$\vec{f}_k = S^j{}_k\, \vec{e}_j$

$y^i = t^i{}_k\, x^k$

$\vec{f}^i = t^i{}_k\, \vec{e}^k$

$y_i = S^k{}_i\, x_k$ $\Longleftrightarrow$

$a_{ij} = \langle \vec{e}_i | \underline{\underline{A}} | \vec{e}_j \rangle$

$\underline{\underline{A}} = a_{ij} |\vec{e}^i\rangle\langle \vec{e}^j|$

Methoden Formeln

$(f_1\ f_2\ f_3) = (e_1\ e_2\ e_3)\ \underline{\underline{S}}$

$\begin{pmatrix} y^1 \\ y^2 \\ y^3 \end{pmatrix} = \begin{pmatrix} \underline{\underline{T}} \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}$

Duale Basis:

$\vec{e}_i \cdot \vec{e}^j = \delta_i{}^j \Longleftrightarrow$

$\begin{pmatrix} - e^1 - \\ - e^2 - \\ - e^3 - \end{pmatrix} \cdot \begin{pmatrix} | & | & | \\ e_1 & e_2 & e_3 \\ | & | & | \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$x^i = \vec{e}_i \vec{x} = \vec{e}_i x^j \vec{e}_j = x^j \delta_{ij} = x^i$

$g_{ij} = \langle \vec{e}_i | \vec{e}_j \rangle \Longrightarrow$

$g_{ij} = g_{ji}$ $\qquad g^{ij} g_{ij} = n$

$\vec{e}^i = g_{ij} \vec{e}_j$ $\qquad \tilde{g}_{ij} = S^k{}_i S^l{}_j\, g_{kl}$

$g^i{}_j = \delta^i_j$

$\|\vec{x}\|^2 = x^i x^j g_{ij} \Longleftrightarrow \vec{v} \cdot \vec{w} = g_{ij} v^i w^j$

$\partial^i = \frac{\partial}{\partial x_i}$ $\qquad \tilde{\partial}^i = \frac{\partial}{\partial \tilde{x}_i}$

$\partial_i = \frac{\partial}{\partial x^i}$ $\Longrightarrow$ $\partial^i x_j = \delta^i_j$ $\qquad \partial^i x_i = \delta^i_i = n$

$\vec{\nabla} = \vec{f}_i \tilde{\partial}^i = \vec{f}^i \partial_i$

$\vec{e}_i \times \vec{e}_j = \varepsilon_{kij} \vec{e}_k$

$\vec{e}_i = \varepsilon_{ijk} \vec{e}_j \vec{e}_k$

$\Longrightarrow$ $\vec{\nabla} \times \vec{x} = \varepsilon_{ijk} \partial_j x_k$

$\vec{\nabla} \cdot \vec{x} = \partial_i x_i$

$\varepsilon_{ijk} a^i{}_1 a^j{}_2 a^k{}_3 = \det(a^i{}_j)$

$\varepsilon_{ijk} \varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

Lineare Abb:

$$\tilde{L}^l{}_i = T^l{}_k L^k{}_j S^j{}_i \qquad \underline{\underline{L}} = L^i{}_j \vec{e}_i \vec{e}^j$$

Bilinear Form:

$$\tilde{B}_{ij} = S^k_i S^l_j B_{kl} \qquad \underline{\underline{B}} = B_{ij} \vec{e}^i \vec{e}^j$$

Gram Schmidt

$$B = \{ \vec{f}_1, \vec{f}_2, \dots \vec{f}_n \}, \qquad P_e(f) = \frac{\langle \vec{f}, \vec{e} \rangle}{\langle \vec{e}, \vec{e} \rangle} \vec{e}$$

$$\vec{e}_1 = \vec{f}_1$$

$$\vec{e}_2 = \vec{f}_2 - P_{e_1}(\vec{f}_2)$$

$$\vec{e}_3 = f_3 - P_{e_1}(\vec{f}_3) - P_{e_2}(\vec{f}_3)$$

Chain rule: $\dfrac{df}{dt} = \sum_i \dfrac{\partial f}{\partial q^i} \dfrac{dq^i}{dt} = \dfrac{\partial f}{\partial q^i} \dfrac{dq^i}{dt}$

Total diff: $df = \dfrac{\partial f}{\partial q^i} dq^i$

Tangential vector: $\left\| \dfrac{\vec{R}}{dt} \right\| = \sqrt{\dfrac{dq^i}{dt} \dfrac{dq^j}{dt} \left(\dfrac{\partial \vec{R}}{\partial q^i} \cdot \dfrac{\partial \vec{R}}{\partial q^j} \right)}$

Jacobi Matrix:

$$\frac{\partial \vec{R}}{\partial r} = \frac{\partial \vec{R}}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial \vec{R}}{\partial y} \frac{\partial y}{\partial r}$$

$$\frac{\partial \vec{R}}{\partial \theta} = \frac{\partial \vec{R}}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial \vec{R}}{\partial y} \frac{\partial y}{\partial \theta}$$

$$\left.\begin{array}{ll} \dfrac{\partial \vec{R}}{\partial r} = \vec{e}_r, & \dfrac{\partial \vec{R}}{\partial x} = \vec{e}_x \\ \dfrac{\partial \vec{R}}{\partial \theta} = \vec{e}_\theta, & \dfrac{\partial \vec{R}}{\partial y} = \vec{e}_y \end{array}\right\} \Rightarrow S = \begin{pmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial x}{\partial \theta} \\ \dfrac{\partial y}{\partial r} & \dfrac{\partial y}{\partial \theta} \end{pmatrix} = J$$

Indexnotation: $\dfrac{\partial \vec{R}}{\partial p^i} = \dfrac{\partial c^j}{\partial p^i} \dfrac{\partial \vec{R}}{\partial c^j}$