$$\Delta u = f \qquad u \in [0,1]$$

Discrete: $$\frac{u_{i+1} - 2u_i + 2u_{i-1}}{\Delta x^2} = f_i$$

$$\Leftrightarrow \begin{pmatrix} -2 & 1 & & \\ 1 & -2 & & \\ & \ddots & \ddots & 1 \\ & & 1 & -2 \end{pmatrix} \begin{pmatrix} u_0 \\ u_1 \\ \vdots \\ u_N \end{pmatrix} = \begin{pmatrix} f_0 \\ \vdots \\ f_N \end{pmatrix}$$

BC: $u_0 = a, \quad u_N = b$

$$\Rightarrow \underbrace{\frac{1}{\Delta x^2} \begin{pmatrix} 1 & & & & \\ 1 & -2 & 1 & & \\ & 1 & -2 & 1 & \\ & & \ddots & \ddots & \ddots \\ & & 1 & -2 & 1 \\ & & & & 1 \end{pmatrix}}_{A^{(1)} \in \mathbb{R}^{(N+1)\times(N+1)}} \begin{pmatrix} u_0 \\ u_1 \\ \vdots \\ u_N \end{pmatrix} = \begin{pmatrix} a \\ f_1 \\ \vdots \\ f_{N-1} \\ b \end{pmatrix} \qquad (1)$$

$$\Rightarrow \frac{1}{\Delta x^2} \underbrace{\begin{pmatrix} -2 & 1 & & \\ 1 & -2 & 1 & \\ & \ddots & \ddots & \ddots \\ & 1 & -2 & 1 \\ & & 1 & -2 \end{pmatrix}}_{A^{(2)} \in \mathbb{R}^{(N-1)\times(N-1)}} \begin{pmatrix} u_1 \\ \vdots \\ u_{N-1} \end{pmatrix} = \begin{pmatrix} f_1 - \frac{a}{\Delta x} \\ f_2 \\ \vdots \\ f_{N-2} \\ f_{N-1} - \frac{b}{\Delta x} \end{pmatrix} \qquad (2)$$

version 1 is easier to implement !!