

Errors & Complexity

Interpol

Lagrange $O(n^2)$

Horner $O(n^2)$ eval: $O(n)$

Error $|f - p_m| \leq C \cdot h^{m+1}$

Error add struct $|f - p_m| \leq C \cdot h^{2m}$

Chebyshev $\|If\| \leq L_n \|f\|_\infty$
 $\|f - If\| \leq (1 + L_n) \min_q \|f - q\|$
 $L_n \leq C \cdot \ln(n)$
 $L = L(l_i)$

Splines $\|f - s\| \leq C \cdot h^4 \cdot \|f^{(4)}\|$

DFT $O(n^2)$, FFT $O(n \log n)$, Conv. $O(n \log n)$

Integr

Quadrature exact for P^n (P^{n+1} if even)

Error $|Sf - T| \leq Ch^2 \|f''\|$

Adapt. $O(h^2)$ if refined points

Gauss exact for P^{2n+1}

Error $|Sf - Qf| \leq h \min_v \|f - v\|_\infty$

Trapez better than Gauss for periodic func!

Cond

abs $\Delta f < K \Delta x$

rel $\frac{\Delta f}{f} < K \frac{\Delta x}{x}$

Gauss Elim

LU ~~$O(n^3)$~~ $O(\frac{2}{3}n^3)$

Crowd $O(\frac{2}{3}n^3)$

Cholesky $O(\frac{1}{3}n^3)$ if spd.

QR Househ. $O(\frac{4}{3}n^3)$ but well cond.

QR Givens $O(\frac{8}{3}n^3)$ but needs only single points

LS

Newton

Conv. $\|\Delta x_{n+1}\| \leq q \|\Delta x_n\|^p$ for $\phi \in C^p$

Newton LS quadratic if $F(x^*) = 0$

Bradylen: superlinear

Eval

Power meth: $|\lambda_e - \lambda_1| \leq C \cdot \left| \frac{\lambda_2}{\lambda_1} \right|^e$

Rayleigh quot.: $d(\text{span } x_1, \text{span } v) \leq C \epsilon^s$
↑
space of evec $\epsilon \in (0, \epsilon_0)$

{ Res error: $\min_x |\lambda - \tilde{\lambda}| \leq C \|v\|^2$ evec-mark.
↓
Bauer Fike $\forall \mu \in \sigma(A + \Delta A) \Rightarrow \min_i |\mu - \lambda_i| \leq \text{cond } T \|\Delta A\|$

Conv. orth iter $\min_{\lambda \in \sigma(A)} |\lambda_e - \lambda| \leq C \cdot \underbrace{\left| \frac{\lambda_{k+1}}{\lambda_k} \right|^e}_{< 1}$

QR ~~$\mathcal{O}(n^3) \forall e \Rightarrow \mathcal{O}(n^4)$~~ 1. Hessenb: $\mathcal{O}(n^3)$, 2. QR: $\mathcal{O}(n^2) \Rightarrow \mathcal{O}(n^3)$

QR Hessenb. $\mathcal{O}(n^2)$

Jacobi: linear, but faster ~~for~~ for $A \approx \text{diag}$, takes less steps than QR

~~QR~~ CG

Conv. $\|\Delta x\| \leq |q(k)|^e \cdot \|\tilde{e}_0\|$

GMRES QR in $\mathcal{O}(n^3)$