Linear maps and matrices

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Let V be a vector space over K (usually $K = \mathbb{R}$ or $K = \mathbb{C}$) with a basis $B = \{\boldsymbol{b}_1, ...\boldsymbol{b}_n\}$ then every $\boldsymbol{v} \in V$ can be expressed as a linear combination of coefficients $\lambda_i \in K$ and basis vectors $\boldsymbol{b}_i$:

$$\boldsymbol{v} = \sum_{i=1}^{n} \lambda_i \cdot \boldsymbol{v}_i \tag{1}$$

Example 1 (Vector expressed in a basis)**.** *Let* $\boldsymbol{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \in \mathbb{R}^2$

$B_1 = \{\boldsymbol{e}_1, \boldsymbol{e}_2\}$ *(the canonical basis) and* $B_2 = \{\boldsymbol{b}_1, \boldsymbol{b}_2\}$ *with* $\boldsymbol{b}_1 := \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ *and* $\boldsymbol{b}_2 := \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Then:

1. *Of course* $\boldsymbol{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$