y(25, A, B, c, 0) = 284$$

$$Y(30, A, B, c, 0) = 397$$

$$Y(40, A, B, c, 0) = 683$$

$$Y(50, A, B, c, 0) = 1049$$

Tier 2:

$$y := 44 \quad n := 10 \quad d := 5 \quad c := 10 \quad s := 9$$

$$A := a(y, n, d, c, s) = 0.4$$

$$B := b(y, n, d, c, s) = 0.21$$

$$Y(5, A, B, c, s) = 26.25$$

$$Y(10, A, B, c, s) = 44$$

$$Y(15, A, B, c, s) = 72.25$$

$$Y(20, A, B, c, s) = 111$$

$$Y(25, A, B, c, s) = 160.25$$

$$Y(30, A, B, c, s) = 220$$

$$Y(40, A, B, c, s) = 371$$

$$Y(50, A, B, c, s) = 564$$

$$Y(5, A, B, c, 0) = 17.25$$

$$Y(10, A, B, c, 0) = 35$$

$$Y(15, A, B, c, 0) = 63.25$$

$$Y(20, A, B, c, 0) = 102$$

$$Y(25, A, B, c, 0) = 151.25$$

$$Y(30, A, B, c, 0) = 211$$

$$Y(40, A, B, c, 0) = 362$$

$$Y(50, A, B, c, 0) = 555$$

Tier 1:

$$y := 19 \quad n := 10 \quad d := 5 \quad c := 5 \quad s := 3$$

$$A := a(y, n, d, c, s) = 0.2$$

$$B := b(y, n, d, c, s) = 0.09$$

$$Y(5, A, B, c, s) = 11.25$$

$$Y(10, A, B, c, s) = 19$$

$$Y(15, A, B, c, s) = 31.25$$

$$Y(20, A, B, c, s) = 48$$

$$Y(25, A, B, c, s) = 69.25$$

$$Y(30, A, B, c, s) = 95$$

$$Y(40, A, B, c, s) = 160$$

$$Y(50, A, B, c, s) = 243$$

$$Y(5, A, B, c, 0) = 8.25$$

$$Y(10, A, B, c, 0) = 16$$

$$Y(15, A, B, c, 0) = 28.25$$

$$Y(20, A, B, c, 0) = 45$$

$$Y(25, A, B, c, 0) = 66.25$$

$$Y(30, A, B, c, 0) = 92$$

$$Y(40, A, B, c, 0) = 157$$

$$Y(50, A, B, c, 0) = 240$$

Tier 0:

$y := 9$ $n := 10$ $d := 5$ $c := 3$ $s := 0$

$A := a(y, n, d, c, s) = 0.2$

$B := b(y, n, d, c, s) = 0.04$

$Y(5, A, B, c, s) = 5$

$Y(10, A, B, c, s) = 9$

$Y(15, A, B, c, s) = 15$

$Y(20, A, B, c, s) = 23$

$Y(25, A, B, c, s) = 33$

$Y(30, A, B, c, s) = 45$

$Y(40, A, B, c, s) = 75$

$Y(50, A, B, c, s) = 113$

$Y(5, A, B, c, 0) = 5$

$Y(10, A, B, c, 0) = 9$

$Y(15, A, B, c, 0) = 15$

$Y(20, A, B, c, 0) = 23$

$Y(25, A, B, c, 0) = 33$

$Y(30, A, B, c, 0) = 45$

$Y(40, A, B, c, 0) = 75$

$Y(50, A, B, c, 0) = 113$

Culture

Tier 3:

$$y := 101 \quad n := 10 \quad d := 5 \quad c := 19 \quad s := 18$$

$$A := a(y, n, d, c, s) = 1.8$$
$$B := b(y, n, d, c, s) = 0.46$$

$Y(5, A, B, c, s) = 57.5$	$Y(5, A, B, c, 0) = 39.5$
$Y(10, A, B, c, s) = 101$	$Y(10, A, B, c, 0) = 83$
$Y(15, A, B, c, s) = 167.5$	$Y(15, A, B, c, 0) = 149.5$
$Y(20, A, B, c, s) = 257$	$Y(20, A, B, c, 0) = 239$
$Y(25, A, B, c, s) = 369.5$	$Y(25, A, B, c, 0) = 351.5$
$Y(30, A, B, c, s) = 505$	$Y(30, A, B, c, 0) = 487$
$Y(40, A, B, c, s) = 845$	$Y(40, A, B, c, 0) = 827$
$Y(50, A, B, c, s) = 1277$	$Y(50, A, B, c, 0) = 1259$

Tier 2:

$$y := 53 \quad n := 10 \quad d := 5 \quad c := 10 \quad s := 9$$

$$A := a(y, n, d, c, s) = 1$$
$$B := b(y, n, d, c, s) = 0.24$$

$Y(5, A, B, c, s) = 30$	$Y(5, A, B, c, 0) = 21$
$Y(10, A, B, c, s) = 53$	$Y(10, A, B, c, 0) = 44$
$Y(15, A, B, c, s) = 88$	$Y(15, A, B, c, 0) = 79$
$Y(20, A, B, c, s) = 135$	$Y(20, A, B, c, 0) = 126$
$Y(25, A, B, c, s) = 194$	$Y(25, A, B, c, 0) = 185$
$Y(30, A, B, c, s) = 265$	$Y(30, A, B, c, 0) = 256$
$Y(40, A, B, c, s) = 443$	$Y(40, A, B, c, 0) = 434$
$Y(50, A, B, c, s) = 669$	$Y(50, A, B, c, 0) = 660$

Tier 1:

$$y := 25 \quad n := 10 \quad d := 5 \quad c := 5 \quad s := 3$$

$$A := a(y, n, d, c, s) = 0.6$$
$$B := b(y, n, d, c, s) = 0.11$$

$Y(5, A, B, c, s) = 13.75$	$Y(5, A, B, c, 0) = 10.75$
$Y(10, A, B, c, s) = 25$	$Y(10, A, B, c, 0) = 22$
$Y(15, A, B, c, s) = 41.75$	$Y(15, A, B, c, 0) = 38.75$
$Y(20, A, B, c, s) = 64$	$Y(20, A, B, c, 0) = 61$
$Y(25, A, B, c, s) = 91.75$	$Y(25, A, B, c, 0) = 88.75$
$Y(30, A, B, c, s) = 125$	$Y(30, A, B, c, 0) = 122$
$Y(40, A, B, c, s) = 208$	$Y(40, A, B, c, 0) = 205$
$Y(50, A, B, c, s) = 313$	$Y(50, A, B, c, 0) = 310$

Tier 0:

$$y:=15 \quad n:=10 \quad d:=5 \quad c:=3 \quad s:=0$$

$$A:=a(y,n,d,c,s)=0.6$$

$$B:=b(y,n,d,c,s)=0.06$$

$$Y(5,A,B,c,s)=7.5$$

$$Y(10,A,B,c,s)=15$$

$$Y(15,A,B,c,s)=25.5$$

$$Y(20,A,B,c,s)=39$$

$$Y(25,A,B,c,s)=55.5$$

$$Y(30,A,B,c,s)=75$$

$$Y(40,A,B,c,s)=123$$

$$Y(50,A,B,c,s)=183$$

$$Y(5,A,B,c,0)=7.5$$

$$Y(10,A,B,c,0)=15$$

$$Y(15,A,B,c,0)=25.5$$

$$Y(20,A,B,c,0)=39$$

$$Y(25,A,B,c,0)=55.5$$

$$Y(30,A,B,c,0)=75$$

$$Y(40,A,B,c,0)=123$$

$$Y(50,A,B,c,0)=183$$

Faith

Tier 3:

$$y := 74 \quad n := 10 \quad d := 5 \quad c := 19 \quad s := 18$$

$$A := a(y, n, d, c, s) = 0$$
$$B := b(y, n, d, c, s) = 0.37$$

$Y(5, A, B, c, s) = 46.25$	$Y(5, A, B, c, 0) = 28.25$
$Y(10, A, B, c, s) = 74$	$Y(10, A, B, c, 0) = 56$
$Y(15, A, B, c, s) = 120.25$	$Y(15, A, B, c, 0) = 102.25$
$Y(20, A, B, c, s) = 185$	$Y(20, A, B, c, 0) = 167$
$Y(25, A, B, c, s) = 268.25$	$Y(25, A, B, c, 0) = 250.25$
$Y(30, A, B, c, s) = 370$	$Y(30, A, B, c, 0) = 352$
$Y(40, A, B, c, s) = 629$	$Y(40, A, B, c, 0) = 611$
$Y(50, A, B, c, s) = 962$	$Y(50, A, B, c, 0) = 944$

Tier 2:

$$y := 38 \quad n := 10 \quad d := 5 \quad c := 10 \quad s := 9$$

$$A := a(y, n, d, c, s) = 0$$
$$B := b(y, n, d, c, s) = 0.19$$

$Y(5, A, B, c, s) = 23.75$	$Y(5, A, B, c, 0) = 14.75$
$Y(10, A, B, c, s) = 38$	$Y(10, A, B, c, 0) = 29$
$Y(15, A, B, c, s) = 61.75$	$Y(15, A, B, c, 0) = 52.75$
$Y(20, A, B, c, s) = 95$	$Y(20, A, B, c, 0) = 86$
$Y(25, A, B, c, s) = 137.75$	$Y(25, A, B, c, 0) = 128.75$
$Y(30, A, B, c, s) = 190$	$Y(30, A, B, c, 0) = 181$
$Y(40, A, B, c, s) = 323$	$Y(40, A, B, c, 0) = 314$
$Y(50, A, B, c, s) = 494$	$Y(50, A, B, c, 0) = 485$

Tier 1:

$$y := 16 \quad n := 10 \quad d := 5 \quad c := 5 \quad s := 3$$

$$A := a(y, n, d, c, s) = 0$$
$$B := b(y, n, d, c, s) = 0.08$$

$Y(5, A, B, c, s) = 10$	$Y(5, A, B, c, 0) = 7$
$Y(10, A, B, c, s) = 16$	$Y(10, A, B, c, 0) = 13$
$Y(15, A, B, c, s) = 26$	$Y(15, A, B, c, 0) = 23$
$Y(20, A, B, c, s) = 40$	$Y(20, A, B, c, 0) = 37$
$Y(25, A, B, c, s) = 58$	$Y(25, A, B, c, 0) = 55$
$Y(30, A, B, c, s) = 80$	$Y(30, A, B, c, 0) = 77$
$Y(40, A, B, c, s) = 136$	$Y(40, A, B, c, 0) = 133$
$Y(50, A, B, c, s) = 208$	$Y(50, A, B, c, 0) = 205$

Tier 0:

$y := 6$ $n := 10$ $d := 5$ $c := 3$ $s := 0$

$A := a(y, n, d, c, s) = 0$

$B := b(y, n, d, c, s) = 0.03$

$Y(5, A, B, c, s) = 3.75$

$Y(10, A, B, c, s) = 6$

$Y(15, A, B, c, s) = 9.75$

$Y(20, A, B, c, s) = 15$

$Y(25, A, B, c, s) = 21.75$

$Y(30, A, B, c, s) = 30$

$Y(40, A, B, c, s) = 51$

$Y(50, A, B, c, s) = 78$

$Y(5, A, B, c, 0) = 3.75$

$Y(10, A, B, c, 0) = 6$

$Y(15, A, B, c, 0) = 9.75$

$Y(20, A, B, c, 0) = 15$

$Y(25, A, B, c, 0) = 21.75$

$Y(30, A, B, c, 0) = 30$

$Y(40, A, B, c, 0) = 51$

$Y(50, A, B, c, 0) = 78$

Gold

Tier 3:

$$y := 122 \quad n := 10 \quad d := 4 \quad c := 29 \quad s := 18$$

$$A := a(y, n, d, c, s) = 3.9$$

$$B := b(y, n, d, c, s) = 0.36$$

$$Y(5, A, B, c, s) = 75.5$$

$$Y(10, A, B, c, s) = 122$$

$$Y(15, A, B, c, s) = 186.5$$

$$Y(20, A, B, c, s) = 269$$

$$Y(25, A, B, c, s) = 369.5$$

$$Y(30, A, B, c, s) = 488$$

$$Y(40, A, B, c, s) = 779$$

$$Y(50, A, B, c, s) = 1142$$

$$Y(5, A, B, c, 0) = 57.5$$

$$Y(10, A, B, c, 0) = 104$$

$$Y(15, A, B, c, 0) = 168.5$$

$$Y(20, A, B, c, 0) = 251$$

$$Y(25, A, B, c, 0) = 351.5$$

$$Y(30, A, B, c, 0) = 470$$

$$Y(40, A, B, c, 0) = 761$$

$$Y(50, A, B, c, 0) = 1124$$

Tier 2:

$$y := 58 \quad n := 10 \quad d := 4 \quad c := 10 \quad s := 9$$

$$A := a(y, n, d, c, s) = 2.3$$

$$B := b(y, n, d, c, s) = 0.16$$

$$Y(5, A, B, c, s) = 34.5$$

$$Y(10, A, B, c, s) = 58$$

$$Y(15, A, B, c, s) = 89.5$$

$$Y(20, A, B, c, s) = 129$$

$$Y(25, A, B, c, s) = 176.5$$

$$Y(30, A, B, c, s) = 232$$

$$Y(40, A, B, c, s) = 367$$

$$Y(50, A, B, c, s) = 534$$

$$Y(5, A, B, c, 0) = 25.5$$

$$Y(10, A, B, c, 0) = 49$$

$$Y(15, A, B, c, 0) = 80.5$$

$$Y(20, A, B, c, 0) = 120$$

$$Y(25, A, B, c, 0) = 167.5$$

$$Y(30, A, B, c, 0) = 223$$

$$Y(40, A, B, c, 0) = 358$$

$$Y(50, A, B, c, 0) = 525$$

Tier 1:

$$y := 29 \quad n := 10 \quad d := 4 \quad c := 5 \quad s := 3$$

$$A := a(y, n, d, c, s) = 1.35$$

$$B := b(y, n, d, c, s) = 0.08$$

$$Y(5, A, B, c, s) = 16.63$$

$$Y(10, A, B, c, s) = 29$$

$$Y(15, A, B, c, s) = 45.13$$

$$Y(20, A, B, c, s) = 65$$

$$Y(25, A, B, c, s) = 88.63$$

$$Y(30, A, B, c, s) = 116$$

$$Y(40, A, B, c, s) = 182$$

$$Y(50, A, B, c, s) = 263$$

$$Y(5, A, B, c, 0) = 13.63$$

$$Y(10, A, B, c, 0) = 26$$

$$Y(15, A, B, c, 0) = 42.13$$

$$Y(20, A, B, c, 0) = 62$$

$$Y(25, A, B, c, 0) = 85.63$$

$$Y(30, A, B, c, 0) = 113$$

$$Y(40, A, B, c, 0) = 179$$

$$Y(50, A, B, c, 0) = 260$$

Tier 0:

$y := 18$ $n := 10$ $d := 4$ $c := 3$ $s := 0$

$A := a(y, n, d, c, s) = 1.1$

$B := b(y, n, d, c, s) = 0.04$

$Y(5, A, B, c, s) = 9.5$

$Y(10, A, B, c, s) = 18$

$Y(15, A, B, c, s) = 28.5$

$Y(20, A, B, c, s) = 41$

$Y(25, A, B, c, s) = 55.5$

$Y(30, A, B, c, s) = 72$

$Y(40, A, B, c, s) = 111$

$Y(50, A, B, c, s) = 158$

$Y(5, A, B, c, 0) = 9.5$

$Y(10, A, B, c, 0) = 18$

$Y(15, A, B, c, 0) = 28.5$

$Y(20, A, B, c, 0) = 41$

$Y(25, A, B, c, 0) = 55.5$

$Y(30, A, B, c, 0) = 72$

$Y(40, A, B, c, 0) = 111$

$Y(50, A, B, c, 0) = 158$

Production

Tier 3:

$$y := 50 \quad n := 10 \quad d := 2 \quad c := 19 \quad s := 18$$

$$A := a(y, n, d, c, s) = 0.9$$

$$B := b(y, n, d, c, s) = 0.04$$

$$Y(5, A, B, c, s) = 42.5$$

$$Y(10, A, B, c, s) = 50$$

$$Y(15, A, B, c, s) = 59.5$$

$$Y(20, A, B, c, s) = 71$$

$$Y(25, A, B, c, s) = 84.5$$

$$Y(30, A, B, c, s) = 100$$

$$Y(40, A, B, c, s) = 137$$

$$Y(50, A, B, c, s) = 182$$

$$Y(5, A, B, c, 0) = 24.5$$

$$Y(10, A, B, c, 0) = 32$$

$$Y(15, A, B, c, 0) = 41.5$$

$$Y(20, A, B, c, 0) = 53$$

$$Y(25, A, B, c, 0) = 66.5$$

$$Y(30, A, B, c, 0) = 82$$

$$Y(40, A, B, c, 0) = 119$$

$$Y(50, A, B, c, 0) = 164$$

Tier 2:

$$y := 26 \quad n := 10 \quad d := 2 \quad c := 10 \quad s := 9$$

$$A := a(y, n, d, c, s) = 0.5$$

$$B := b(y, n, d, c, s) = 0.02$$

$$Y(5, A, B, c, s) = 22$$

$$Y(10, A, B, c, s) = 26$$

$$Y(15, A, B, c, s) = 31$$

$$Y(20, A, B, c, s) = 37$$

$$Y(25, A, B, c, s) = 44$$

$$Y(30, A, B, c, s) = 52$$

$$Y(40, A, B, c, s) = 71$$

$$Y(50, A, B, c, s) = 94$$

$$Y(5, A, B, c, 0) = 13$$

$$Y(10, A, B, c, 0) = 17$$

$$Y(15, A, B, c, 0) = 22$$

$$Y(20, A, B, c, 0) = 28$$

$$Y(25, A, B, c, 0) = 35$$

$$Y(30, A, B, c, 0) = 43$$

$$Y(40, A, B, c, 0) = 62$$

$$Y(50, A, B, c, 0) = 85$$

Tier 1:

$$y := 13 \quad n := 10 \quad d := 2 \quad c := 5 \quad s := 3$$

$$A := a(y, n, d, c, s) = 0.45$$

$$B := b(y, n, d, c, s) = 0.01$$

$$Y(5, A, B, c, s) = 10.38$$

$$Y(10, A, B, c, s) = 13$$

$$Y(15, A, B, c, s) = 15.88$$

$$Y(20, A, B, c, s) = 19$$

$$Y(25, A, B, c, s) = 22.38$$

$$Y(30, A, B, c, s) = 26$$

$$Y(40, A, B, c, s) = 34$$

$$Y(50, A, B, c, s) = 43$$

$$Y(5, A, B, c, 0) = 7.38$$

$$Y(10, A, B, c, 0) = 10$$

$$Y(15, A, B, c, 0) = 12.88$$

$$Y(20, A, B, c, 0) = 16$$

$$Y(25, A, B, c, 0) = 19.38$$

$$Y(30, A, B, c, 0) = 23$$

$$Y(40, A, B, c, 0) = 31$$

$$Y(50, A, B, c, 0) = 40$$

Tier 0:

$y := 6$ $n := 10$ $d := 2$ $c := 3$ $s := 0$

$A := a(y, n, d, c, s) = 0.3$

$B := b(y, n, d, c, s) = 0$

$Y(5, A, B, c, s) = 4.5$

$Y(10, A, B, c, s) = 6$

$Y(15, A, B, c, s) = 7.5$

$Y(20, A, B, c, s) = 9$

$Y(25, A, B, c, s) = 10.5$

$Y(30, A, B, c, s) = 12$

$Y(40, A, B, c, s) = 15$

$Y(50, A, B, c, s) = 18$

$Y(5, A, B, c, 0) = 4.5$

$Y(10, A, B, c, 0) = 6$

$Y(15, A, B, c, 0) = 7.5$

$Y(20, A, B, c, 0) = 9$

$Y(25, A, B, c, 0) = 10.5$

$Y(30, A, B, c, 0) = 12$

$Y(40, A, B, c, 0) = 15$

$Y(50, A, B, c, 0) = 18$