EPFL Interview

Edyn basics

- Coulomb force: $\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \vec{e}_r$
- E field: $\vec{E} = \frac{\vec{F}}{q}$ $(D = \varepsilon \cdot E)$
- Elektr. flux: $\phi_{el} = \int_{\partial\Omega} \vec{E} \cdot \vec{e}_n = \int_{\Omega} \operatorname{div} \vec{E}$

  $\left(\Rightarrow \phi_{el} = \frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_{\Omega} \rho \right.$

  $\left. \Rightarrow \operatorname{div} \vec{E} = \frac{\rho}{\varepsilon_0} \right)$ Mx1 / Gauss-law

- Potential (if E cons.): $-\nabla \phi := \vec{E}$

  $\left(\Rightarrow -\Delta \phi = \frac{\rho}{\varepsilon_0}, \right)$ $\phi_1 - \phi_2 =: U = \int_C \vec{E} \cdot d\vec{s}$

- Lorentz force: $\vec{F}_L := q\vec{E} + q(\vec{v} \times \vec{B})$
- B field & mag. flux: $\phi_m = \int \vec{B} \cdot d\vec{A}$ $(B = \mu H)$
- closed surface mag. flux: $\oint_{\partial\Omega} \vec{B} \cdot d\vec{A} = 0$

  $\Rightarrow \operatorname{div} \vec{B} = 0$ Mx3 / Gauss for Magn.

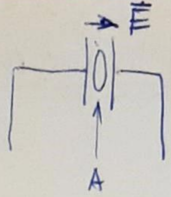
- Ampere's law (from Experiments): $\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I = \mu_0 \int j \cdot d\vec{A}$

  $\Rightarrow \operatorname{curl} \vec{B} = \mu_0 \cdot j$ Mx4 / Ampere's law

  (in static fields: $\oint_C \vec{E} \cdot d\vec{s} = 0 \Rightarrow \operatorname{rot} \vec{E} = 0$ Mx2)

- B field of stationary currents (Biot Savart): $\vec{B}(r) = -\frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \vec{e}_r}{r^2}$
- Faraday's law of Ind. (Experim.): $\left. \begin{array}{l} U_{IND} = -\frac{d\phi_m}{dt} \\ U = \int_C E \cdot d\vec{s} \end{array} \right\} \Rightarrow \operatorname{rot} E = -\partial_t \vec{B}$ Mx2 / Faraday

• Displacement current:



area A: here $j=0$

$\Rightarrow \oint_c B\,ds = \mu_0 I \Rightarrow B=0$ ↯

introduce j_d .. displ. curr. density

$j_d = \varepsilon_0 \frac{\partial E}{\partial t}$

$\Rightarrow \oint_c \vec{B}\cdot d\vec{s} = \mu_0 \int j + j_d \Rightarrow$

$\text{curl}\, B = \mu_0 j + \mu_0 \varepsilon_0 \partial_t \vec{E}$ Mx 4 / Ampere

• Maxwell:

$$\begin{cases} \text{div}\,\vec{E} = \frac{\rho}{\varepsilon_0} & \text{.. Gauss} \\ \text{div}\,\vec{B} = 0 & \text{.. Gauss} \\ \text{curl}\,\vec{E} = -\partial_t \vec{B} & \text{.. Faraday Induct} \\ \text{curl}\,\vec{B} = \mu_0 j + \varepsilon_0 \mu_0 \partial_t \vec{E} & \text{.. Amp. circuit} \end{cases}$$

• Wave equ. in Vacuum $j=0, \rho=0$

$Mx \Rightarrow \text{curl}(\text{curl}\, E) = -\text{curl}(\partial_t B)$

$= -\partial_t(\varepsilon_0 \mu_0 \partial_t \vec{E}) = -\varepsilon_0 \mu_0 \partial_t^2 E$

use $\text{curl}(\text{curl}\, E) = \nabla(\underbrace{\nabla\cdot E}_{=\frac{\rho}{\varepsilon_0}=0}) - \underbrace{\nabla\cdot(\nabla E)}_{=\Delta E}$

$\Rightarrow \Delta\vec{E} = \varepsilon_0 \mu_0 \partial_t^2 \vec{E}$.

(and similar: $\Delta B = \varepsilon \mu \partial_t^2 B$)

sol: $E = E_0 \cdot e^{i(\vec{k}\vec{r} - \omega t)}$

• in cyl. coord: $E(x,y,z) = E(R(x,y,z), \varphi(..), z(..))$

$\partial_x E = \partial_R E \cdot \underbrace{\partial_x R} + \partial_\varphi E \cdot \underbrace{\partial_x \varphi} + \underbrace{\partial_z E}_{=0}\, \underbrace{\partial_x z}_{=0}$

$\partial_x \sqrt{x^2+y^2}$ $\quad \partial_x \arctan\left(\frac{y}{x}\right) = \frac{1}{1+\left(\frac{y}{x}\right)^2}$

$\left(\frac{-y}{x^2+y^2} = -\frac{R\sin\varphi}{R^2} = -\frac{\sin\varphi}{R}\right)$

usw

$\rightsquigarrow \partial_R^2 E + \frac{1}{R}\partial_R E + \frac{1}{R^2}\partial_\varphi E + \partial_z^2 E = \nabla_{zyl} E$

Kinetic Plasma Th.

f_s ... density of particles of type „s"

- particle cons: $\partial_t f_s + \nabla_{6D} \cdot (\vec{u}_{6D} f_s) = 0$

$$\Rightarrow \partial_t f_s + \partial_{\vec{r}} \cdot (\vec{v} f_s) + \partial_{\vec{v}} \left(\frac{F}{m_s} f_s\right)$$

$$F = F_{SR} + F_{LR} = F_{coll} + F_{Lorentz}$$

- Boltzmann: $\Rightarrow \partial_t f_s + \vec{v} \cdot \partial_{\vec{r}} f_s + \underbrace{\frac{q}{m}(E + \vec{v} \times B)}_{F_{Lorentz}} \cdot \partial_{\vec{v}} f_s = (\partial_t f)_c$

„Boltzmann"

- large n in Debye cube / neglect collisions

$$\Rightarrow \partial_t f_s + \vec{v} \cdot \partial_{\vec{r}} f_s + \frac{q}{m}(E + \vec{v} \times B) \cdot \partial_{\vec{v}} f_s = 0$$

„Vlasov"

~~MHD~~ Fluid eqn

take n^{th} moments of Boltzmann (multiply by $\vec{v}^n$ and $\int$)

$v^0 = 1$: $\int \partial_t f_s \cdot 1 + \int v \cdot \partial_r f_s \cdot 1 + \int \frac{q}{m}(E + v \times B) \partial_v f_s \cdot 1 = \int C \cdot 1$

$$\Rightarrow \frac{\partial n}{\partial t} + \nabla \cdot (n \vec{u}) = 0$$

$v^1 = v$: $\int \partial_t f_s \cdot v + \int \cdot v + \dots = \int C \cdot v$

$$\Rightarrow mn \partial_t u + mn (u \cdot \nabla) u = qn (E + u \times B) - \nabla \cdot P + \Sigma R$$

v^2: $\int \partial_t f_s v^2 + \dots = \dots$

$$\Rightarrow \frac{3}{2} n \partial_t T + P : \nabla u + \nabla \cdot Q = \Sigma \partial_t W$$

heat flux (Q), therm. energy (W)

non-closed, as higher moments missing

use Braginskii

MHD

Assume:
- lengths >> λ_{Dby}, $\omega \ll \omega_{gyro}$, mean fluid vel = ion vel. / $m_e \to 0$
- 2 species: e^+, e^-

eqn:

$$\partial_t \rho + \nabla \cdot (\rho \vec{u}) = 0$$

$$\rho D_t u - \underbrace{J}_{=qnu} \times B + \nabla P_i + \nabla P_e = 0$$

;

usw

Ideal MHD

eqn.

$$\partial_t \rho + \nabla \cdot (\rho u) = 0 \quad \text{... cont.}$$

$$\rho D_t u - J \times B + \nabla P = 0 \quad \text{... moment.}$$

$$E + u \times B = 0 \quad \text{.. ohm}$$

$$\partial_t \left(\frac{P}{\rho^\gamma} \right) = 0 \quad \text{.. energy / eq. of state}$$

$$\left.\begin{aligned} \nabla \cdot B &= 0 \\ \nabla \times E &= -\partial_t B \\ \nabla \times B &= \mu_0 \cdot I \end{aligned}\right\} \text{Mx } 2,3,4$$

Assume:
- high collision.
- small r_{gyro}
- low resistivity
- magnetized plasma
  - to use Braginskii for RHS
- neglect RHS